On Matching Condition for SUSY-preserving and -breaking Parameters

Summer Institute 2006 @ APCTP, POSTECH, Pohang, Korea
2006. 8. 29

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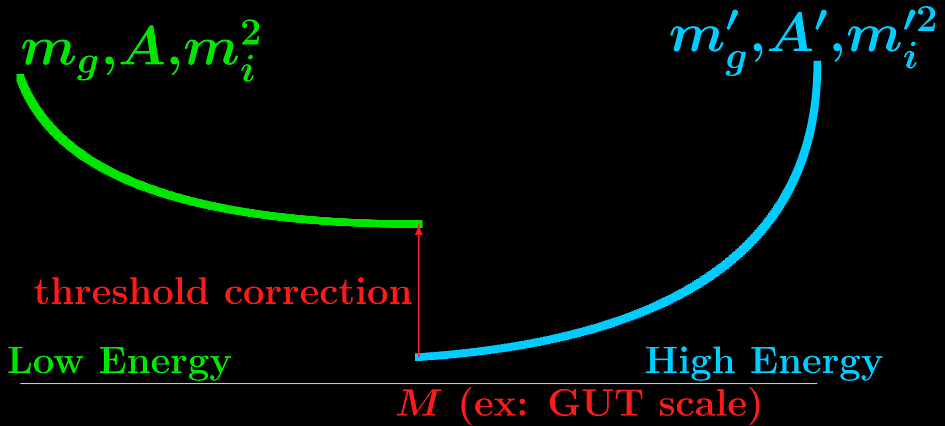
CONTENTS

1 MOTIVATION

◇ In general, RG running and matching of the soft parameters m_g, A, m_i^2



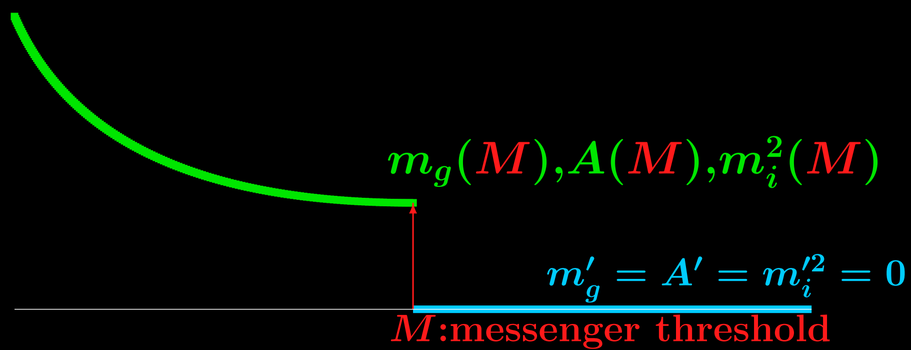
At the threshold scale M,
H.E. theory $\longrightarrow$ L.E. theory.

"threshold correction" of two type

▷ heavy chiral superfield

▷ heavy vector superfield

◇ Gauge Mediation (simplest example) Giudice & Rattazzi, 1998



H.E.side ;

$$m'_g(\mu) = 0, A'(\mu) = 0, m'^2_i(\mu) = 0$$

No soft parameter in high energy.

two type messenger

▷ chiral messenger

▷ vector messenger Goto, Hisano & Murayama, 1994

2 Matching Condition and General Relation

◇ Running mass parameters of heavy superfields are given like this:

▷ chiral superfield: $M_R^2(\mu) = |M_H|^2 / Z'_\Phi(\mu) Z'_{\bar{\Phi}}(\mu)$

▷ vector superfield: $M_R^2(\mu) = |M_H|^2 Z'_X(\mu)\alpha'(\mu), \quad \alpha' \equiv g'^2/8\pi^2$

M_H:holomorphic mass, Z_i:wave function renor. of heavy superfield i

◇ self-consistent definition

▷ running mass parameters evaluated at $\mu = M_R$:

$M_R \equiv M_R(\mu = M_R)$

◇ Matching Condition at tree level:

$$\begin{aligned} \alpha^{-1}(M_R) &= \alpha'^{-1}(M_R) \\ \ln Z_i^{-1}(M_R) &= \ln Z_i'^{-1}(M_R) \end{aligned}$$

at $\mu = M_R$

$\Longrightarrow$

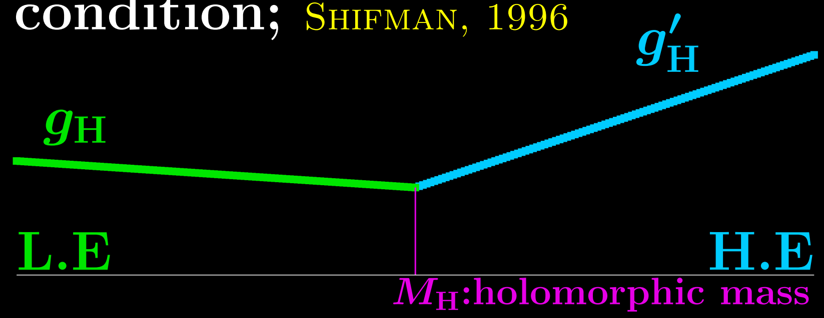
$$\alpha_H(M_H) = \alpha'_H(M_H)$$

at $\mu = M_H$

∗ Consistent with Shifman's matching condition; Shifman, 1996

"exact" RG equation for g_H:

$$\frac{1}{g_H^2(\mu)} = \frac{1}{g_H^2(\Lambda)} - \frac{b}{8\pi^2} \ln\left(\frac{\Lambda}{\mu}\right)$$



ARKANI-HAMED, GIUDICE, LUTY & RATTAZZI, 1998

KAZAKOV & VELIZHANIN, 2000

◇ Soft parameters can be introduced as

▷ gaugino mass: $\frac{1}{g_{\rm H}^2} \longrightarrow \frac{1-\theta^2 m_{g_{\rm H}}}{g_{\rm H}^2}$

▷ scalar mass: $Z_i \longrightarrow Z_i(1-\theta^2\bar{\theta}^2 m_i^2)$

▷ scalar coupling: $M_{ij} \longrightarrow M_{ij}(1-\theta^2 B_{ij})$, $y_{ijk} \longrightarrow y_{ijk}(1-\theta^2 A_{ijk})$

◇ superspace extension (renormalized parameters)

It is well-known that soft RGE is calculated from SUSY RGE by superspace extension:

$$\ln\hat{\alpha} = \ln\alpha + \left[\theta^2 m_g + \text{H.c.}\right] + \theta^2\bar{\theta}^2\left(|m_g|^2 + \Delta_g\right)$$
$$\ln\hat{Z}_i^{-1} = \ln Z_i^{-1} + \left[\theta^2 A_i + \text{H.c.}\right] + \theta^2\bar{\theta}^2 m_i^2, \qquad A_{ijk} \equiv A_i + A_j + A_k$$

We argue the threshold correction can be calculated by the same method.

3 Formulae for Soft Threshold Effects

3.1 Superspace Extension of Heavy Superfield Mass

◇ We perform the superspace extension to the mass of heavy superfield:

▷ holomorphic mass: $\hat{M}_{\rm H} = M_{\rm H} + \theta^2 F$

▷ real threshold mass:

$$\ln \hat{M}_{\rm R}(\theta, \bar{\theta}) = \ln M_{\rm R} + \left[\theta^2 B_{\rm R} + {\rm H.c.}\right] + \theta^2 \bar{\theta}^2 Y_{\rm R}$$

◇ To determine $B_{\rm R}$ and $Y_{\rm R}$, we require the superspace extension of

Physical Threshold Mass

$$\hat{M}^2_{\rm R}(\mu = \hat{M}_{\rm R}) = \left|\hat{M}_{\rm H}\right|^2 / \hat{Z}'_{\Phi}(\mu = \hat{M}_{\rm R}) \hat{Z}'_{\bar{\Phi}}(\mu = \hat{M}_{\rm R})$$

◇ The results are

$$B_{\rm R} = \frac{F/M_{\rm H} + A'_{\Phi}(M_{\rm R}) + A'_{\bar{\Phi}}(M_{\rm R})}{2 - \gamma'_{\Phi}(M_{\rm R}) - \gamma'_{\bar{\Phi}}(M_{\rm R})}, \quad \gamma'_{\Phi} = -\frac{d \ln Z'_{\Phi}}{d \ln \mu}, \quad \gamma'_{\bar{\Phi}} = -\frac{d \ln Z'_{\bar{\Phi}}}{d \ln \mu},$$

$$Y_{\rm R} = \frac{m'^2_{\Phi}(M_{\rm R}) + m'^2_{\bar{\Phi}}(M_{\rm R}) + \frac{d}{d \ln \mu} \left. \frac{\left|F/M_{\rm H} + A'_{\Phi} + A'_{\bar{\Phi}}\right|^2}{2 - \gamma'_{\Phi} - \gamma'_{\bar{\Phi}}} \right|_{\mu = M_{\rm R}}}{2 - \gamma'_{\Phi}(M_{\rm R}) - \gamma'_{\bar{\Phi}}(M_{\rm R})}$$

3.2 Matching Condition at Tree-Level

◇ Superspace extended matching condition at tree level:

$$\hat{\alpha}^{-1}(\hat{M}_{\rm R}) = \hat{\alpha}'^{-1}(\hat{M}_{\rm R}), \quad \ln \hat{Z}_i^{-1}(\hat{M}_{\rm R}) = \ln \hat{Z}_i'^{-1}(\hat{M}_{\rm R})$$

▷ Gaugino mass can be obtained by taking θ^2-comp. of α

$$\Delta m_\alpha = -B_{\rm R} \Delta\gamma_\alpha,$$

$$\Delta m_\alpha \equiv m_\alpha(M_{\rm R}) - m'_\alpha(M_{\rm R}), \quad \Delta\gamma_\alpha \equiv \gamma_\alpha(M_{\rm R}) - \gamma'_\alpha(M_{\rm R}), \quad \gamma_\alpha \equiv \beta/\alpha$$

We can check that this formula correctly reproduce the one-loop result by Hisano, Murayama & Goto (1994).

▷ In the same way, we can derive the formula for ΔA_{ijk} from superspace extension of Yukawa coupling

$$\ln \hat{y}_{ijk} = \ln y_{ijk} + \theta^2 A_{ijk} + \theta^4 \frac{1}{2}(m_i^2 + m_j^2 + m_k^2) \ .$$

However, comparing with the previous results by Polonsky & Pomarol (1995), there is a discrepancy.

3.3 Matching Condition at One Loop

The reason will be:

"wave functions do not match at tree level"

◇ We must expand matching condition to one-loop

▷ for wave function renormalization

$$\ln \hat{Z}_i^{-1}(\hat{M}_{\mathrm{R}}) = \ln \hat{Z}_i'^{-1}(\hat{M}_{\mathrm{R}}) + \frac{\Delta d}{4}\hat{\alpha}_y'(\hat{M}_{\mathrm{R}}), \quad \alpha_y' \equiv |y'|^2/8\pi^2$$

◇ By one-loop matching condition of $\ln Z^{-1}$, threshold correction to A-parameter is such as

$$\Delta A = (A' - 2B_{\mathrm{R}})\frac{\Delta d}{4}\alpha_y'$$

3.4 Application of Our Formula

◇ **EXAMPLE:**

We apply this formula to the minimal SU(5) where heavy superfields are Σ_1, Σ_3, H_c (, X, Y).

We find the top A-parameter is given by

$$\begin{aligned}\Delta A_t &= \Delta A_t^{(\Sigma_1)} + \Delta A_t^{(\Sigma_3)} + \Delta A_t^{(H_c)} \\ &= \frac{1}{4}\Big[-4\left\{A_{\psi\psi\overline{\mathcal{H}}}(M_{H_c}) - 2B_{\mathrm{R}}^{(\overline{H}_c)}\right\}\alpha_{y_t}(M_{H_c}) - 2\left\{A_{\psi\phi\mathcal{H}}(M_{H_c}) - 2B_{\mathrm{R}}^{(H_c)}\right\}\alpha_{y_b}(M_{H_c}) \\ &\quad -2\left\{A_{\psi\psi\overline{\mathcal{H}}}(M_{H_c}) - 2B_{\mathrm{R}}^{(\overline{H}_c)}\right\}\alpha_{y_t}(M_{H_c}) - 4\left\{A_{\psi\phi\mathcal{H}}(M_{H_c}) - 2B_{\mathrm{R}}^{(H_c)}\right\}\alpha_{y_b}(M_{H_c}) \\ &\quad -6\left\{A_{\mathcal{H}\Sigma\overline{\mathcal{H}}}(M_{H_c}) - 2B_{\mathrm{R}}^{(H_c)}\right\}\alpha_\lambda(M_{H_c}) - 2\left\{A_{\mathcal{H}\Sigma\overline{\mathcal{H}}}(M_{\Sigma_3}) - 2B_{\mathrm{R}}^{(\Sigma_3)}\right\}\alpha_\lambda(M_{\Sigma_3}) \\ &\quad -\frac{3}{5}\left\{A_{\mathcal{H}\Sigma\overline{\mathcal{H}}}(M_{\Sigma_1}) - 2B_{\mathrm{R}}^{(\Sigma_1)}\right\}\alpha_\lambda(M_{\Sigma_1})\Big] \\ &\quad + \text{(RG-running between each threshold)}\end{aligned}$$

This result "almost" coincides with P & P results.

[NOTE]

The contributions from B_{R} seem to be absent in their results. We would like to study this point more carefully.

4 Summary

◇ Renormalized parameters on superspace:

$$\ln \hat{\alpha} = \ln \alpha + \left[\theta^2 m_g + \text{H.c.}\right] + \theta^2\bar{\theta}^2\left(|m_g|^2 + \Delta_g\right)$$
$$\ln \hat{Z}_i^{-1} = \ln Z_i^{-1} + \left[\theta^2 A_i + \text{H.c.}\right] + \theta^2\bar{\theta}^2 m_i^2$$

◇ Physical threshold masses

▷ heavy chiral superfield mass: $M_{\mathrm{R}}(M_{\mathrm{R}}) = \sqrt{|M_{\mathrm{H}}|^2 / Z'_{\Phi}(M_{\mathrm{R}}) Z'_{\bar{\Phi}}(M_{\mathrm{R}})}$

▷ heavy vector superfield mass: $M_{\mathrm{R}}(M_{\mathrm{R}}) = \sqrt{|M_{\mathrm{H}}|^2 Z'_X(M_{\mathrm{R}})\alpha'(M_{\mathrm{R}})}$

◇ Superfield extension of threshold mass:

$$\ln \hat{M}_{\mathrm{R}}(\theta, \bar{\theta}) = \ln M_{\mathrm{R}} + \left[\theta^2 B_{\mathrm{R}} + \text{H.c.}\right] + \theta^2\bar{\theta}^2 Y_{\mathrm{R}}$$

◇ Matching conditon at one-loop (heavy chiral superfield)

$$\hat{\alpha}^{-1}(\hat{M}_{\mathrm{R}}) = \hat{\alpha}'^{-1}(\hat{M}_{\mathrm{R}})$$
$$\ln \hat{Z}_i^{-1}(\hat{M}_{\mathrm{R}}) = \ln \hat{Z}_i'^{-1}(\hat{M}_{\mathrm{R}}) + \frac{\Delta d}{4}\alpha'_y(\hat{M}_{\mathrm{R}})$$