A_4 flavour symmetry breaking scheme for understanding quark and neutrino mixing angles

Yong-Yeon Keum
National Taiwan University

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Motivation

Motivations:

- The explanation of flavour is one of the most profound goals in the construction of standard model (SM) extensions.
- There are several aspects to the overall puzzle:
  - Why three families of quarks and leptons are?
  - Why the specific mixing patterns was observed?
  - Can we understand quark and lepton mass eigenvalues?
  - Why are neutrinos so light?
- A priori, it is not clear if these aspects of the flavour problem should be treated organically, or they can be solved piecemeal.
- In this talk, we want to discuss that the observed mixing angle patterns suggest an underlying flavour symmetry breaking structure based on the discrete group A_4, while also incorporating the see-saw explanation for why neutrinos are especially light.

Mixing Matrices in SM.

- 3 families of quarks and leptons
  - $Q_i = (u_i, d_i)$,u_i^c, d_i^c;
  - $L_i = (\nu_i, l_i)$, l_i^c, $[i = 1, 2, 3]$
  - (Notation: all fermions are left handed)
- one scalar Higgs doublet

$$\Phi = (\phi^+, \phi^0) \tag{1}$$

- Allowed Yukawa couplings under $SU(2)_L$ x $U(1)_Y$ gauge symmetry:
  - $h_{ij}^u (d_i \phi^+ + u_i \phi^0) u_j^c$;
  - $h_{ij}^d (d_i \bar{\phi}^0 - u_i \phi^-) d_j^c$;
  - $f_{ij} (l_i \bar{\phi}^0 - \nu_i \phi^-) l_j^c$.
- Let $< \phi^0 >= v$, then

$$\mathcal{M}_u = h_{ij}^u v \;=\; V_L^u \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} (V_R^u)^\dagger, \tag{2}$$

$$\mathcal{M}_d = h_{ij}^d v^* \;=\; V_L^d \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} (V_R^d)^\dagger. \tag{3}$$

Mixing Matrices

The observed quark mixing matrix is

$$V_{CKM} = (V_L^u)^\dagger V_L^d \simeq \begin{pmatrix} 0.976 & 0.22 & 0.003 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{pmatrix} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{4}$$

Similarly,

$$\mathcal{M}_l = f_{ij} v^* = U_L^l \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} (U_R^l)^\dagger, \tag{5}$$

whereas the neutrino mass matrix is either
(A) Dirac case,

$$(\mathrm{A}) : \mathcal{M}_\nu^D = U_L^\nu \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} (U_R^\nu)^\dagger, \tag{6}$$

or (B) Majorana case,

$$(\mathrm{B}) : \mathcal{M}_\nu^M = U_L^\nu \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} (U_L^\nu)^T. \tag{7}$$

Neutrino-Mixing

Flavor eigenstate need not be Mass eigenstate:

e.g.

$$
\begin{array}{ccccc}
|\nu_e> & = & \cos\theta\ |\nu_1> & + & \sin\theta\ |\nu_2> \\
\uparrow & & \uparrow & & \uparrow \\
\textit{neutrinos} & & e^{iE_1 t} & & e^{i(p+\frac{m_2^2}{2\,p})t} \\
\textit{associated} & & = e^{i\sqrt{p^2+m_1^2}t} & & \\
\textit{with } e^- & & \simeq e^{i(p+\frac{m_1^2}{2\,p})t} & &
\end{array}
$$

Probability that it remains ν_e after time t;

$$
\begin{aligned}
P(\nu_e \to \nu_e) &= |\ <\nu_e\,|\nu_e(t)>\ |^2 \\
&= |(cos\theta <\nu_1| + sin\theta <\nu_2|)(cos\theta|\nu_1> + sin\theta\ e^{i(\frac{m_2^2-m_1^2}{2p})t}\,|\nu_2>)|^2 \\
&= |cos^2\theta + sin^2\theta\, e^{i\frac{\delta m_{21}^2}{2p}t}|^2
\end{aligned} \tag{8}
$$

Evidently, experiments can only determine $\delta m_{21}^2 \equiv m_2^2 - m_1^2$ and θ,

but 'NOT' m_2^2 nad m_1^2, separately

Just as for charged leptons (& for quarks) there appears to be hierarchy:

$$
\begin{array}{lcll}
\Delta m_{sol}^2 = m_2^2 - m_1^2 & \sim & 8\cdot 10^{-5} eV^2 & \Leftarrow \quad \text{sign determination by MSW effect} \\
\Delta m_{atm}^2 = m_3^2 - m_2^2 & \sim & \pm 2.4\cdot 10^{-3} eV^2 &
\end{array} \tag{9}
$$

The observed lepton mixing matrix is

$$U_{MNS} = (U_L^l)^\dagger U_L^\nu \simeq \begin{pmatrix} 0.85 & 0.52 & 0.053 \\ -0.33 & 0.62 & -0.72 \\ -0.40 & 0.59 & 0.70 \end{pmatrix} \simeq \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} . \quad (10)$$

It is clear that U_{MNS} is very different from V_{CKM} and the study of non-Abelian discrete family symmetries may help us understand why.

Simple ideas in search of a model

- Tribimaximal mixing:
  Assumption: there are no light sterile neutrinos, and the LSND anomaly has a non-oscillation explanation.

  The solar, atmospheric, reactor and accelerator neutrino data allow a simple form for the NMSP matrix: tribimaximal mixing;

$$U_{MNSP} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}. \quad (11)$$

  A special case of bi-large mixing.

  Harrison, Perkins and Scott; He and Zee; Low and Volkas; others

  Precusor: Wolfenstein(late 70's)

The matrix elements are square roots of simple fractions.

Is there a gauge model using flavour symmetry that produces this nice, simple mixing matrix?
But the CKM matrix is not similarly nice! Is tribimaximal mixing just a leptonic mirage?

$$\text{But} \quad U_{CKM} \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{12}$$

Maybe a symmetry induces tribimaximal NMSP and diagonal CKM, then small breaking of that symmetry generates the off-diagonal CKM elements while also distorting NMSP from exact tribimaximal form, making e.g. $\theta_{13} \neq 0$?

Exact flavour Symmetry

- Exact flavour symmetries need not constrain mass eigenvalues:
  Harrison, Perkins and Scott introduce mass matrices of "circulant" form:

$$M_{circ} = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix} \tag{13}$$

For definiteness, let's say this is for charged leptons
$l = (e, \mu, \tau) : \bar{l}_L M_{circ} l_R + H.C.$

- Three arbitray mass parameters: a,b and c.
- Row 2 is cyclic perm of row 1; row 3 is cyclic perm of row 2.
- Cyclic permuation symmetry C_3 of $(e, \mu, \tau)_{L,R}$ produces a circulant mass matrix.
- The mass eigenvalues are arbitray.

But ... circulant matrices are diagonalised by matrices containing certain specific numbers, independent of a, b, c, and hence also the mass eigenvalues:

$$M_{circ} M^{\dagger}_{circ} = M^{\dagger}_{circ} M_{circ} = \begin{pmatrix} \alpha & \beta & \beta^* \\ \beta^* & \alpha & \beta \\ \beta & \beta^* & \alpha \end{pmatrix} \tag{14}$$

implies the unitary left=right diagonalisation matrices

$$U_L = U_R = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^* \\ 1 & \omega^* & \omega \end{pmatrix} \tag{15}$$

where $\omega \equiv e^{i\,2\pi/3}$.
These are "trimaximal" mixing matrices.

- "Form-diagonalisable" matrices:
  Circulant matrices are of "form-diagonalisable" type: the diagonalisable matrices care only about the **form** of the mass matrix, not the specific **values** of the parameters.
  Orthogonal idea to relating mixing angles to mass ratios.
  The simplest example:

$$\begin{pmatrix} a & b \\ b & a \end{pmatrix} \tag{16}$$

gives towfold maximal mixing, as used in the mirror matter model for pairwise active-mirror neutrino mixing.

The Harrison-Perkins-Scott way to get tribimaximal mixing:

$$M_l = \begin{pmatrix} a & b & c \\ c & a & b \\ b & c & a \end{pmatrix} \qquad M_\nu = \begin{pmatrix} x & 0 & y \\ 0 & z & 0 \\ y & 0 & x \end{pmatrix} \tag{17}$$

- C_3 symmetry for charged leptons
- $Z_2 \otimes Z_2$: $\nu_e \leftrightarrow \nu_\tau$, $\nu_\mu \leftrightarrow -\nu_\mu$ for neutrinos
- Matrices form-diagonalisable; masses arbitrary

- The trimaximal charged-lepton and the ν_e/ν_τ twofold-maximal left-diagonalisation matrices to give tribimaximal MNSP(with phases).

Our ideas

We extended their idea as follows:

- Suppose all charged fermions have circulant mass matrices.
- Neutrinos have the different form from the other three because ν mass generation is different, e.g. Majorana mass term are there.
- In the limit of these flavour symmetries, MNSP is tribimaximal and CKM is diagonal.
- The angles in the diagonalisation matrices are generally 3-fold or 2-fold maximal. They cancel exactly in forming CKM because of the common 3-fold cyclic symmetry. They do not in MNSP because the electrically neutral neutrinos have a different flavour symmetry.
- After symmetry breaking, deviations from diagonal CKM and tribimaximal MNSP then arise from corrections.

We show how this conjecture can be realised in the A_4 symmetry scheme.

The scheme and tree-level results

The symmetry group of our scheme is $G \otimes X$(auxilliary sym.), where

$$G = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes A_4, \tag{18}$$

The three families of quarks and leptons are placed in the following representations of G:

$$\begin{aligned}
&Q_L \sim \left(3, 2, \tfrac{1}{3}\right)(\underline{\mathbf{3}}) && \ell_L \sim (1, 2, -1)(\underline{\mathbf{3}}) \\
&u_R \oplus u'_R \oplus u''_R \sim \left(3, 1, \tfrac{4}{3}\right)(\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'') && \nu_R \sim (1, 1, 0)(\underline{\mathbf{3}}) \\
&d_R \oplus d'_R \oplus d''_R \sim \left(3, 1, -\tfrac{2}{3}\right)(\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'') && e_R \oplus e'_R \oplus e''_R \sim (1, 1, -2)(\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'')
\end{aligned} \tag{19}$$

The Higgs field assignments are

$$\Phi \sim (1, 2, -1)(\underline{\mathbf{3}}), \quad \phi \sim (1, 2, -1)(\underline{\mathbf{1}}), \quad \chi \sim (1, 1, 0)(\underline{\mathbf{3}}). \tag{20}$$

The additional symmetry $U(1)_X$:
ℓ_L, e_R, e'_R, e''_R and ϕ carry $X = 1$, while all other fields have $X = 0$.
This non-flavour symmetry ensures that the $G_{SM} \otimes A_4$ invariant Yukawa term $\overline{\ell}_L \nu_R \Phi$ is absent from the Lagrangian. This additional symmetry prevent the unwanted Yukawa term.

The $G \otimes X$ invariant Yukawa Lagrangian is

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} = \ & \lambda_u \ (\overline{Q}_L\Phi)_{\underline{\mathbf{1}}}\, u_R + \lambda'_u(\overline{Q}_L\Phi)_{\underline{\mathbf{1}}'}\, u''_R + \lambda''_u(\overline{Q}_L\Phi)_{\underline{\mathbf{1}}''}\, u'_R + \\ & + \ \lambda_d(\overline{Q}_L\tilde{\Phi})_{\underline{\mathbf{1}}}\, d_R + \lambda'_d(\overline{Q}_L\tilde{\Phi})_{\underline{\mathbf{1}}'}\, d''_R + \lambda''_d(\overline{Q}_L\tilde{\Phi})_{\underline{\mathbf{1}}''}\, d'_R + \\ & + \ \lambda_\nu(\overline{\ell}_L\nu_R)_{\underline{\mathbf{1}}}\, \phi + M[\overline{\nu}_R(\nu_R)^c]_{\underline{\mathbf{1}}} + \lambda_\chi[\overline{\nu}_R(\nu_R)^c]_{\underline{\mathbf{3}}_s}\cdot\chi + \\ & + \ \lambda_e(\overline{\ell}_L\tilde{\Phi})_{\underline{\mathbf{1}}}\, e_R + \lambda'_e(\overline{\ell}_L\tilde{\Phi})_{\underline{\mathbf{1}}'}\, e''_R + \lambda''_e(\overline{\ell}_L\tilde{\Phi})_{\underline{\mathbf{1}}''}\, e'_R + h.c. \end{aligned} \quad (21)$$

where $\tilde{\Phi} \equiv i\tau_2\Phi^*$.

Only twelve parameters describe the masses and mixings of nine Dirac and six Majorana fermions.

When writing out the charged-fermion $f = u, d, e$ Yukawa invariants explicitly, each of the three mass matrix terms has the form:

$$\left(\ \overline{f}_{1L}, \overline{f}_{2L}, \overline{f}_{3L}\ \right)\begin{pmatrix} \lambda v_1 & \lambda' v_1 & \lambda'' v_1 \\ \lambda v_2 & \omega\lambda' v_2 & \omega^2\lambda'' v_2 \\ \lambda v_3 & \omega^2\lambda' v_3 & \omega\lambda'' v_3 \end{pmatrix}\begin{pmatrix} f_R \\ f''_R \\ f'_R \end{pmatrix} + h.c. \quad (22)$$

where $\langle\Phi^0\rangle = (v_1, v_2, v_3)$ is the vacuum expectation value (VEV) pattern for Φ, the v_i are taken to be relatively real, and the λ's have a suppressed subscript f. The numerical subscripts $1, 2, 3$ denote A_4 components.

For the special VEV pattern

$$v_1 = v_2 = v_3 \equiv v \tag{23}$$

each of these mass matrices M_f factorises as per

$$M_f = U(\omega) \begin{pmatrix} \sqrt{3}\lambda_f v & 0 & 0 \\ 0 & \sqrt{3}\lambda'_f v & 0 \\ 0 & 0 & \sqrt{3}\lambda''_f v \end{pmatrix}, \tag{24}$$

so that the left-diagonalisation matrices $V_L^{u,d,e}$ are identical and equal to the unitary "trimaximal mixing matrix"

$$U(\omega) = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \tag{25}$$

Notice that all nine mass eigenvalues are a priori arbitrary, despite the totally prescribed diagonalisation matrices. This is an example of "form diagonalisability".

The process here is a complete contrast to the popular strategy of relating mixing angles to mass ratios.

One immediately finds that, at this order, the chosen A_4 structure of the field content and the $\langle \Phi \rangle$ vacuum forces the CKM matrix to be the identity:

$$V_{CKM} = V_L^{d\dagger} V_L^u = U(\omega)^\dagger U(\omega) = 1. \tag{26}$$

The vacuum is a very special one, as it induces the breakdown

$$A_4 \rightarrow Z_3 \cong C_3 = \{1, c, a\}, \tag{27}$$

where $\cong$ denotes "isomorphism".

Now to one of our main points: It is quite possible that the reason why the observed CKM matrix is nearly the identity is the hierarchical breaking

$$A_4 \rightarrow C_3 \rightarrow \text{nothing}, \tag{28}$$

with the small mixing angles generated by higher-order effects after the relatively weak subsequent breaking of the residual C_3.

Neutrino Sector

The neutrino Dirac mass matrix is different from that of the charged-leptons, being derived from the Yukawa term $\overline{\ell}_L \nu_R \phi$.

The Dirac mass matrix is proportional to the 3×3 identity matrix,

$$M_\nu^D = \lambda_\nu \, v_\phi \, 1 \equiv m_\nu^D \, 1, \tag{29}$$

The right-handed Majorana Mass term:

- The right-handed neutrino bare Majorana mass term is similarly trivial, being M times the identity.
- The required non-trivial structure is supplied by the Yukawa coupling to χ, which expanded out is

$$\lambda_\chi \left(\overline{\nu}_{1R}, \overline{\nu}_{2R}, \overline{\nu}_{3R} \right) \begin{pmatrix} 0 & \chi_3 & \chi_2 \\ \chi_3 & 0 & \chi_1 \\ \chi_2 & \chi_1 & 0 \end{pmatrix} \begin{pmatrix} (\nu_{1R})^c \\ (\nu_{2R})^c \\ (\nu_{3R})^c \end{pmatrix}. \tag{30}$$

We now make our second key assumption about A_4 breaking: we want

$$\langle\chi_1\rangle = \langle\chi_3\rangle = 0, \quad \langle\chi_2\rangle \equiv v_\chi \neq 0, \tag{31}$$

so that the full 6×6 neutrino mass matrix is

$$\begin{pmatrix} 0 & 0 & 0 & m_\nu^D & 0 & 0 \\ 0 & 0 & 0 & 0 & m_\nu^D & 0 \\ 0 & 0 & 0 & 0 & 0 & m_\nu^D \\ m_\nu^D & 0 & 0 & M & 0 & M_\chi \\ 0 & m_\nu^D & 0 & 0 & M & 0 \\ 0 & 0 & m_\nu^D & M_\chi & 0 & M \end{pmatrix}, \tag{32}$$

where $M_\chi \equiv \lambda_\chi v_\chi$.

Note that M and M_χ are in general complex numbers with a relative phase difference.

In the see-saw limit $|M|, |M_\chi| \gg m_\nu^D$, the effective 3×3 mass matrix M_L:

$$M_L = -M_\nu^D M_R^{-1} (M_\nu^D)^T = -\frac{(m_\nu^D)^2}{M} \begin{pmatrix} \frac{M^2}{M^2 - M_\chi^2} & 0 & -\frac{M M_\chi}{M^2 - M_\chi^2} \\ 0 & 1 & 0 \\ -\frac{M M_\chi}{M^2 - M_\chi^2} & 0 & \frac{M^2}{M^2 - M_\chi^2} \end{pmatrix}, \tag{33}$$

The diagonalisation matrix is simply

$$V_L^\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}. \qquad (34)$$

The MNSP matrix, at this order, is then

$$V_{MNSP} = V_L^{e\dagger} V_L^\nu = U(\omega)^\dagger V_L^\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\omega}{\sqrt{6}} & \frac{\omega}{\sqrt{3}} & -\frac{e^{i\pi/6}}{\sqrt{2}} \\ -\frac{\omega^2}{\sqrt{6}} & \frac{\omega^2}{\sqrt{3}} & \frac{e^{-i\pi/6}}{\sqrt{2}} \end{pmatrix} \qquad (35)$$

which, up to phases, is tribimaximal and hence fits the neutrino oscillation data well. In the neutrino sector, the flavour breaking pattern driven by $\langle \chi \rangle$ is

$$A_4 \to Z_2 = \{1, r_2\}. \qquad (36)$$

Comment: Z_2 subgroup does not commute with the C_3 subgroup of the charged-fermion sector. The neutrino and charged-fermion sectors form "parallel worlds" of flavour symmetry breaking. These parallel symmetry breaking worlds cannot be sequestered from each other completely, of course. For the theory as a whole, A_4 is completely broken.

Corrections after flavour symmerty breaking

After flavour symmetry breaking, higher-order and radiative effects will in general create terms that violate A_4.

We will divide these higher-order effects into two classes:
those that involve effects *within each sector*, $\longrightarrow$ preserve C_3 and Z_2, respectively,

and those that involve *interactions between the two sectors* $\longrightarrow$ violate A_4 completely.

Corrections within the charged fermion sector

After A_4 spontaneously breaks to C_3 in this sector, higher-order effects will generate Yukawa terms that violate A_4 but respect C_3.
Under $C_3 = \{1, c, a\}$, the triplets Q_L, ℓ_L and Φ transform as per

$$c : (1, 2, 3) \to (3, 1, 2) \quad \text{and} \quad a : (1, 2, 3) \to (2, 3, 1), \tag{37}$$

The A_4 singlets f_R become C_3 singlets ($f = u, d, e$ as before), while the non-trivial one-dimensional A_4-plets f'_R and f''_R transform thus:

$$f'_R \begin{cases} \xrightarrow{c} \omega\ f'_R \\ \xrightarrow{a} \omega^2 f'_R \end{cases} \quad \text{and} \quad f''_R \begin{cases} \xrightarrow{c} \omega^2 f''_R \\ \xrightarrow{a} \omega\ f''_R \end{cases} \tag{38}$$

The previously allowed A_4 invariant $(\overline{f}_{1L}\Phi_1^0 + \overline{f}_{2L}\Phi_2^0 + \overline{f}_{3L}\Phi_3^0)\ f_R$ (where the Φ_i^0 generically denote the charge-neutral fields within Φ and $\tilde{\Phi}$) is now supplemented with the following terms that violate A_4 but respect C_3:

$$\begin{array}{c} (\overline{f}_{1L}\Phi_2^0 + \overline{f}_{2L}\Phi_3^0 + \overline{f}_{3L}\Phi_1^0)\ f_R \\ (\overline{f}_{1L}\Phi_3^0 + \overline{f}_{2L}\Phi_1^0 + \overline{f}_{3L}\Phi_2^0)\ f_R. \end{array} \tag{39}$$

Similarly, the A_4 invariants $(\overline{f}_{1L}\Phi_1^0 + \omega\overline{f}_{2L}\Phi_2^0 + \omega^2\overline{f}_{3L}\Phi_3^0)\ f''_R$ and $(\overline{f}_{1L}\Phi_1^0 + \omega^2\overline{f}_{2L}\Phi_2^0 + \omega\overline{f}_{3L}\Phi_3^0)\ f'_R$ are joined by

$$\begin{array}{c} (\overline{f}_{1L}\Phi_2^0 + \omega\overline{f}_{2L}\Phi_3^0 + \omega^2\overline{f}_{3L}\Phi_1^0)\ f''_R \\ (\overline{f}_{1L}\Phi_3^0 + \omega\overline{f}_{2L}\Phi_1^0 + \omega^2\overline{f}_{3L}\Phi_2^0)\ f''_R \end{array}$$

$$(\overline{f}_{1L}\Phi_2^0 + \omega^2\overline{f}_{2L}\Phi_3^0 + \omega\overline{f}_{3L}\Phi_1^0)\ f'_R$$
$$(\overline{f}_{1L}\Phi_3^0 + \omega^2\overline{f}_{2L}\Phi_1^0 + \omega\overline{f}_{3L}\Phi_2^0)\ f'_R. \qquad (40)$$

Each of the new terms comes, generically, with a different coupling constant.

It is interesting, though, that despite all these new Yukawa terms, the mass matrices retain the form of Eq. 24 once the C_3-preserving VEV pattern $\langle\Phi_1^0\rangle = \langle\Phi_2^0\rangle = \langle\Phi_3^0\rangle \equiv v$ is used. This means that the left-diagonalisation matrices for the u, d and e sectors remain trimaximal, and hence the CKM matrix remains trivial.

It is the C_3 subgroup of A_4 that is responsible for preventing quark mixing. The origin of CKM mixing must then arise from C_3 breaking.

Corrections within the neutrino sector

It is easy to see that the minus sign associated with the unbroken Z_2 transformations keeps the $(1,2)$, $(2,1)$, $(2,3)$ and $(3,2)$ entries of both the neutrino Dirac and right-handed Majorana mass matrices zero.

However, these two matrices need not be proportional to the identity any longer, after A_4 symmetry breaking.

$$M_\nu^D = \lambda_\nu \langle\phi\rangle \left.\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right|_{\rm bare} + \left.\begin{pmatrix} \epsilon_{11} & 0 & \epsilon_{13} \\ 0 & \epsilon_{22} & 0 \\ \epsilon_{31} & 0 & \epsilon_{33} \end{pmatrix}\right|_{\rm h.o.} \quad (41)$$

and the bare right-handed Majorana mass matrix is

$$M \left.\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right|_{\rm bare} + \left.\begin{pmatrix} \epsilon'_{11} & 0 & \epsilon'_{13} \\ 0 & \epsilon'_{22} & 0 \\ \epsilon'_{31} & 0 & \epsilon'_{33} \end{pmatrix}\right|_{\rm h.o.} \quad (42)$$

where subscript "h.o." stands for "higher order".

The $\nu_R - \chi$ coupling terms also contain the independent Z_2 invariants

$$\begin{aligned} &\overline{\nu}_{2R}(\nu_{3R})^c\chi_{1,3}, \;\; \overline{\nu}_{3R}(\nu_{1R})^c\chi_2, \;\; \overline{\nu}_{1R}(\nu_{2R})^c\chi_{3,1}, \\ &\overline{\nu}_{1R}(\nu_{1R})^c\chi_2, \;\; \overline{\nu}_{2R}(\nu_{2R})^c\chi_2, \;\; \overline{\nu}_{3R}(\nu_{3R})^c\chi_2. \end{aligned} \quad (43)$$

The new terms involve corrections to the (i,i) and $(1,3)=(3,1)$ entries in the right-handed Majorana mass matrix.

In total then, the right-handed Majorana mass terms are

$$\begin{pmatrix} M & 0 & M_\chi \\ 0 & M & 0 \\ M_\chi & 0 & M \end{pmatrix}\Bigg|_{\rm bare} + \begin{pmatrix} \epsilon''_{11} & 0 & \epsilon''_{13} \\ 0 & \epsilon''_{22} & 0 \\ \epsilon''_{31} & 0 & \epsilon''_{33} \end{pmatrix}\Bigg|_{\rm h.o.} \tag{44}$$

It is obvious then that the effective M_L is additively corrected from Eq. 33 by

$$M_L \to M_L + \begin{pmatrix} \delta_{11} & 0 & \delta_{13} \\ 0 & \delta_{22} & 0 \\ \delta_{13} & 0 & \delta_{33} \end{pmatrix}\Bigg|_{\rm h.o.}, \tag{45}$$

where, in general, the δ_{ij} are complex. The neutrino left-diagonalisation matrix is now corrected to

$$V_L^\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & e^{i\alpha_3} \end{pmatrix}, \tag{46}$$

where

$$\theta = \frac{\pi}{4} + \delta \tag{47}$$

and $\delta \ll 1$.

The phases α_i can be absorbed into the neutrino mass eigenstate fields, but the phase β, given by

$$\beta = \mathrm{Arg}(M + \delta_{33}) - \mathrm{Arg}(M + \delta_{11}), \tag{48}$$

is important because it will contribute to CP violation in neutrino oscillations.

The MNSP matrix becomes

$$V_{MNSP} = U(\omega)^\dagger V_L^\nu = \frac{1}{\sqrt{3}} \begin{pmatrix} c + se^{i\beta} & 1 & ce^{i\beta} - s \\ c + \omega se^{i\beta} & \omega^2 & \omega ce^{i\beta} - s \\ c + \omega^2 se^{i\beta} & \omega & \omega^2 ce^{i\beta} - s \end{pmatrix}, \tag{49}$$

where $c \equiv \cos\theta$ and $s \equiv \sin\theta$.

A nonzero U_{e3} element is generated, and there are other small deviations from exact tribimaximal mixing.

CP violation in neutrino oscillations is generated at this level. The Jarlskog invariant is

$$\mathrm{Im}[V_{11}\, V_{12}^*\, V_{21}^*\, V_{22}] = \frac{1}{9}(\cos 2\theta - \sin 2\theta \sin\beta) \sin\left(\frac{2\pi}{3}\right) \tag{50}$$

where the V_{ij} denote the entries of V_{MNSP}.

Interactions between the sectors after FS breaking

Neglecting interactions between the charged-fermion (plus Φ) sector and the neutrino (plus χ and ϕ) sector,

- The neutrino-world A_4 breaking is sufficient to generate realistic neutrino mass and mixing phenomenology while at the same time explaining why the dominant mixing pattern is tribimaximal.
- The charged-fermion world, however, has an interesting residual C_3 symmetry that prevents the generation of quark mixing. This is pleasing at lowest order because of the known fact that CKM angles are small; clearly, however, the C_3 subgroup must be slightly broken to achieve a fully realistic quark sector.

The full theory does in fact have broken C_3, as the neutrino sector does not respect it. This suggests that one should look to C_3 breaking mediated to the quark sector from the neutrino/χ sector as the natural source for quark mixing.

The details of this breaking mechanism depends on the specific dynamics.

The effective operators relevant for quark mixing are at dimension-five:

$$\overline{Q}_L\, u_R\, \Phi\, \chi, \quad \overline{Q}_L\, u'_R\, \Phi\, \chi, \quad \overline{Q}_L\, u''_R\, \Phi\, \chi$$
$$\overline{Q}_L\, d_R\, \tilde{\Phi}\, \chi, \quad \overline{Q}_L\, d'_R\, \tilde{\Phi}\, \chi, \quad \overline{Q}_L\, d''_R\, \tilde{\Phi}\, \chi, \qquad (51)$$

plus similar operators with $\Phi \to \phi$. The VEV of χ communicates C_3 breaking to the quarks through these operators. Each operator is suppressed either by a high mass scale M_{inter} that characterises the dynamical interactions between the two sectors (or a set of such scales), or by small coupling constants controlling those interactions.

On the detail model study of an allowed renormalisable Higgs potentials, I want to leave it for you to read our paper: hep-ph/0601001.

Conclusions

- We discussed an A_4 flavour symmetry structure that fits very well with observed patterns of quarks and neutrino mixing, while leaving mass eigenvalues arbitrary.
- The required structure splits naturally into two sectors: the neutrino/χ/ϕ domain and the charged-fermion/Φ domain. Different spontaneous flavour breaking patterns occur in these parallel worlds of symmetry breaking.
- The charged-fermion sector has $A_4 \to C_3$, while the neutrinos see $A_4 \to Z_2$. Radiative or higher-order effects *within* each sector that violate A_4 but preserve the respective subgroup were examined.
- The C_3 symmetry prevents the generation of quark mixing, while a small and interesting deviation from tribimaximal mixing is allowed by the Z_2. The latter includes a small but nonzero U_{e3} and has CP violation in neutrino oscillations.
- For quark mixing to be induced, C_3 has to be broken. We explored the natural possibility that the neutrino sector communicates its C_3 breaking to the quarks, and showed through an effective operator analysis that generically this does induce a realistic CKM matrix.
- Finding the most elegant possible underlying dynamics remains a goal for the future.
- We believe that the proposed flavour symmetry structure is a promising base from which to explore the fundamental origin of quark and lepton mixing angles.

Backup-Slide-0

Basic A_4 properties:

The A_4 group: **The discrete symmetry group of the rotations that leave a tetrahedron invariant, or the group of the even permutations of four objects.**

It has 12 elements and 4 inequivalent irreducible representations: $1, \underline{\mathbf{1}}', \underline{\mathbf{1}}''$ and **3**.

The twelve representation matrices for $\underline{\mathbf{3}}$ are conveniently taken to be the 3×3 identity matrix 1, the reflection matrices $r_1 \equiv \mathrm{diag}(1,-1,-1)$, $r_2 \equiv \mathrm{diag}(-1,1,-1)$ and $r_3 \equiv \mathrm{diag}(-1,-1,1)$, the cyclic and anticyclic matrices

$$c = a^{-1} \equiv \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad a = c^{-1} \equiv \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad (52)$$

respectively, as well as $r_i c r_i$ and $r_i a r_i$.

Under the group element corresponding to $c(a)$, $\underline{\mathbf{1}}' \to \omega(\omega^2)\underline{\mathbf{1}}'$ and $\underline{\mathbf{1}}'' \to \omega^2(\omega)\underline{\mathbf{1}}''$, where $\omega = e^{i2\pi/3}$ is a complex cube-root of unity, with both being unchanged under the r_i.

The basic non-trivial tensor products are

$$\underline{\mathbf{3}} \otimes \underline{\mathbf{3}} = \underline{\mathbf{3}}_s \oplus \underline{\mathbf{3}}_a \oplus \underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'', \quad \text{and} \quad \underline{\mathbf{1}}' \otimes \underline{\mathbf{1}}' = \underline{\mathbf{1}}'', \qquad (53)$$

where $s(a)$ denotes symmetric (antisymmetric) product.

Let (x_1, x_2, x_3) and (y_1, y_2, y_3) denote the basis vectors for two $\underline{\mathbf{3}}$'s. Then

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{3}}_s} = (x_2 y_3 + x_3 y_2 \, , \, x_3 y_1 + x_1 y_3 \, , \, x_1 y_2 + x_2 y_1), \tag{54}$$

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{3}}_a} = (x_2 y_3 - x_3 y_2 \, , \, x_3 y_1 - x_1 y_3 \, , \, x_1 y_2 - x_2 y_1), \tag{55}$$

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}} = x_1 y_1 + x_2 y_2 + x_3 y_3, \tag{56}$$

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}'} = x_1 y_1 + \omega\, x_2 y_2 + \omega^2\, x_3 y_3, \tag{57}$$

$$(\underline{\mathbf{3}} \otimes \underline{\mathbf{3}})_{\underline{\mathbf{1}}''} = x_1 y_1 + \omega^2\, x_2 y_2 + \omega\, x_3 y_3, \tag{58}$$

in an obvious notation.

Backup-Slide-1

Higgs Potential in Non-Susy case:

The $G_{SM} \otimes A_4$-invariant, renormalisable Higgs potential terms consistent with the discrete Z_2 subgroup of $U(1)_X$ are given by

$$\begin{aligned} V(\Phi) &= \mu_\Phi^2(\Phi^\dagger\Phi)_{\underline{\mathbf{1}}} + \lambda_1^\Phi(\Phi^\dagger\Phi)_{\underline{\mathbf{1}}}(\Phi^\dagger\Phi)_{\underline{\mathbf{1}}} + \lambda_2^\Phi(\Phi^\dagger\Phi)_{\underline{\mathbf{1}}'}(\Phi^\dagger\Phi)_{\underline{\mathbf{1}}''} \\ &+ \lambda_3^\Phi(\Phi^\dagger\Phi)_{\underline{\mathbf{3}}_s}(\Phi^\dagger\Phi)_{\underline{\mathbf{3}}_s} + \lambda_4^\Phi(\Phi^\dagger\Phi)_{\underline{\mathbf{3}}_a}(\Phi^\dagger\Phi)_{\underline{\mathbf{3}}_a} \\ &+ i\lambda_5^\Phi(\Phi^\dagger\Phi)_{\underline{\mathbf{3}}_s}(\Phi^\dagger\Phi)_{\underline{\mathbf{3}}_a}. \end{aligned} \tag{59}$$

$$\begin{aligned} V(\chi) &= \mu_\chi^2(\chi\chi)_{\underline{\mathbf{1}}} + \delta^\chi(\chi\chi\chi)_{\underline{\mathbf{1}}} + \lambda_1^\chi(\chi\chi)_{\underline{\mathbf{1}}}(\chi\chi)_{\underline{\mathbf{1}}} + \lambda_2^\chi(\chi\chi)_{\underline{\mathbf{1}}'}(\chi\chi)_{\underline{\mathbf{1}}''} \\ &+ \lambda_3^\chi(\chi\chi)_{\underline{\mathbf{3}}}(\chi\chi)_{\underline{\mathbf{3}}}. \end{aligned} \tag{60}$$

$$V(\phi) = \mu_\phi^2(\phi^\dagger\phi) + \lambda^\phi(\phi^\dagger\phi)^2 \tag{61}$$

$$\begin{aligned} V(\Phi,\chi) &= \delta_s^{\Phi\chi}(\Phi^\dagger\Phi)_{\underline{\mathbf{3}}_s}\chi + i\delta_a^{\Phi\chi}(\Phi^\dagger\Phi)_{\underline{\mathbf{3}}_a}\chi + \lambda_1^{\Phi\chi}(\Phi^\dagger\Phi)_{\underline{\mathbf{1}}}(\chi\chi)_{\underline{\mathbf{1}}} \\ &+ \lambda_2^{\Phi\chi}(\Phi^\dagger\Phi)_{\underline{\mathbf{1}}'}(\chi\chi)_{\underline{\mathbf{1}}''} + \lambda_2^{\Phi\chi*}(\Phi^\dagger\Phi)_{\underline{\mathbf{1}}''}(\chi\chi)_{\underline{\mathbf{1}}'} \\ &+ \lambda_3^{\Phi\chi}(\Phi^\dagger\Phi)_{\underline{\mathbf{3}}_s}(\chi\chi)_{\underline{\mathbf{3}}} + i\lambda_4^{\Phi\chi}(\Phi^\dagger\Phi)_{\underline{\mathbf{3}}_a}(\chi\chi)_{\underline{\mathbf{3}}}. \end{aligned} \tag{62}$$

$$\begin{aligned} V(\Phi,\phi) &= \lambda_1^{\Phi\phi}(\Phi^\dagger\Phi)_{\underline{\mathbf{1}}}(\phi^\dagger\phi) + \lambda_2^{\Phi\phi}(\Phi^\dagger\phi)(\phi^\dagger\Phi) + \lambda_3^{\Phi\phi}(\Phi^\dagger\phi)(\Phi^\dagger\phi) \\ &+ \lambda_3^{\Phi\phi*}(\phi^\dagger\Phi)(\phi^\dagger\Phi). \end{aligned} \tag{63}$$

$$V(\phi,\chi) = \lambda^{\phi\chi}(\phi^\dagger\phi)(\chi\chi)_{\underline{\mathbf{1}}}. \tag{64}$$

There is no renormalizable term simultaneously involving Φ, ϕ and χ allowed by the Z_2 subgroup of $U(1)_X$, that is, $V(\Phi, \chi, \phi) = 0$.

The total potential is given by

$$V = V(\Phi) + V(\chi) + V(\phi) + V(\Phi, \chi) + V(\Phi, \phi) + V(\phi, \chi) + V(\Phi, \chi, \phi). \tag{65}$$

Total vacuum structure with the conditions:

$$< \Phi_1 >=< \Phi_2 >=< \Phi_3 >= v \tag{66}$$

$$< \chi_1 >=< \chi_3 >= 0, \qquad < \chi_2 >= v_{\chi_2} \neq 0.$$

$$\tag{68}$$

Taking the derivative of $< V >$ w.r.t. χ_1 and χ_3, and taking into account the VEV structure, we obtain

$$\frac{\partial < V >}{\partial \chi_1}|_{VEV} = \frac{\partial < V >}{\partial \chi_3}|_{VEV} = 2\delta_s^{\Phi\chi}\ |v|^2 + 4\lambda_3^{\Phi\chi})\ |v|^2 v_{\chi_2} \tag{69}$$

Consider the derivatives w.r.t. v_j:

$$\frac{\partial < V >}{\partial v_1}|_{VEV} = \frac{\partial < V >}{\partial v_3}|_{VEV} = \frac{\partial < V(\Phi) + V(\Phi, \phi) >}{\partial v_1}|_{VEV} + \delta_s^{\Phi\chi}\ v\ v_{\chi_2} + (\lambda_1^{\Phi,\chi} - \lambda_2^{\Phi\chi})\ v\, v_{\chi_2}^2 \tag{70}$$

On the other hand,

$$\frac{\partial < V >}{\partial v_2}|_{VEV} = \frac{\partial < V(\Phi) + V(\Phi, \phi) >}{\partial v_1}|_{VEV} + (\lambda_1^{\Phi,\chi} + 2\ \lambda_2^{\Phi\chi})\ v\, v_{\chi_2}^2 \tag{71}$$

From above relations, we have an extremum:

$$v_{\chi_2} = -\frac{\delta_s \Phi\chi}{2\,\lambda_3^{\Phi\chi}} = \frac{\delta_s^{\Phi\chi}}{2\lambda_2^{\Phi\chi}}, \tag{72}$$

and we are lead to

$$3\ \lambda_2^{\Phi\chi} = -2\lambda_3^{\Phi\chi} \tag{73}$$

which is not obtained from the symmetry.

A Supersymmetric dynamical completion:

A model with $A_4 \otimes Z_{12} \otimes Z_2$ The chiral superfield content is

$$\begin{array}{lll} Q_L \sim (\underline{\mathbf{3}}, 1, -1), & u_R^c,\ d_R^c \sim (\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'', \omega_{12}^7, -1), & \\ \ell_L \sim (\underline{\mathbf{3}}, 1, -1), & e_R^c \sim (\underline{\mathbf{1}} \oplus \underline{\mathbf{1}}' \oplus \underline{\mathbf{1}}'', \omega_{12}^7, -1), & \nu_R \sim (\underline{\mathbf{3}}, \omega_{12}^4, -1), \\ \Phi_{u,d} \sim (\underline{\mathbf{3}}, \omega_{12}^5, 1), & \phi_u \sim (\underline{\mathbf{1}}, \omega_{12}^8, 1), & \phi_d \sim (1, \omega_{12}^4, 1) \\ \chi \sim (\underline{\mathbf{3}}, \omega_{12}^4, 1), & \chi' \sim (\underline{\mathbf{3}}, \omega_{12}^2, 1), & \\ s \sim (\underline{\mathbf{1}}, \omega_{12}^4, 1), & s' \sim (\underline{\mathbf{1}}, \omega_{12}^2, 1), & s'' \sim (\underline{\mathbf{1}}, \omega_{12}^8, 1), \end{array} \tag{74}$$

where $\omega_{12}^{12} = 1$.

The Z_{12} charges are chosen in such a way that, first, communication between χ and $\Phi_{u,d}$ is forbidden, so as to avoid the troublesome terms in $V(\Phi, \chi)$, but, second, the χ^3 and $\Phi_u \Phi_d \chi'$ terms are allowed so that the desired VEV alignment can be enforced.
The Z_2 charge disallows terms of the type $\nu_R^c \chi^2$ which can cause the fermion partner of χ to mix with neutrinos and therefore destroy the pattern of the neutrino mass matrix.

With the above, we have

$$\begin{aligned} \ell L &= Q_L \Phi_u U_R^c + Q_L \Phi_d D_R^c \\ &\quad + L_L \phi_u \nu_R^c + L_L \phi_d E_R^c \qquad (75) \\ &\quad + \nu_R^c \nu_R^c s + \nu_R^c \nu_R^c \chi \qquad (76) \end{aligned}$$

$\nu_R^c \nu_R^c s$ generates universal Majorana masses once $< s >= v_s \neq 0$.

The superpotential for the Higgs multiplets is

$$
\begin{aligned}
W &= a_1 \chi^3 + a_2 \chi^2 s + a_3 s^3 + a_4 \phi_u \phi_d + a_5 (\Phi_u \Phi_d)_{3s} \chi' + a_6 (\Phi_u \Phi_d)_{3a} \chi' \\
&+ a_7 \Phi_u \Phi_d s' + a_8 \chi'^2 s'' + a_9 s s'' + a_{10} s'^2 s'' + a_{11} s''^3. \qquad (77)
\end{aligned}
$$

From this structure, it is evident that all supersymmetric $V(\Phi, \chi)$ terms are absent from the F-term contributions, while the D-term contributions are also safe because, of course, they cannot involve the gauge singlet χ.

The D-term potential is:

$$
\begin{aligned}
V_D &= \frac{g_2^2}{8} [2\{(\Phi_u^{\alpha\dagger} \Phi_u^{\beta})_1 + (\Phi_d^{\alpha\dagger} \Phi_d^{\beta})_1 + \phi_u^{\alpha\dagger} \phi_u^{\beta} + \phi_d^{\alpha\dagger} \phi_d^{\beta}\}^2 \\
&- \{(\Phi_u^\dagger \Phi_u)_1 + (\Phi_d^\dagger \Phi_d)_1 + \phi_u^\dagger \phi_u + \phi_d^\dagger \phi_d\}^2] \\
&+ \frac{g_1^2}{8} [(\Phi_u^\dagger \Phi_u)_1 - (\Phi_d^\dagger \Phi_d)_1 + \phi_u^\dagger \phi_u - \phi_d^\dagger \phi_d]^2 \qquad (78)
\end{aligned}
$$

where α and β sum over the SU(2) index.

In the supersymmetric limit, the desired VEV structure cannot be obtained, but supersymmetry has to be broken in any case. To this end, we follow the usual soft supersymmetry breaking approach by adding to the potential all soft-breaking terms that preserve $A_4 \otimes Z_{12} \otimes Z_2$. These terms are given by

$$\begin{aligned} V_{soft} &= b_1\chi^3 + b_2\chi^2 s + b_3 s^3 + b_4\phi_u\phi_d + b_5(\Phi_u\Phi_d)_{3s}\chi' + b_6(\Phi_u\Phi_d)_{3a}\chi' \\ &+ b_7\Phi_u\Phi_d s' + b_8\chi'^2 s'' + b_9 s s'' + b_{10}s'^2 s'' + b_{11}s''^3 + H.C. \\ &+ c_1\chi^\dagger\chi + c_2 s^\dagger s + c_3 s'^\dagger s' + c_4 s''^\dagger s'' + c_5\phi_u^\dagger\phi_u + c_6\phi_d^\dagger\phi_d + c_7\chi'^\dagger\chi' \\ &+ c_8\Phi_u^\dagger\Phi_u + c_9\Phi_d^\dagger\Phi_d. \end{aligned} \tag{79}$$

The total potential is given by

$$V_{total} = V_{susy} + V_D + V_{soft} \tag{80}$$

We have checked that the total Higgs potential resulting from above admits the required forms for the VEVs as extrema for the case where all the Higgs potential parameters are real. Terms from $a_{1,2}$, $b_{1,2}$ and c_1 allow two solutions for the VEV pattern of χ, namely, that all component VEVs are equal or that only one of the them is nonzero. The latter is the desired one. Terms from $a_{5,6,7,8}$, $b_{5,6,7,8}$ and $c_{7,8,9}$ force the component VEVs to be equal, if nonzero, in each of the fields χ', Φ_u and Φ_d.