Single-electron sources: from electron optics to thermodynamic machines

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UNIT OF EXCELLENCE MARÍA DE MAEZTU

AGENCIA ESTATAL DE INVESTIGACIÓN

Contents

Part 1: Single-electron sources

How one can control the single-electron excitation in solids?

How one can measure it?

Applications

Part 2: Towards quantum heat engines

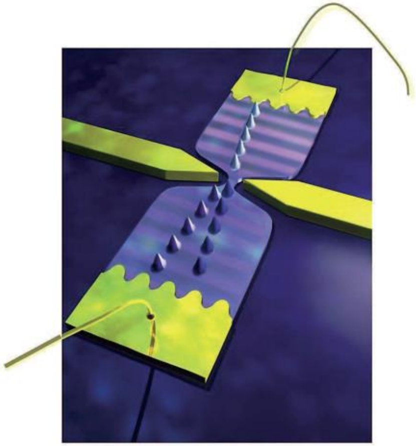
Single electron source in view of quantum heat engine

Can we archive high efficiency?

Single electron sources use AC voltage !

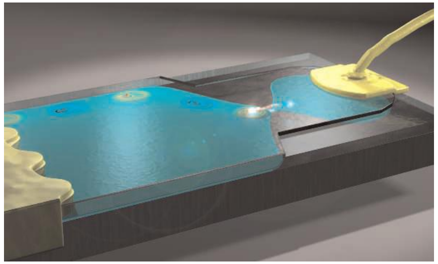
Cold electron (< 1 meV)

Leviton



T. Jullien, et al.,
Nature **514**, 603 (2014)

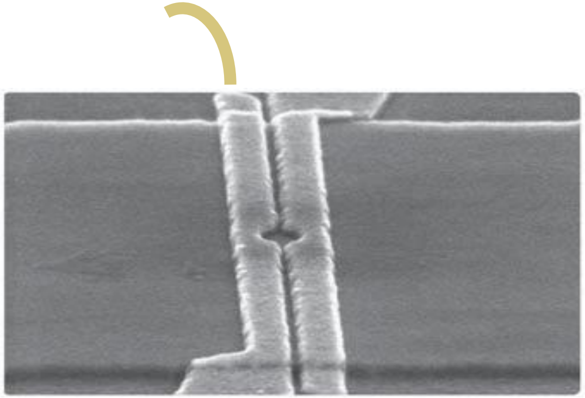
Mesoscopic capacitor



S. Giblin, Perspectives,
Science, **316**, 1130 (2007)

Hot electron (> 10 meV)

Quantum-dot(QD) pump



S.P. Giblin, et al.,
Nat. Commun. **10**, 1038 (2012)

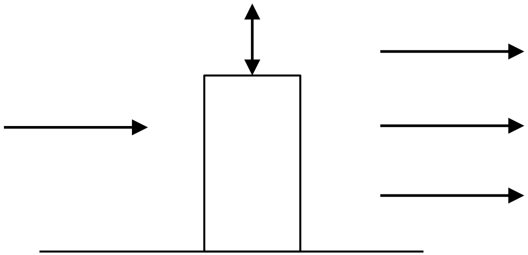
Control:
How do they generate single-electron excitation ?

Floquet scattering matrix are useful for describing AC driven system

E.g,

Oscillation with Ω

$e^{ikx-iEt}$



$E_n = E + n\hbar\Omega$

New energies after photon absorption/emission

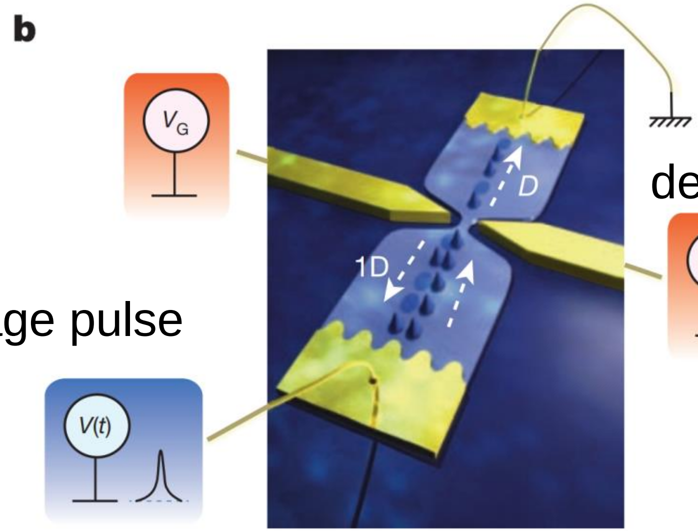
The amplitude from ingoing to outgoing state: $S_{\alpha\beta}(E_n, E)$

This describes the responses! (For noninteracting electron)

(charge/energy currents, number of electron/hole excitations)

M Moskalets, M Büttiker, PRB 66, 205320 (2002)

Leviton setup



Voltage pulse

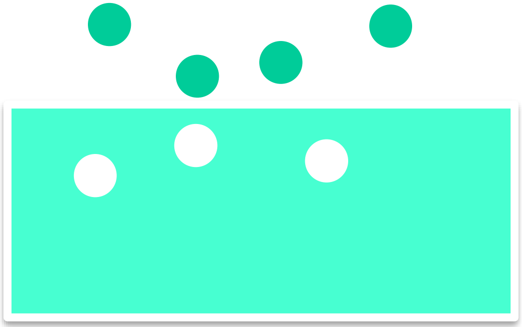
detector

Transferred charge :
(assuming spins polarized)

$$I(t) = \frac{e^2}{h} V(t), \qquad \Delta q = \frac{e^2}{h} \int V(t) dt$$

One electron transferred when $\int V(t) dt = \frac{h}{e}$

But usually excites many e-h pairs!



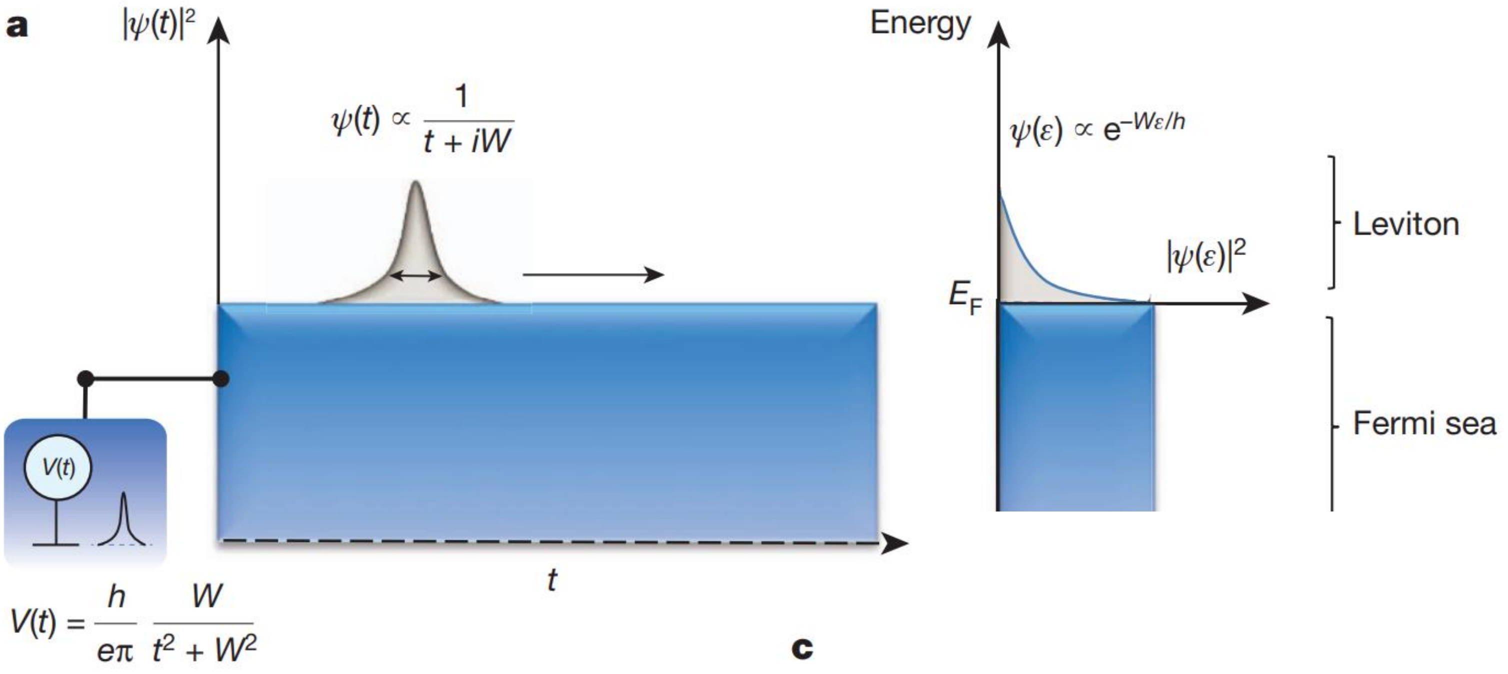
$N_e = 4$

$N_h = 3$

$$\Delta q = e(N_e - N_h) = e$$

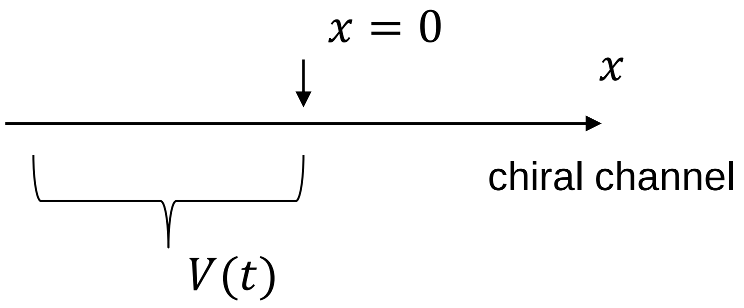
L.S. Levitov and H. Lee, J. Math. Phys. 37, 4845 (1996)

When applying Lorentzian pulse, only one electron is excited. (zero temp.+no interaction.)



J. Keeling, I. Klich, and L. S. Levitov, PRL 97, 116403 (2006)
J. Dubois, et al., Nature 502, 659 (2013)

Let’s obtain the Floquet scattering matrix.



Linear dispersion: $K = \frac{p^2}{2m} \to (\text{const}) + vp$

Time-dependent Schrodinger equation:

$$i\hbar\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)\psi(x,t) = V(t)\Theta(-\mathrm{x})\psi(x,t)$$

Solution: $\psi(x,t) = e^{-\frac{i}{\hbar}E\left(t-\frac{x}{v}\right)}\left[\Theta(-x)e^{-i\phi(t)} + \Theta(x)e^{-i\phi(t-x/v)}\right]$

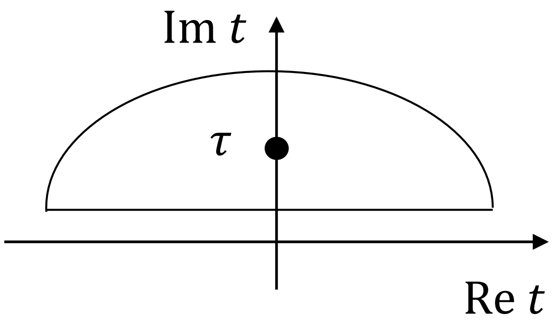
$$\phi(t) = \frac{1}{\hbar}\int_{-\infty}^{t} dt' eV(t')$$

Fourier transform of $\mathrm{e}^{-i\phi(t-x/v)}$ determines the Floquet scattering matrix.

Let's calculate the fourier transform for Lorentzian $V(t) = \frac{h}{\pi e}\frac{\tau}{t^2+\tau^2}$

$$\phi(t) = \frac{1}{\hbar}\int_{-\infty}^{t} dt' eV(t') = 2\arctan\left(\frac{t}{\tau}\right) + \pi \qquad \to\ e^{-i\phi(t)} = -\frac{\tau - it}{\tau + it} = \frac{t + i\tau}{t - i\tau}$$

Fourier transform is obtained by contour integration



$$S(E,E') = \int dt\, e^{-i\phi(t)} e^{i(E-E')t/\hbar}$$

$$S(E,E') = h\delta(E - E') - 2h\tau\, \mathrm{e}^{-\tau(E-E')}\, \theta(E - E')$$

➔ No hole creation!

Number of electron excitations: $$N_e = \int_{E'<E_F<E} \frac{dE'}{h}\frac{dE}{h}|S(E,E')|^2 = 1$$

Rank-1 property

Why one single electron excitation?

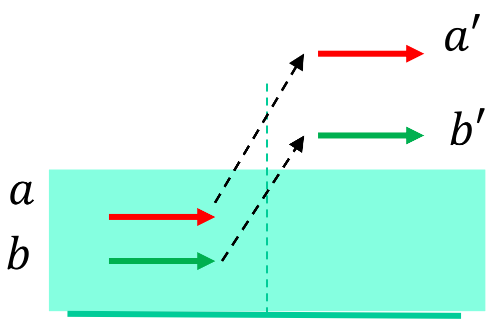
Rank-1 property of Floquet scattering matrix

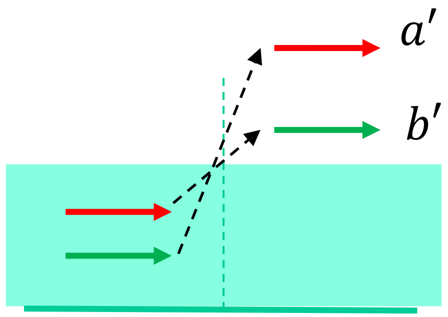
i.e., $S_+(E,E') = -2\tau\, e^{-\tau(E-E')}$
$= \langle E|\phi_+\rangle\langle\phi_-|E'\rangle$

Consider if there is two or more electrons excited

Then, focus on a pair

Two process are possible:

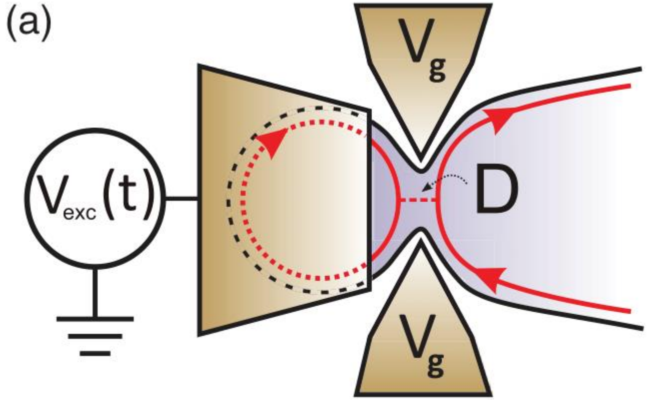




amplitudes: $\langle a'|\phi_+\rangle\langle\phi_-|a\rangle\langle b'|\phi_+\rangle\langle\phi_-|b\rangle$ $\quad -\langle b'|\phi_+\rangle\langle\phi_-|a\rangle\langle a'|\phi_+\rangle\langle\phi_-|b\rangle$

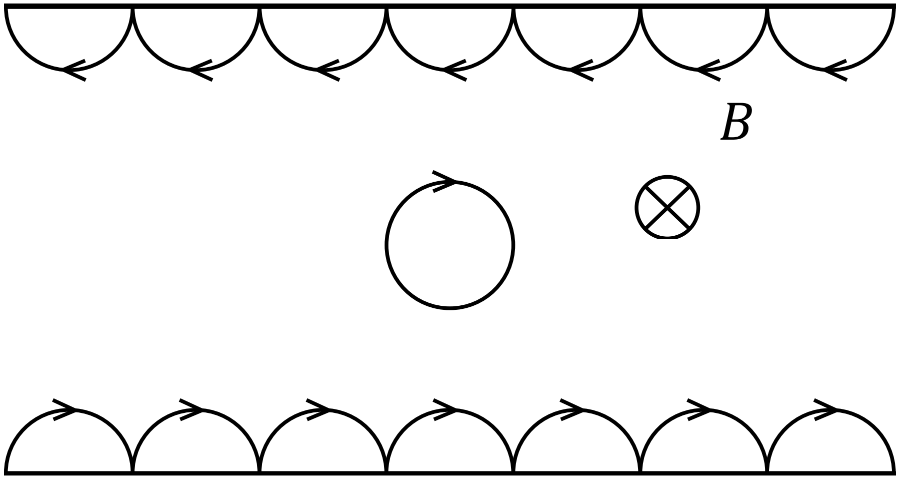
They cancel due to Fermionic statistics!

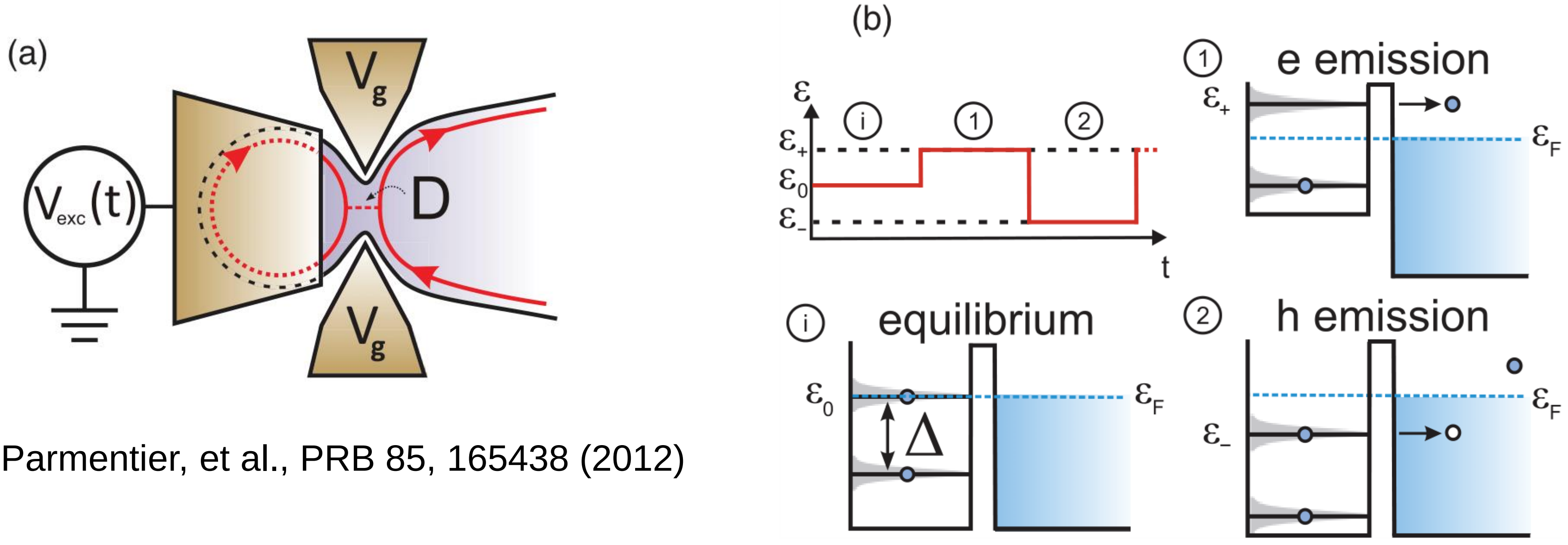
Another type of cold electron source: Mesoscopic capacitor



F. D. Parmentier, et al., PRB 85, 165438 (2012)

Edge states





F. D. Parmentier, et al., PRB 85, 165438 (2012)

When a QD level rises linearly in time, it only transfers a single electron excitation !

[J. Keeling, A. V. Shytov, and L. S. Levitov, PRL 101, 196404 (2008)]

$$S(E,E') = h\delta(E-E') - h\Theta(E-E')\frac{\gamma}{c}\exp\left[-\frac{(E-E')}{2c\gamma^{-1}} + \frac{i}{\hbar}\frac{E^2-E'^2}{2c}\right]$$

Exponential decay

c : rate of the rise
γ^{-1} : life time

Phase gain

$$\int_{t'}^{t} d\tau\, c\tau = c\frac{t^2-t'^2}{2}$$

$$t = \frac{E}{c}\,, \qquad t' = \frac{E'}{c}$$

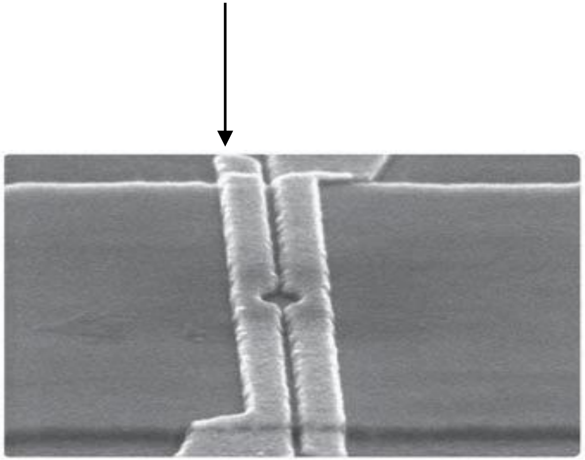
Θ function ➔ No holes!

S is Rank-1 ➔ Single electron excitation!

Wave function: Lorentzian convoluted with energy-time correlated wave

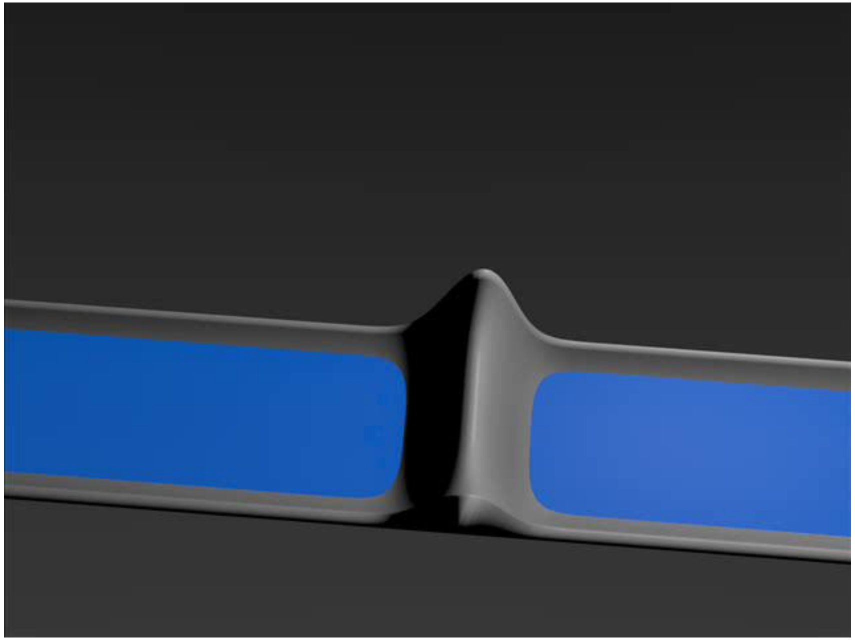
Hot electron source: Quantum-dot pump

Time dependent voltage



S.P. Giblin, et al.,
Nat. Commun. **10**, 1038 (2012)

Dynamic quantum dot *pumps* single electron



Barrier rises ➔ Capture
Quantum dot potential also rises ➔ Emission

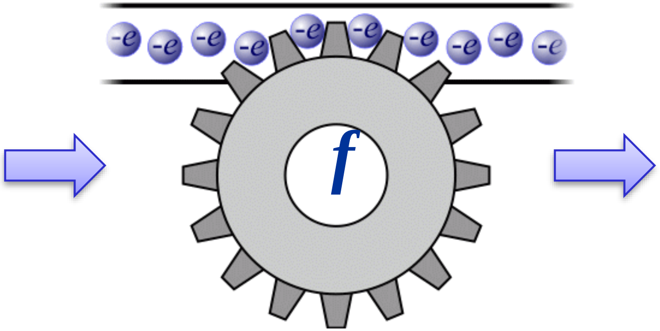
- Quantum-dot pump is highly accurate! (error: 0.1 ppm)

Proposed redefinition of ampere

“The current in the direction of flow of a particular number (~ 6.2415093×10^{18}) of elementary charges per second"

Time

Device



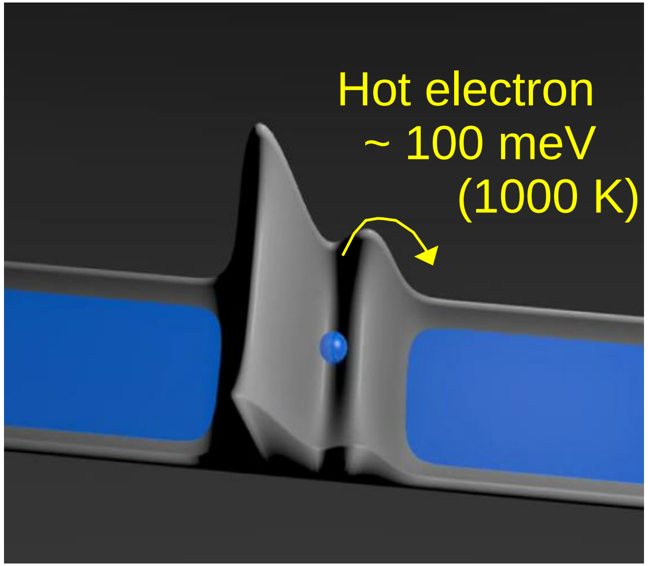
Current

Quantum Current Standard

Hot electron is generated because of the barriers !

Oscillating barrier generates e-h excitations

- ➔ But they are near Fermi-level
- ➔ Energetically separated from pumped electron
- ➔ No need to fine tune the AC voltage shape



But difficult to control emitted ***wave function*** in simple forms

- ➢ Capture : usually complicated nonequilibrium dynamics
- ➢ Emission : Nonadiabatic excitation, tunneling

Proposal of Gaussian source

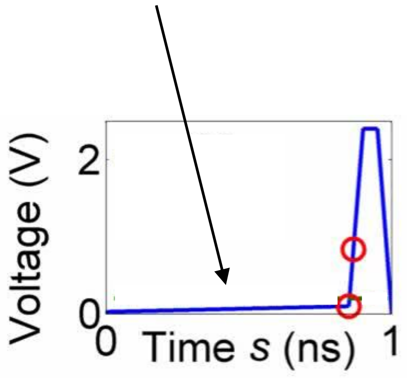
2) Strong magnetic field (~10T)

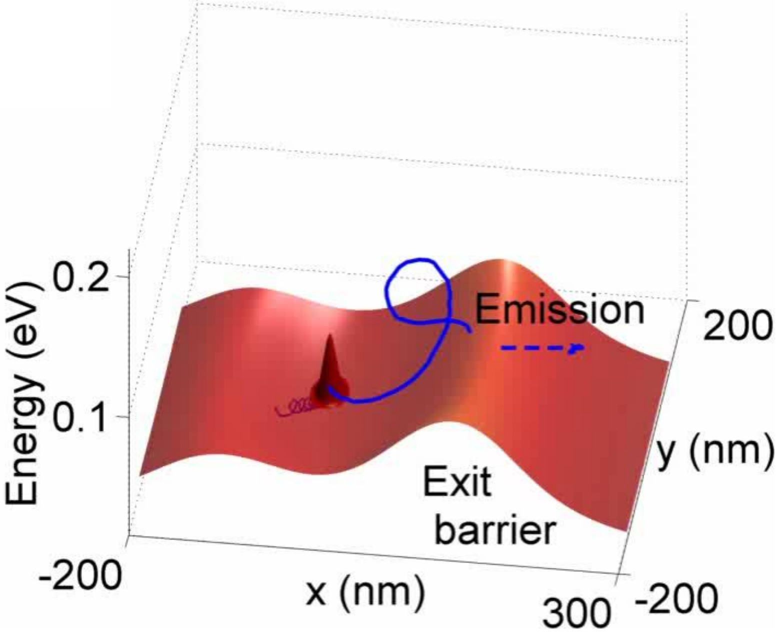
➔ Additional Magnetic confinement

➔ Formation of Coherent state (~10nm)

1) Engineered time-dependent voltage

➔Adiabatic capturing





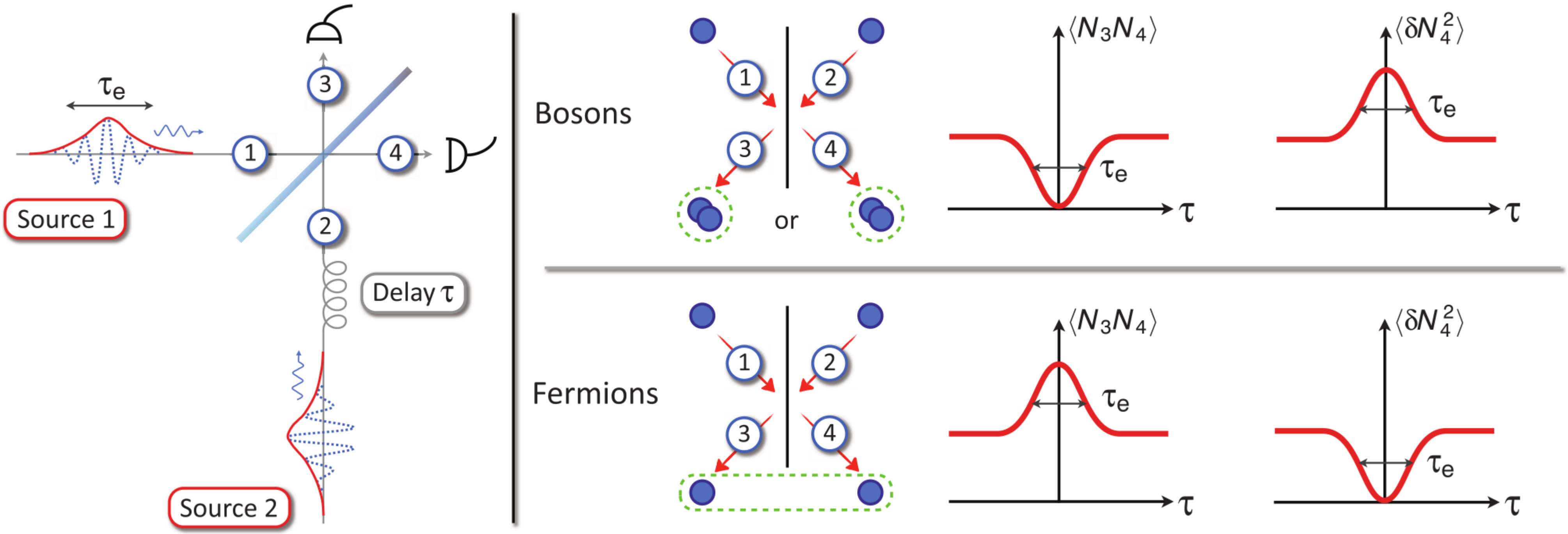
Gaussian packet emitted!

S. Ryu, M. Kataoka, H.-S. Sim, PRL (2016)

Measurements

Cold electrons are measured by quantum tomography.

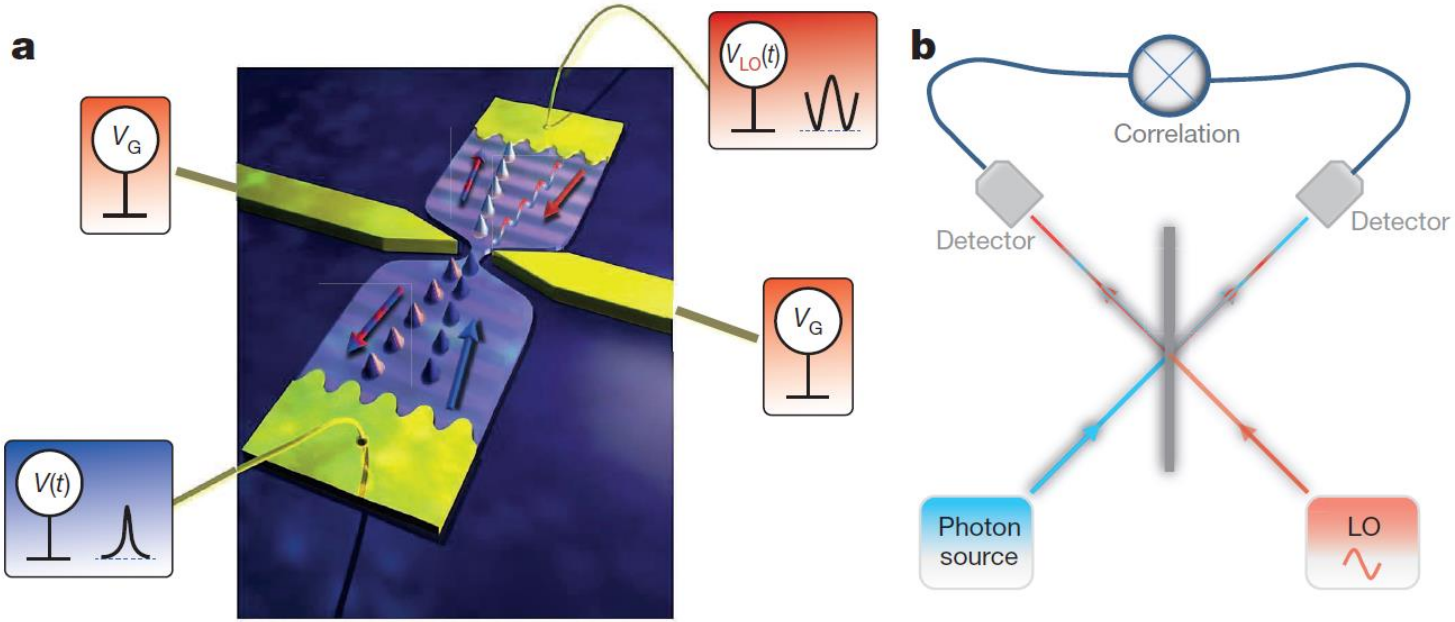
They are based on Hong-Ou-Mandel effect



$$\langle \delta N_4^2 \rangle \propto 1 - |\langle \varphi_1 | \varphi_2 \rangle|^2$$

E. Bocquillon, et al., Science 339, 1054 (2013)

For quantum tomography, a target is collided with a reference state.

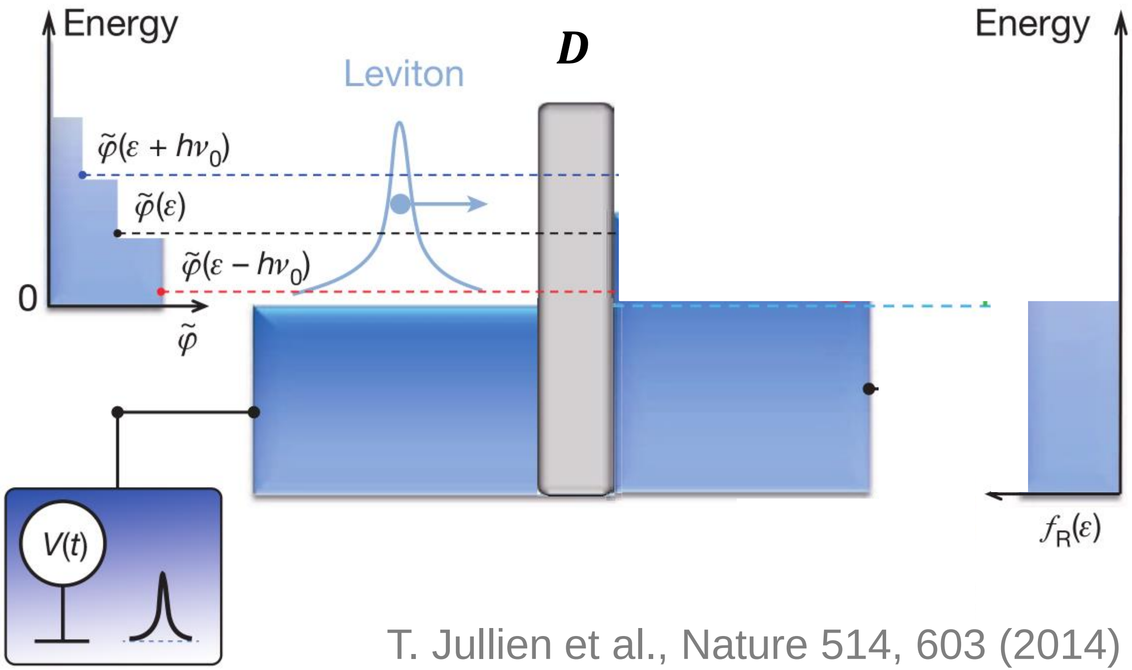


T. Jullien et al., Nature 514, 603 (2014)

Bunching (antibunching) depends on $\left|\langle \psi_{\rm target} | \psi_{\rm ref} \rangle\right|^2$

For simplicity, assume a pure state $\tilde{\varphi}$, zero temperature.

ν_0: Repeating freq.



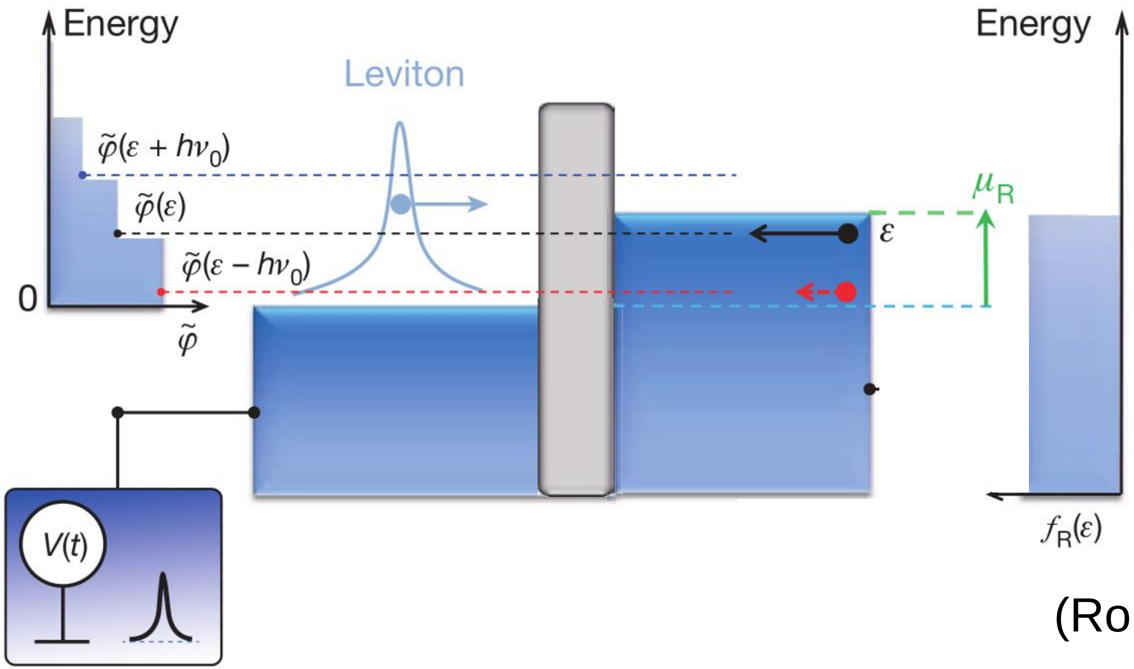
T. Jullien et al., Nature 514, 603 (2014)

When chemical potentials are same, $\langle \delta N^2 \rangle = D(1-D)$

Partitioning noise

D : Transmission prob. (constant)

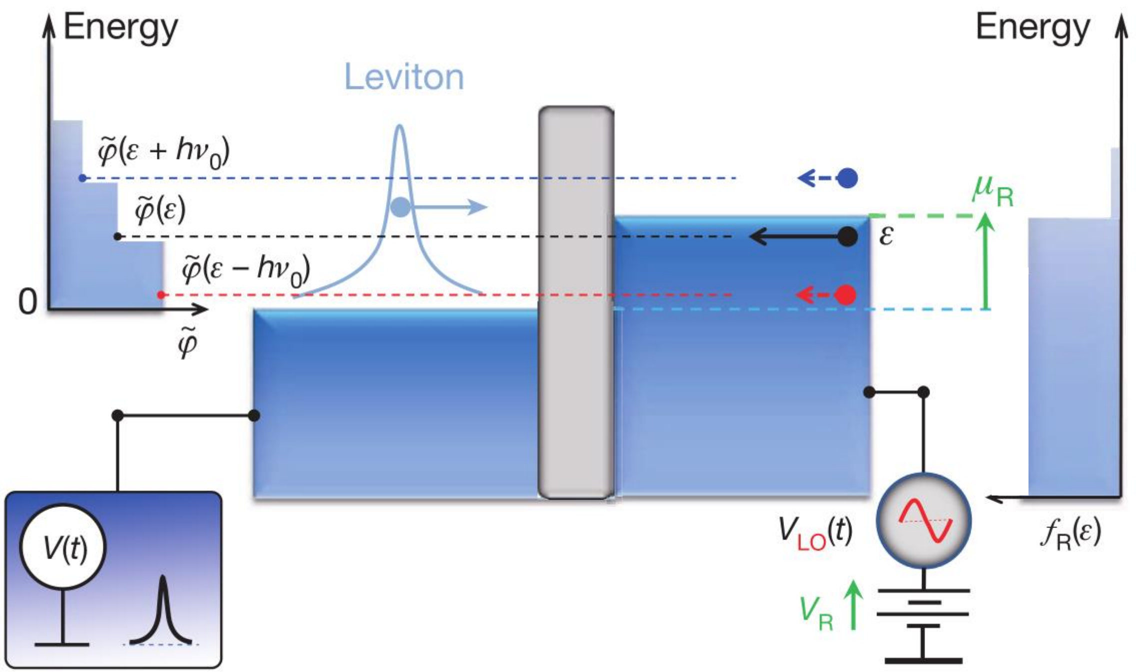
When applying DC voltage to the right reservoir, the probability density $|\tilde{\varphi}(\varepsilon)|^2$ is obtained.



(Rough sketch of the protocol)

$$\frac{d\langle \delta N^2 \rangle}{d\mu_R} \propto |\langle \varepsilon = \mu_R | \tilde{\varphi} \rangle|^2 \text{ due to the anti-bunching !}$$

Off-diagonal terms $\tilde{\varphi}^*(\varepsilon)\tilde{\varphi}(\varepsilon')$ can be obtained similarly, but with harmonic driving to the right res.



Effect of weak harmonic driving:

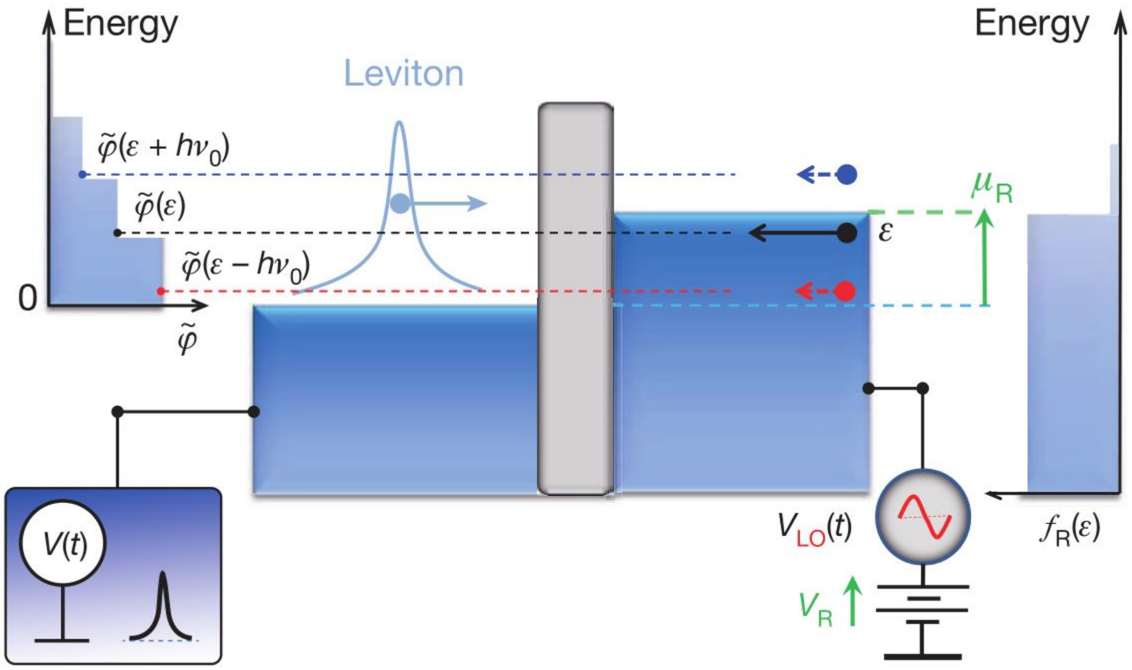
$\eta\, e^{i2\pi\nu\tau}$

$1 - \eta^2$

$-\eta\, e^{-i2\pi\nu\tau}$

$\eta\, h\nu \cos 2\pi\nu(t - \tau)$

(Can be easily driven by using that F.T of $e^{\sin t}$ is Besell function)



The antibunching overlap is

$$\varphi^*(\varepsilon)\left(1-\eta^2\right)$$
$$+\varphi^*(\varepsilon+h\nu)\eta e^{i2\pi\nu\tau}$$
$$-\varphi^*(\varepsilon-h\nu)\eta e^{-i2\pi\nu\tau}$$

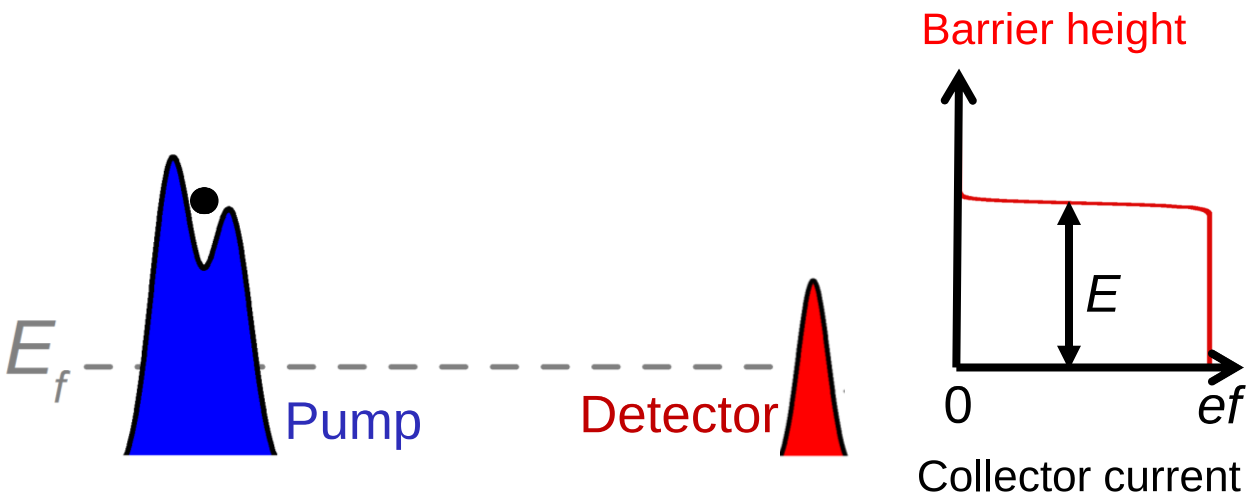
The derivative of the noise $\frac{d\langle\delta N^2\rangle}{d\mu_R}$

$$\propto |\varphi(\mu_R)|^2$$
$$+\eta\,\mathrm{Re}\Big[\varphi^*(\mu_R+h\nu)\varphi(\mu_R)e^{i2\pi\nu\tau}$$
$$-\varphi^*(\mu_R-h\nu)\varphi(\mu_R)e^{-i2\pi\nu\tau}\Big]$$

By tuning ν and τ, real and imaginary part of the off-diagonal parts can be obtained!

Hot electron: energy/time spectroscopy demonstrated in experiments

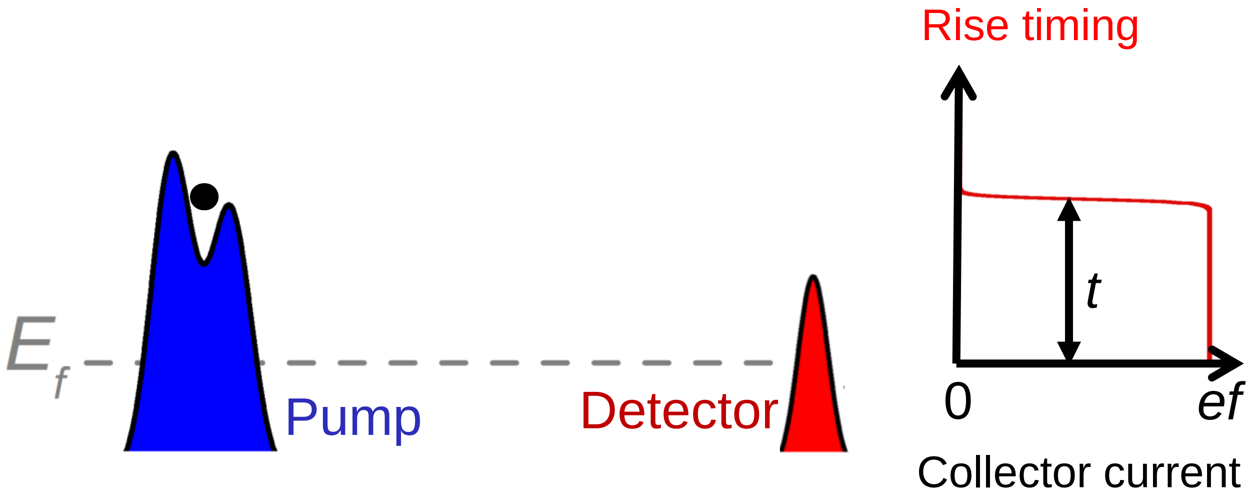
J. D. Fletcher et. al., Phys. Rev. Lett. 111, 216807 (2013).



Energy resolution is better for sharply changing transmission

***Dynamic* potential barrier** can work as an arrival time detector.

J. D. Fletcher et. al., Phys. Rev. Lett. 111, 216807 (2013).



Time resolution is better for fast rising barrier

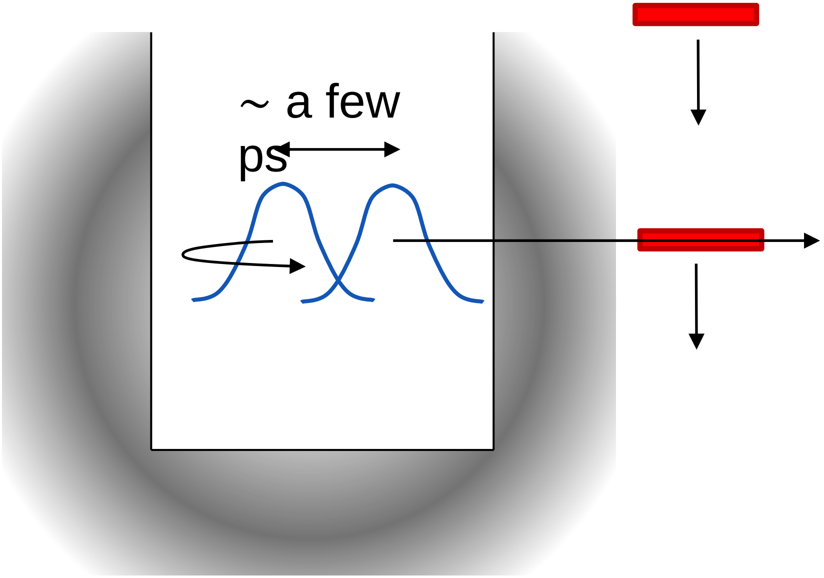
Fast electron oscillations in the quantum-dot has also been demonstrated

Target

(Oscillating motion in a cavity)

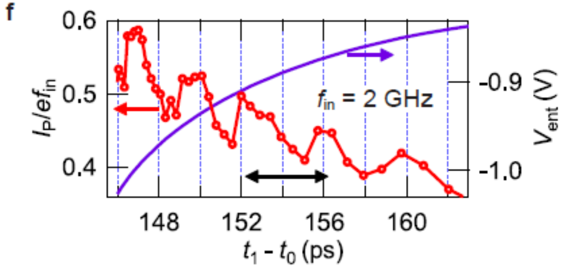
Detector

(Dynamic resonant level)



No current

Finite current (resonant tunneling)

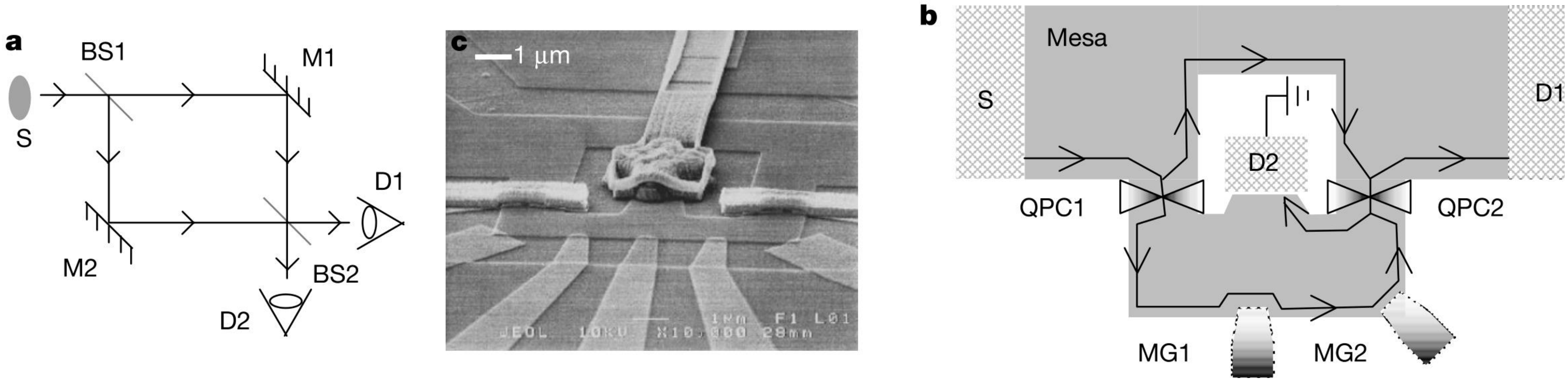


G. Yamahata, Sungguen Ryu, et al, Nat. nanotechnol. (2019)

Applications

Electron quantum optics experiments show wave nature of electrons !

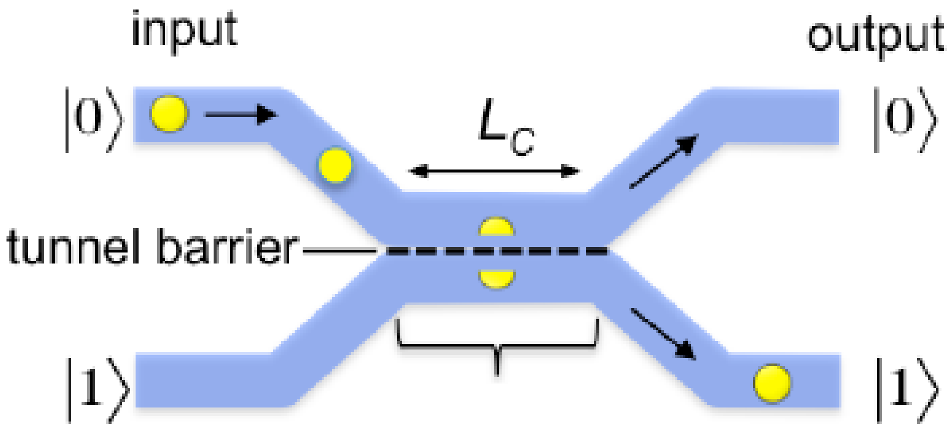
- Test bed for checking coherence for further quantum information processing using electrons
- E.g., first realization of Mach-Zehnder interferometer using continuous stream of electron



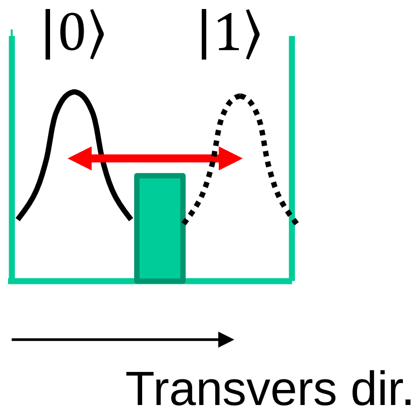
Yang Ji, Nature **422** 415–418 (2003

More electron quantum optics using single electron sources are under progress.

Application in quantum information processing: Flying qubit proposal

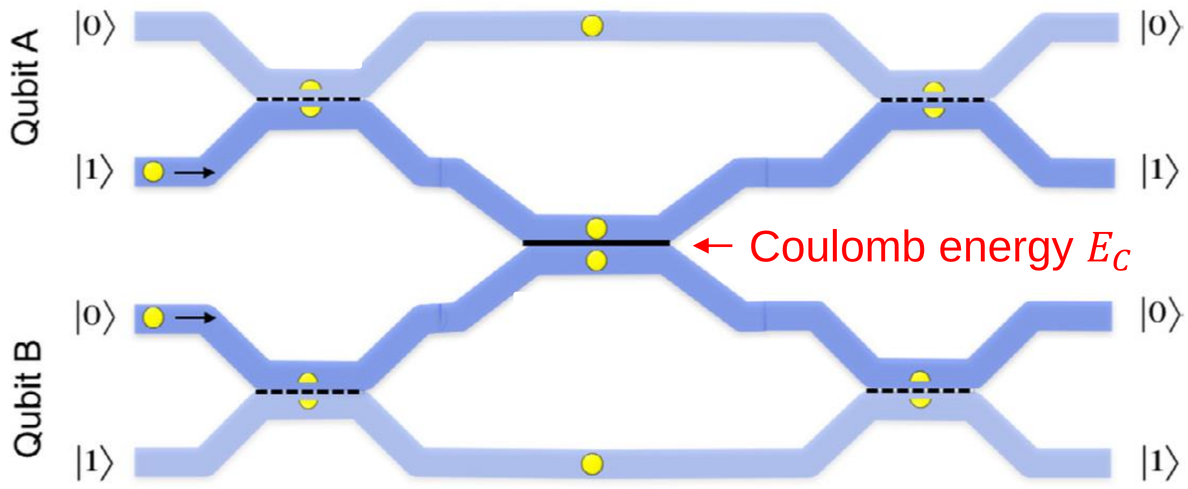


Single-qubit operation



C. Bäuerle et al., Rep. Prog. Phys. (2018)

Two-qubit operation is realized by using Coulomb interaction



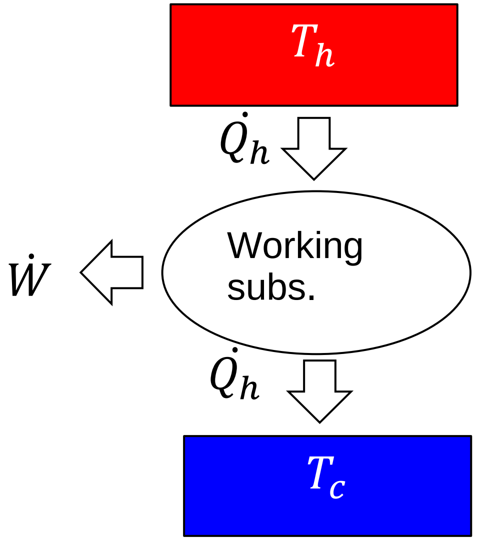
C. Bäuerle et al., Rep. Prog. Phys. (2018)

Only $|10\rangle \rightarrow e^{-i\mathrm{E}_C t}|10\rangle$: Controlled-phase gate!

Part 2: Towards quantum heat engines

Heat engines convert heat to useful work.

- Efficiency $\eta = \frac{W}{Q_h}$



Thermodynamics has been developed by asking a question:

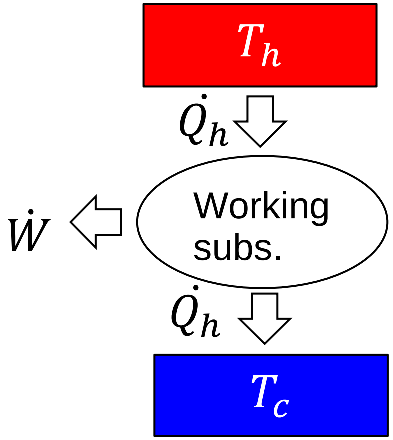
What is the **fundamental limit for efficiency** of heat engines?

Traditional engines consists of macroscopic moving parts (piston).

→ Working substance is near equilibrium.

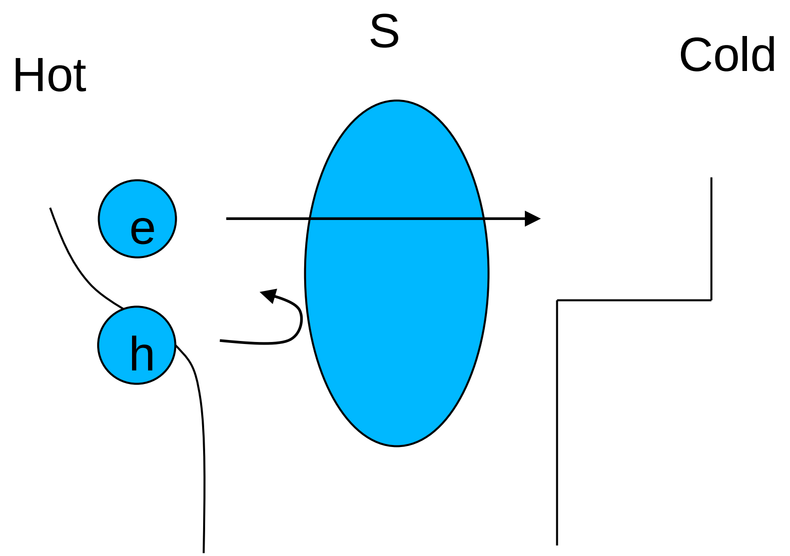
Carnot efficiency: $\eta \leq \eta_C$ $\eta_C = 1 - \frac{T_c}{T_h}$ → Universal

Consequence of:
- 2nd law: $\dot{S} \geq 0$
- Clausius relation $\dot{S} = \frac{\dot{Q}_h}{T_h} + \frac{\dot{Q}_c}{T_c}$



Scheme of heat engine

Thermoelectric devices promote sustainable energy system! (No macroscopic moving parts)

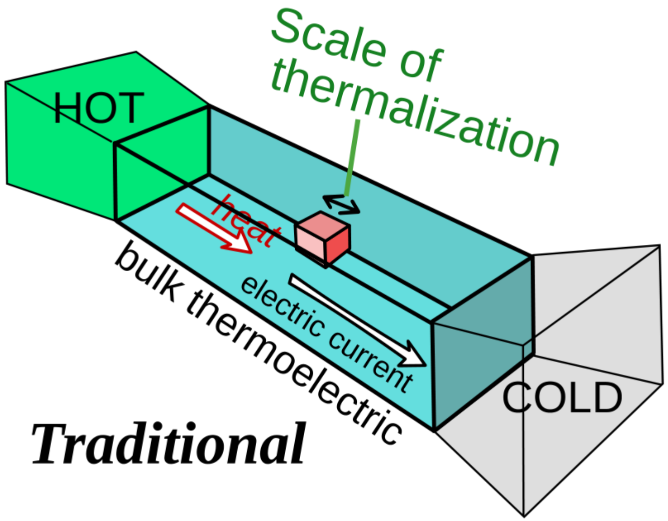


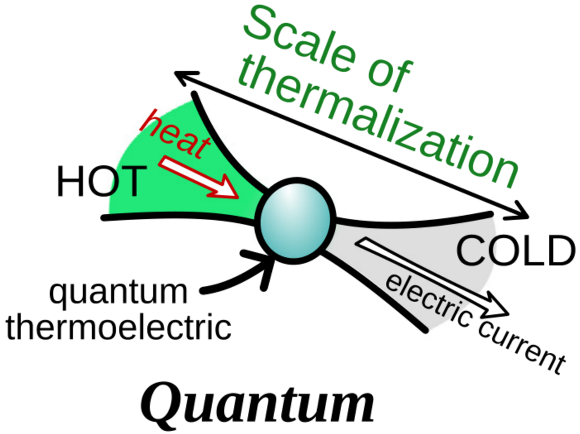
Electron-hole asymmetry generates thermoelectricity!

Quantum thermodynamics consider micro-engines.

E.g., classical vs quantum thermoelectrics

[G. Benenti et al., Phys. Rep. 2017]





- Local equilibrium
- Boltzman transport theory

- Nonequilibrium
- Scattering theory

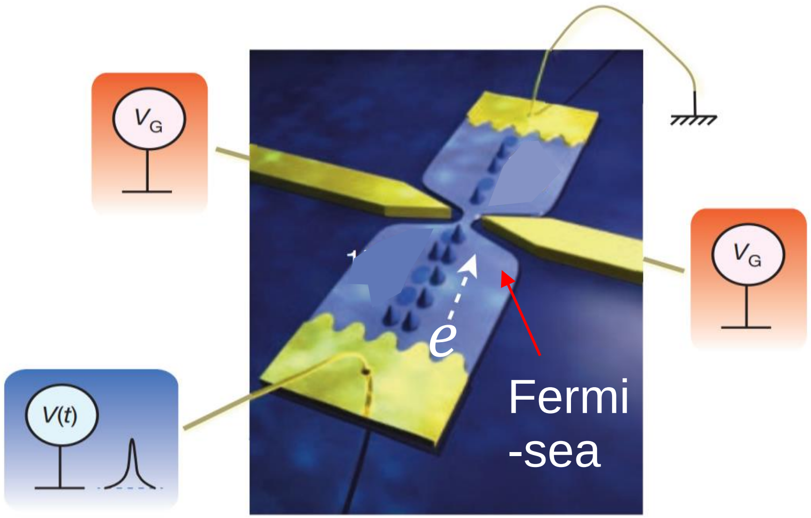
The quantum **coherence/correlation** can be built up!

How the quantumness affects the functionality?

A pioneering work showed that coherence can be also used as resource!

[M.O. Scully, et al., Science (2003)]

- Showed efficiency **beyond Carnot limit.**
- Important to clarify what is { heat, work, entropy production }



[J. Dubois et al, Nature (2013)]

Single-electron sources have shown impacts in

- Metrology
- Quantum information
- Electron quantum optics

However, clarifying heat/energy/entropy currents are only of recent interest.

Fast AC driving induce strong nonequilibrium effect **beyond adiabatic response**.

Can one enhance engine efficiency using the AC driving?

An AC driven quantum device can have efficiencies beyond the Carnot limit.

S Ryu, R López, L Serra, D Sánchez, Nat. Commun. 13, 1 (2022)

Thanks to:

i) AC driving

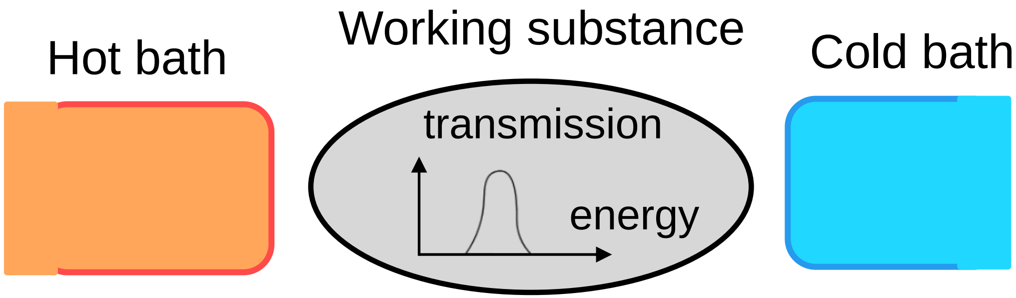
- Violation of the Clausius relation (nonequilibrium)

ii) Chirality

- No power injection by AC

2nd law is not violated when adopting ***Shannon entropy flow*** as entropy production

In the absence of AC driving:



Energy dependent transmission

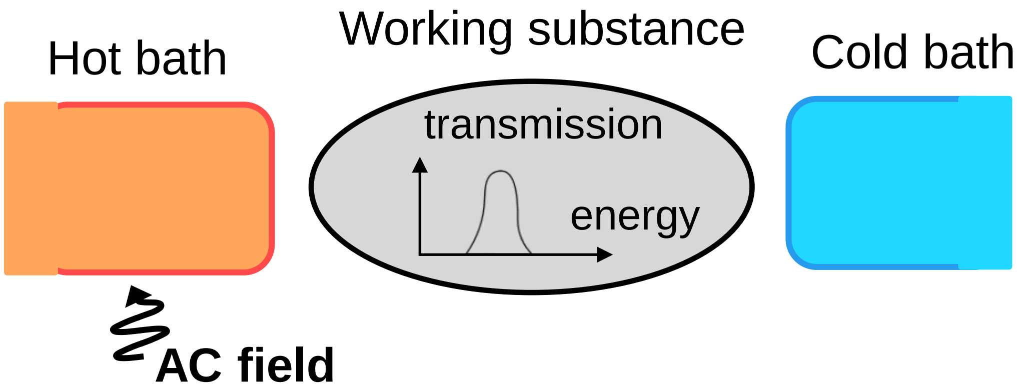
→ *e-h* asymmetry

→ thermoelectric current I_e^R

→ power generation $P_e = (\mu_R - \mu_L) I_e^R / e$

➢ *efficiency* $\eta = \frac{P_e}{\dot{Q}_H}$

With AC driving:



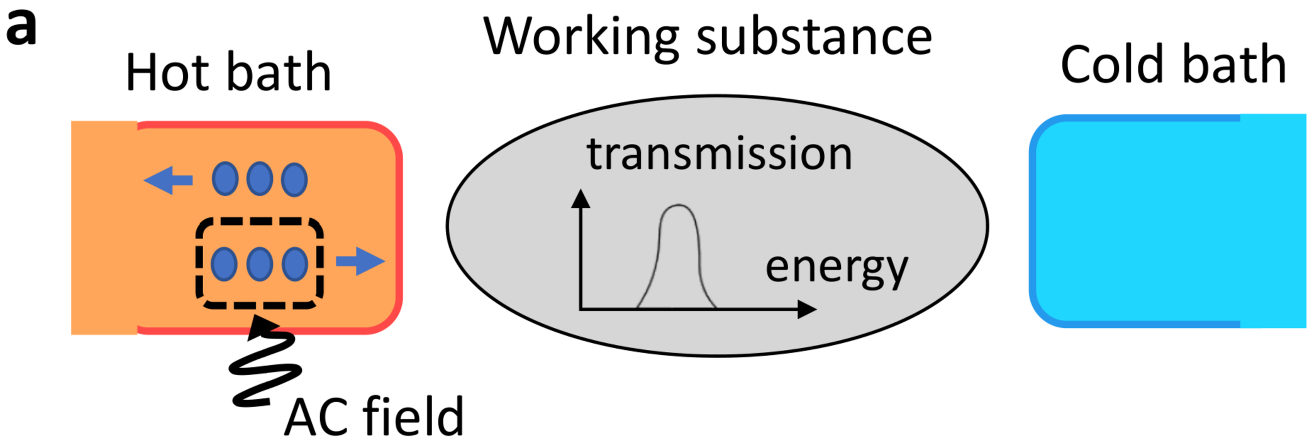
Dynamical e-h asymmetry (Levitons) ➔ enhanced thermoelectric current

But AC field injects power P_{in} to electrons (working substance) ➔ Less power output

- efficiency with AC: $\eta = \frac{P_e - P_{\mathrm{in}}}{\dot{Q}_H}$

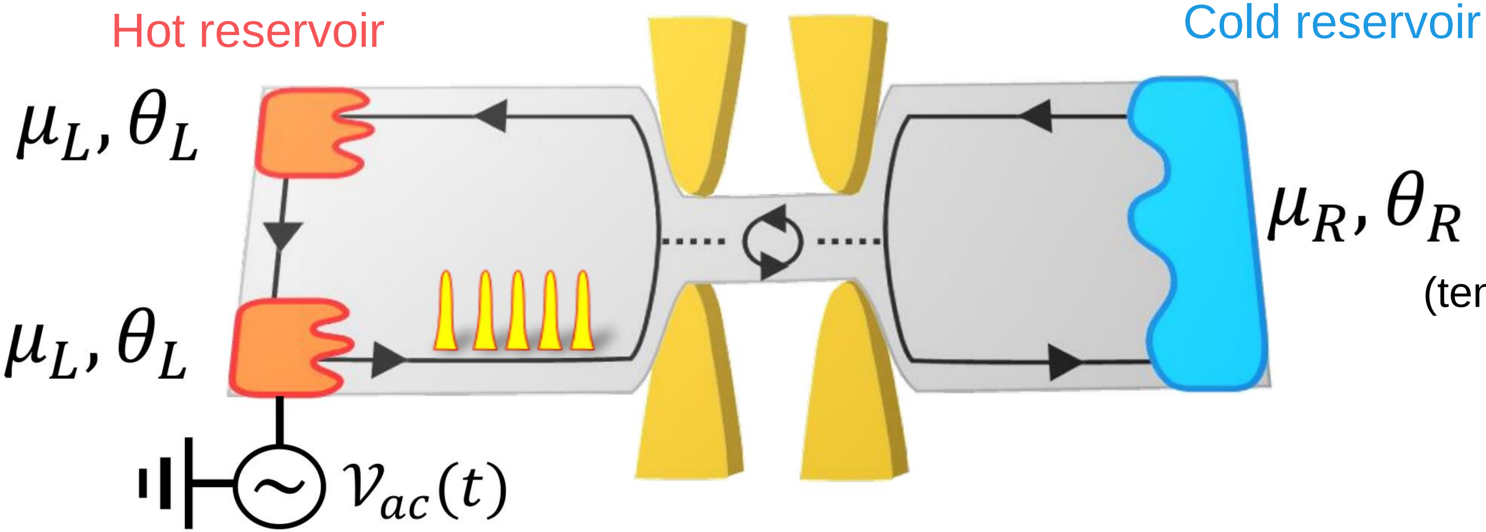
We couldn't find substantial efficiency enhancement… ☹

We find remarkable efficiency enhancement when using selective AC voltage!



This completely avoids any power injection by AC driving!

Realization using chiral conductor



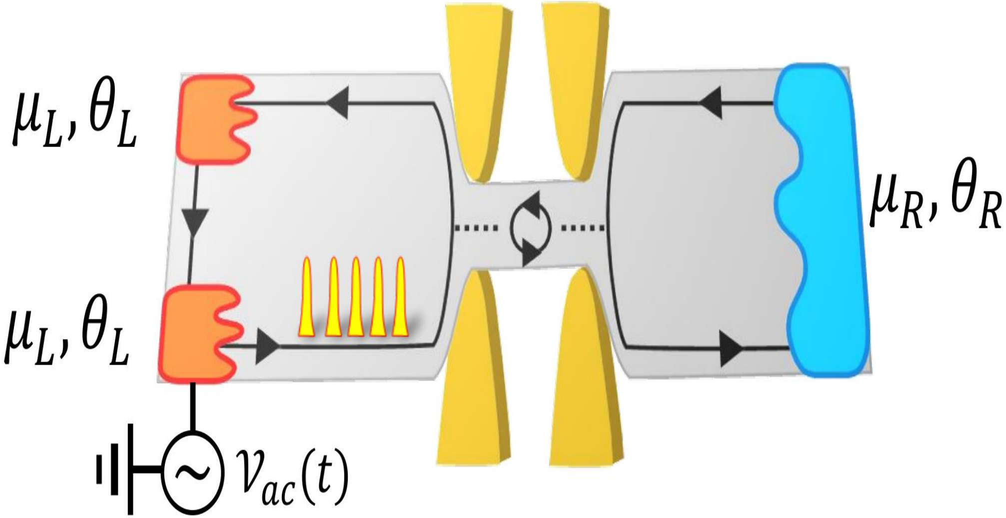
(temperature)

$\mathcal{V}_{ac}(t)$: Lorentzian pulses which excite Levitons

We only consider time-averaged currents of charge/heat/energy.

We use Floquet scattering matrix formalism to determine the currents and powers

[M. Moslaket and M. Büttiker, PRB (2002)]



E.g., transmission from L to R with ph. absorb.

i) absorption of n photons with amplitude

$$a_n = \frac{\Omega}{2\pi}\int_0^{2\pi/\Omega} dt\, e^{-i\frac{e}{\hbar}\int_{-\infty}^{t} dt' V_{ac}(t')} e^{in\Omega t}$$

phase factor due to AC $\quad$ n-th mode

(Ω: AC frequency)

$$\mathcal{E} \to \mathcal{E}_n = \mathcal{E} + n\hbar\Omega$$

ii) transmission with $\mathcal{E}_n$

➢ $S_{RL}(\mathcal{E}_n, \mathcal{E}) = a_n t_{\text{static}}(\mathcal{E}_n)$

R to L: $\quad S_{LR}(\mathcal{E}_n, \mathcal{E}) = \delta_{n,0} t'_{\text{static}}(\mathcal{E})$

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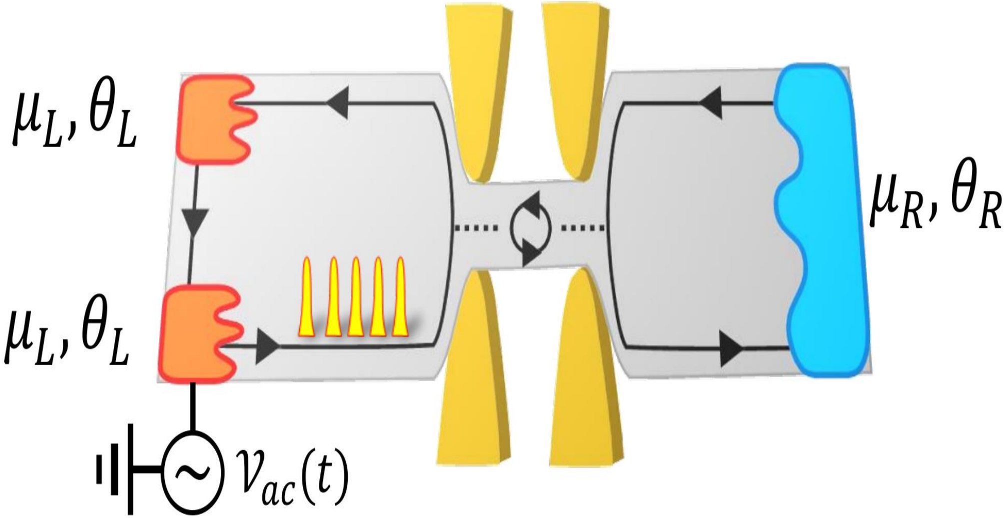

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phase factor due to AC — n-th mode

(Ω: AC frequency)

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➢ $S_{RL}(\mathcal{E}_n, \mathcal{E}) = a_n t_{\mathrm{static}}(\mathcal{E}_n)$

R to L: $S_{LR}(\mathcal{E}_n, \mathcal{E}) = \delta_{n,0} t'_{\mathrm{static}}(\mathcal{E})$

Remarkably, the **power injection** by AC voltage **vanishes** !

$$P_{in} = \sum_{\alpha,\beta} \int \frac{d\mathcal{E}}{h} \sum_n n\hbar\Omega \left|S_{\alpha\beta}(\mathcal{E}_n, \mathcal{E})\right|^2 f_\beta(\mathcal{E})$$

Mean energy change in photoassisted scattering from β to α *Fermi-distribution of reservoir β*

Due to the chirality, the *mean energy change* is
- $e\overline{\mathcal{V}_{ac}} = \boldsymbol{0}$ for the left input,
- $\boldsymbol{0}$ for the right input

➢ Valid regardless of the form of $\mathcal{V}_{ac}(t)$!

For nonchiral conductors, the power injection is generally nonvanishing.

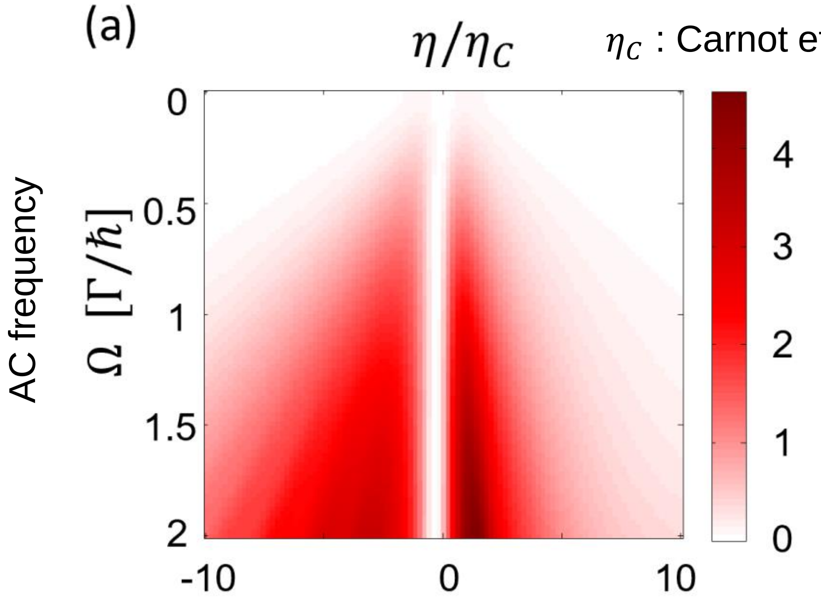
E.g., for adiabatic driving, we find that it satisfies the Joule's law: $P_{in} \propto \overline{\mathcal{V}_{ac}^2} > 0$

➢ No significant efficiency enhancement

AC driving enhances the efficiency when reaching nonadiabatic regime $\hbar\Omega > \Gamma$

Γ = resonance level broadening $\equiv 1$

Color plot of efficiency



η_C : Carnot efficiency

Average chemical potential μ $[\Gamma]$

(measured from the resonance level)

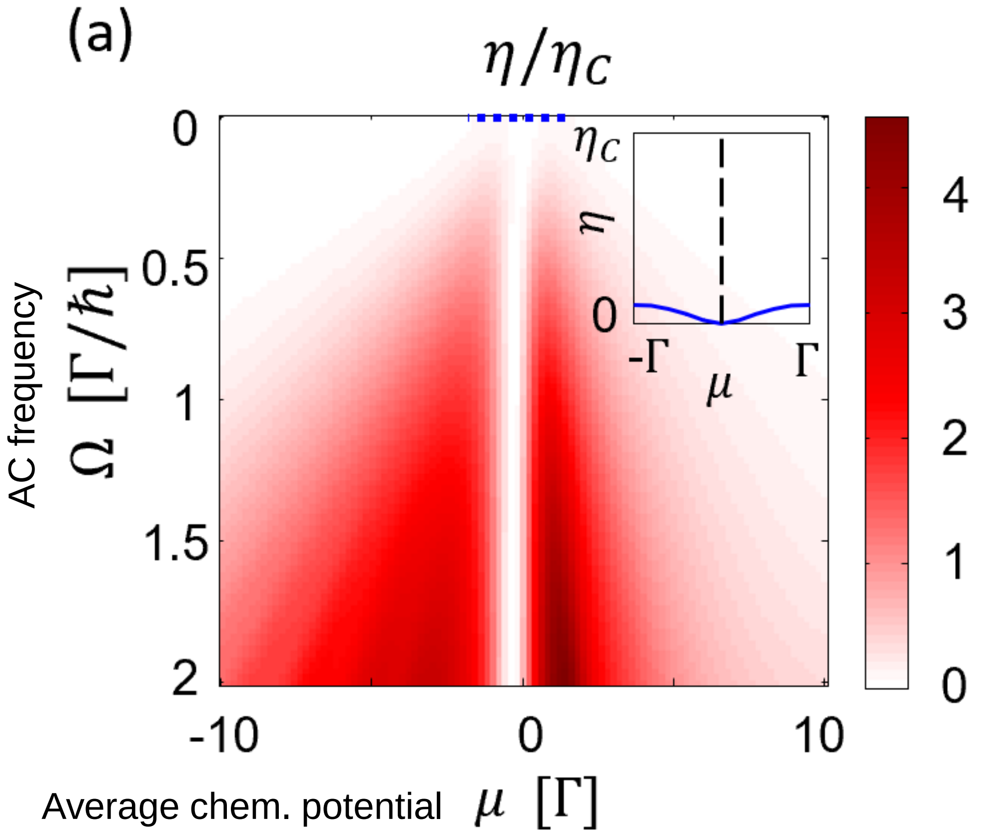
Bath temperatures are chosen in:

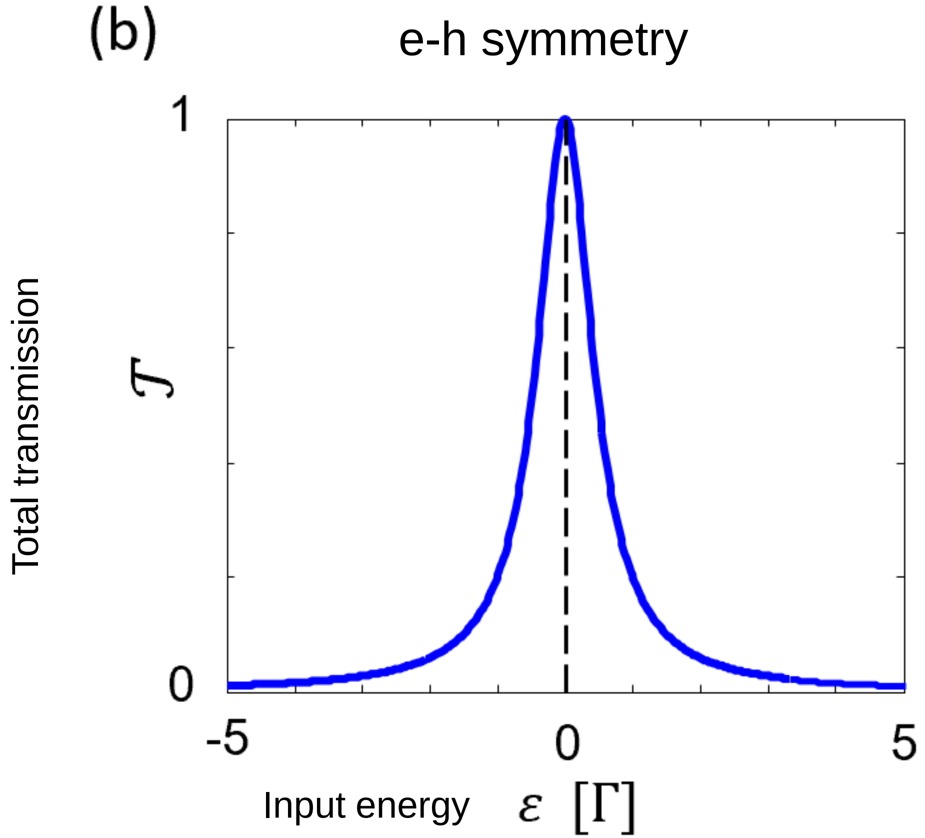
- Coherent regime: $\theta = 0.25\Gamma/k_B$
- Linear regime: $\theta_L - \theta_R = 0.1\theta$

Chemical potential difference is chosen to generate maximum power

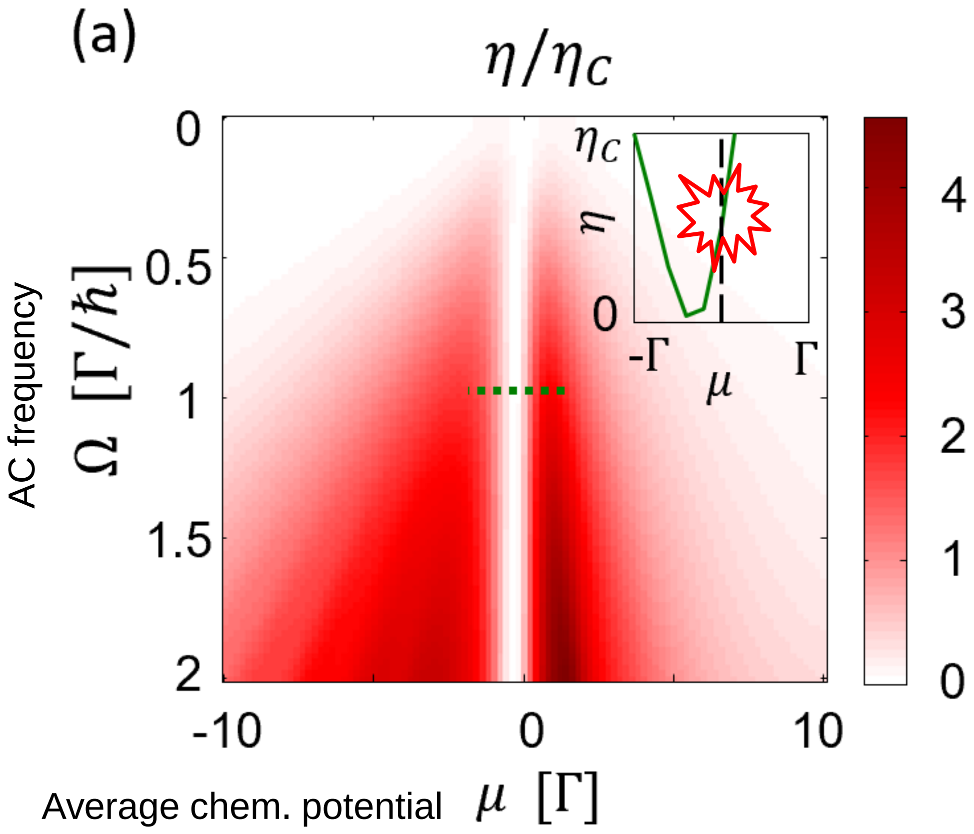
Notably, at $\mu = 0$, the driving realizes a *photoassisted thermoelectric engine*

Static case:



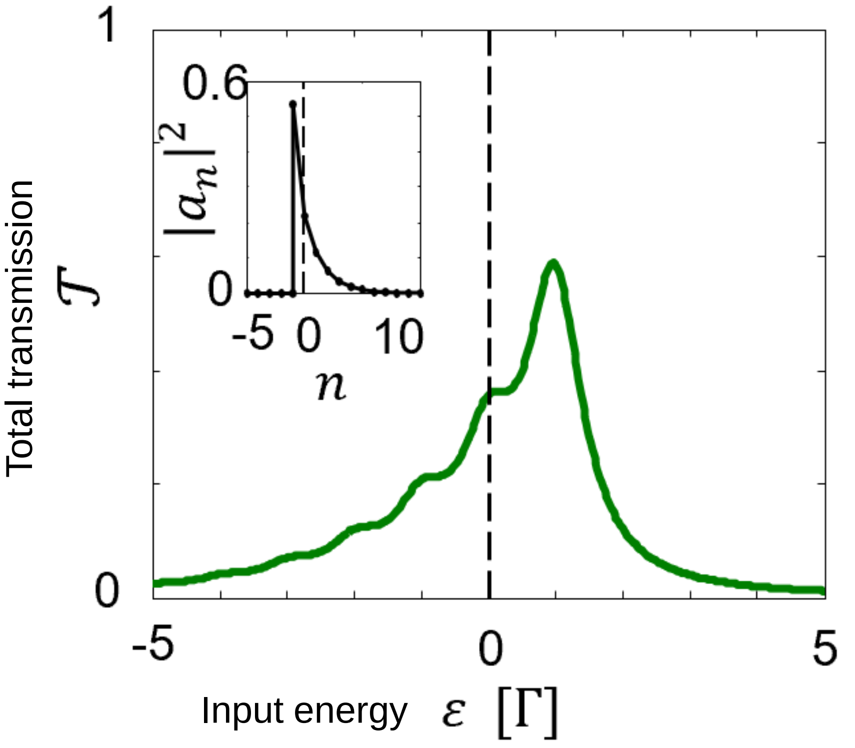


AC driven case:



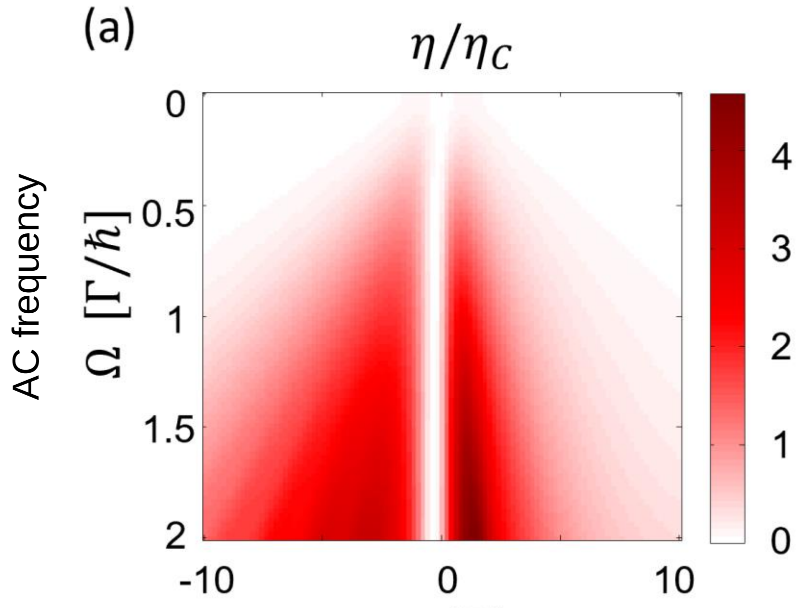
e-h asymmetry dynamically induced (Levitons)

(b)

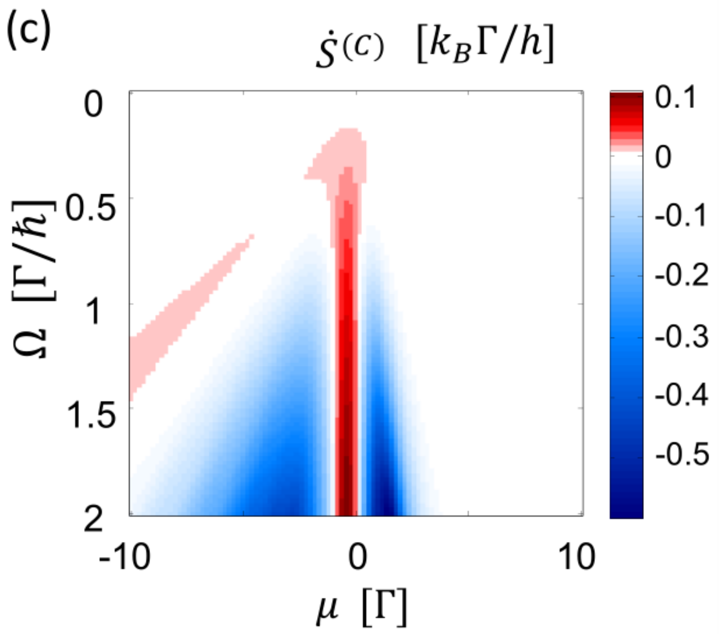


Remarkably, the efficiencies in the AC driven case become even larger than the Carnot limit

This occurs when the entropy production $\dot{S}^{(C)} = \frac{\dot{Q}_L}{\theta_L} + \frac{\dot{Q}_R}{\theta_R}$ assuming the Clausius relation is *negative*



Average chem. potential μ [Γ]



The seeming violation of the second law of thermodynamics is resolved when adopting the Shannon entropy flow as the entropy production,

[A. Bruch, et al., PRL (2018)]

binary Shannon entropy function

$$\dot{S} = \frac{k_B}{h} \sum_{\alpha=L,R} \int d\mathcal{E} \left(-\sigma[f_\alpha(\mathcal{E})] + \sigma\left[f_\alpha^{(\mathrm{out})}(\mathcal{E}) \right] \right) \geq 0$$

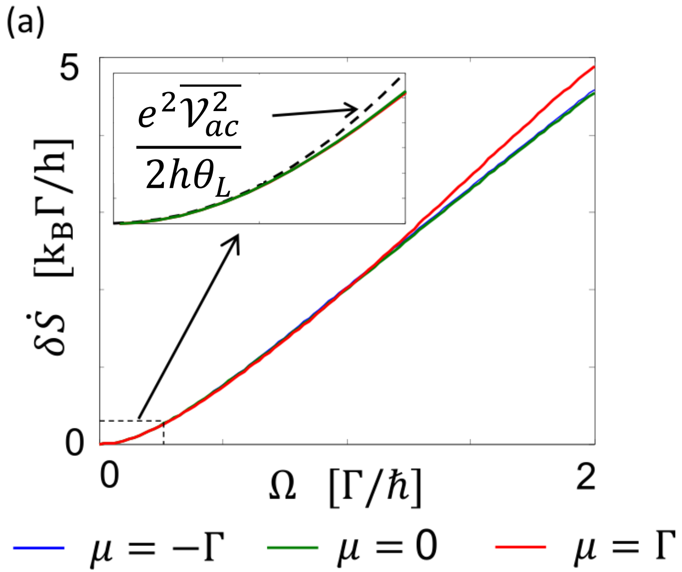
Ingoing distribution

Outgoing distribution

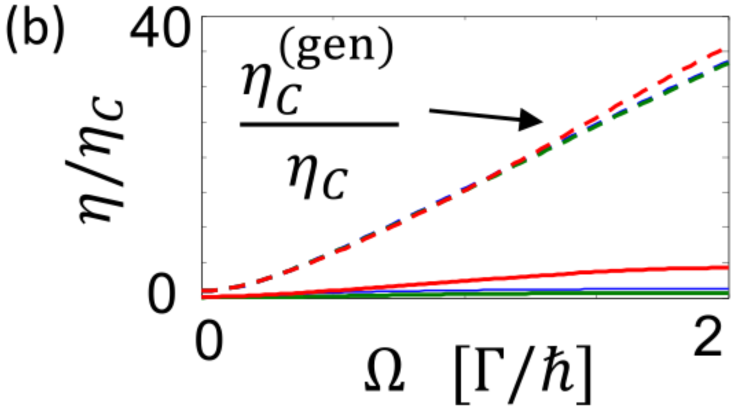
The deviation $\delta\dot{S} \approx \dot{S} - \dot{S}^{(C)}$ quantifies the nonequilibrium effect by AC

$$\delta\dot{S} \approx \frac{e^2\overline{\mathcal{V}_{ac}^2}}{2h\theta_L}$$ ← *energy uncertainty*

In the linear regime
and small energy uncertainty ($\ll k_B\theta_L$)



A new upper bound $\eta_C^{(gen)}$



$\eta_C^{(\mathrm{gen})}$ is not universal,

hence there is room to tailor large efficiency

Our system extracts work even when bath temperatures are equal $\theta_L = \theta_R = \theta$

- Violating Kelvin-Planck statement

We find the pumped current to be $\frac{e}{2h}\overline{\mathcal{V}_{ac}^2}\frac{d|t_{\text{static}}|^2}{d\mathcal{E}}$ (In weak driving, coherent regime)

$$\mathcal{V}_{ac}^{(\text{rms})} \ll k_B\theta \ll \Gamma$$

- A universal relation:

The power generation beyond the Carnot limit is possible for

and any AC voltage with nonvanishing fluctuation

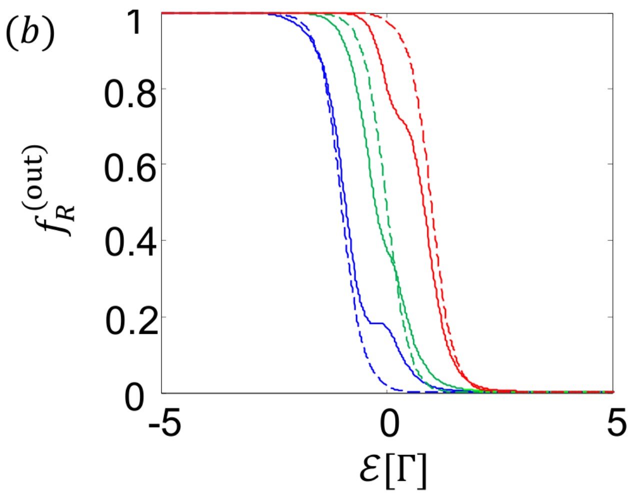
any local scatterer of energy-dependent transmission

One may define an effective temperature of the left reservoir, phenomenologically.

Effective temperature measures the broadening of a Fermi-Dirac-like distribution

AC voltage rearranges the distribution of electrons in energy in a more uncertain way due to $\overline{\mathcal{V}_{ac}^2}$, without injecting any net work over a period.

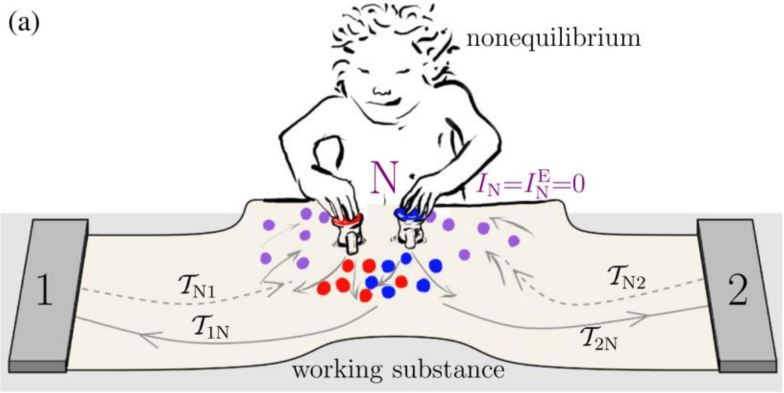
However, such effective temperature is not valid for **nonadiabatic** AC driving because it strongly departs from a Fermi-Dirac distribution.



Alternative interpretation:

AC voltage as a nonequilibrium demon

[R. Sánchez, et al., PRL (2019)]



[R. Sánchez, et al., PRL (2019)]

Demon (AC voltage)

- rearranges electron energy distribution (more uncertainty)
- without energy injection (demon condition)

Our setup does not need a fine tuning for the demon condition

- $P_{in} = 0$ regardless of the AC voltage profile !

Conclusion

- AC driven chiral conductor can exhibit efficiency beyond the Carnot limit
  due to the negative entropy production when the Clausius relation is assumed.

- The seeming violation of 2nd law is resolved
  when employing entropy production based on Shannon entropy flow.

- Chiral transport is crucial for efficiency enhancement;
  Nonchiral conductors do not exhibit efficiencies beyond Carnot's
  as AC injects power which diminishes the generated power.

- We expect experimental realizations as the regime is approachable.

THANK YOU

for your attention