# Nanomechanical Systems & Cavity Nano Optomechanics

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Technical University of Munich (TUM), Germany

The 12th School of Mesoscopic Physics: Hybrid Quantum Systems

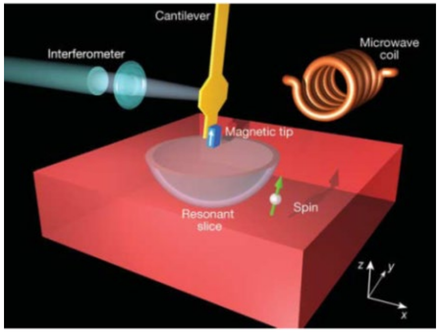
Changeup Ground, POSTECH, May 18-20, 2023

# Why study nano- and cavity optomechanical systems?

## Because of a broad range of fascinating scientific applications…

### … as classical sensors

force sensing:



single $e^-$ spin (*zeptonewton sensitivity*)

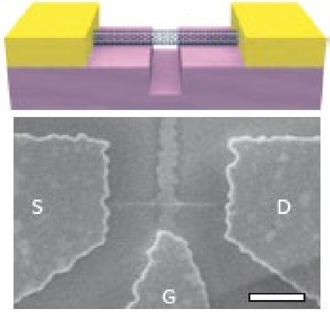
−▶ nano MRI?

*Rugar et al., Nature 430, 329 (2004)*
*Moser et al., Nature Nano 8, 493 (2013)*

mass sensing / mass spectrometry:

single proton (*yoctogram sensitivity*)

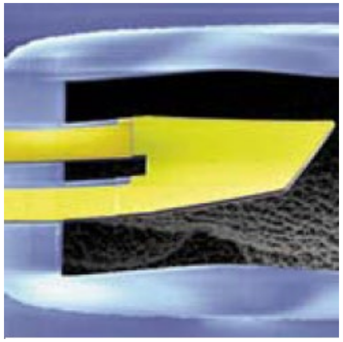
−▶ artificial nose?



*Chaste et al., Nature Nano 7, 301 (2012)*
*Hanay et al., Nature Nano 7, 602 (2012)*

### … for quantum technologies

quantum ground state:



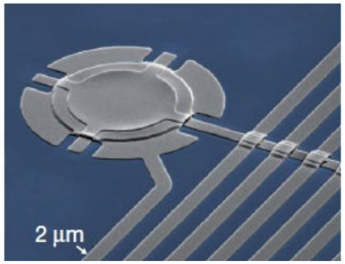
Science BREAKTHROUGH OF THE YEAR

−▶ "the first quantum machine"

*O'Connell et al., Nature 464, 679 (2010)*

*Teufel et al., Nature 475, 359 (2011)*
*Chan et al., Nature 478, 89 (2011)*

quantum devices:



−▶ quantum transducers, quantum sensors

*Palomaki et al., Science 342, 710 (2013)*

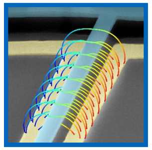
QUANTUM FLAGSHIP

The quantum technologies roadmap: a European community view

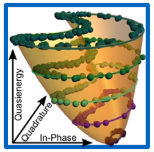
*Acín et al., New J. Phys. 20, 080201 (2018)*

# OUTLINE

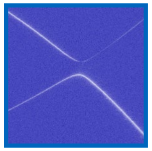
## PART 1: (HIGH Q) NANOMECHANICAL SYSTEMS



1. An introduction to cavity optomechanics:
   Radiation-pressure induced dynamical backation

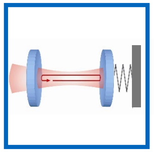


2. The membrane-in-the-middle configuration:
   A vibrating membrane inside a Fabry-Pérot cavity



3. Cavity optomechanics with van der Waals materials:
   Radiation pressure backaction on a flake of hBN

## PART 2: CAVITY OPTOMECHANICS



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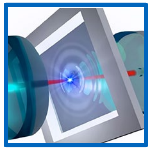

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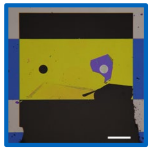

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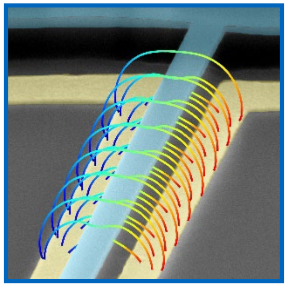
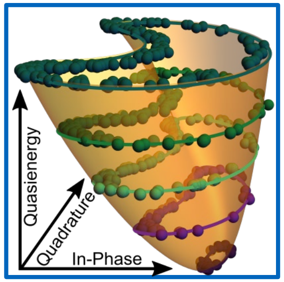
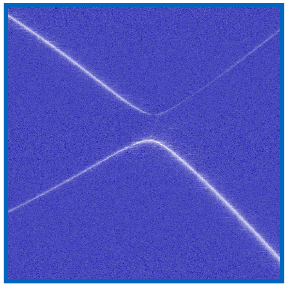


1. High Q nanomechanical string resonators:
   A well-controlled model system for dynamical phenomena

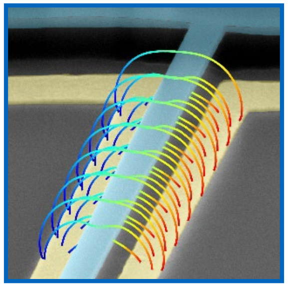


2. Nonlinear response of a single nanomechanical mode:
   A new type of frequency comb



3. Coherent control of a nanomechanical two-mode system:
   Enhanced Ramsey spectroscopy for fast sening applications

# OUTLINE PART 1



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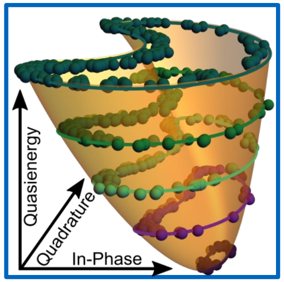

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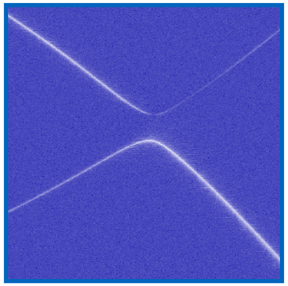

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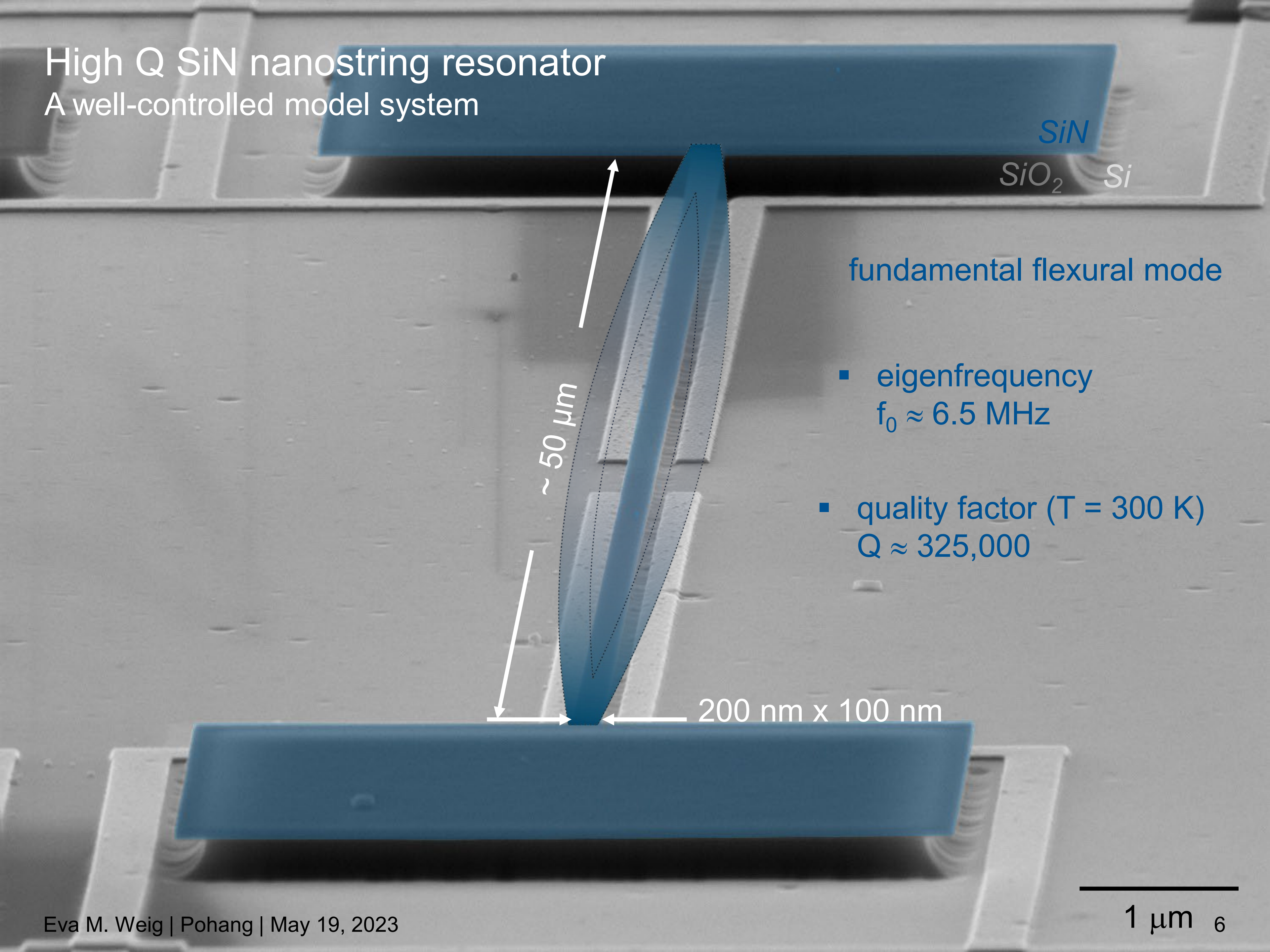
# High Q SiN nanostring resonator

## A well-controlled model system



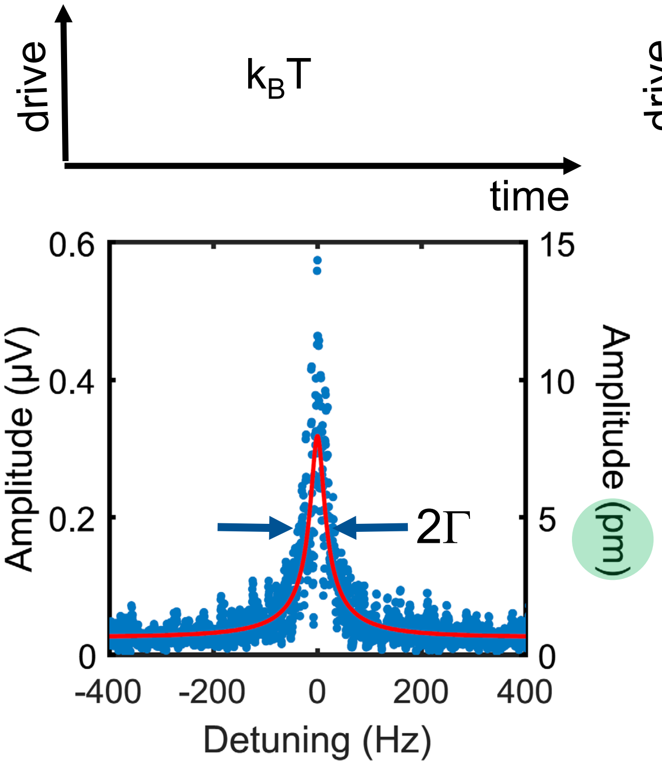
fundamental flexural mode

- eigenfrequency  
  $f_0 \approx 6.5$ MHz
- quality factor (T = 300 K)  
  Q ≈ 325,000

# Characterization of high Q nanostring resonators

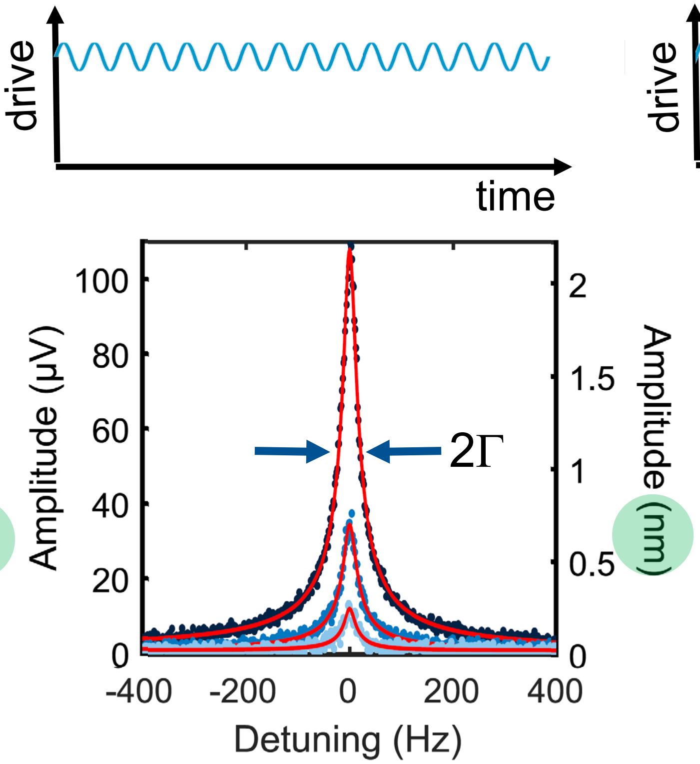
## Weakly damped and very coherent

### Noise spectrum (spectrum analyzer)



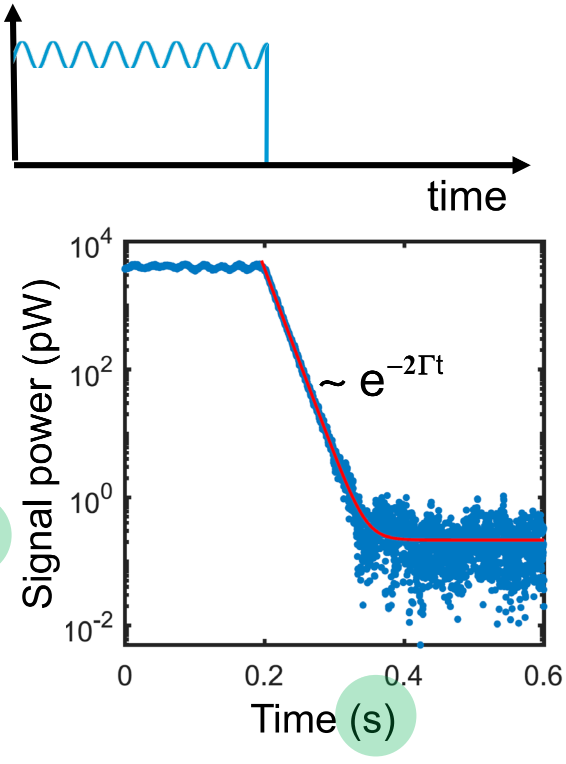
- eigenfrequency $\omega_0/2\pi$
- (energy) dissipation $2\Gamma/2\pi$
- calibration V ↔ m

### Response measurement (VNA, fast lockin)



- $\omega_0/2\pi \approx 6$ MHz
- $2\Gamma/2\pi \approx 20$ Hz
- $Q = \omega_0/2\Gamma \approx 300{,}000$

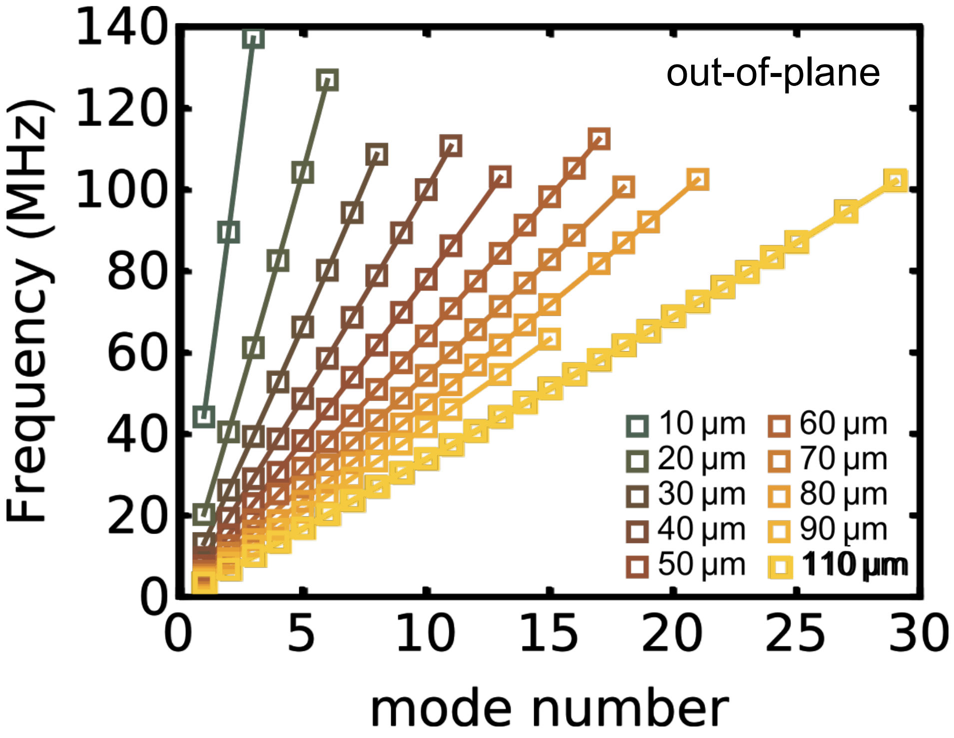
### Ringdown measurement (oscilloscope, fast lockin)

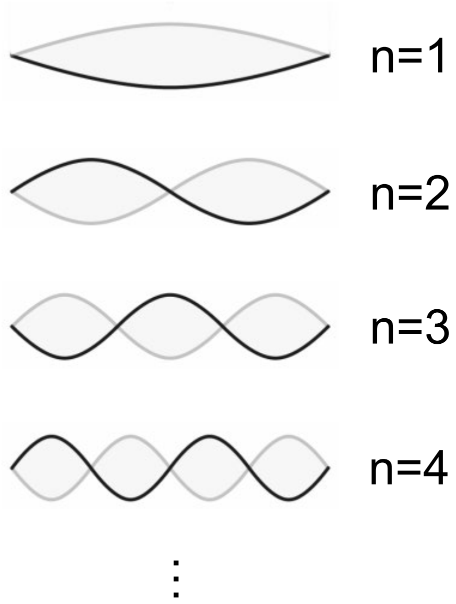


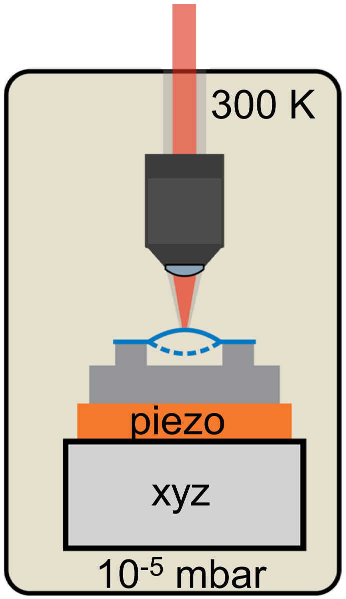
- $2\Gamma/2\pi$ (more accurate)
- coherence time $\tau = (2\Gamma)^{-1} \approx 8$ ms

# Eigenfrequency spectrum of a nanostring resonator

## For a doubly clamped string with strong built-in tensile stress





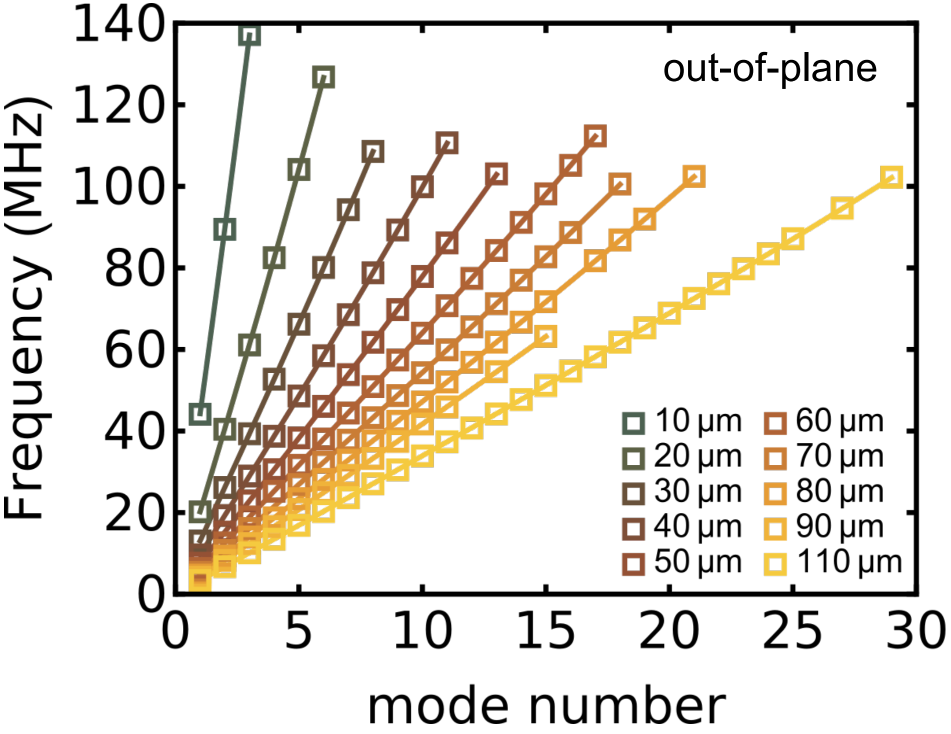


*Bückle, Klass, Nägele, Braive, Weig, Phys. Rev. Appl. 15, 034063 (2021)*
*Klaß, Doster, Bückle, Braive, Weig, Appl. Phys. Lett. 121, 083501 (2022)*

# Length-dependent tensile stress

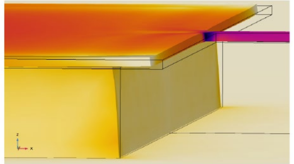
## For a doubly clamped string with strong built-in tensile stress

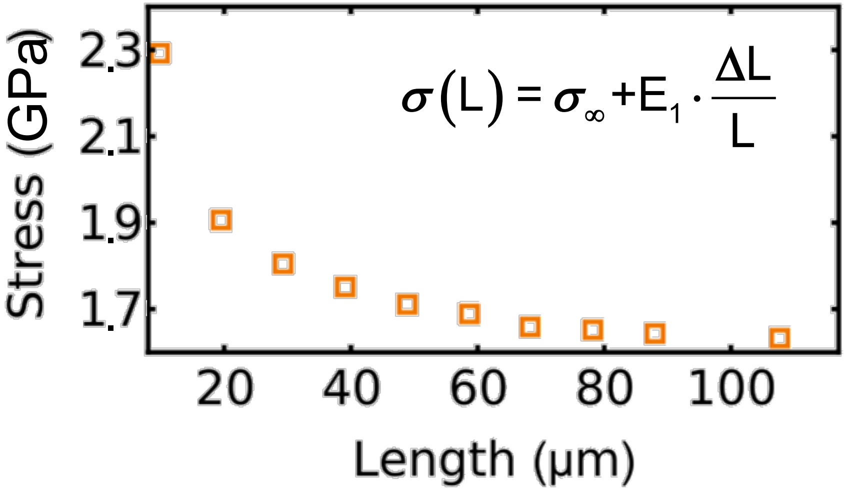
$$f_n = \frac{n^2\pi}{2L^2}\sqrt{\frac{Eh^2}{12\rho}+\frac{\sigma L^2}{n^2\pi^2\rho}} \approx \frac{n}{2L}\sqrt{\frac{\sigma}{\rho}}$$



Tensile stress:

- Depends on length



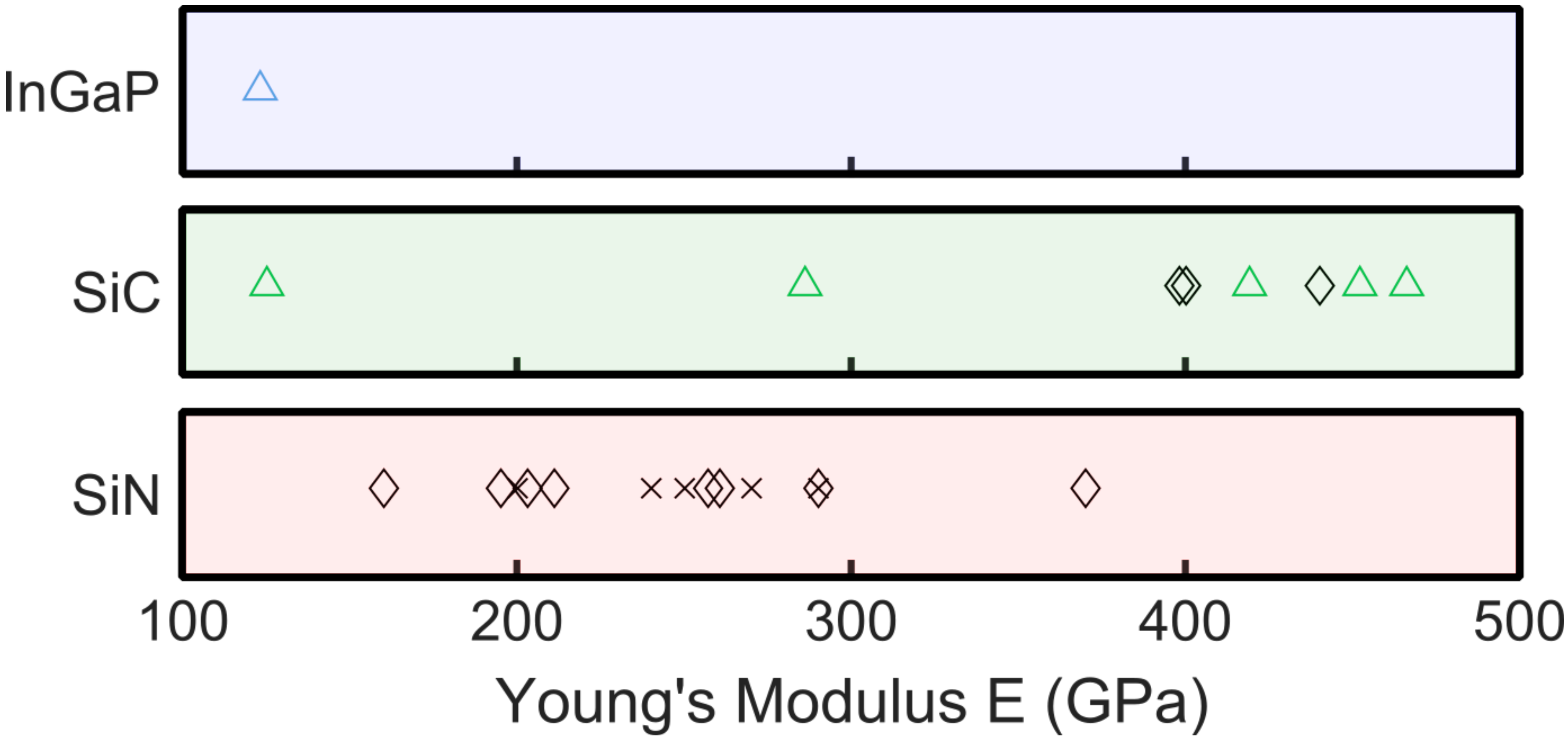


$$\sigma(L) = \sigma_\infty + E_1 \cdot \frac{\Delta L}{L}$$

*Bückle, Klass, Nägele, Braive, Weig, Phys. Rev. Appl. 15, 034063 (2021)*

# Young‘s Modulus

A simple literature parameter?

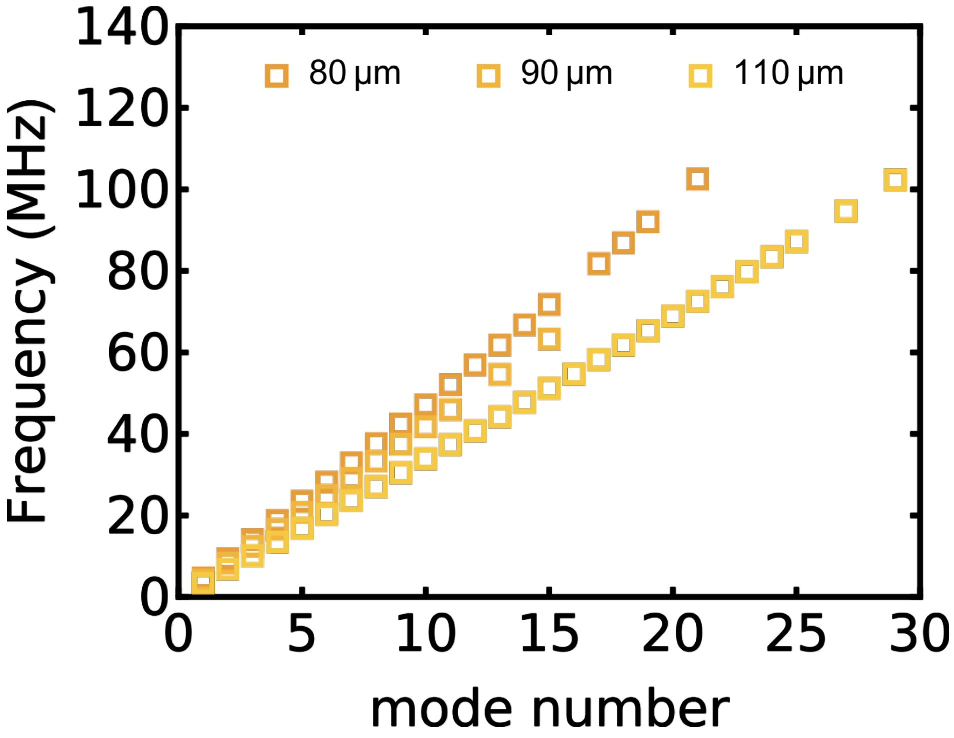


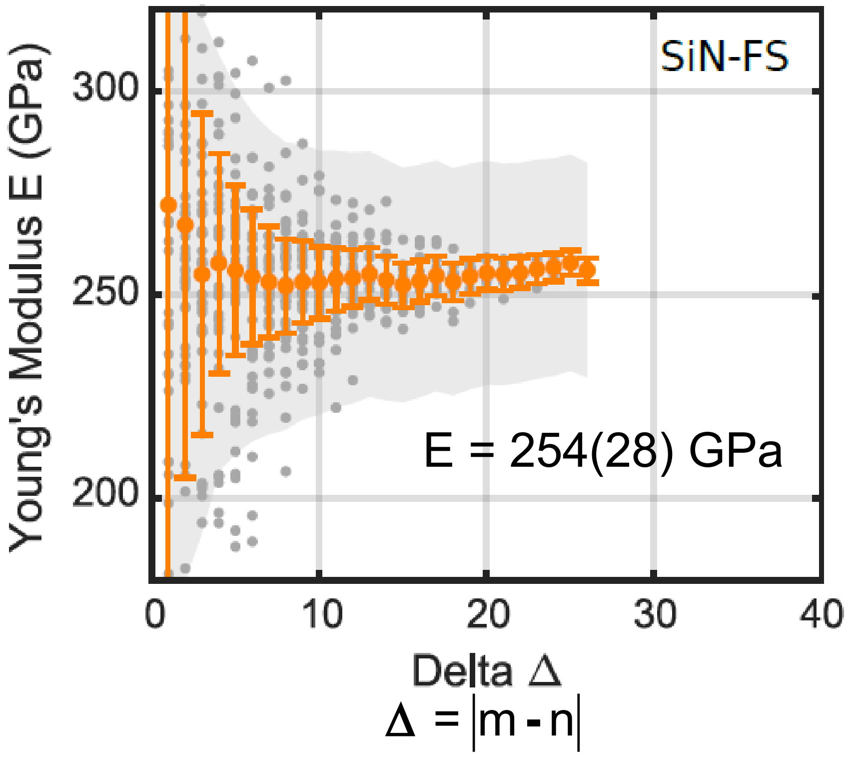
*Klaß, Doster, Bückle, Braive, Weig, Appl. Phys. Lett. 121, 083501 (2022)*

# Young‘s modulus E of a prestressed string resonator

## Direct *differential* determination from out-of-plane eigenfrequency

$$f_n = \frac{n^2\pi}{2L^2}\sqrt{\underbrace{\frac{Eh^2}{12\rho}}_{\text{too small}} + \frac{\sigma L^2}{n^2\pi^2\rho}} \approx \frac{n}{2L}\sqrt{\frac{\sigma}{\rho}} \quad \Longrightarrow \quad \underbrace{E}_{\text{just right}} = \frac{48L^4\rho}{\pi^2h^2\left(n^2-m^2\right)}\left(\frac{f_n^{\,2}}{n^2}-\frac{f_m^{\,2}}{m^2}\right)$$





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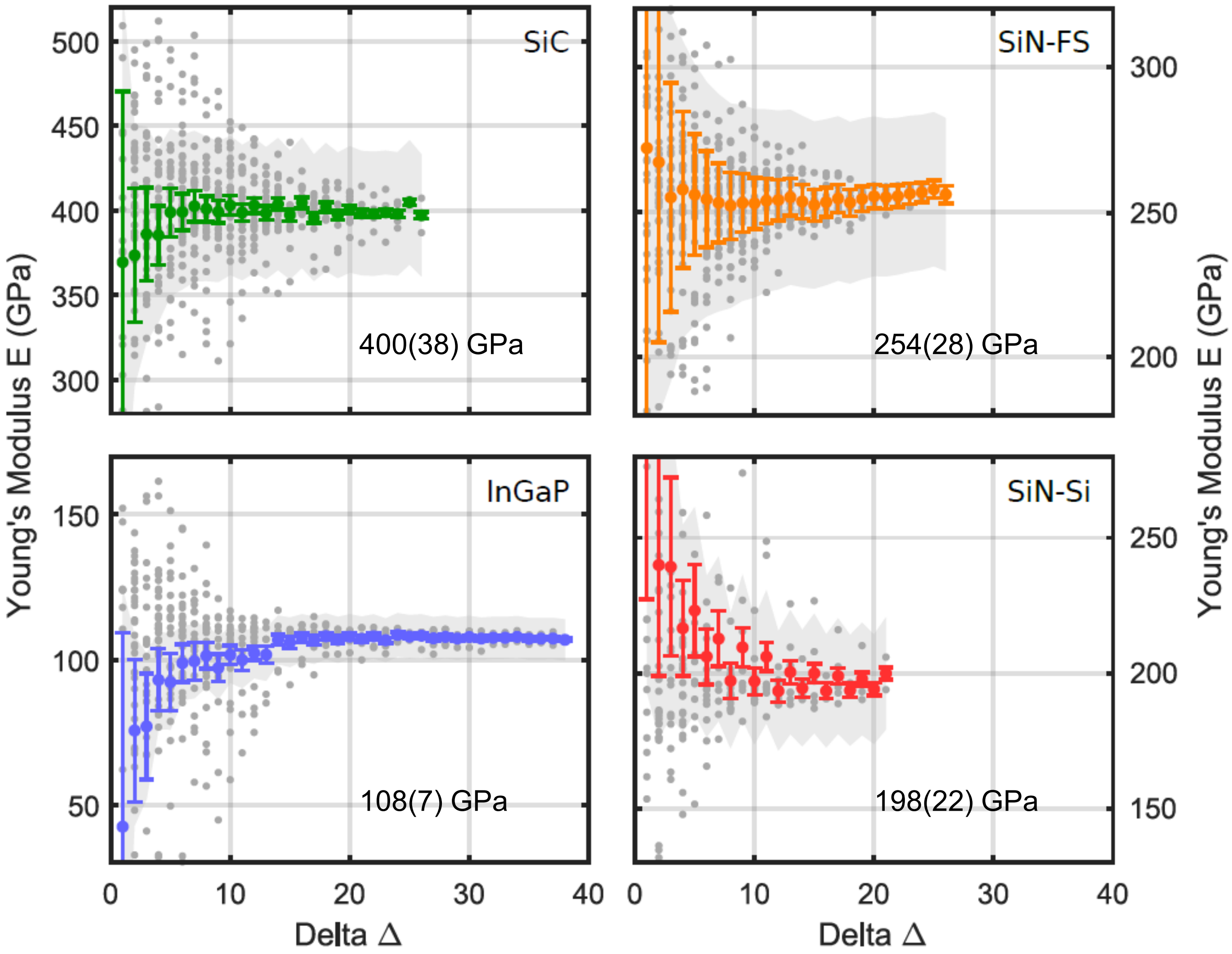
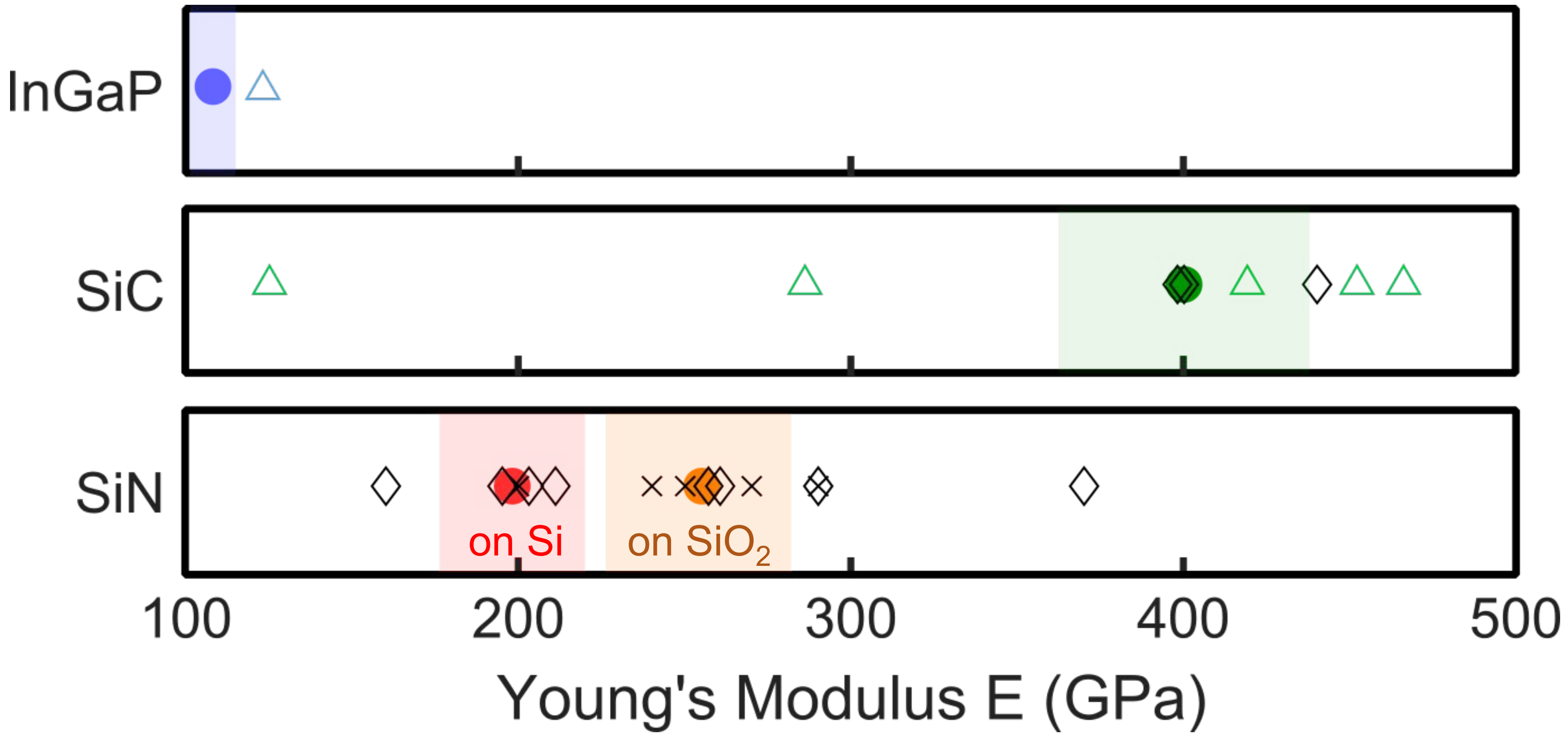


*Klaß, Doster, Bückle, Braive, Weig, Appl. Phys. Lett. 121, 083501 (2022)*

# Young‘s Modulus

Literature values do not replace a measurement!



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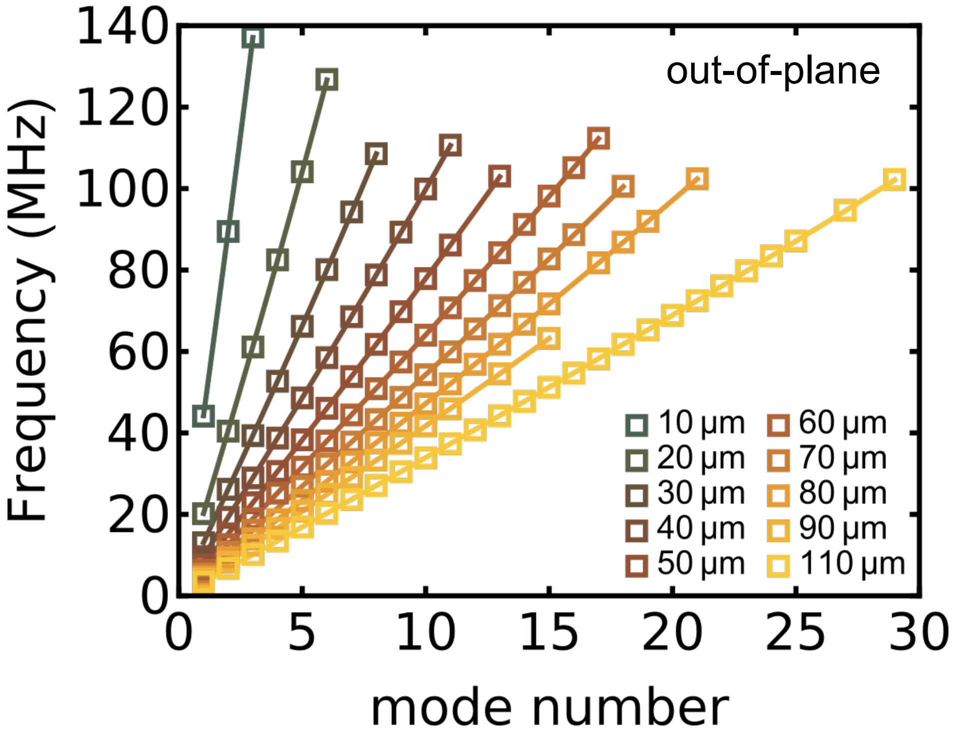
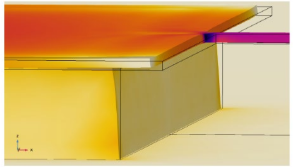


Tensile stress:

- Depends on length



Young‘s modulus:

- Precise determination

*Bückle, Klass, Nägele, Braive, Weig, Phys. Rev. Appl. 15, 034063 (2021)*
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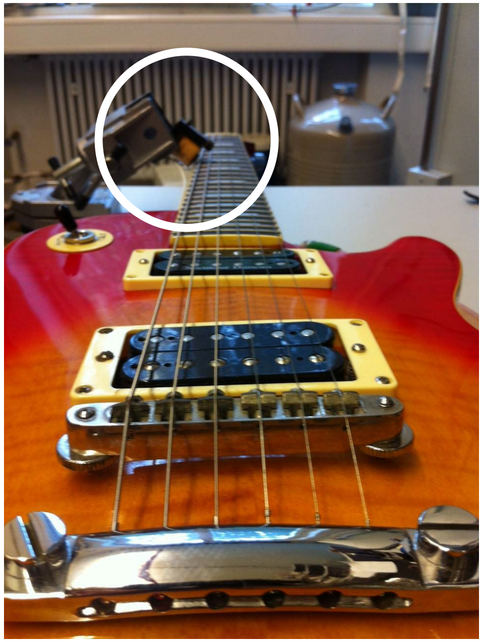
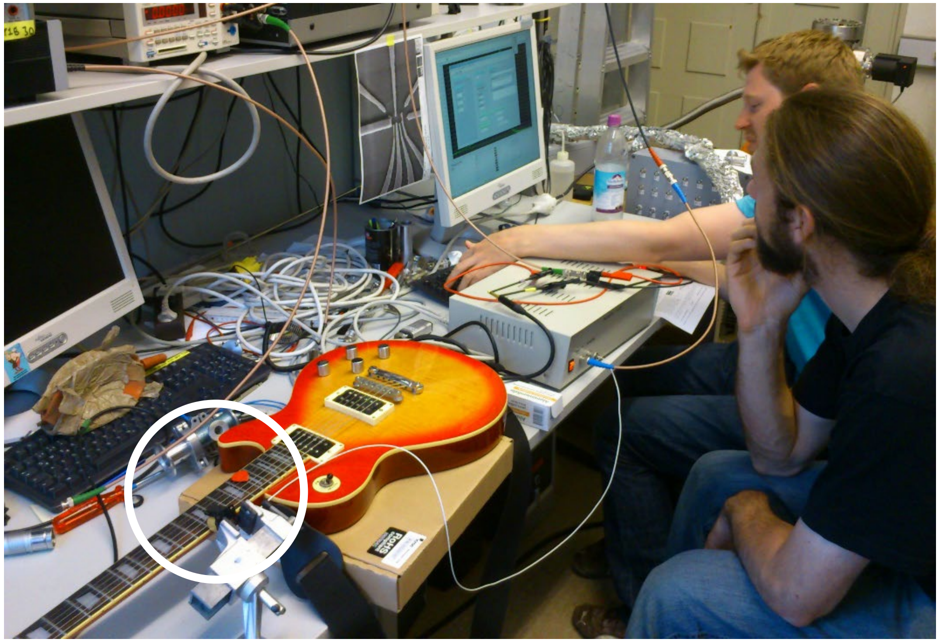
# Some real guitar string physics

## Measured on the lower E string of Johannes‘ Epiphone Les Paul





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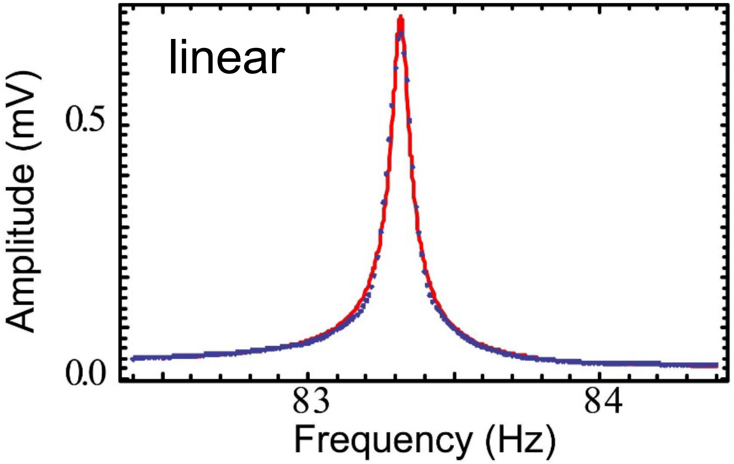
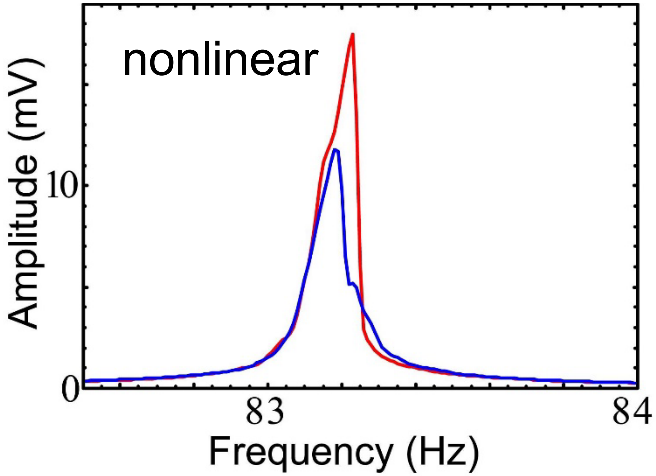




guitar specs:

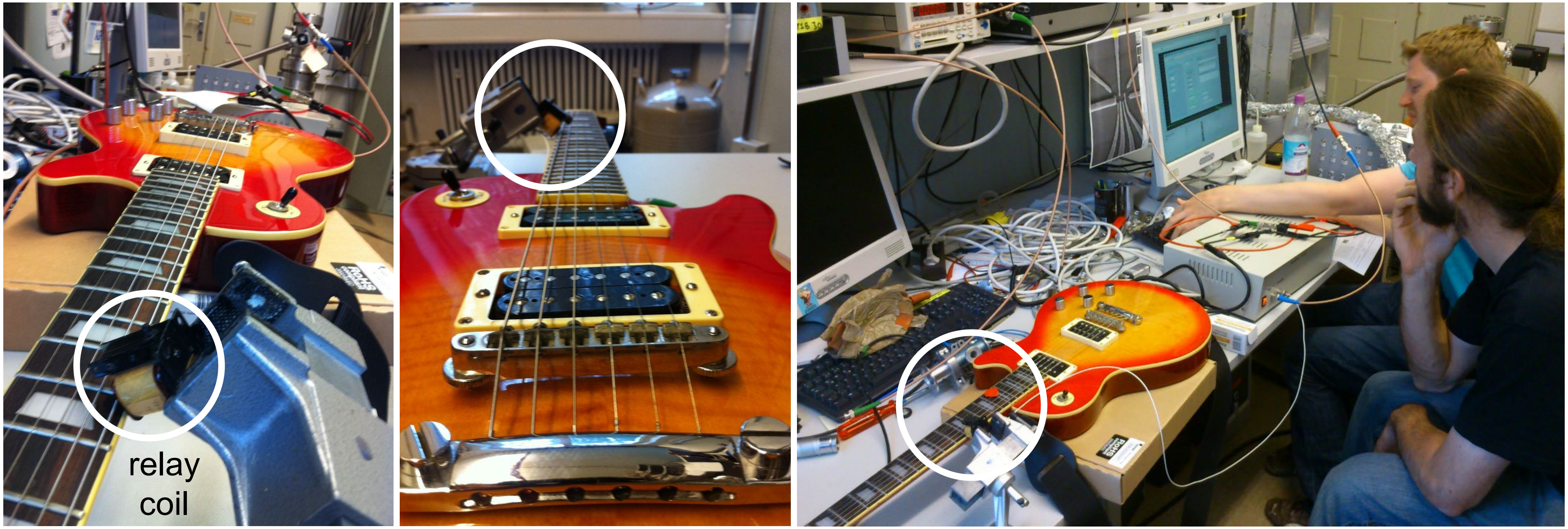
L ~ 1 m

f ~ 100 Hz

L/w ~ 2,000

$\sigma$ ~ 1.5 GPa

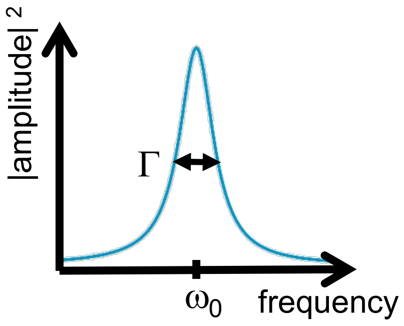
Q ~ 2,000



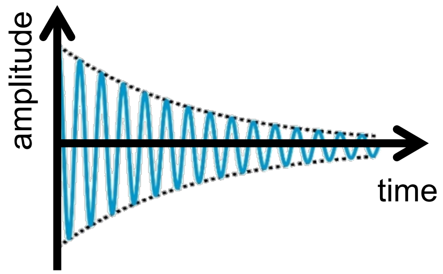
# Q

## A measure of the relative dissipation in a resonator

Q



$$Q = \frac{\text{stored vibrational energy}}{\text{dissipated energy}}$$



relative linewidth $\omega_0/\Gamma$

number of free oscillations

# High stress = High Q

Tensile stress increases the stored energy of a string resonator

(Intrinsic) loss arises from anelasticity, i.e. delay between internal strains and stresses:

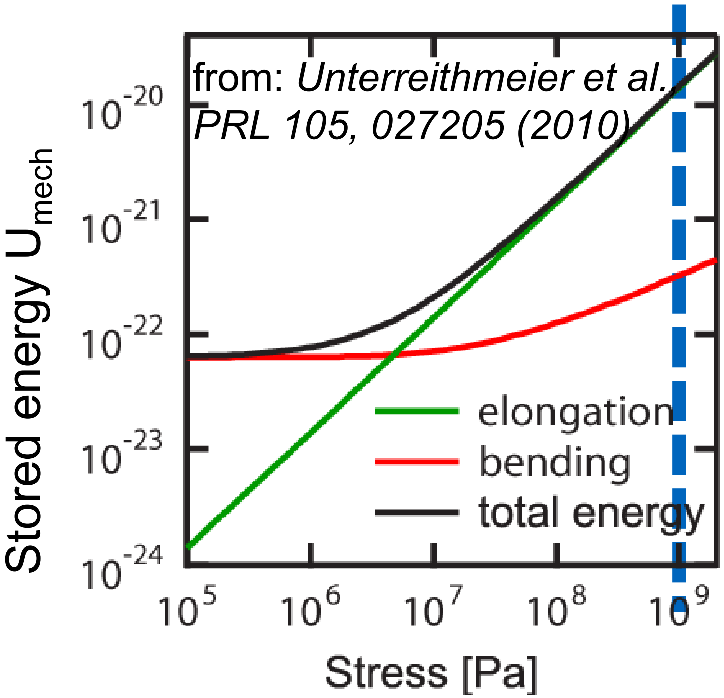
Zener model assumes complex Young‘s modulus $E = E_1 + i\,E_2$

*dominated by*
*bending-induced elongation ($\sigma$)*

$$Q = 2\pi \cdot \frac{U_{mech}}{\Delta U_{diss}} = \underbrace{\frac{E_1}{E_2}}_{= Q_{intr}} \cdot \underbrace{\frac{1}{\frac{h}{L}\frac{E_1}{12\sigma}\left(2+\frac{n^2\pi^2 h}{L}\frac{E_1}{12\sigma}\right)}}_{\gg 1}$$

*“dissipation dilution“*

*dominated by*
*beam bending ($E_1$)*

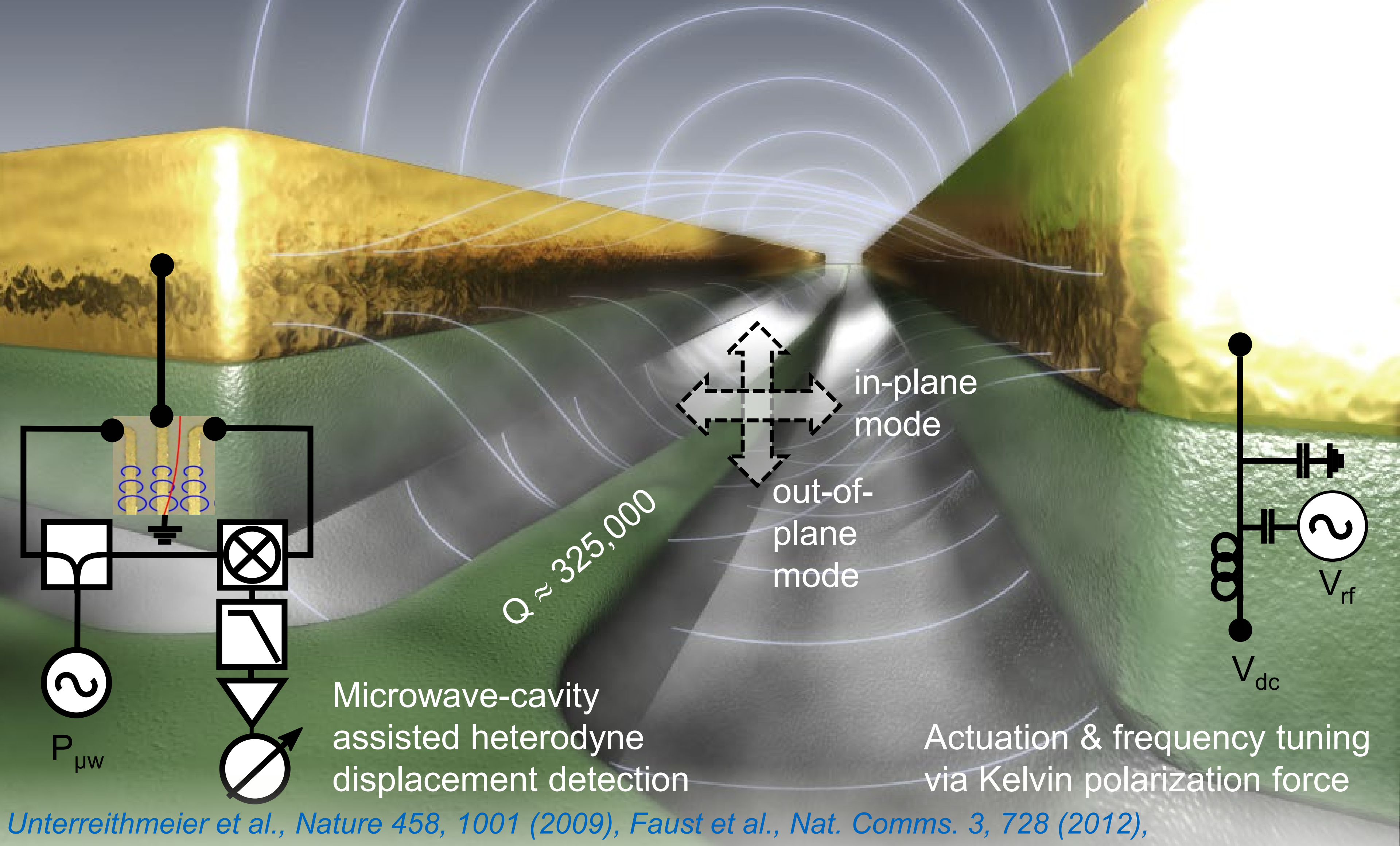


from: *Unterreithmeier et al. PRL 105, 027205 (2010)*

*Gonzales & Saulson, J. Ac. Soc. Am. 96, 207 (1994); Yu et al., Phys. Rev. Lett. 108, 083603 (2012)*

# Dielectric gradient field transduction

## An integrated platform to control high Q nanomechanical resonators



Microwave-cavity assisted heterodyne displacement detection

Actuation & frequency tuning via Kelvin polarization force

*Unterreithmeier et al., Nature 458, 1001 (2009), Faust et al., Nat. Comms. 3, 728 (2012), Rieger et al., Appl. Phys. Lett. 101, 103110 (2012)*

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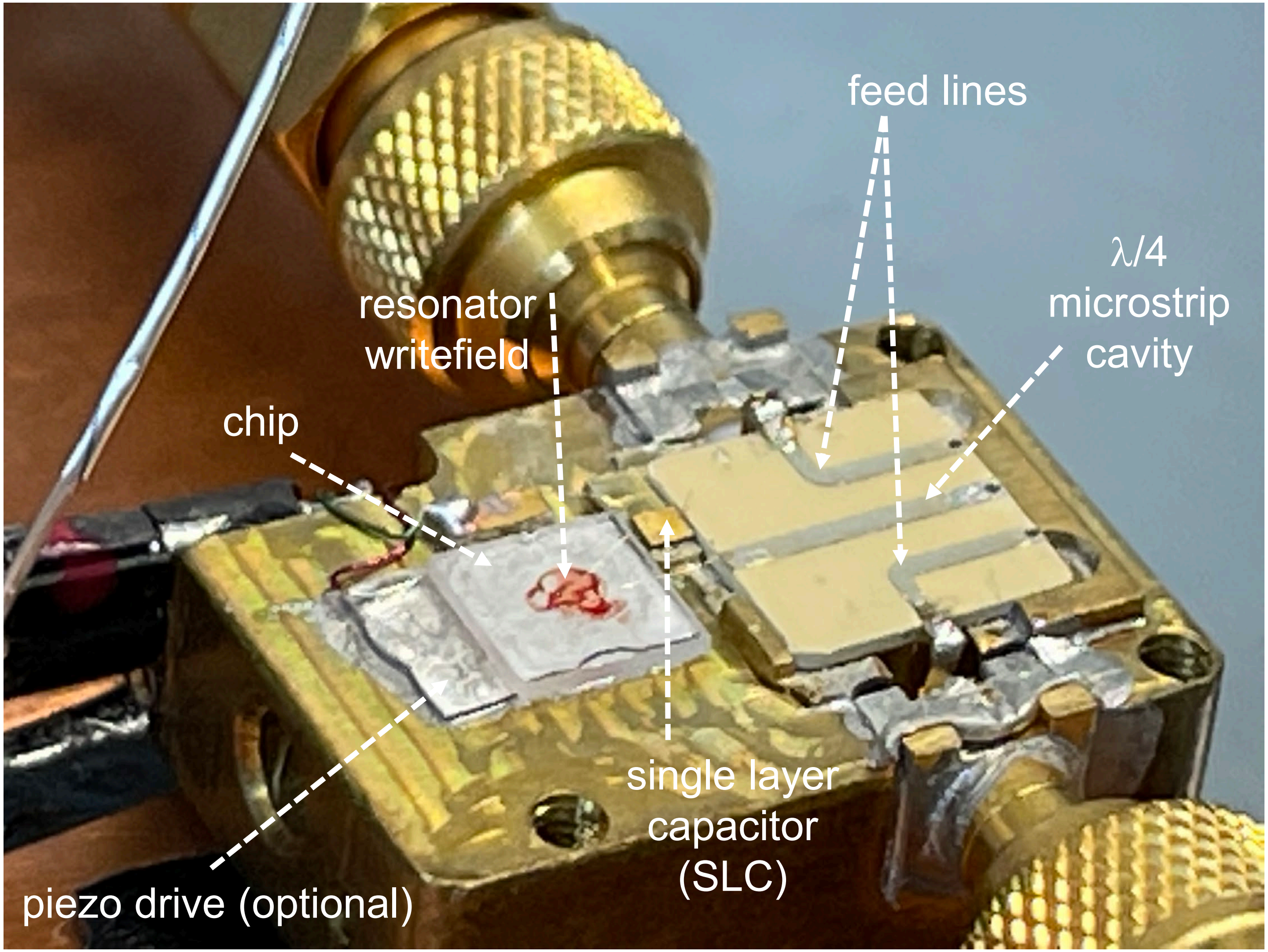
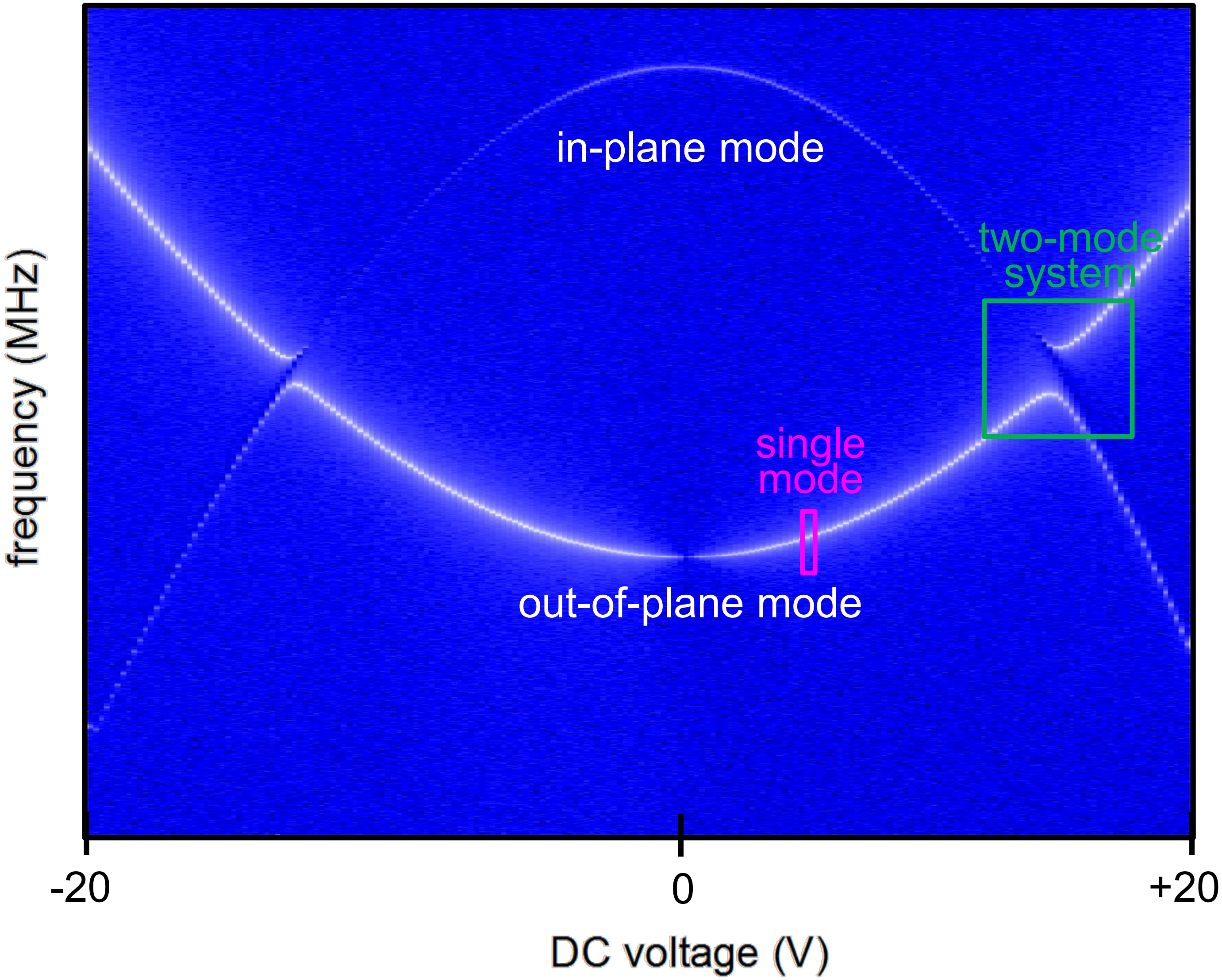
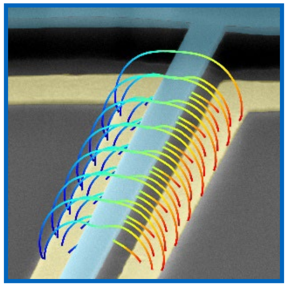


# Dielectric eigenfrequency tuning of in- & out-of-plane mode

## Single-mode and two-mode regime



# OUTLINE



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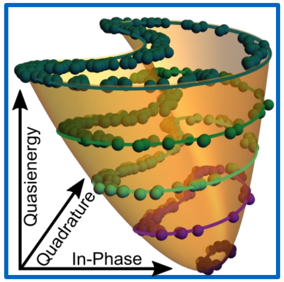

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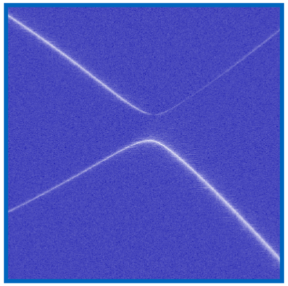

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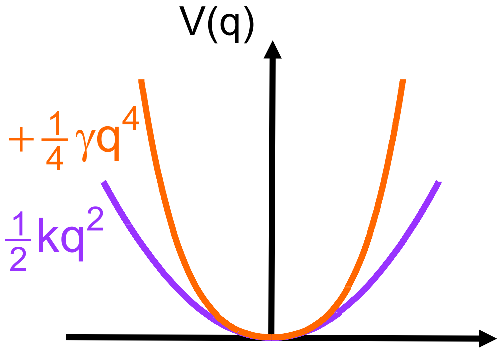
   Enhanced Ramsey spectroscopy for fast sening applications

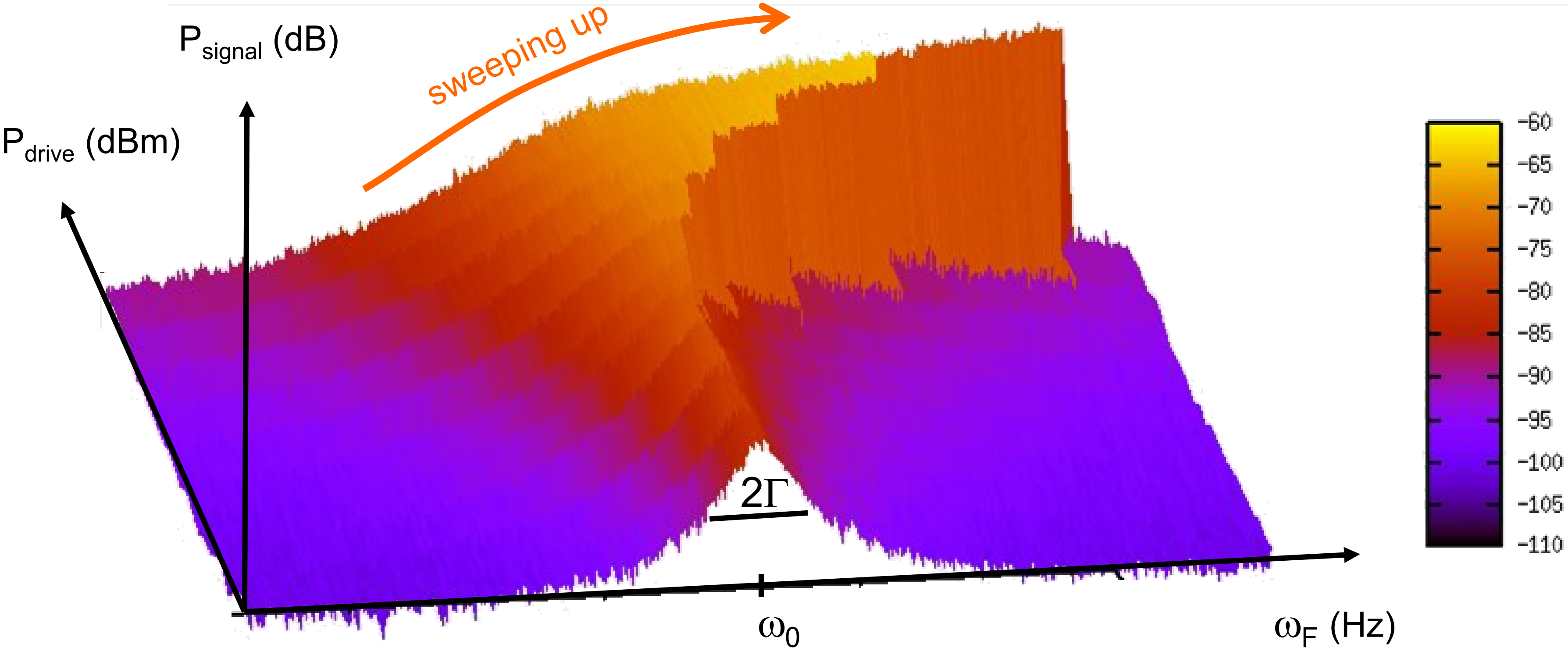
# Transition from the linear to the nonlinear regime

## Duffing model describes response for relatively weak drive

$$\ddot{q}(t)+2\Gamma\cdot\dot{q}(t)+\omega_0{}^2\cdot q(t) \qquad\qquad = F\cdot\cos(\omega_F t)$$

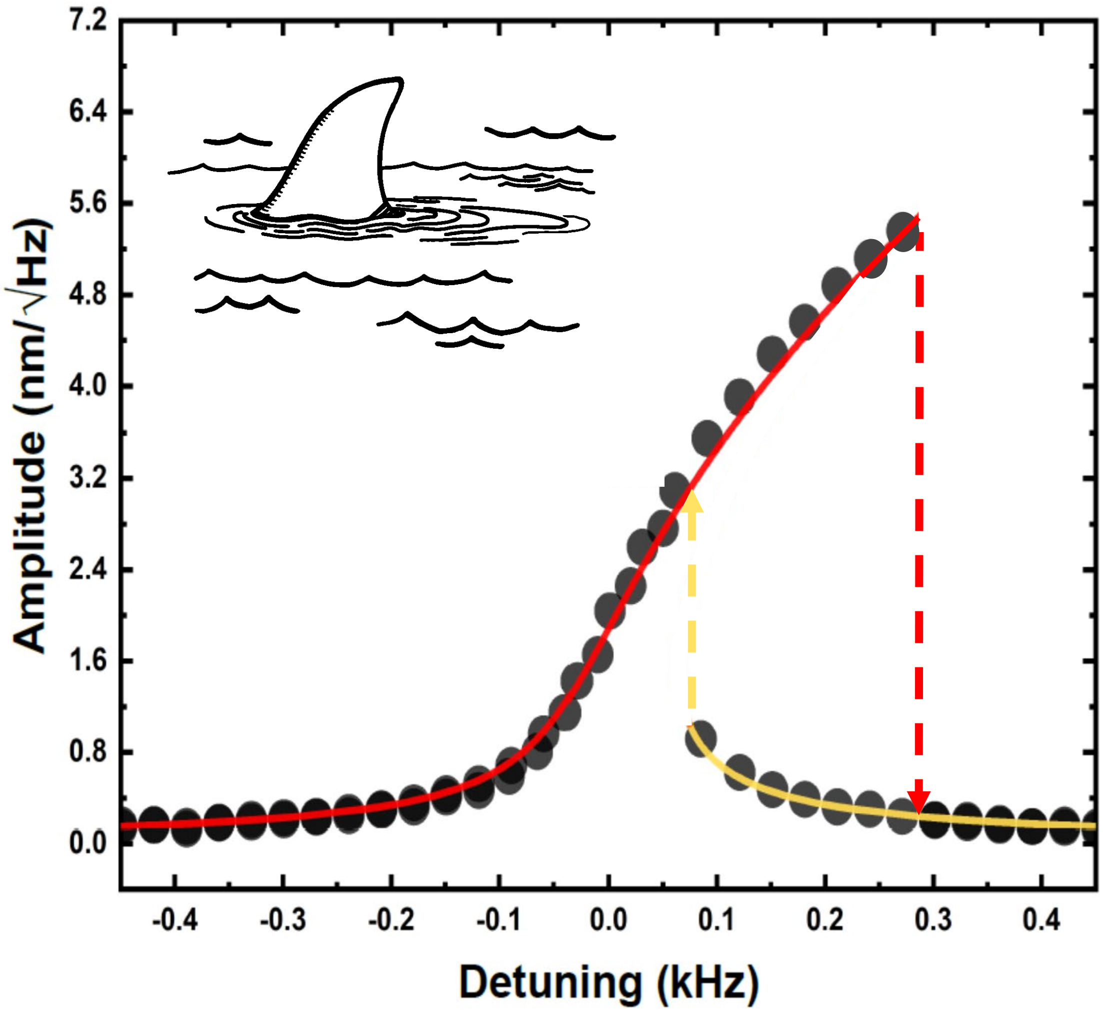
Duffing





# Nonlinear Duffing response of a nanomechanical resonator

## Amplitude-dependent eigenfrequency, hysteresis and bistability



OOP mode at $U_{DC}$ = 5 V with $P_{drive}$ = -30 dBm

Duffing model:

$$\ddot{q}(t) + 2\Gamma \cdot \dot{q}(t) + \omega_0{}^2 \cdot q(t) + \gamma \cdot q^3(t) = F \cdot \cos(\omega_F t)$$

- Hysteretic response curve

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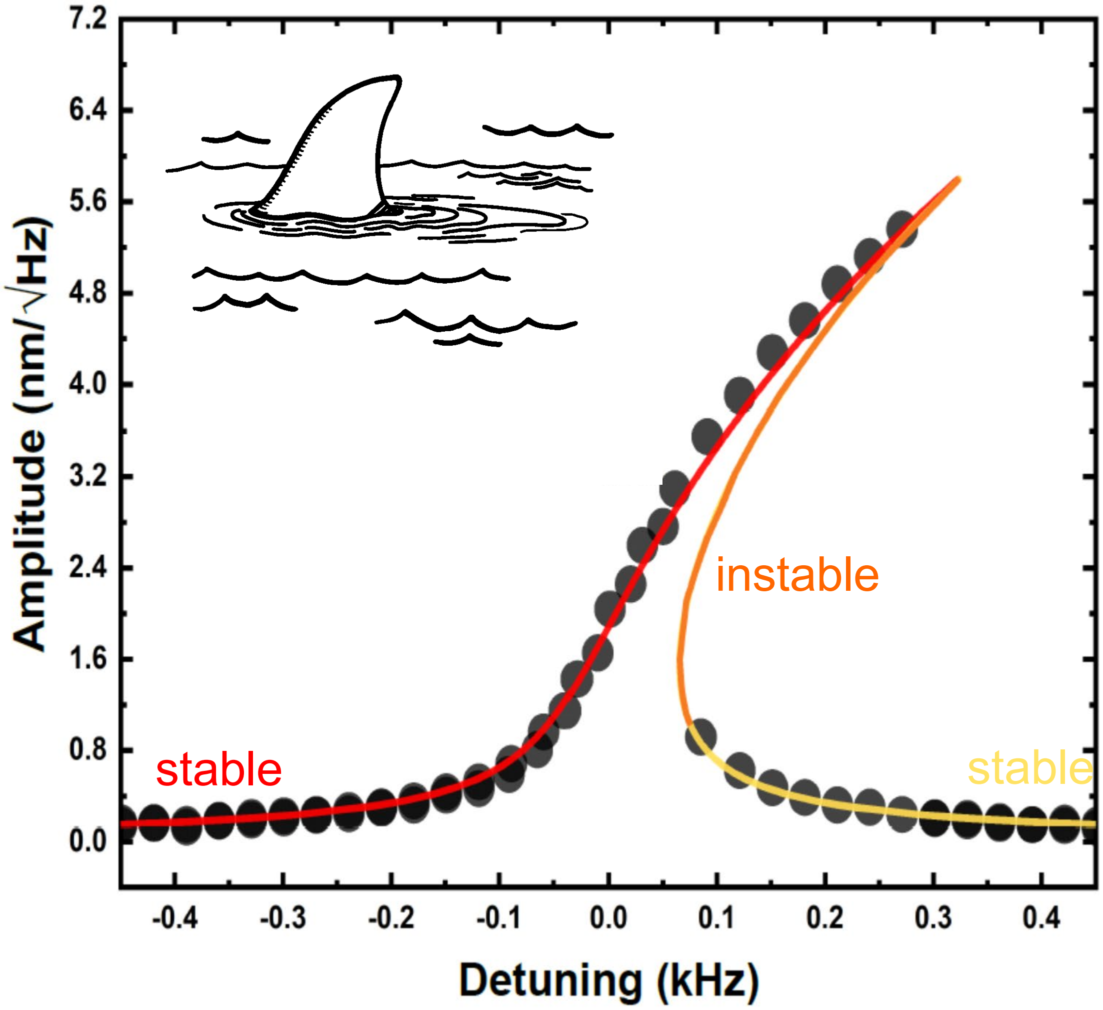


OOP mode at $U_{DC}$ = 5 V with $P_{drive}$ = -30 dBm

Duffing model:

$$\ddot{q}(t) + 2\Gamma \cdot \dot{q}(t) + \omega_0{}^2 \cdot q(t) + \gamma \cdot q^3(t) = F \cdot \cos(\omega_F t)$$

- Hysteretic response curve
- Up to 3 amplitude solutions (2 stable, 1 instable)
- (Mostly geometric) nonlinearity:

$\gamma = 1.59 \cdot 10^{26}\ m^{-2}s^{-2}$

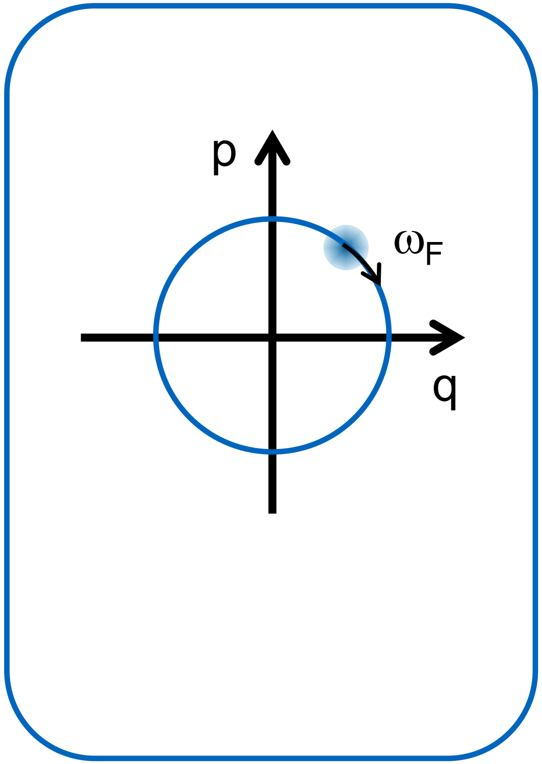
# The driven Duffing resonator in the rotating frame

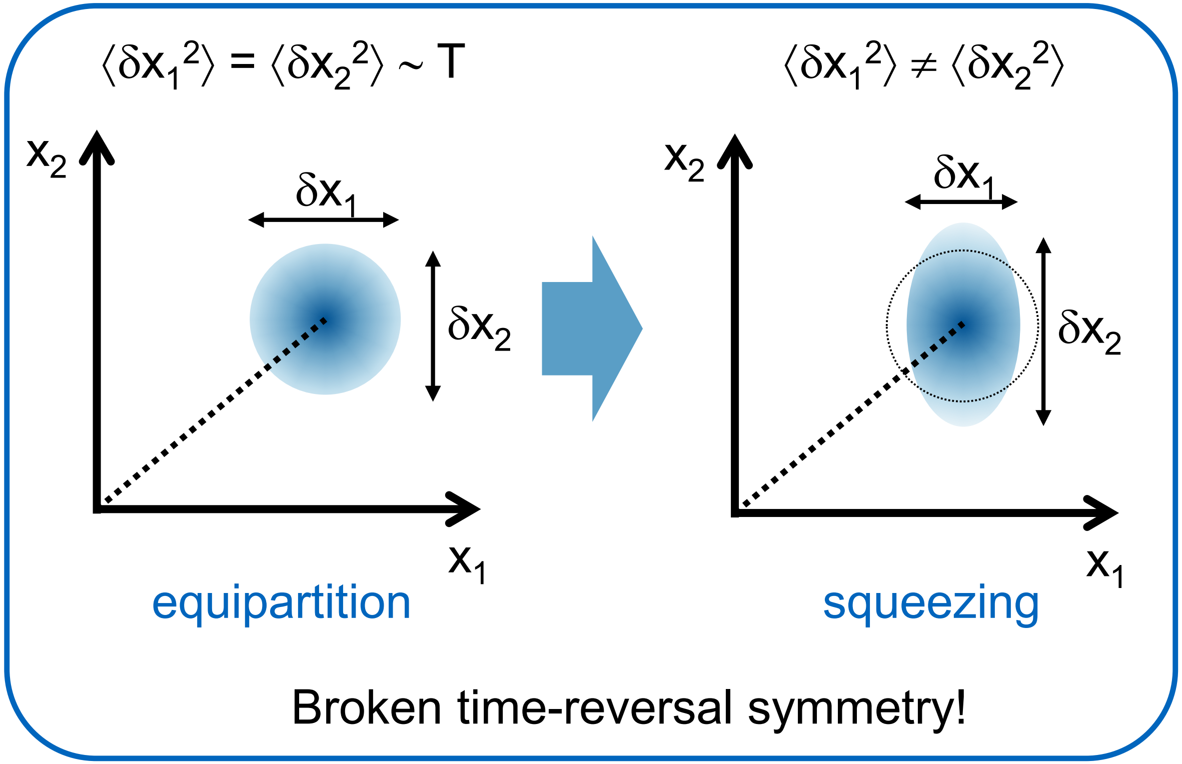
## Squeezing of thermomechanical fluctuations

$$\ddot{q}(t)+2\Gamma\cdot\dot{q}(t)+\omega_0{}^2\cdot q(t)+\gamma\cdot q^3(t)=F\cdot\cos(\omega_F t)+\xi(t)$$

Laboratory frame:

Rotating frame:





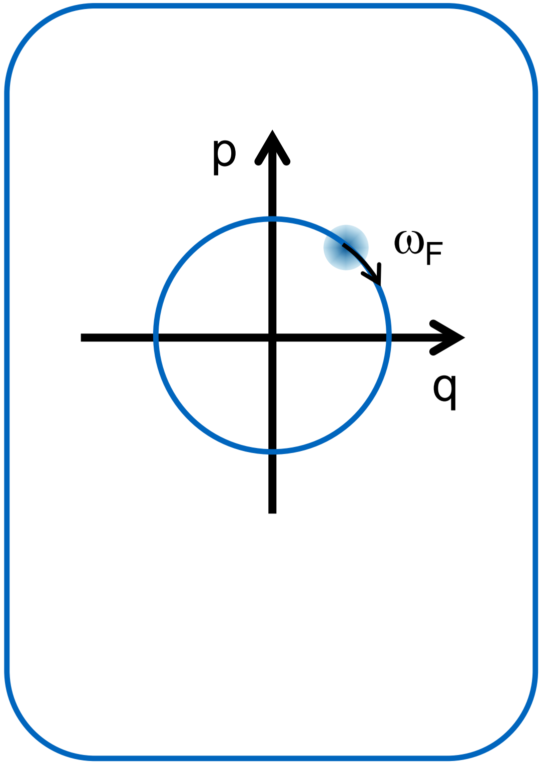
Broken time-reversal symmetry!

# The driven Duffing resonator in the rotating frame

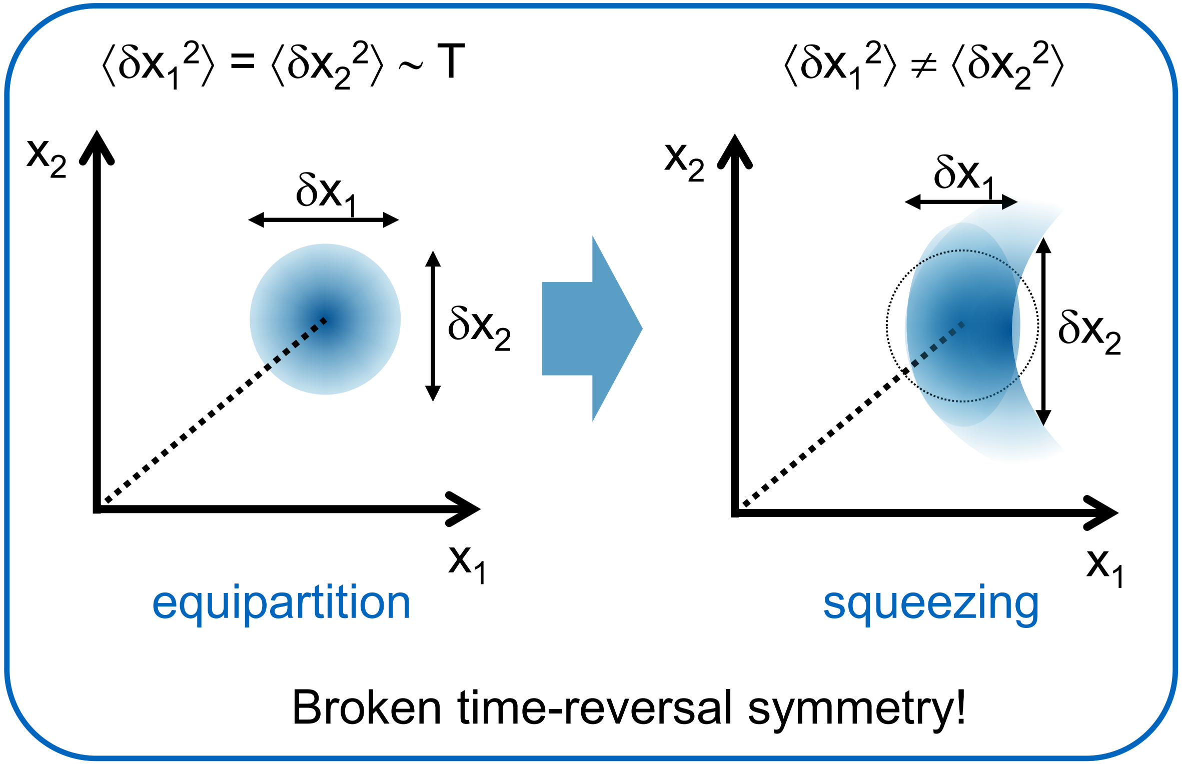
## Squeezing of thermomechanical fluctuations

$$\ddot{q}(t) + 2\Gamma \cdot \dot{q}(t) + \omega_0^2 \cdot q(t) + \gamma \cdot q^3(t) = F \cdot \cos(\omega_F t) + \xi(t)$$

Laboratory frame:



Rotating frame:



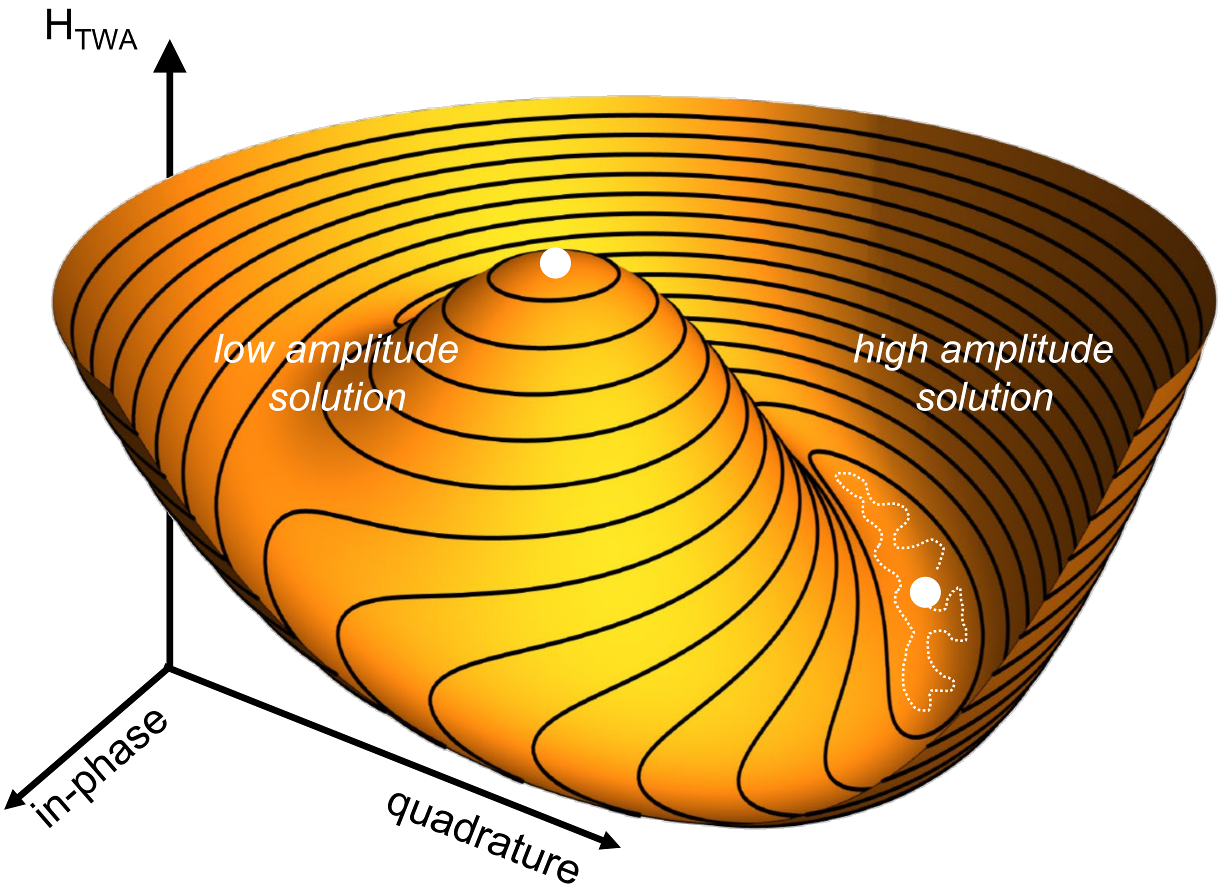
Broken time-reversal symmetry!

High Q resonators:
Direct homodyne measurement of squeezed state is impeded by frequency flucutations

see *Fong et al., Phys. Rev. B 85, 161410(R) (2012)*

# The driven Duffing resonator in the rotating frame

## Thermal noise drives system out of the stable states



- Extrema represent stable states of forced vibration
- Thermal noise kicks system out of stable state
- Curvature sets (slow) oscillation frequency

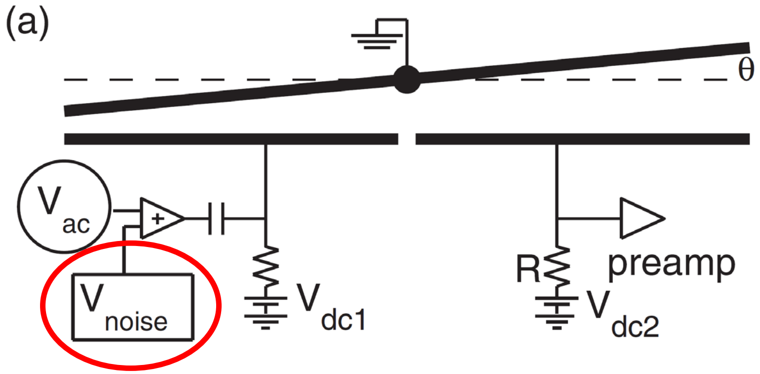
- In the lab frame, the frequency $\omega_{hi,lo}$ is mixed into two satellite peaks around $\omega_F$
  (with additional noise injection, see: *Stambaugh et al., Phys. Rev. Lett. 97, 110602 (2006)*)

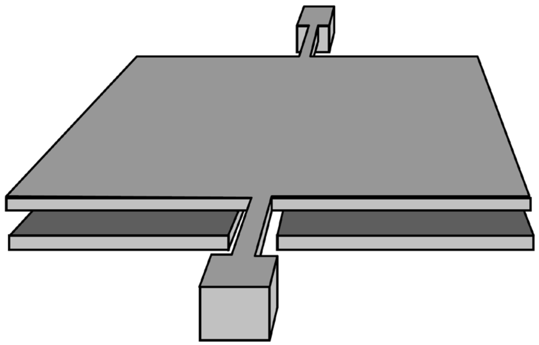
*Dykman & Krivoglaz, Phys. Stat. Sol. (b) 48, 497 (1971)*

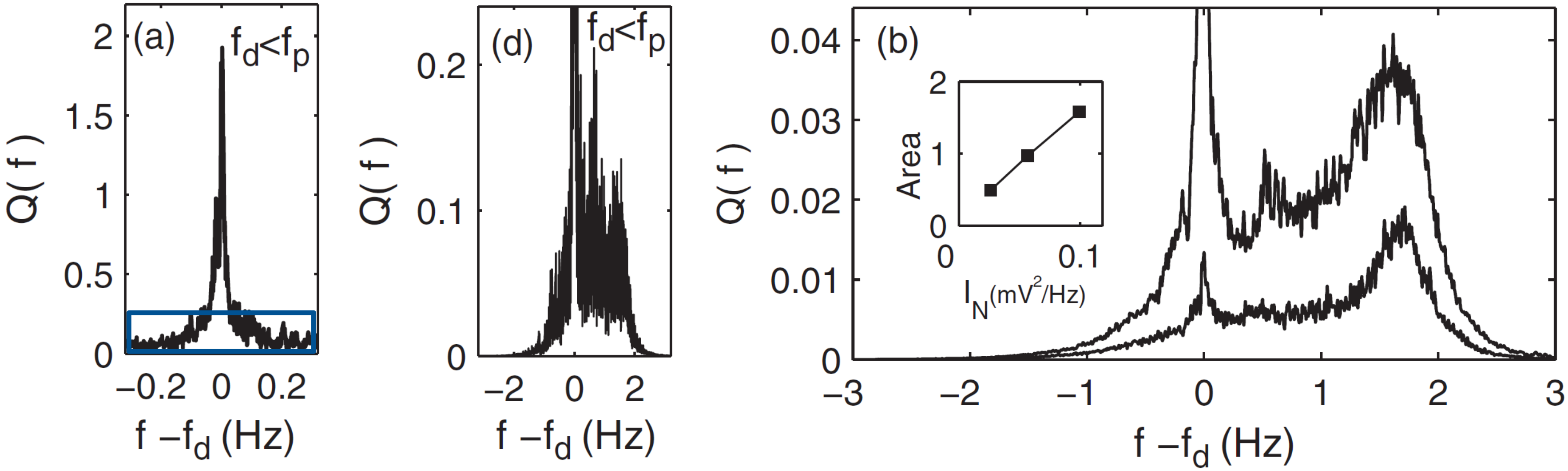
# Thermal noise induced satellite peaks in the power spectrum

## Under strong resonant drive at fundamental eigenfrequency $\omega_0$

First observation in the group of Ho Bun Chan…. under injection of white noise:



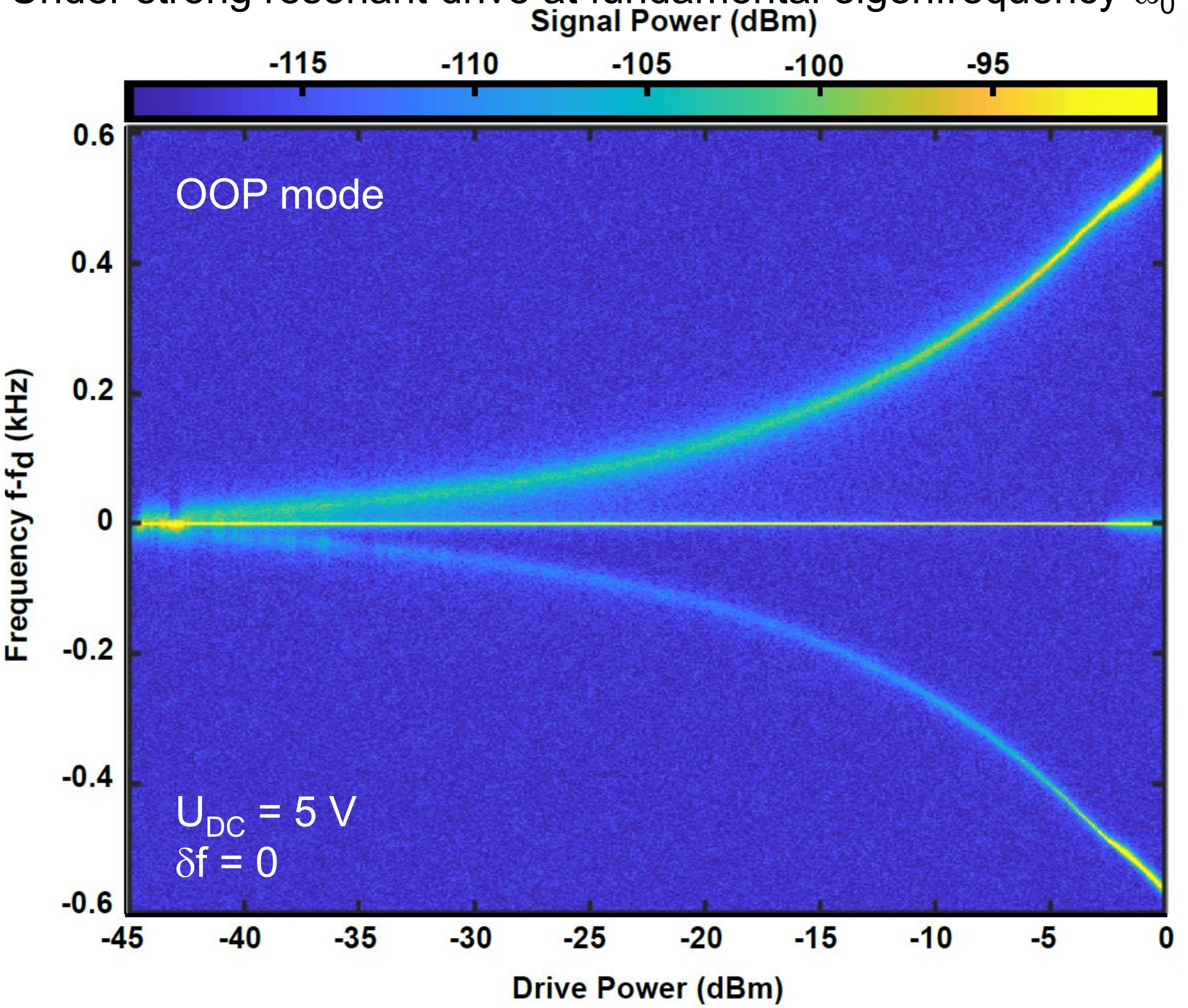




*Stambaugh et al., Phys. Rev. Lett. 97, 110602 (2006)*

# Thermal noise induced satellite peaks in the power spectrum

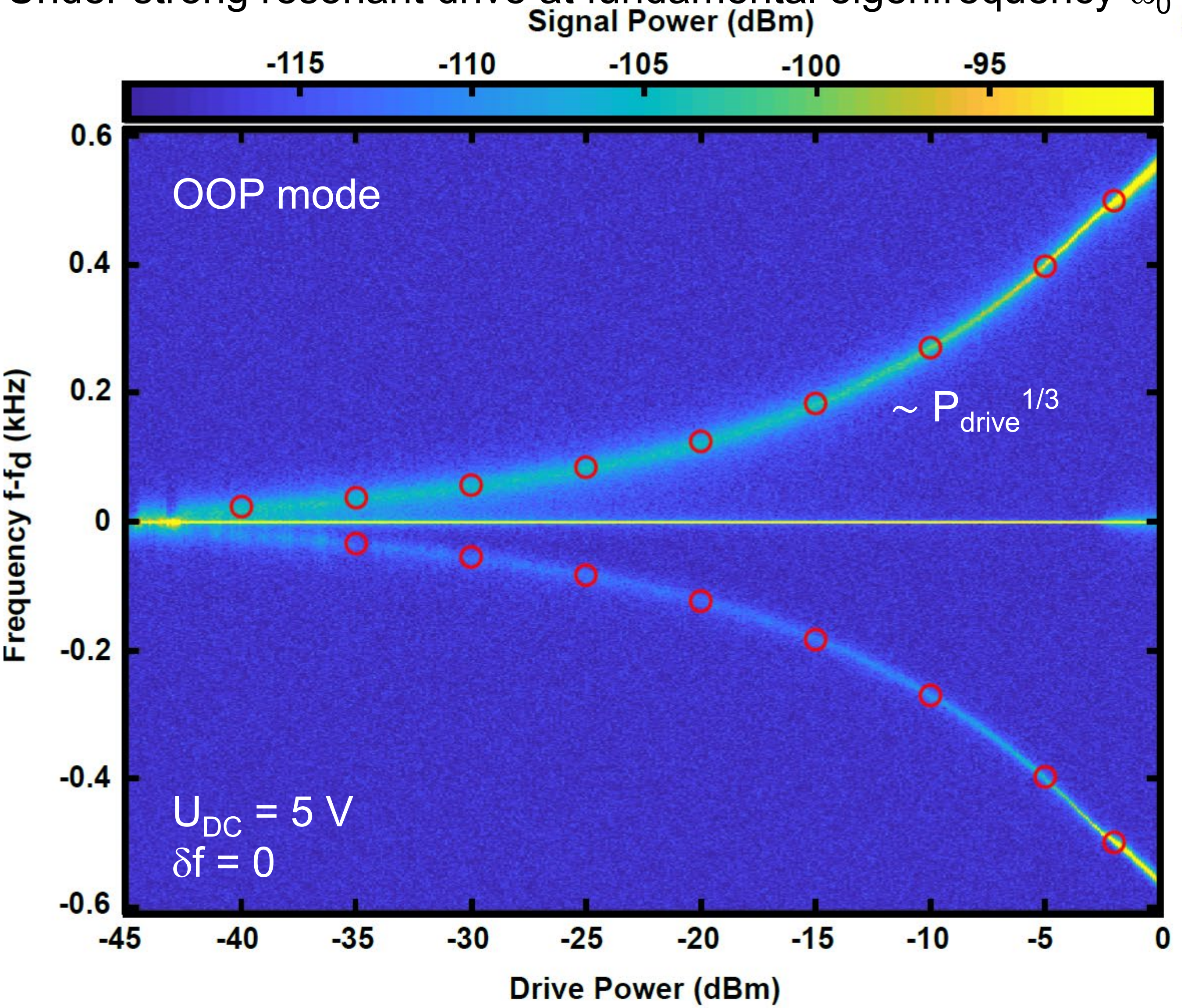
## Under strong resonant drive at fundamental eigenfrequency $\omega_0$



*Huber, Rastelli, Seitner, Kölbl, Belzig, Dykman, Weig, Phys. Rev. X 10, 021066 (2020)*

# Thermal noise induced satellite peaks in the power spectrum

## Under strong resonant drive at fundamental eigenfrequency $\omega_0$



*Huber, Rastelli, Seitner, Kölbl, Belzig, Dykman, Weig, Phys. Rev. X 10, 021066 (2020)*

# Thermal noise induced satellite peaks in the power spectrum

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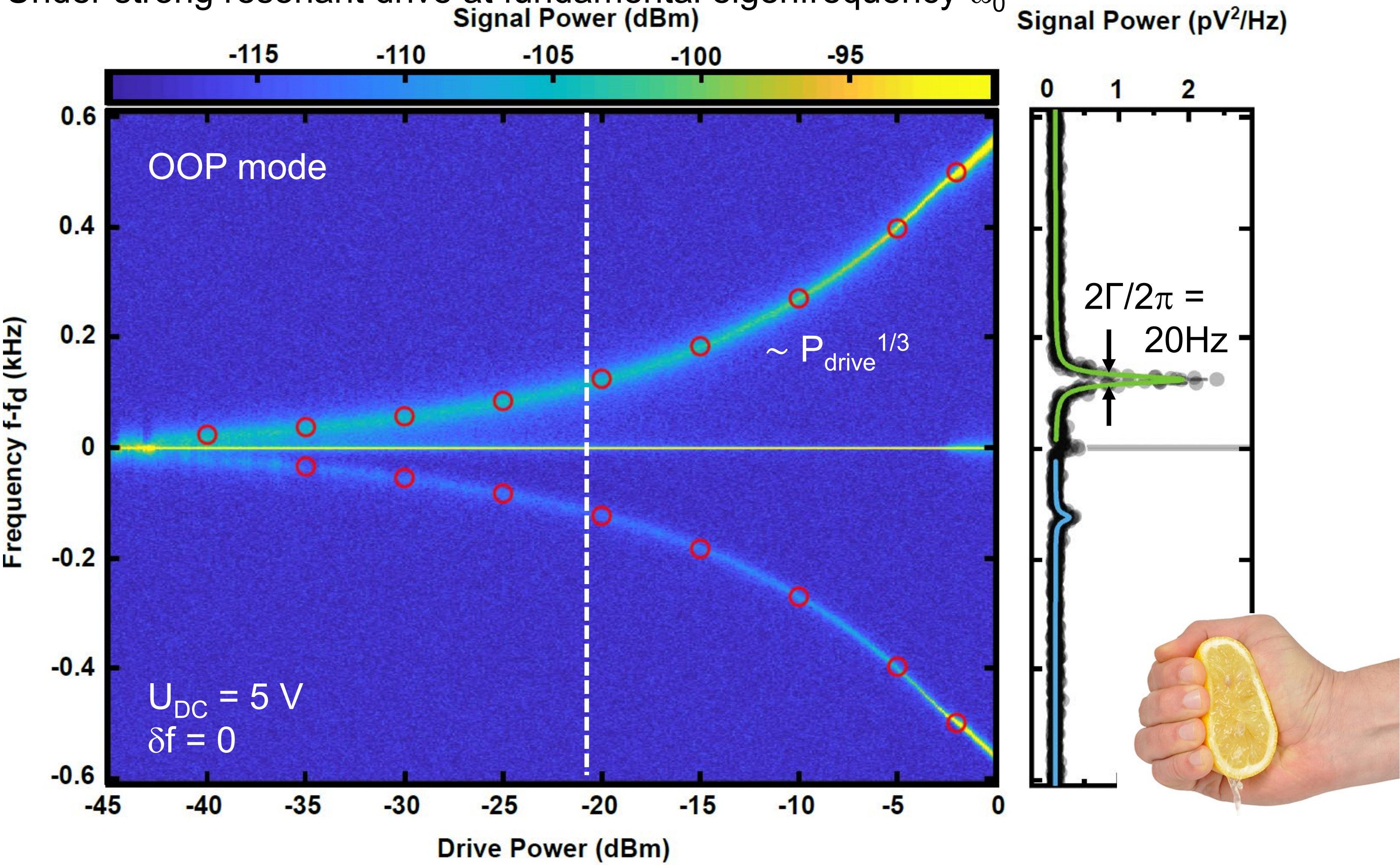
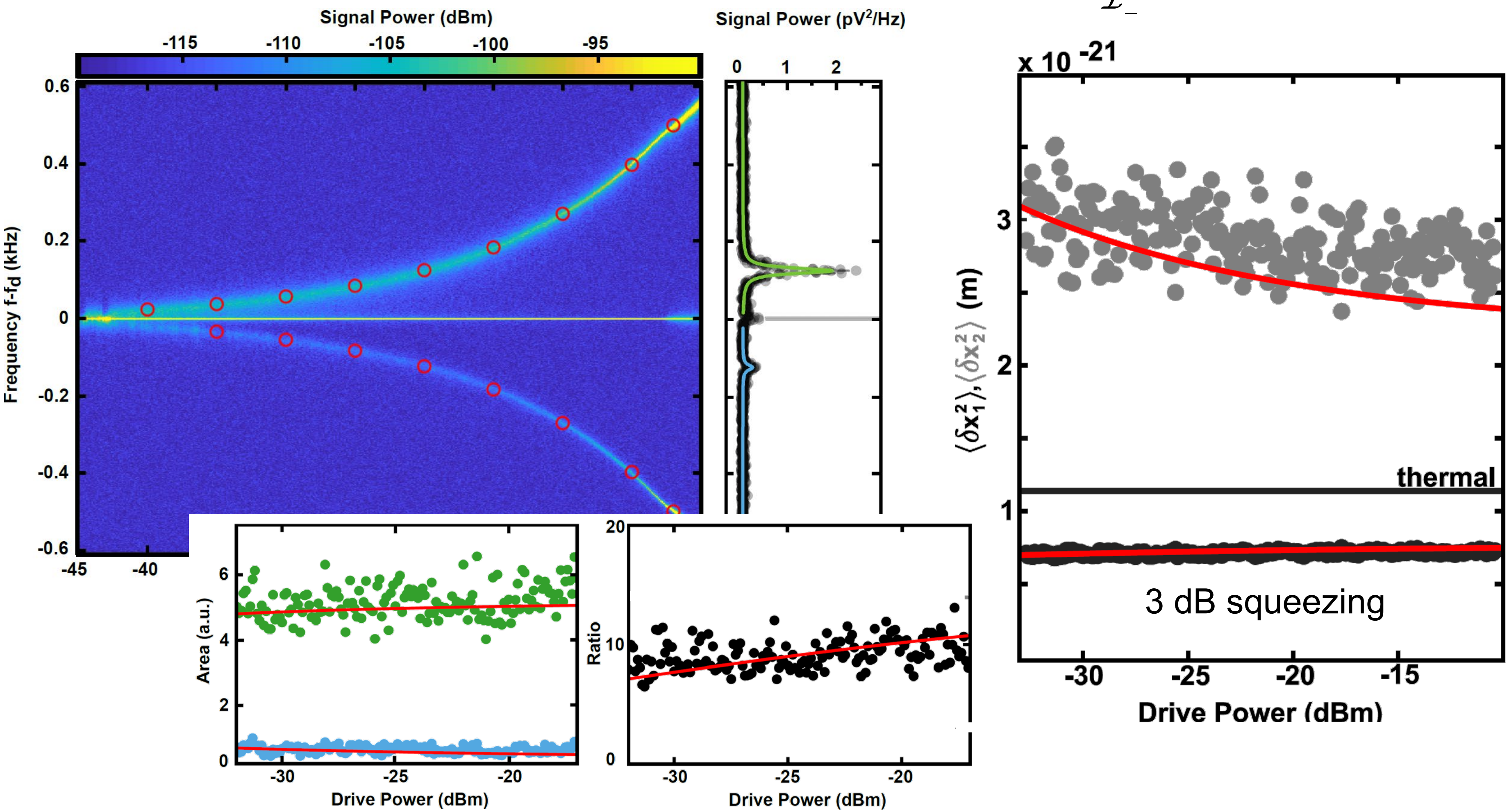


*Huber, Rastelli, Seitner, Kölbl, Belzig, Dykman, Weig, Phys. Rev. X 10, 021066 (2020)*

# Spectral evidence of squeezing

## Homodyne measurement not feasible for high Q resonators

Ratio of satellite areas encodes squeezing parameter $\phi$: $$\frac{\mathcal{I}_+}{\mathcal{I}_-} = \tanh^{-2}(\phi)$$



*Huber, Rastelli, Seitner, Kölbl, Belzig, Dykman, Weig, Phys. Rev. X 10, 021066 (2020)*

# Homodyne detection of squeezed states

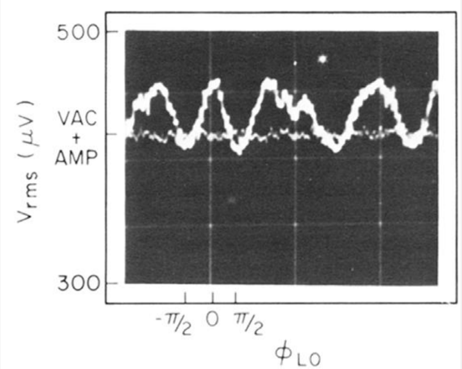
## in quantum optics, microwave circuits and mechanical resonators

The origins of squeezing:

Thermomechanical squeezing:

*quantum squeezing*

### Squeezed light:



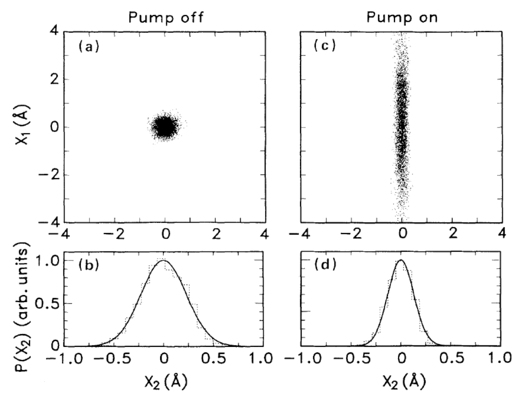
$1^{st}$ generation of squeezing: Na atoms in optical cavity

*Slusher et al., PRL 55, 2409 (1985)*

*classical squeezing*

### Nanomechanical parametric amplifier:

Electrically controlled Si micro-cantilever, $f_0$ = 33 kHz

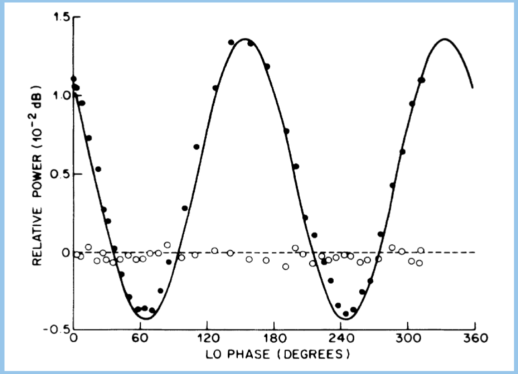


*Rugar et al., PRL 67, 699 (1991)*

*classical squeezing*

### Squeezed microwave radiation:

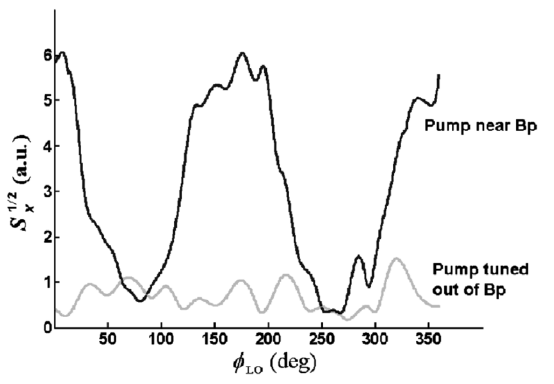
Josephson parametric amplifier



*Yurke et al., PRL 60, 764 (1988)*

*classical squeezing*

### Nanomechanical Duffing resonator:

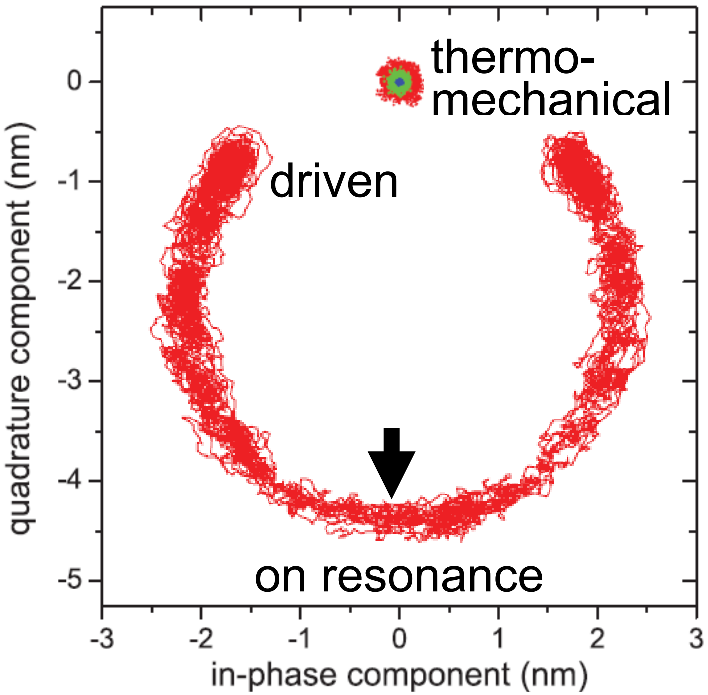


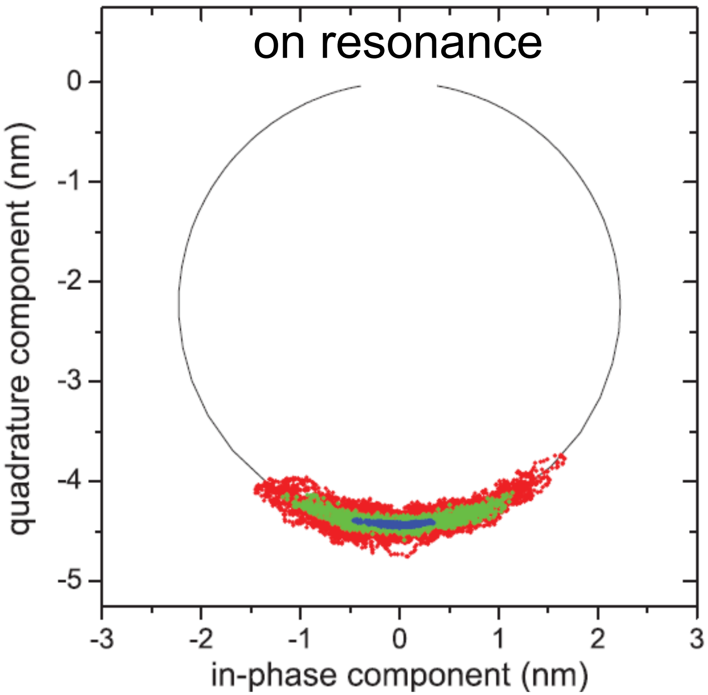
under additional white noise heating

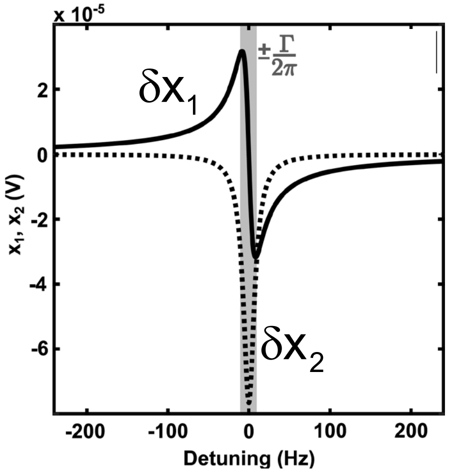
*Almog et al., PRL 98, 078103 (2007)*

# Homodyne detection of high Q resonators

## A standard procedure is getting tricky under (small) frequency fluctuations







$$\delta x_1 \approx \frac{\delta\Omega \cdot F}{2\Gamma^2\omega_0}$$

for $\omega_F \approx \omega_0$

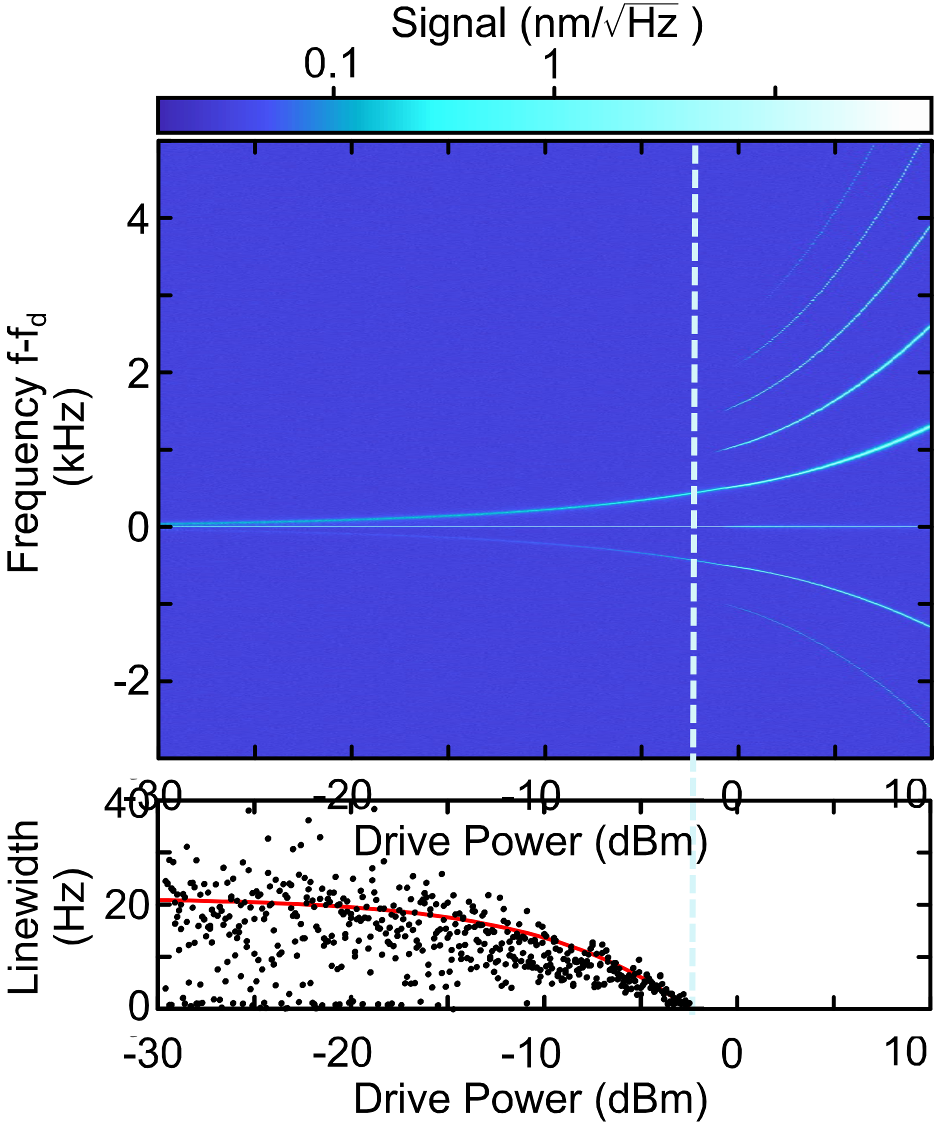
The in-phase quadrature is extremely sensitive to frequency fluctuations…

see *Fong et al., Phys. Rev. B 85, 161410(R) (2012)*

… but small frequency fluctuations only negligibly distort the spectral peaks

# Emergence of a frequency comb at higher drive powers

## From stable state of forced vibration to self-sustained oscillation



- Effective decrease in damping caused by resonantly induced friction force

$$F_{RIFF} = -\eta \left\langle F \cos(\omega_d t) \dot{q} \right\rangle q$$

- Negative nonlinear damping in the rotating frame
- Induced by the drive and proportional to the amplitude of the vibrations in the rotating frame

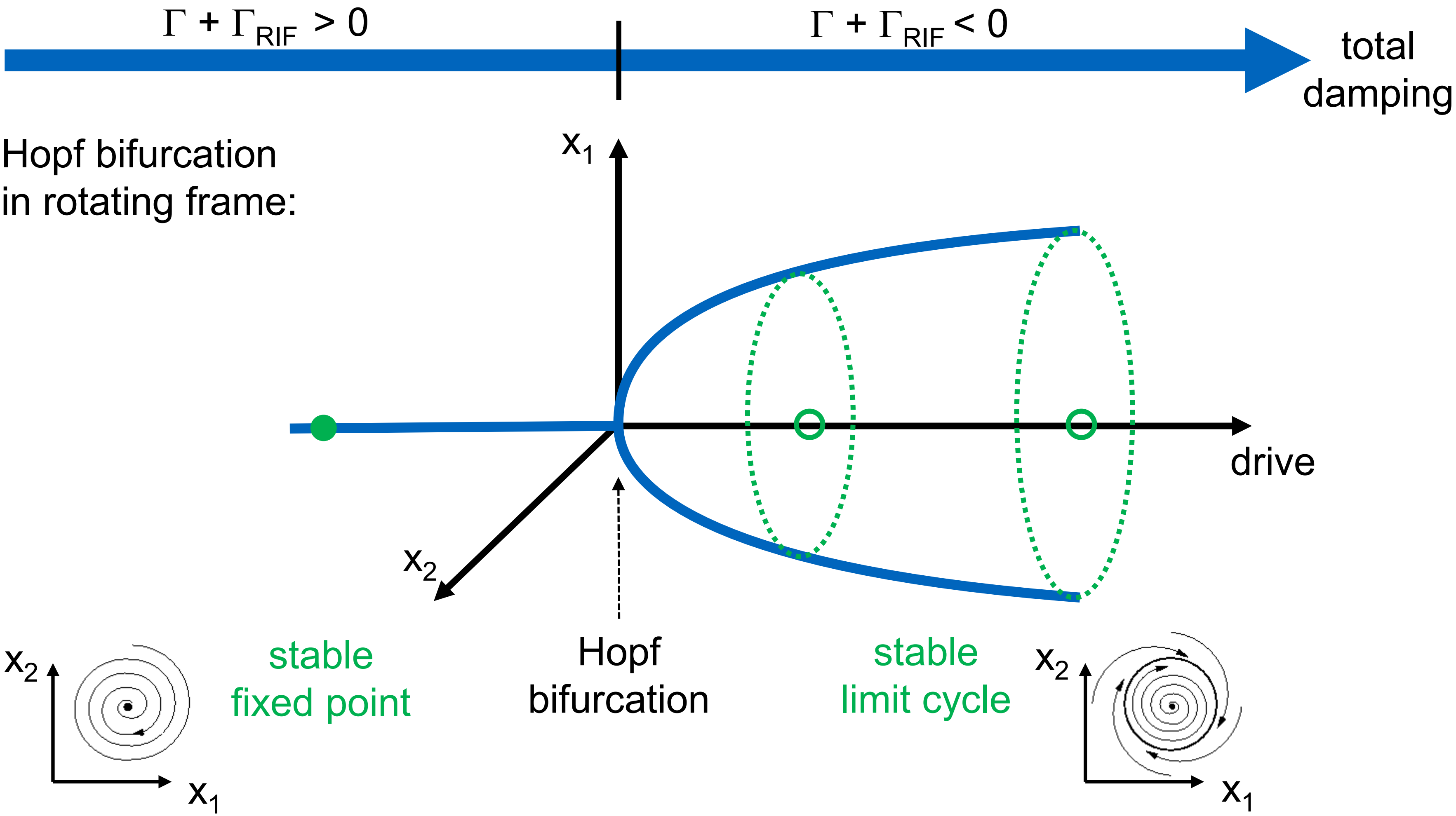
theory: *Dykman, Rastelli, Roukes, Weig, Phys. Rev. Lett. 122, 254301 (2019)*

see also:
*Bousse et al., JMEMS 29, 954 (2020)*

*Ochs, Boness, Rastelli, Seitner, Belzig, Dykman, Weig, Phys. Rev. X 12, 041019 (2022)*

# Negative, resonantly induced nonlinear friction $\Gamma_{\mathrm{RIF}}$

## From stable state of forced vibration to self-sustained oscillation



Hopf bifurcation
in rotating frame:

stable
fixed point

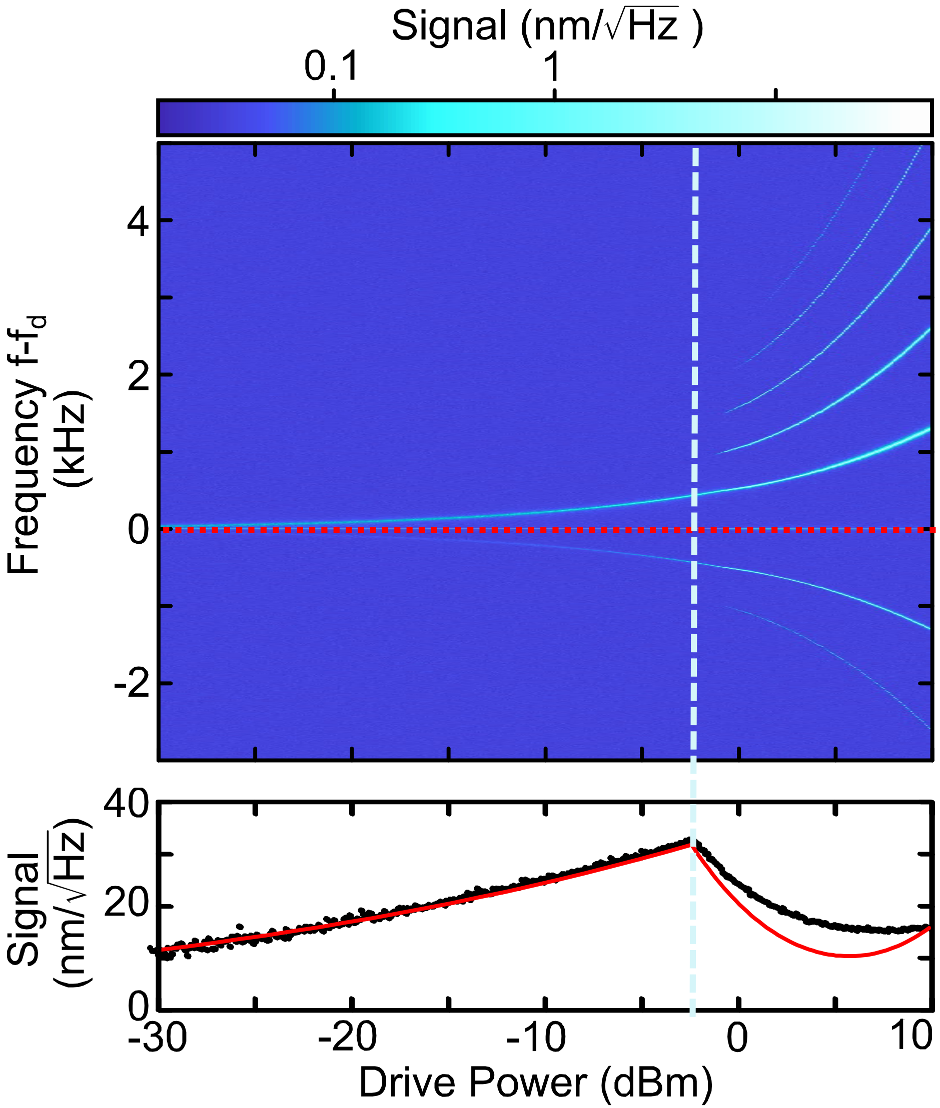
Hopf
bifurcation

stable
limit cycle

theory: *Dykman, Rastelli, Roukes, Weig, Phys. Rev. Lett. 122, 254301 (2019)*
*Ochs, Boness, Rastelli, Seitner, Belzig, Dykman, Weig, Phys. Rev. X 12, 041019 (2022)*

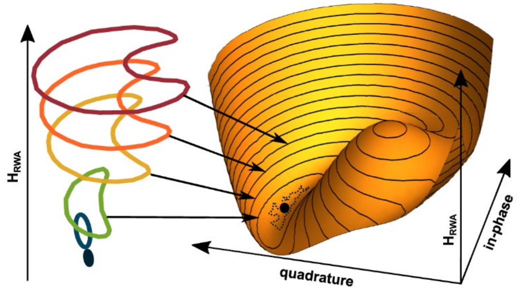
# Negative, resonantly induced nonlinear friction $\Gamma_{RIF}$

## From stable state of forced vibration to self-sustained oscillation



- $\Gamma + \Gamma_{RIF} = 0$:
  Hopf bifurcation in rotating frame
- $\Gamma + \Gamma_{RIF} \lesssim 0$:
  Limit cycle in rotating frame
  $\Rightarrow$ Two ultra-narrow satellites

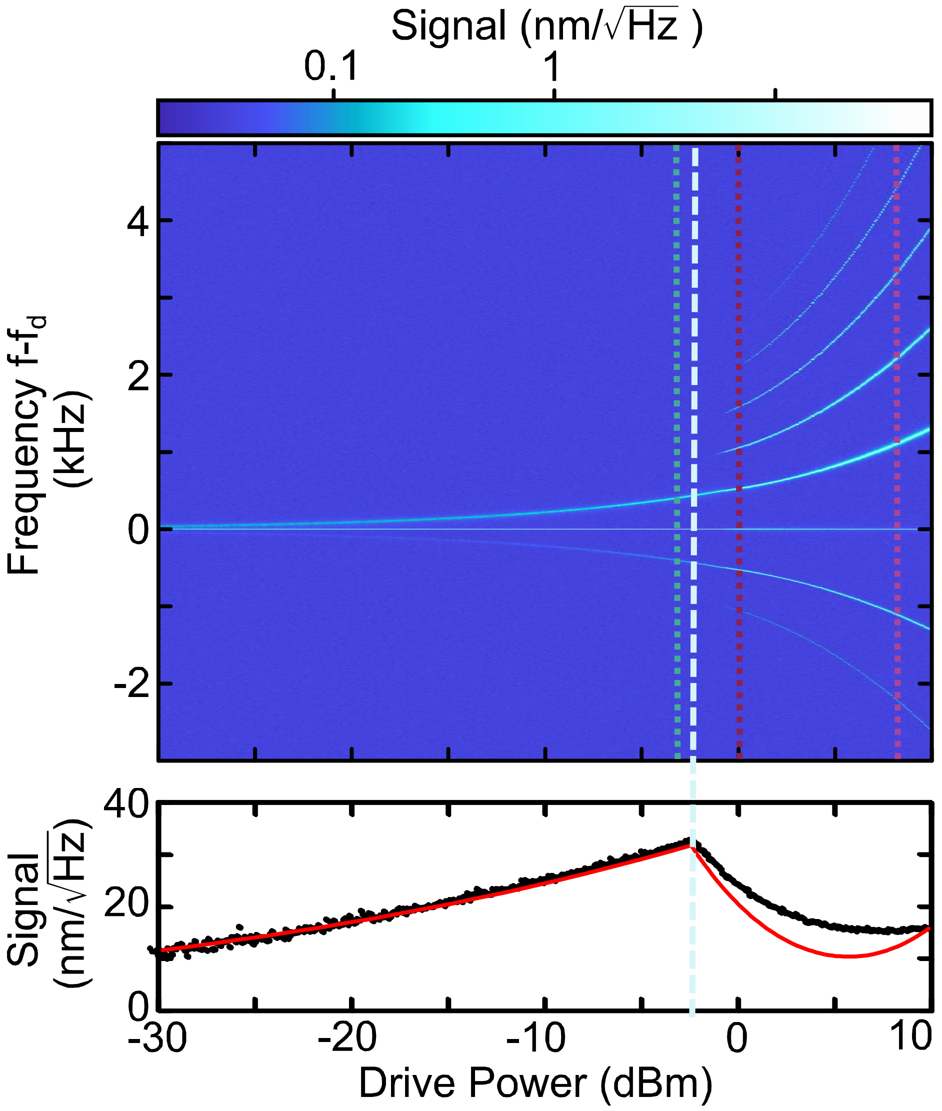
  see also: *Houri et al., Phys. Rev. Appl. 16, 064015 (2021)*
- $\Gamma + \Gamma_{RIF} < 0$:
  Limit cycle at higher quasienergy
  $\Rightarrow$ Non-sinusoidal trajectory yields higher order satellites

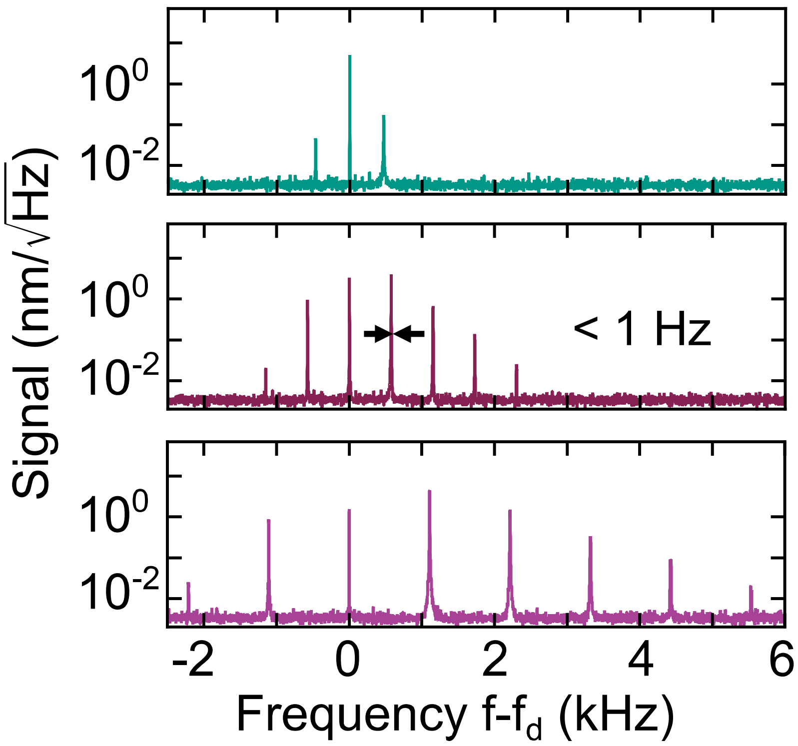


*Ochs, Boness, Rastelli, Seitner, Belzig, Dykman, Weig, Phys. Rev. X 12, 041019 (2022)*

# Negative, resonantly induced nonlinear friction $\Gamma_{RIF}$

## From stable state of forced vibration to self-sustained oscillation



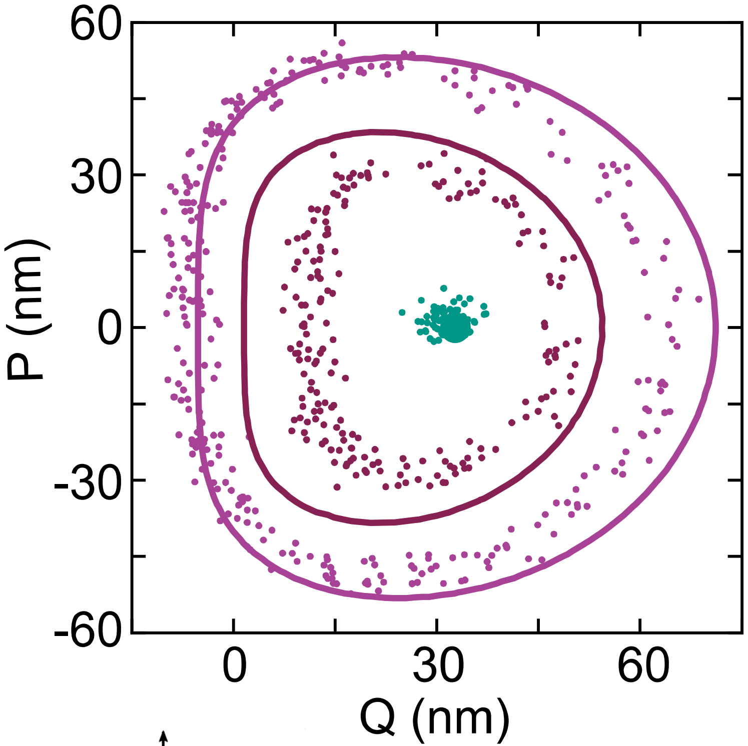


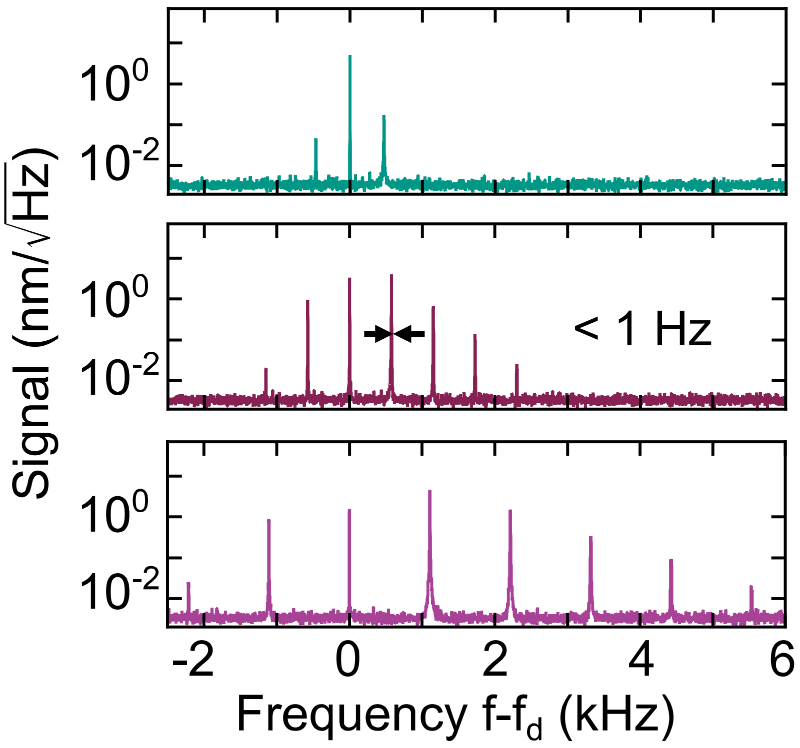
*Ochs, Boness, Rastelli, Seitner, Belzig, Dykman, Weig, Phys. Rev. X 12, 041019 (2022)*

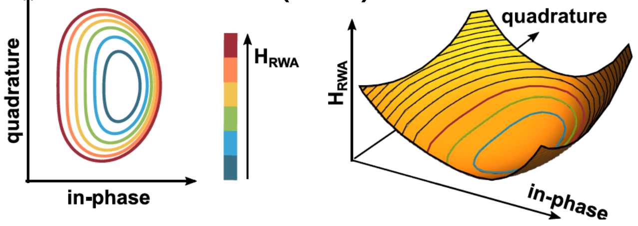
# Mapping the Hamiltonian function in the rotating frame

## Limit cycle rises to higher quasienergy for increasing drive power

Resonant drive:





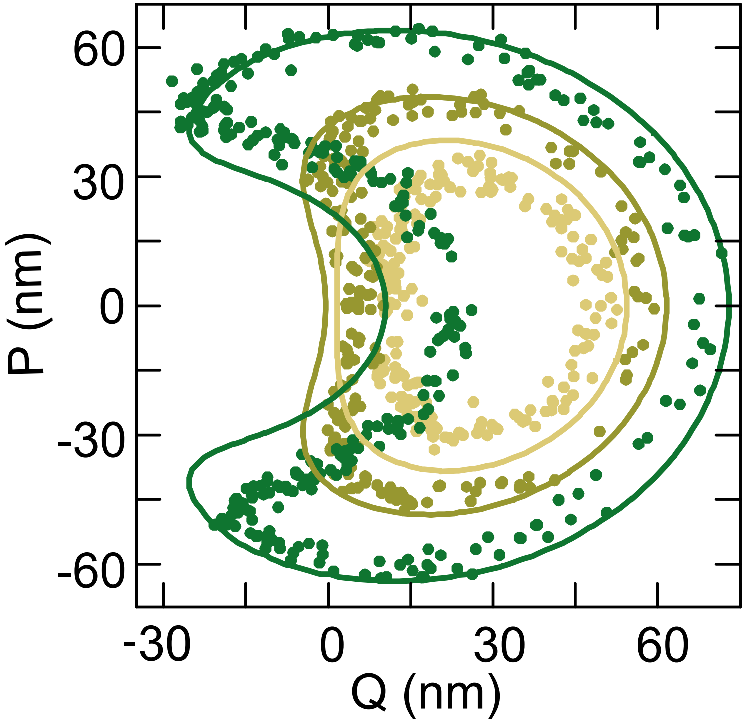


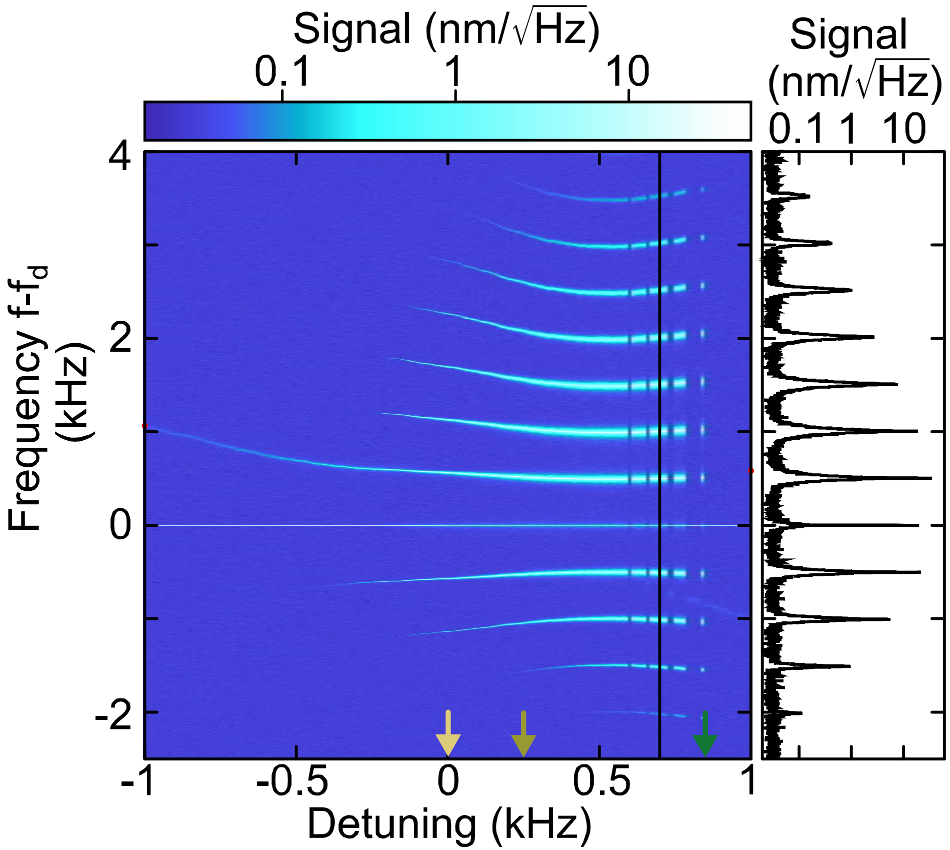
*Ochs, Boness, Rastelli, Seitner, Belzig, Dykman, Weig, Phys. Rev. X 12, 041019 (2022)*

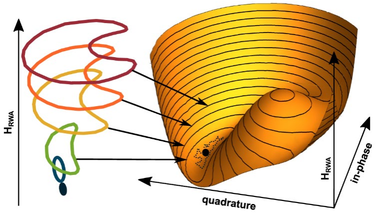
# Phonic frequency comb for detuned driving

Higher anharmonicity of trajectories gives rise to larger number of comb lines

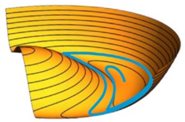
Variable detuning, drive power 1 dBm:



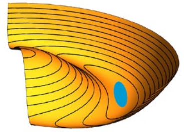




HI solution



LO solution



no instability!

*Ochs, Boness, Rastelli, Seitner, Belzig, Dykman, Weig, Phys. Rev. X 12, 041019 (2022)*

# Optical frequency combs as “rulers of light“

## Optical spectrum consisting of discrete, equidistant lines

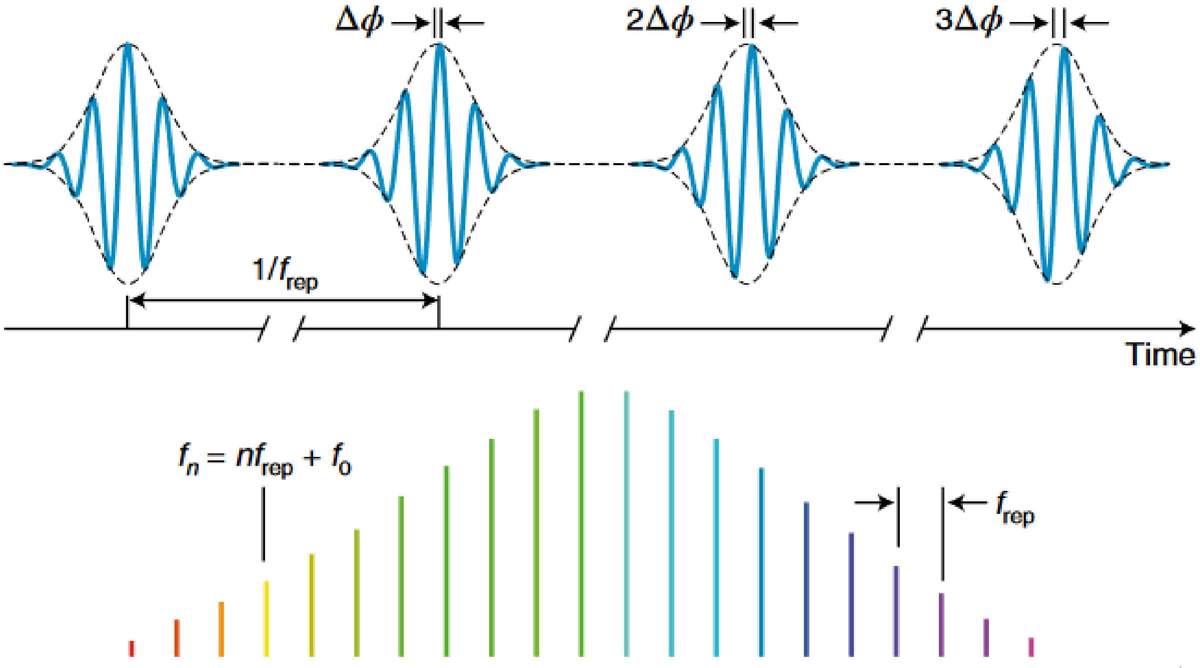


*image: Picqué & Hänsch, Nature Photonics 13, 146 (2019)*

- Link between microwave and optical part of electromagnetic spectrum
- Precise determination of unknown optical frequencies
  (optical atomic clocks, precision spectroscopy, …)

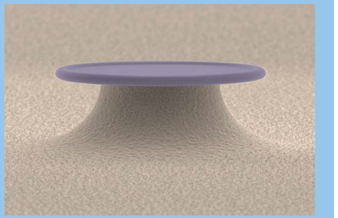
*Review: Fortier & Baumann, Communications Physics 2, 153 (2019)*

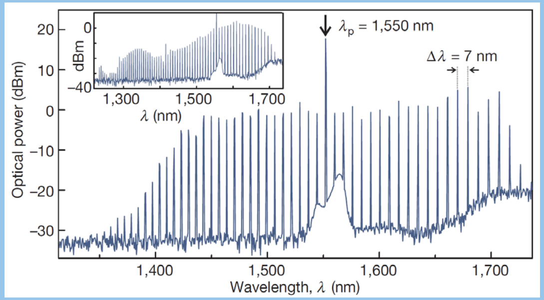
# Frequency combs in driven microresonators

## Induced by parametric interactions

### Optical Kerr combs in WGM resonators:

*e.g. silica microtoroid*

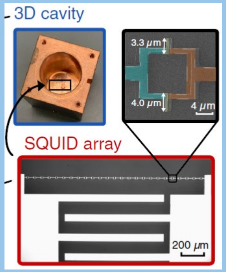


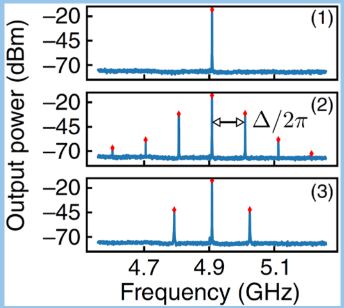


*Del'Haye et al., Nature 450, 1214 (2007)*

### Combs in microwave resonators:

*e.g. circuit QED*



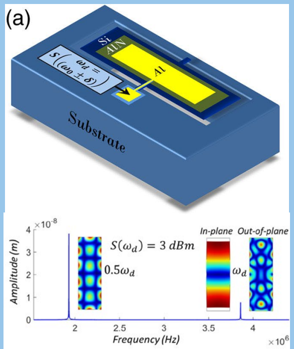


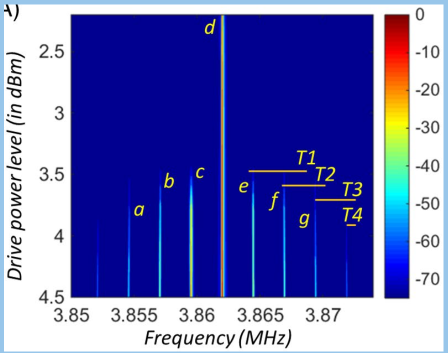
*images: Lu, Phys. Rev. Appl. (2021)*

*Khan et al., PRL 120, 153601 (2018)*

### Phononic combs:

*e.g. nonlinearly coupled mechan. modes*

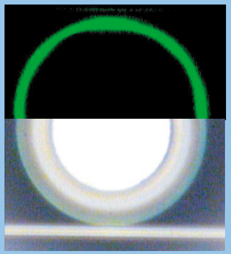


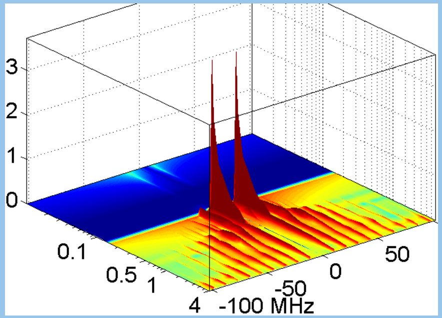


*Ganesan et al., PRL 118, 033903 (2017)*

### Optomechanical combs:

*e.g. cavity optomechanical system*



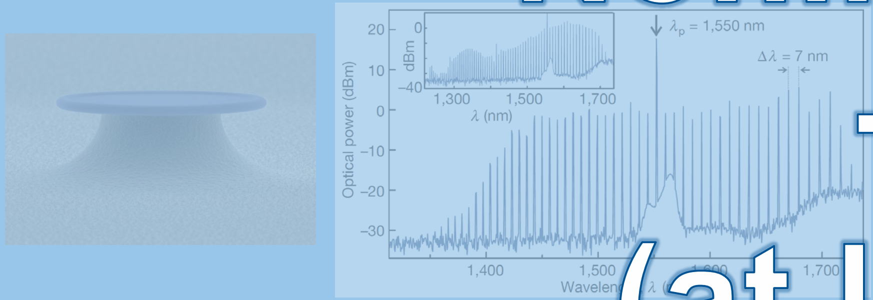


*Carmon et al., PRL 94, 223902 (2005)*

# Frequency combs in driven microresonators

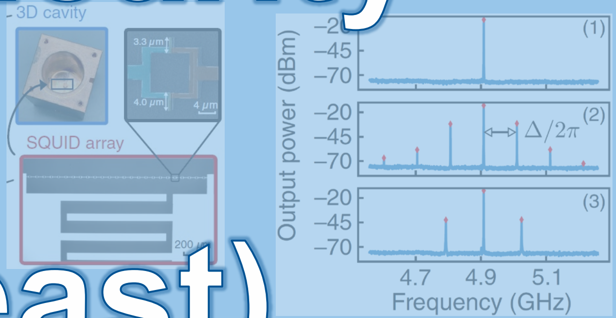
## Induced by parametric interactions

Optical Kerr combs in WGM resonators:
*e.g. silica microtoroid*



*Del'Haye et al., Nature 450, 1214 (2007)*

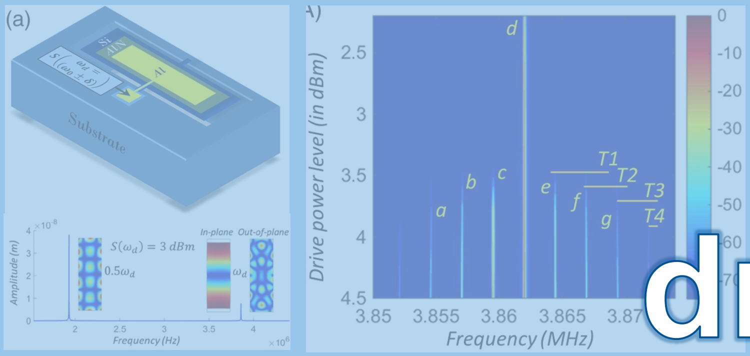
Combs in microwave resonators:
*e.g. circuit QED*



*images: Lu, Phys. Rev. Appl. (2021)*

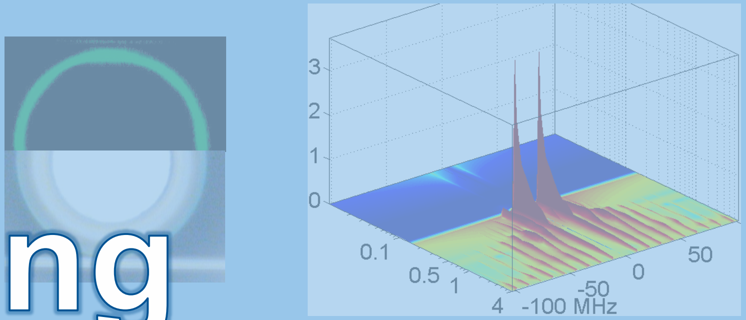
*Khan et al., PRL 120, 153601 (2018)*

Phononic combs:
*e.g. nonlinearly coupled mechanical modes*



*Ganesan et al., PRL 118, 033903 (2017)*

Optomechanical comb:
*e.g. cavity optomechanical system*



*Carmon et al., PRL 94, 223902 (2005)*

**Nonlinearity**

**+**

**(at least)**

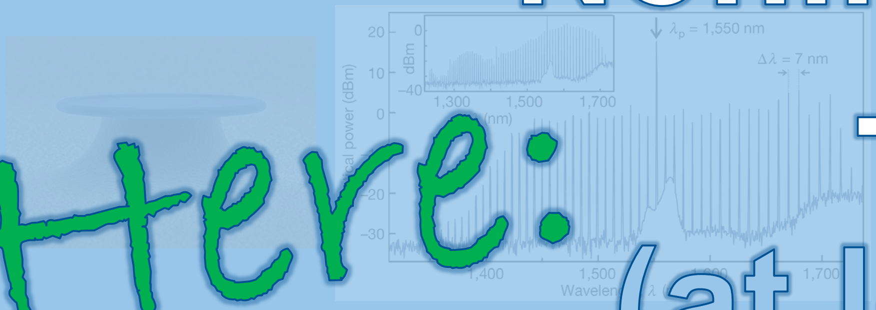
**two coupled modes**

**+**

**driving**

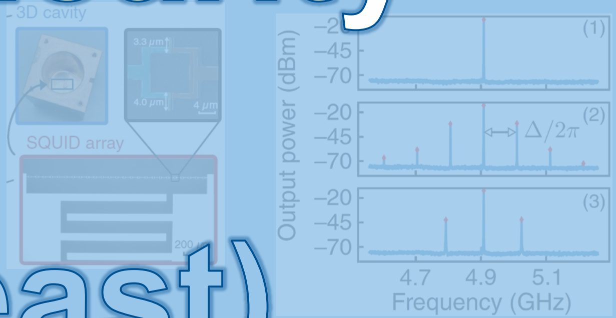
# Frequency combs in driven microresonators
## Induced by parametric interactions

Optical Kerr combs in WGM resonators:
*e.g. silica microtoroid*



*Del'Haye et al., Nature 450, 1214 (2007)*

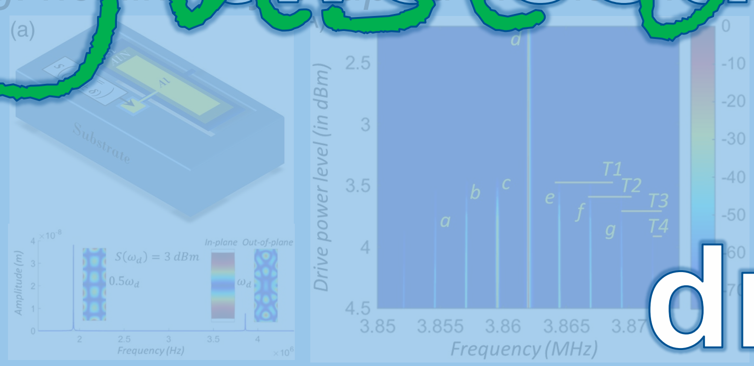
Combs in microwave resonators:
*e.g. circuit QED*



*images: Lu, Phys. Rev. Appl. (2021)*

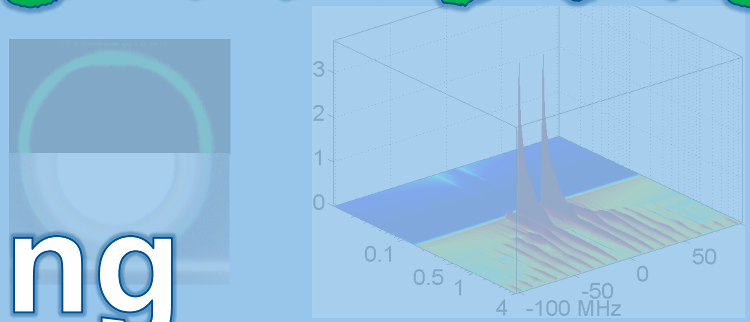
*Khan et al., PRL 120, 153601 (2018)*

Phononic combs:
*e.g. nonlinearly coupled [illegible] modes*



*Ganesan et al., PRL 118, 033903 (2017)*

Optomechanical combs:
*e.g. cavity [illegible]*



*Carmon et al., PRL 94, 223902 (2005)*

**Nonlinearity**

**+**

**(at least)**

**two coupled modes**

Here:

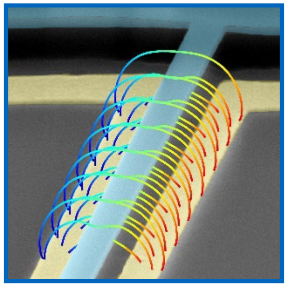
Just one mode

**+**

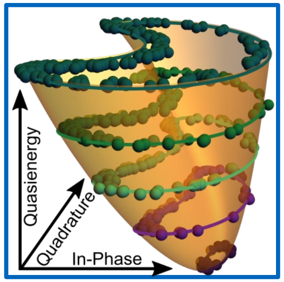
**driving**

*Ochs, Boness, Rastelli, Seitner, Belzig, Dykman, Weig, Phys. Rev. X 12, 041019 (2022)*

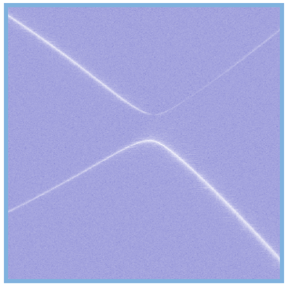
# OUTLINE



1. High Q nanomechanical string resonators:
   A well-controlled model system for dynamical phenomena



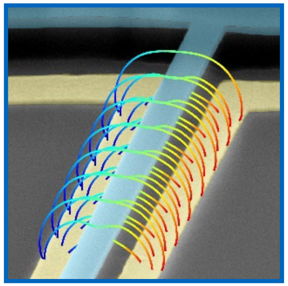
2. Nonlinear response of a single nanomechanical mode:
   A new type of frequency comb



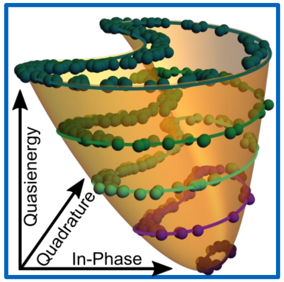
3. Coherent control of a nanomechanical two-mode system:
   Enhanced Ramsey spectroscopy for fast sening applications

   *[skipped for the sake of time]*

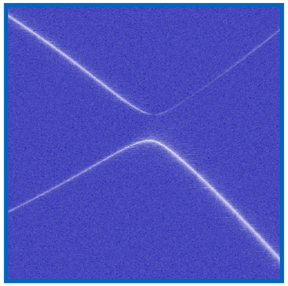
# SUMMARY (HIGH Q) NANOMECHANICAL SYSTEMS



1. High Q nanomechanical string resonators:
   - A well-controlled model system
   - Toolbox for nonlinear dynamics and coherent control

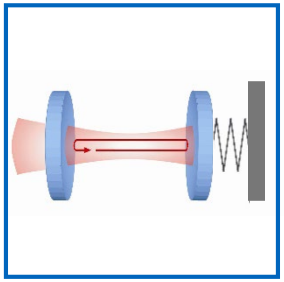


2. Nonlinear response of a single nanomechanical mode:
   - Spectral signatures of squeezing
   - Frequency comb from a single resonantly driven mode



3. Coherent control of a nanomechanical two-mode system:
   - Clasical toy model for coherent Bloch sphere dynamics
   - Iterative adaptive Ramsey protocol for fast sensing

# PART 2: CAVITY OPTOMECHANICS



1. An introduction to cavity optomechanics:
   Radiation-pressure induced dynamical backation

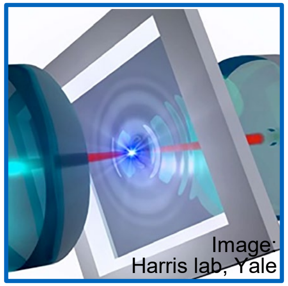
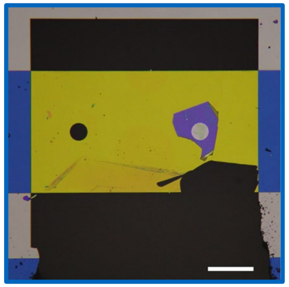


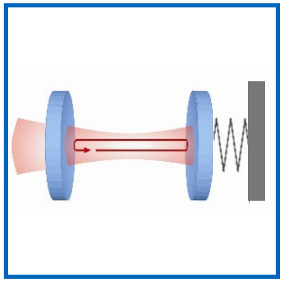
Image:
Harris lab, Yale

2. The membrane-in-the-middle configuration:
   A vibrating membrane inside a Fabry-Pérot cavity



3. Cavity optomechanics with van der Waals materials:
   Radiation pressure backaction on a flake of hBN

# PART 2: CAVITY OPTOMECHANICS



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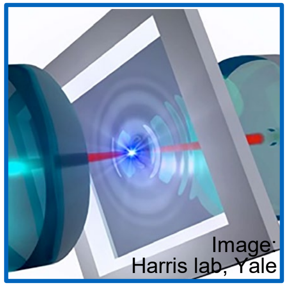

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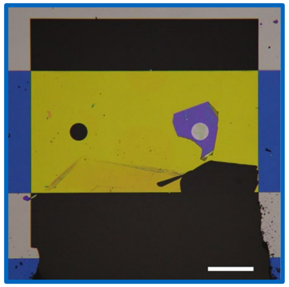


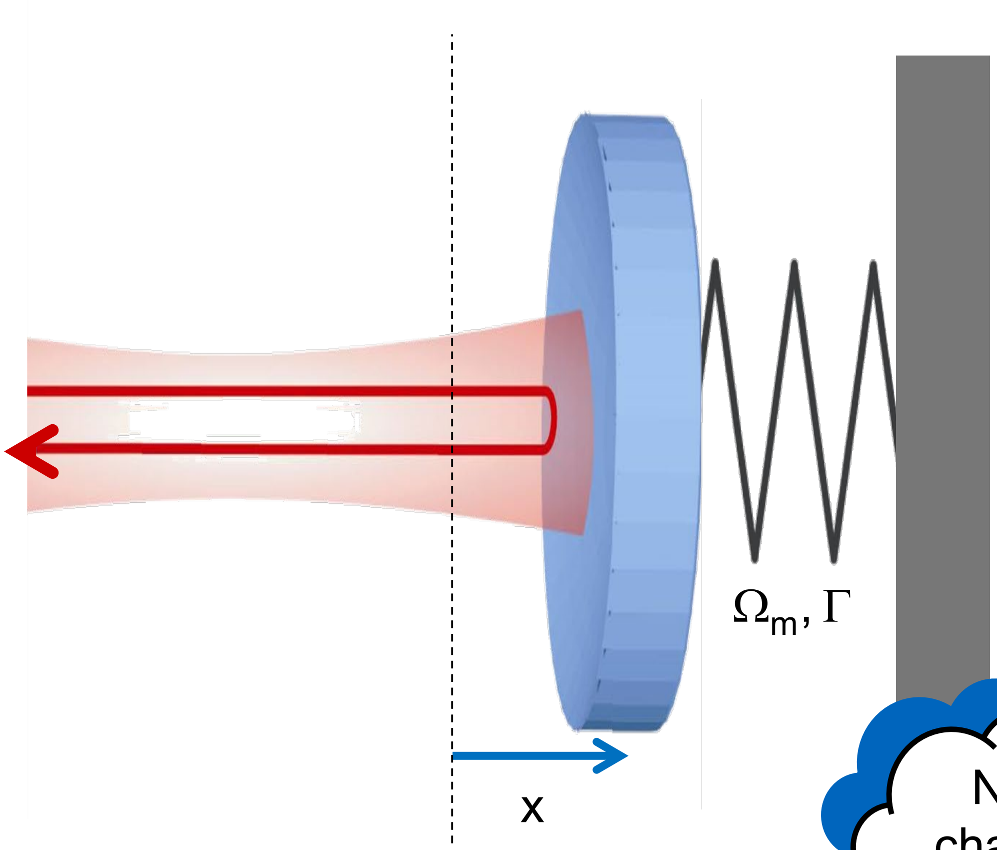
3. Cavity optomechanics with van der Waals materials:
   Radiation pressure backaction on a flake of hBN

# How to measure the position of a (macroscopic) resonator

## Use it as a mirror

Measure the phase shift of the reflected light:

$\varphi = 2kx$



Notice change of notation!

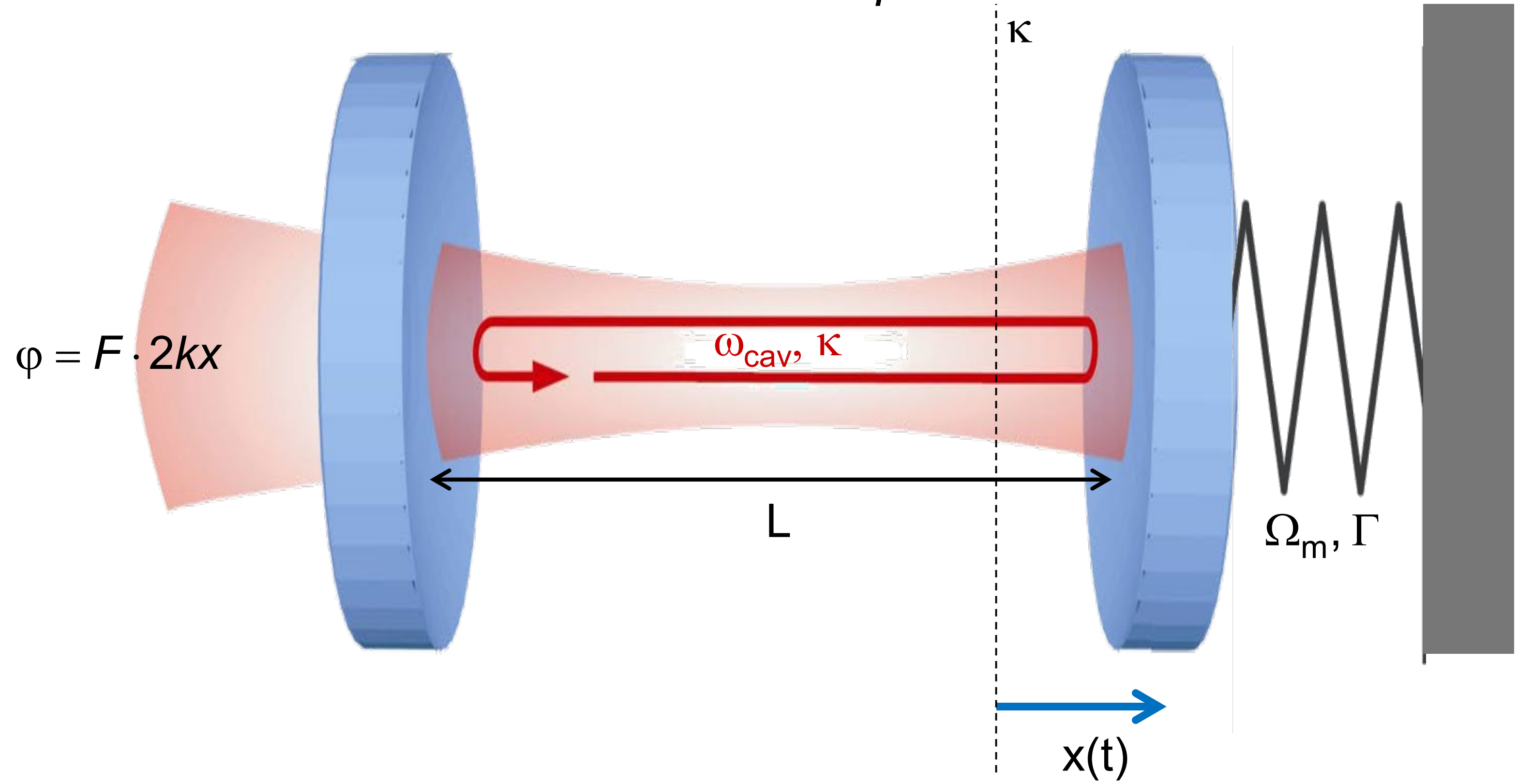
# How to measure the position of the resonator more precisely

Build an optical cavity with finesse $F$

Fabry-Pérot cavity: frequency & modes: $\frac{M}{2}\cdot\lambda = L \Rightarrow \omega_{cav} = \frac{2\pi c}{\lambda} = \frac{\pi c}{L}\cdot M\,,\ M = 1,2,\ldots$

free spectral range: $\Delta\omega_{FSR} = \pi\frac{c}{L}$

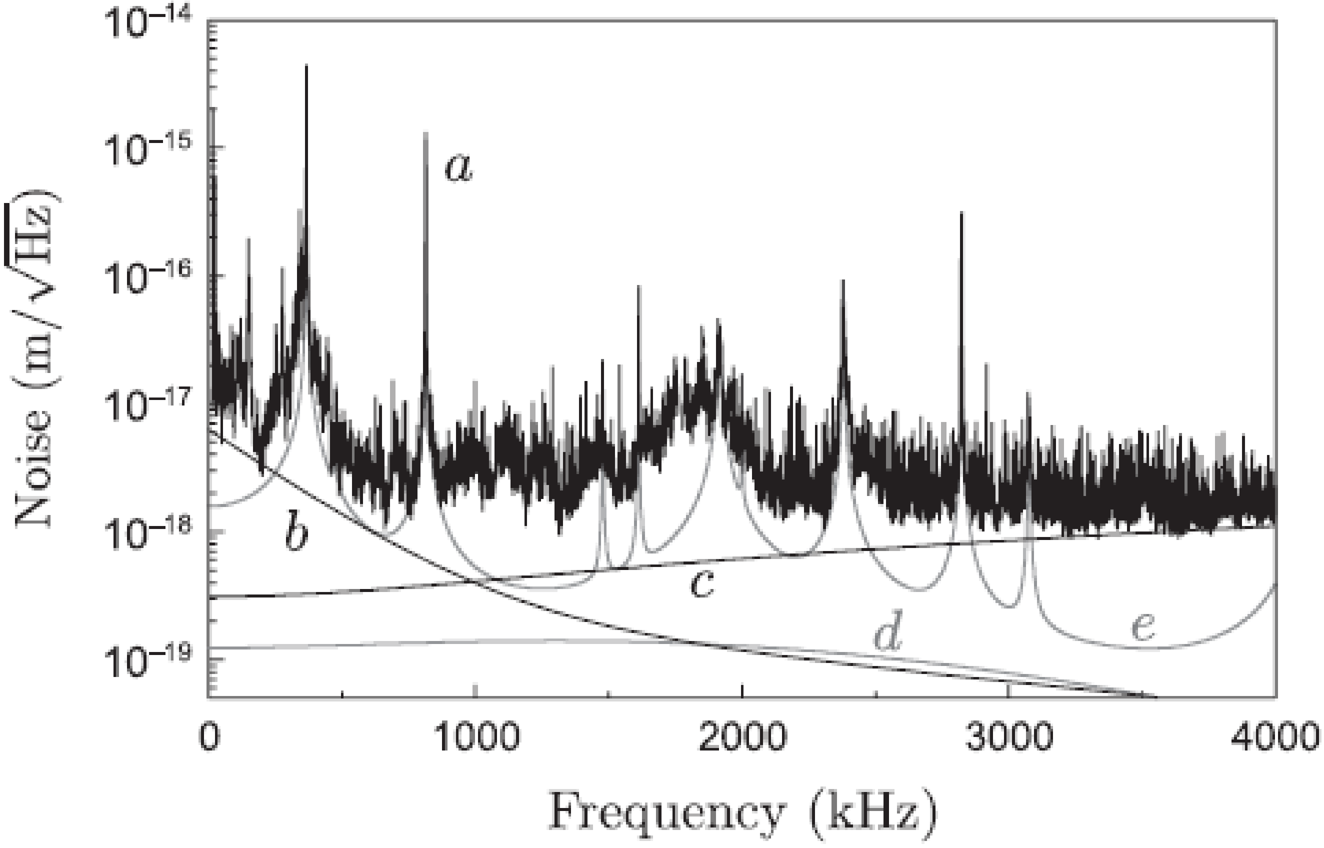
finesse: $F = \frac{\Delta\omega_{FSR}}{\kappa}$



# How to measure the position of a resonator even more precisely

## Build an optical cavity

How well does it work?



- + extremely high displacement sensitivity $\sqrt{S_x}$ below 1 am/√Hz
- + only limited by quantum phase noise (shot noise)
- \- limited to macro- and microscale resonators due to optical diffraction limit

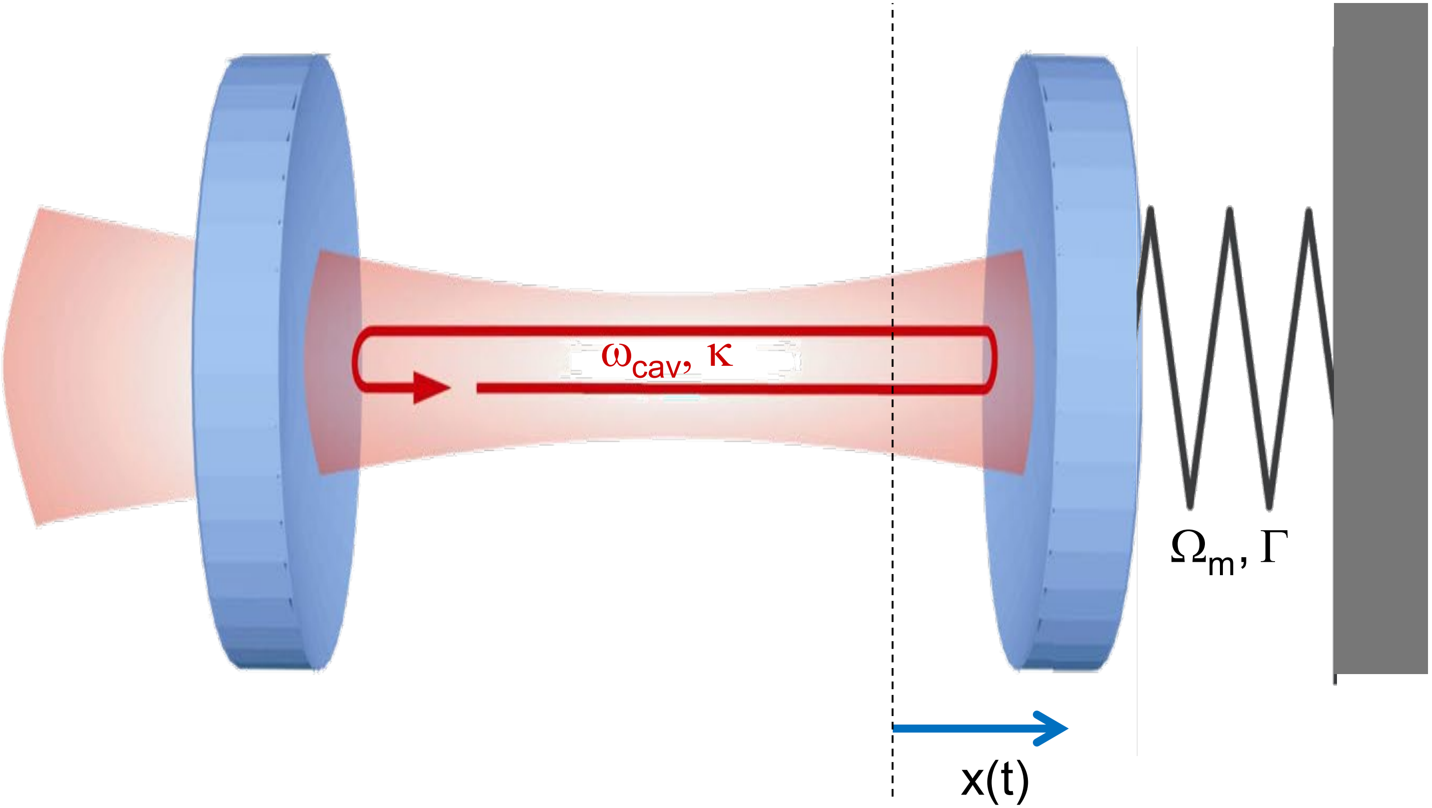
*O. Arcizet et al., Phys. Rev. Lett. 97, 133601 (2006)*

# What is an optomechanical system?

## A general definition

Parametric coupling between the mechanical displacement of a vibration mode and the energy stored inside a radiation mode:

dispersive coupling: $\omega_{\mathrm{cav}} = \omega_{\mathrm{cav}}\left(x\right)$



*see also:*

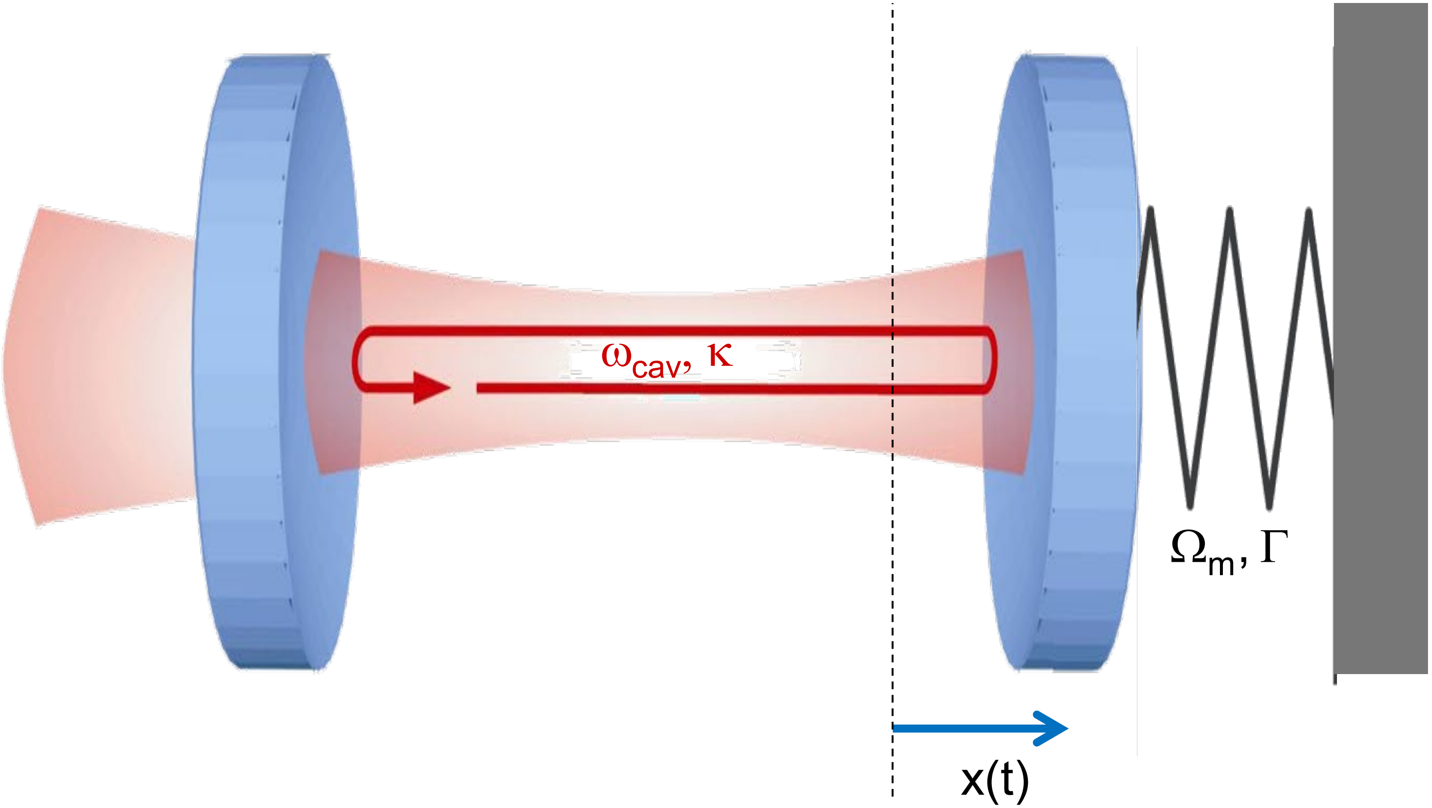
*M. Aspelmeyer, T.J. Kippenberg, F. Marquardt, Cavity Optomechanics, Rev. Mod. Phys. 86, 1391 (2014)*

# Parametrizing optomechanical coupling

## The linear regime

The frequency pull parameter G:

$$\omega_{cav}(x) \approx \omega_{cav} + x \cdot \frac{\partial \omega_{cav}(x)}{\partial x} + \ldots = \omega_{cav} - G \cdot x + \ldots \quad \text{with} \quad G = -\frac{\partial \omega_{cav}(x)}{\partial x}$$



*see also:*
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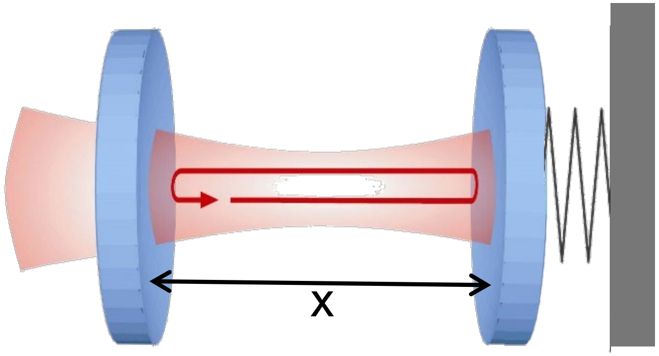
# Example: Fabry-Pérot cavity

## Frequency pull parameter and radiation pressure force

Frequency pull parameter:

$$G = -\frac{\partial \omega_{cav}(x)}{\partial x}$$

$$= -\frac{\partial}{\partial x}\left(\frac{\pi c}{x} \cdot M\right) = -\left(-\frac{\pi c M}{x^2}\right) = \frac{\omega_{cav}(x)}{x}$$



Energy stored in optical cavity mode:

$$E(x) = N\hbar\omega_{cav} = \frac{N\hbar\pi c M}{x}$$

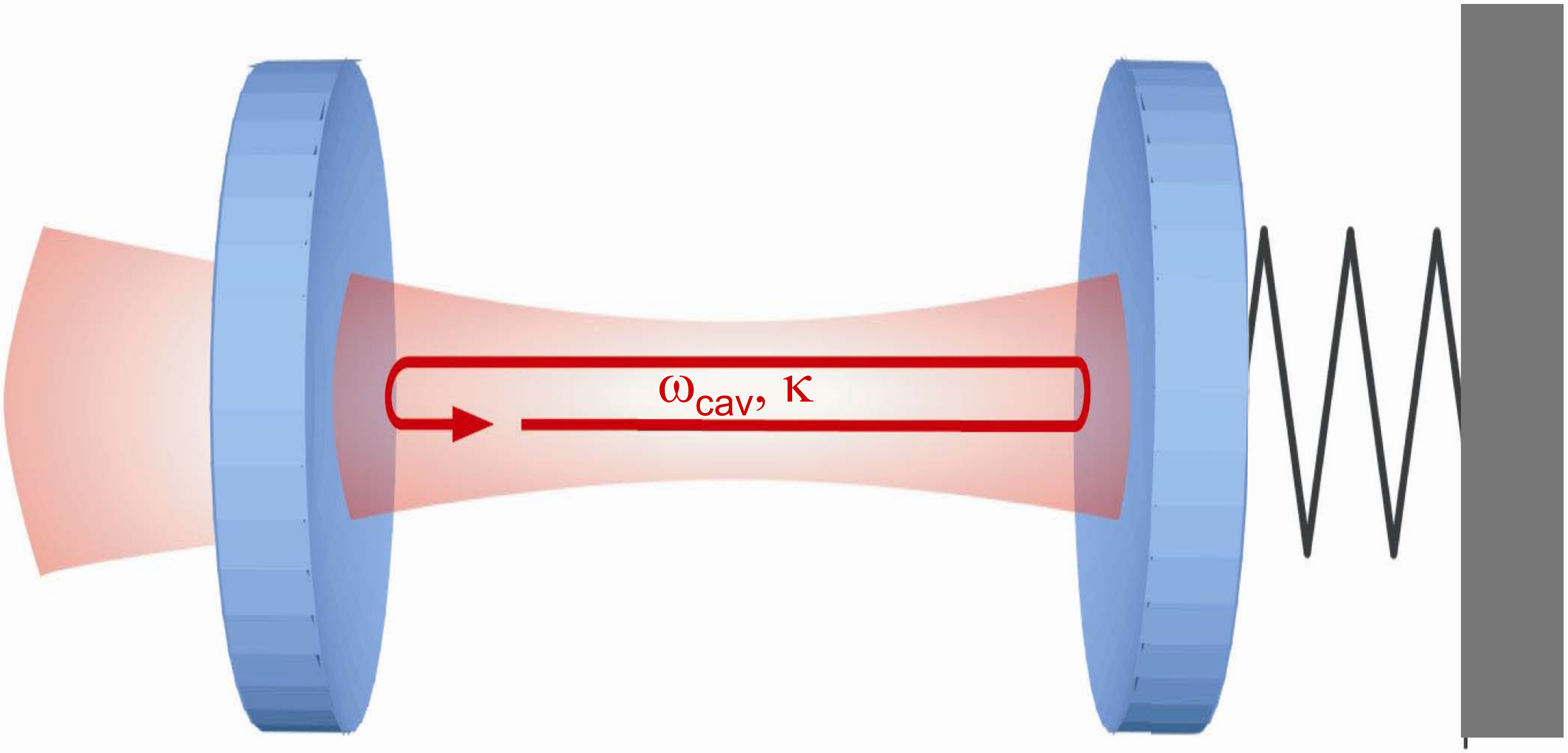
Radiation pressure force:

$$F_{rad} = -\frac{\partial E}{\partial x} = -N\hbar\frac{\partial \omega_{cav}}{\partial x} = N\hbar G = \frac{N\hbar\omega_{cav}}{x}$$

*see also:*
*M. Aspelmeyer, T.J. Kippenberg, F. Marquardt, Rev. Mod. Phys. 86, 1391 (2014)*

# Cavity optomechanics

## An (incomplete) overview of a rapidly growing field



# Cavity optomechanics

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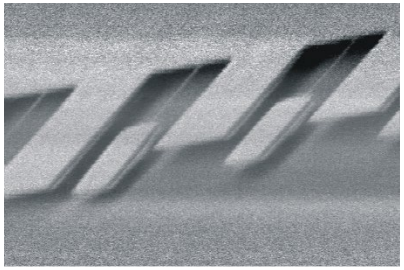
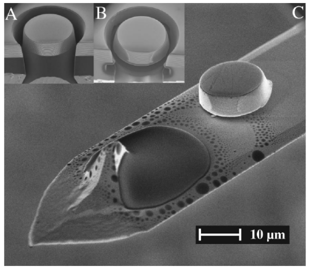
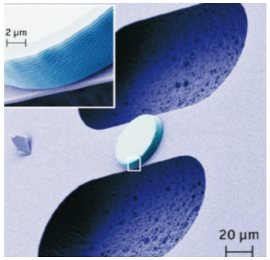

Karrai (Munich)



Bouwmeester (UCSB)



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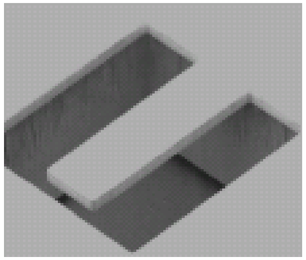
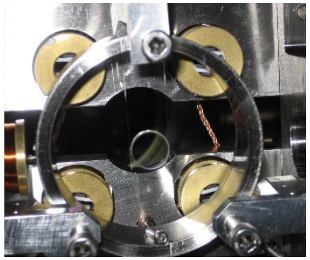
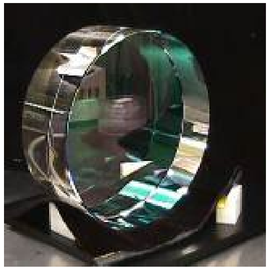
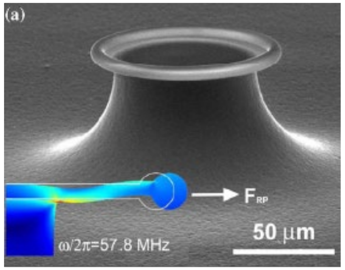

Heidmann (Paris)


Mavalvala (MIT)


LIGO



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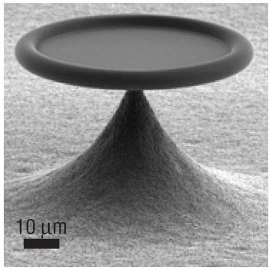

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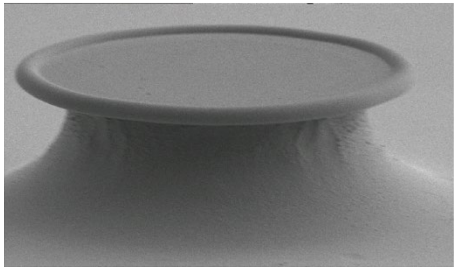
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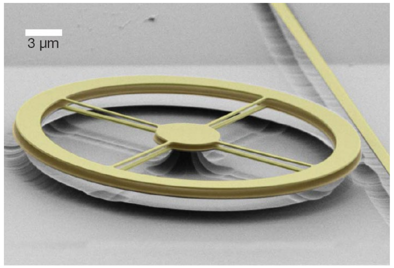

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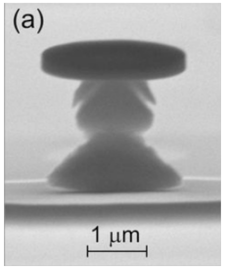

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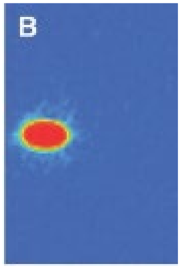
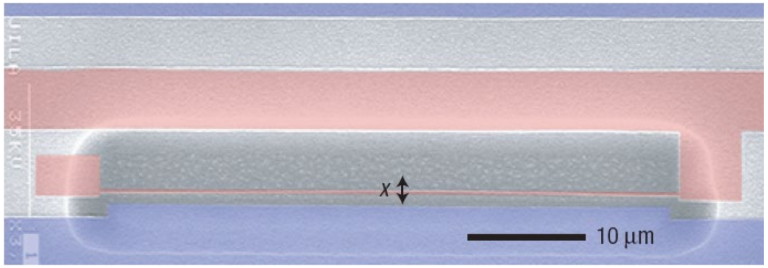
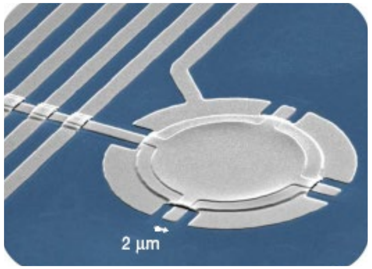


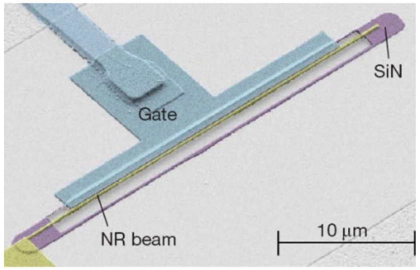
Stamper-Kurn



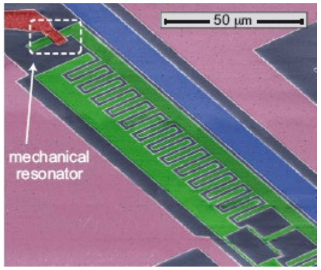
Lehnert (JILA Boulder)



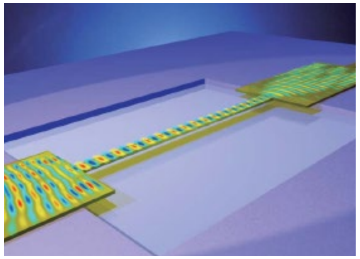
Teufel (NIST Boulder)

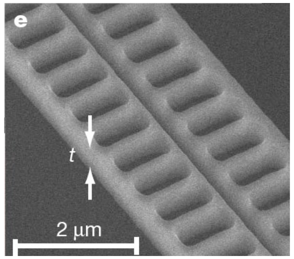


Schwab (Caltech)

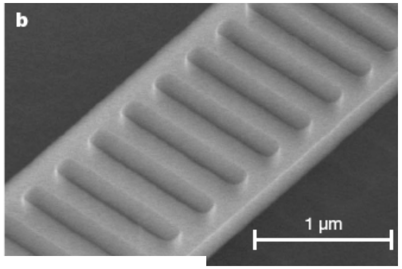


Sillanpää (Aalto)


Tang (Yale)

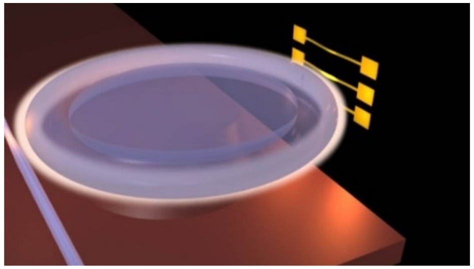


Painter (Caltech)



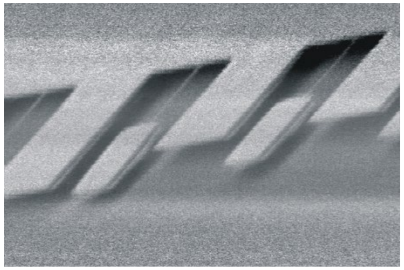
Painter (Caltech)

Harris (Yale)


Weig/Kotthaus/Kippenberg

# Cavity optomechanics

## An (incomplete) overview of a rapidly growing field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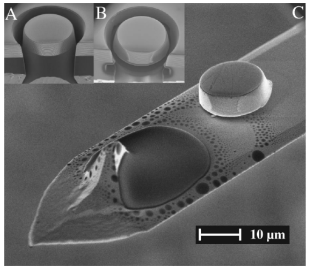

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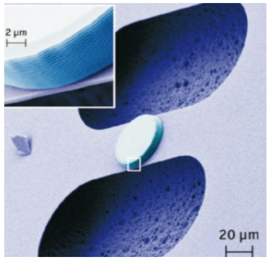

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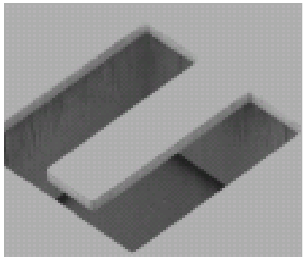
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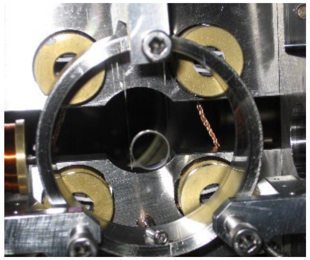
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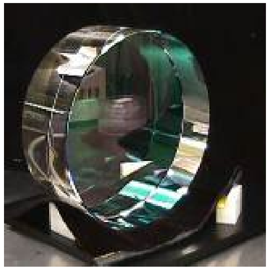
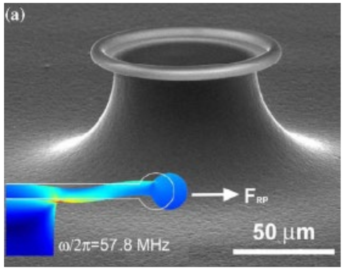

LIGO



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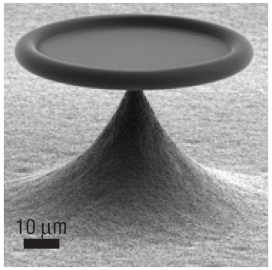

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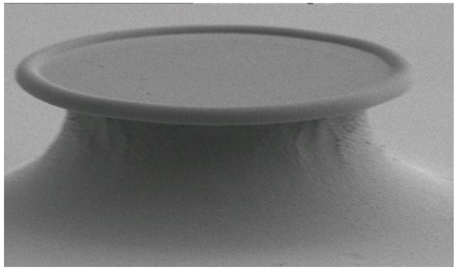
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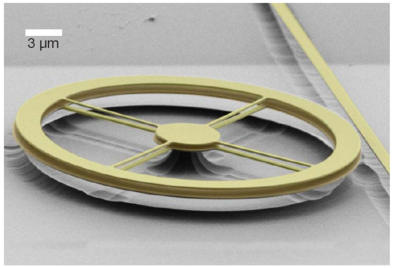

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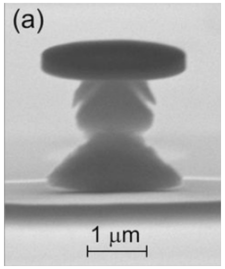

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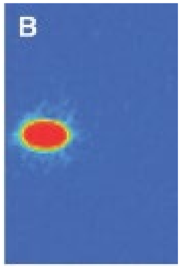

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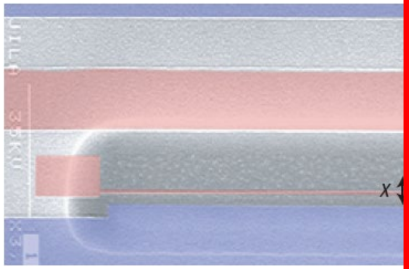

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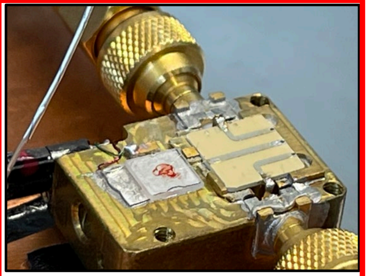
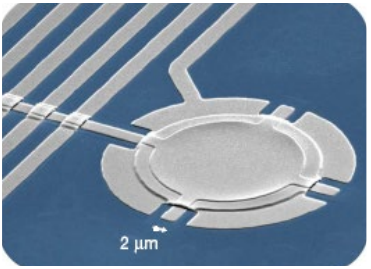

Weig (TUM)



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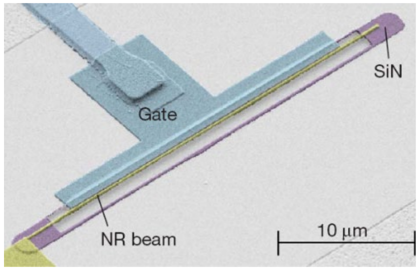

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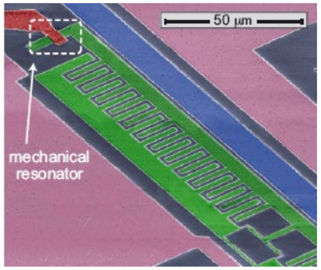

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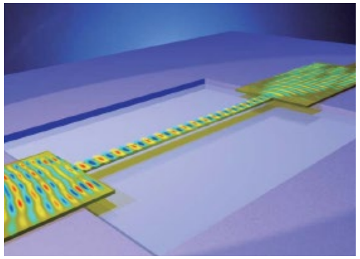
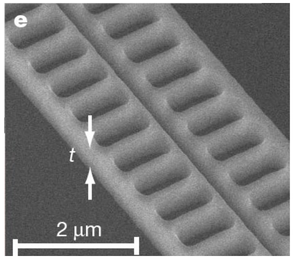
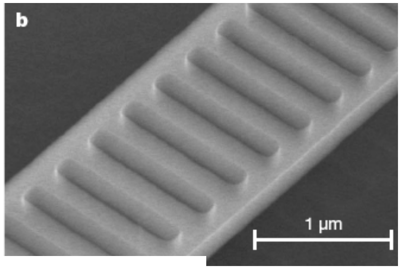

Tang (Yale)

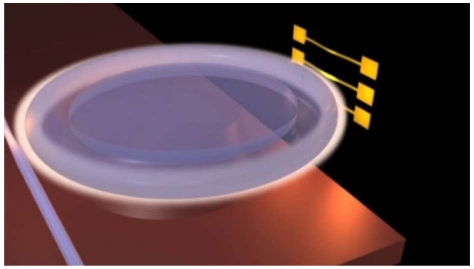


Painter (Caltech)



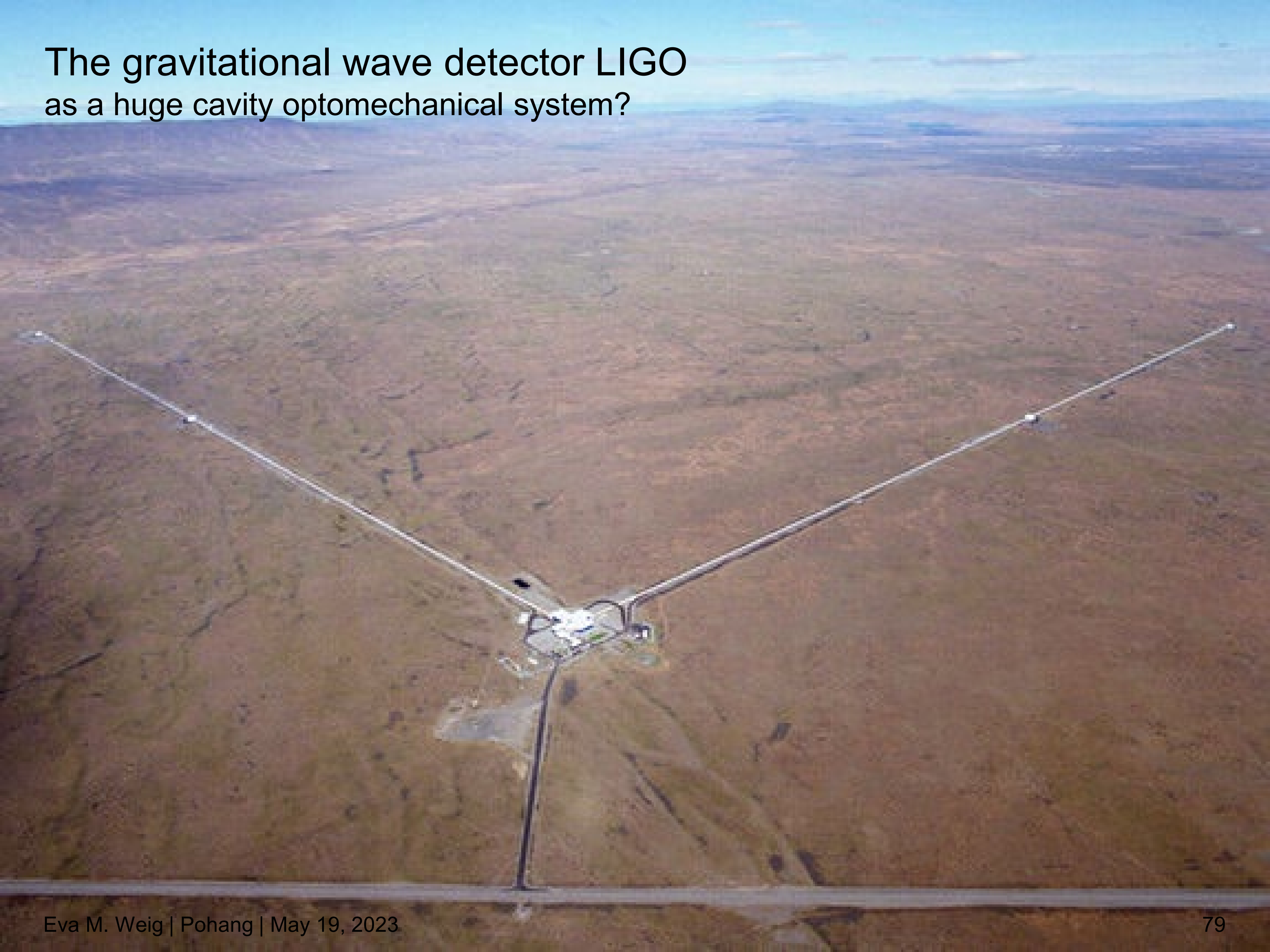
Painter (Caltech)

Harris (Yale)


Weig/Kotthaus/Kippenberg

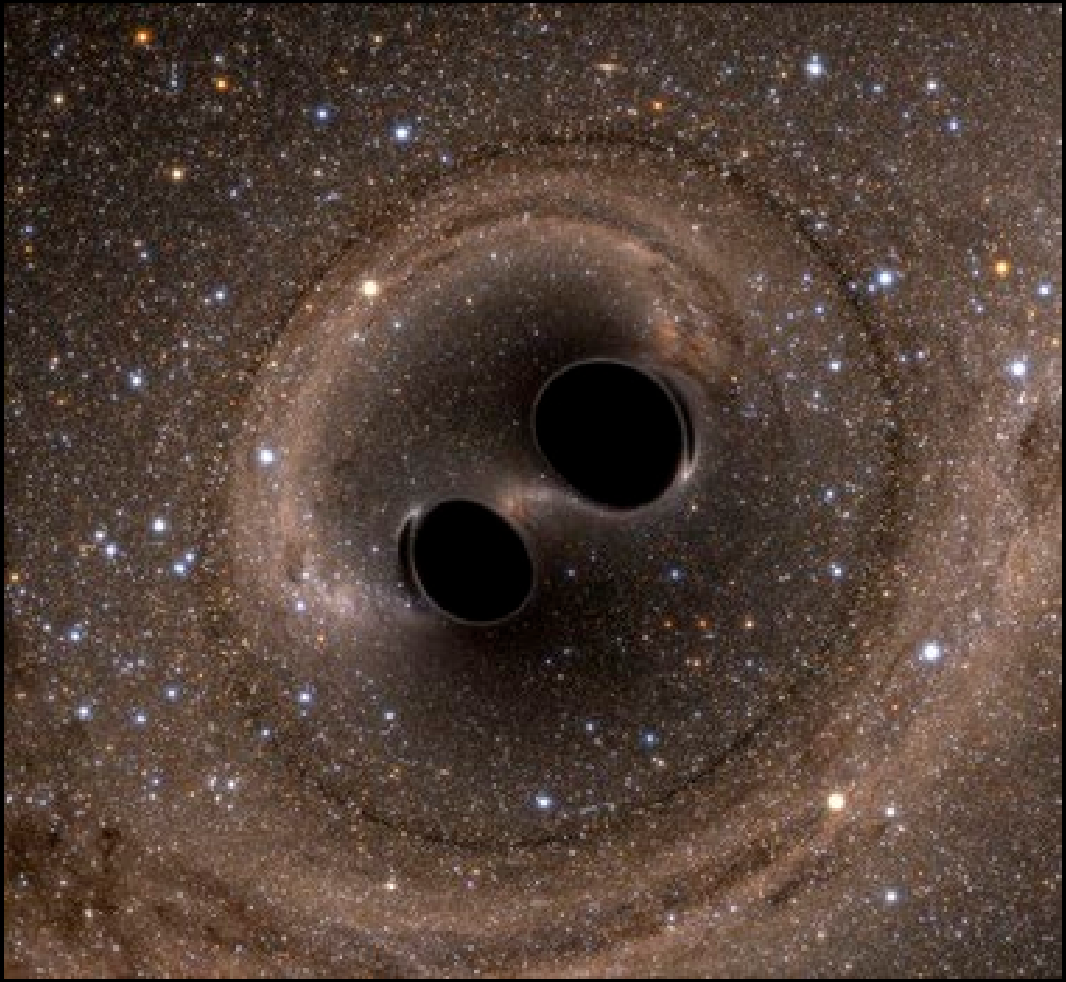
# The gravitational wave detector LIGO

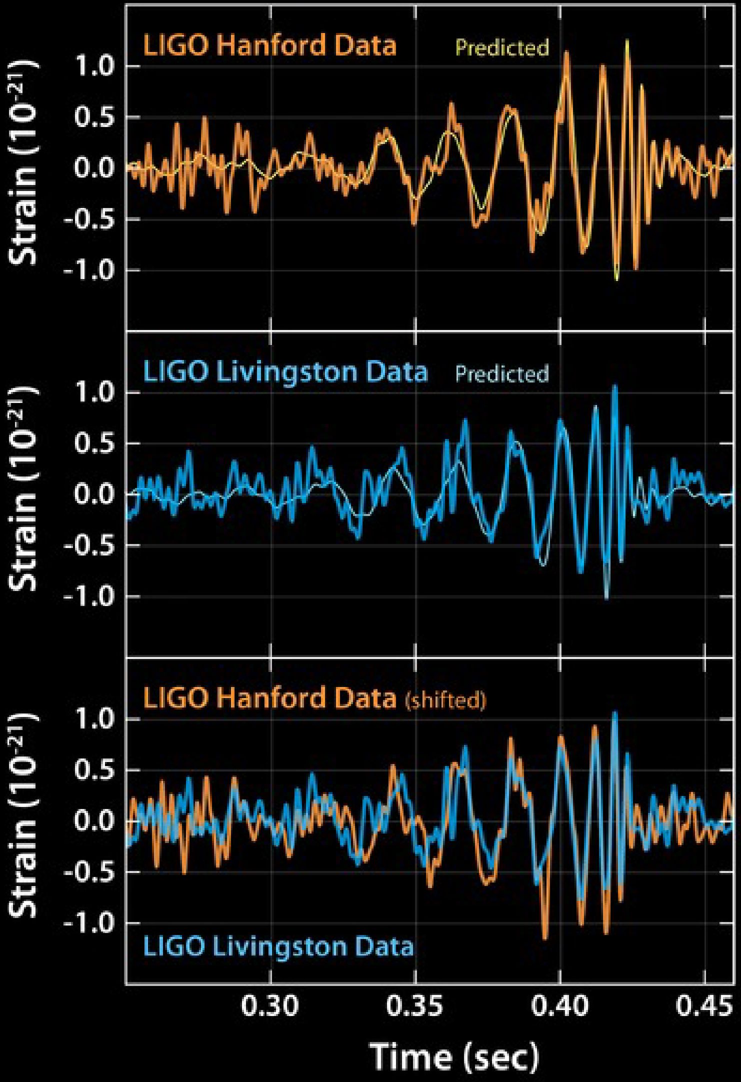
as a huge cavity optomechanical system?



# The gravitational wave detector LIGO

## Detecting binary black hole merger GW150914

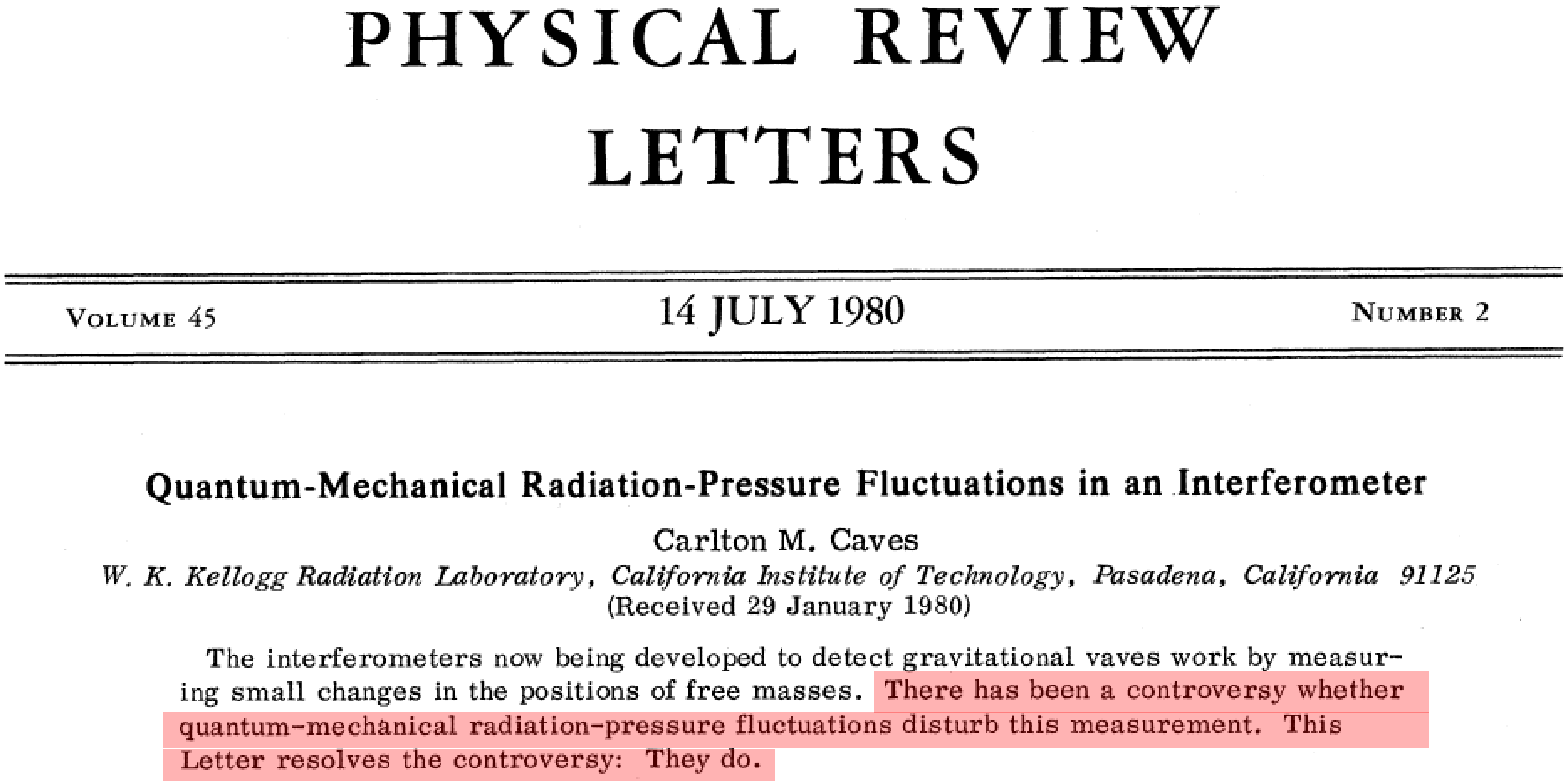




*LIGO Scientific Collaboration, Phys. Rev. Lett. 116, 061102 (2016)*

# Cavity optomechanics as a limiting factor:

## Quantum theory of measurement & gravitational wave detection



PHYSICAL REVIEW LETTERS

VOLUME 45 14 JULY 1980 NUMBER 2

### Quantum-Mechanical Radiation-Pressure Fluctuations in an Interferometer

Carlton M. Caves
*W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125*

(Received 29 January 1980)

The interferometers now being developed to detect gravitational vaves work by measuring small changes in the positions of free masses. There has been a controversy whether quantum-mechanical radiation-pressure fluctuations disturb this measurement. This Letter resolves the controversy: They do.

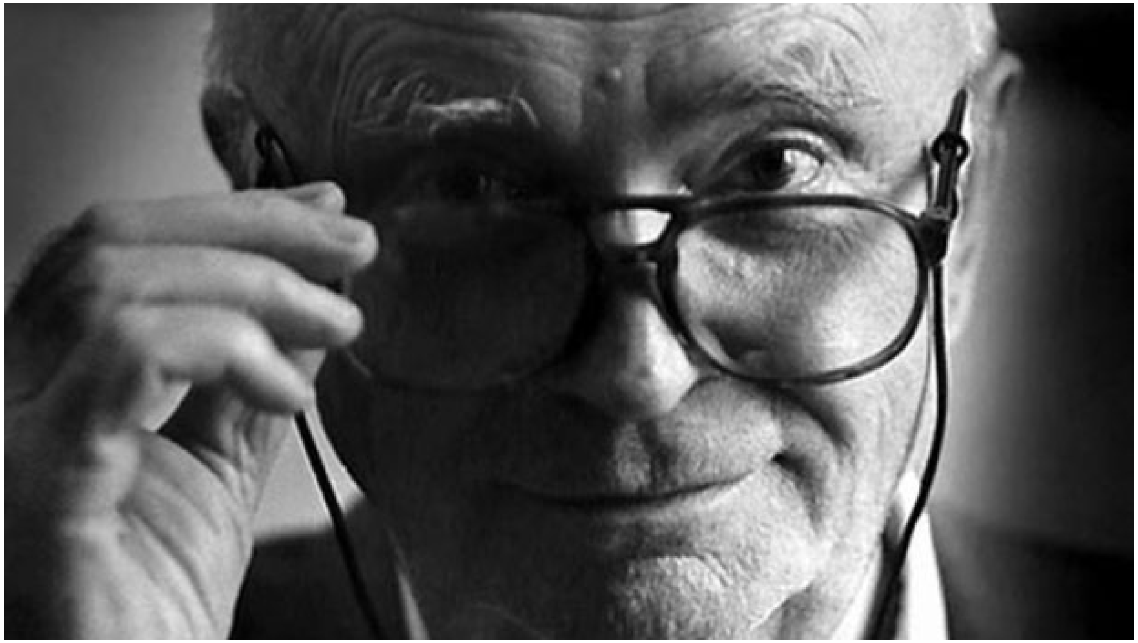
*C. M. Caves, Phys. Rev. Lett. 45, 75 (1980).*

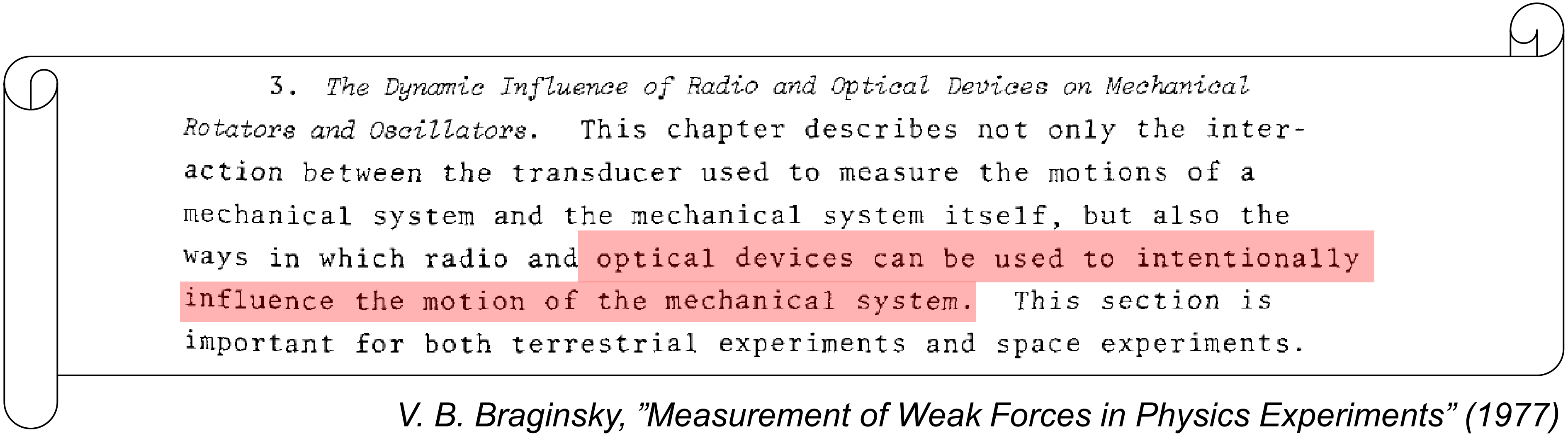
# Light-induced backaction

described by Vladimir Braginsky from Moscow State University

The mechanical back-action of light can lead to

- light-induced rigidity  
  (“optical spring effect”)
- light-induced instability  
  (“optomech. self-oscillation”)
- light-induced friction  
  (“optomechanical cooling”)



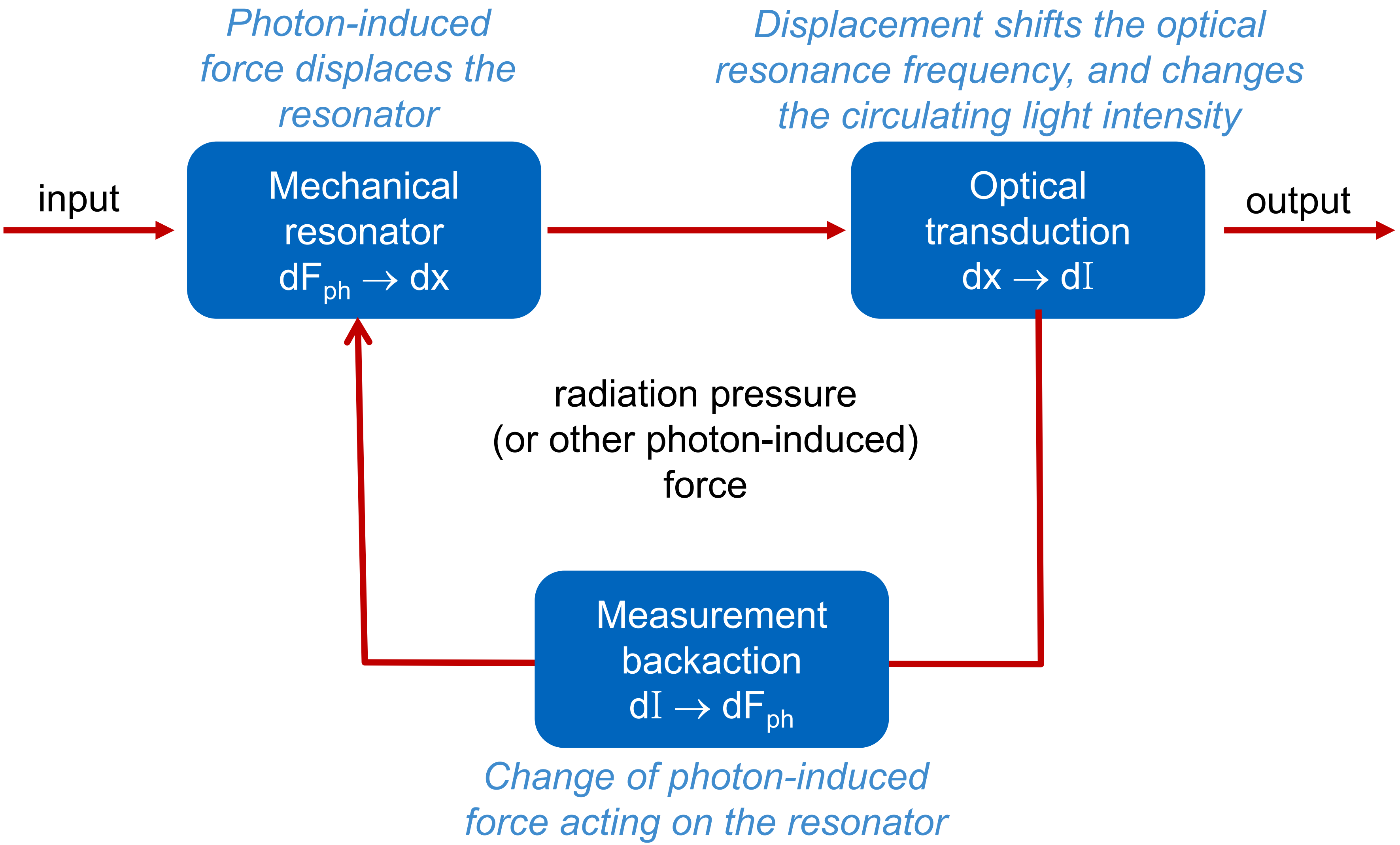


3. *The Dynamic Influence of Radio and Optical Devices on Mechanical Rotators and Oscillators.* This chapter describes not only the interaction between the transducer used to measure the motions of a mechanical system and the mechanical system itself, but also the ways in which radio and optical devices can be used to intentionally influence the motion of the mechanical system. This section is important for both terrestrial experiments and space experiments.

*V. B. Braginsky, ”Measurement of Weak Forces in Physics Experiments” (1977)*

# Dynamical backaction

## As an intrinsic feedback loop



*Photon-induced force displaces the resonator*

*Displacement shifts the optical resonance frequency, and changes the circulating light intensity*

input

Mechanical resonator $dF_{ph} \rightarrow dx$

Optical transduction $dx \rightarrow dI$

output

radiation pressure (or other photon-induced) force

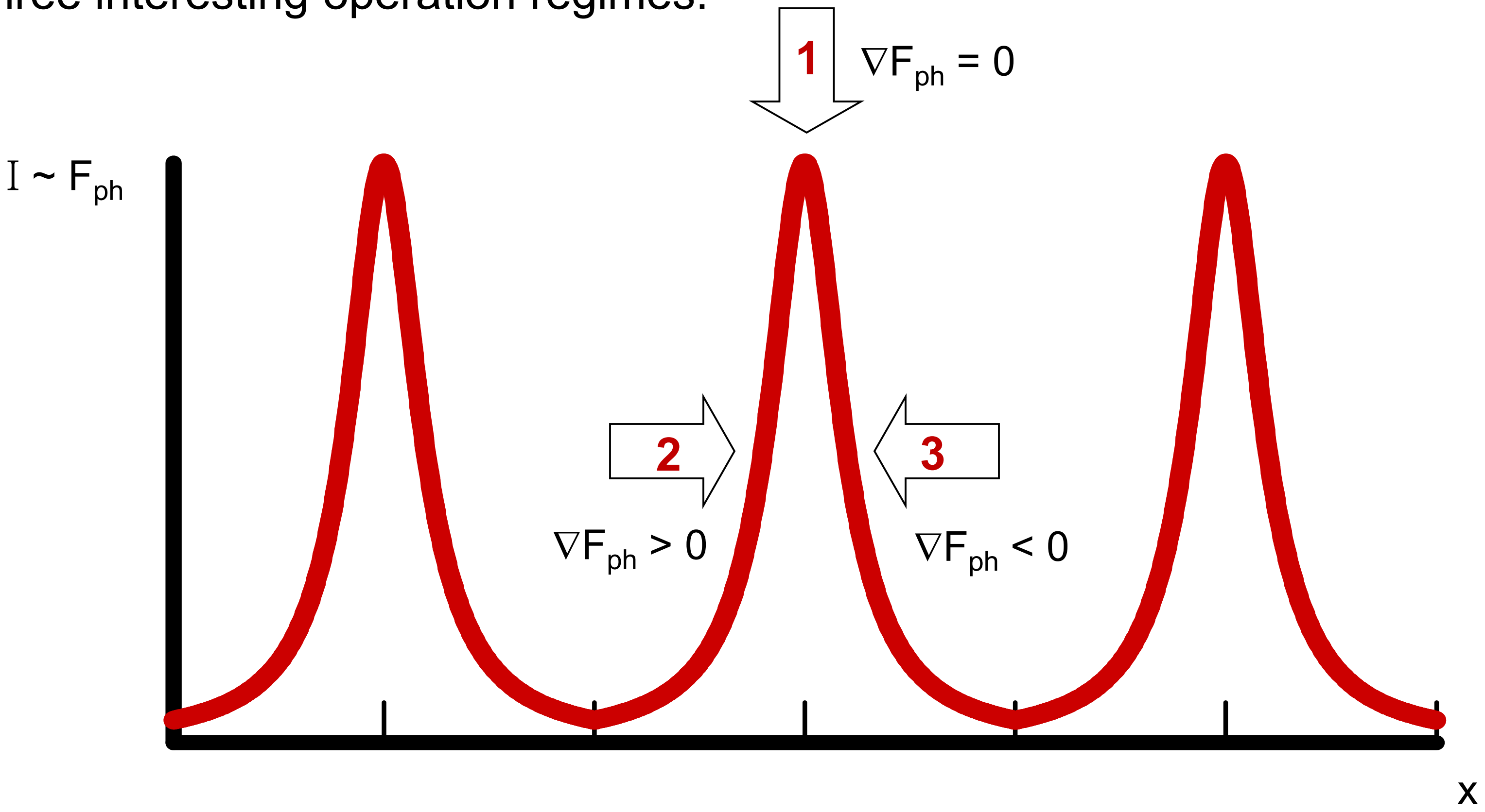
Measurement backaction $dI \rightarrow dF_{ph}$

*Change of photon-induced force acting on the resonator*

# How to make the most of dynamical backaction?

Detuning of the cavity determines what will happen

Three interesting operation regimes:



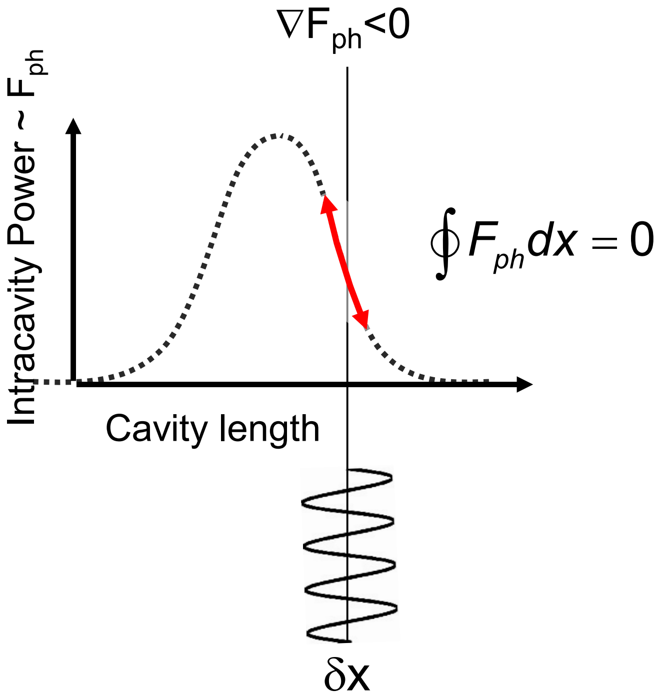
1: cavity on resonance:
no backaction, used e.g. for displacement sensing of resonator

2,3: detuned cavity:
strong backaction effects, use to manipulate resonator dynamics

# Dynamical backaction on nanomechanical resonators

with instantaneous cavity response ($\tau = 0$)

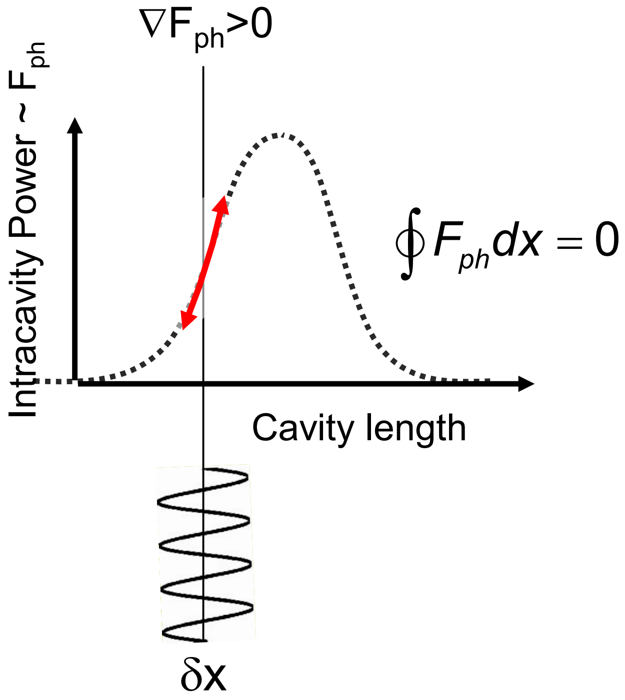
## Negative force gradient



$$\oint F_{ph} dx = 0$$

- area under closed path vanishes
- no work performed on system
- nothing happens

## Positive force gradient



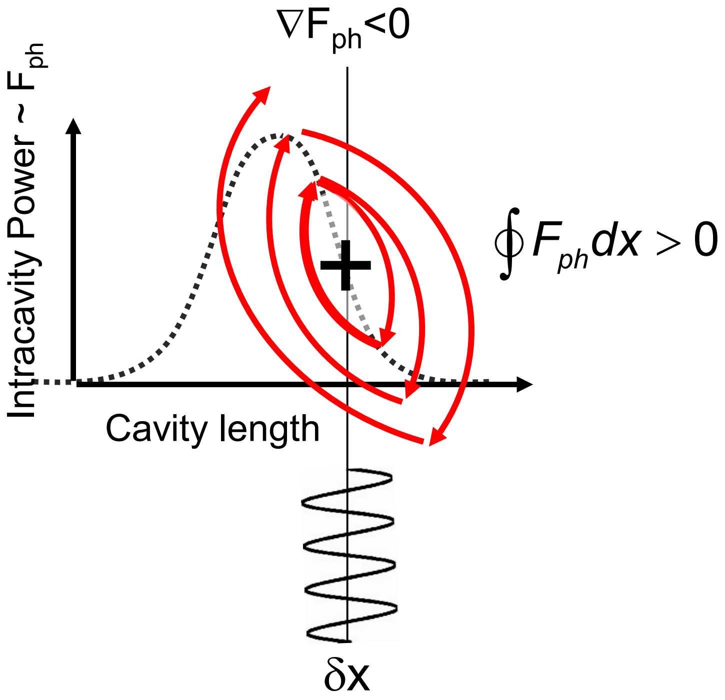
$$\oint F_{ph} dx = 0$$

- area under closed path vanishes
- no work performed on system
- nothing happens

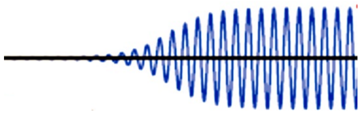
# Dynamical backaction on nanomechanical resonators

with finite cavity response ($\tau > 0$)

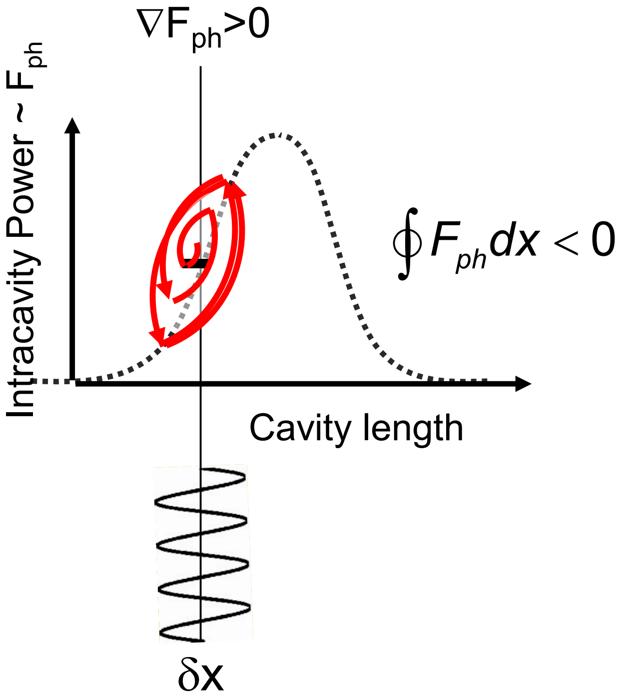
## Negative force gradient



- positive work performed on resonator, i.e. energy gain of mechanical system
- “Optomechanical pumping”



## Positive force gradient

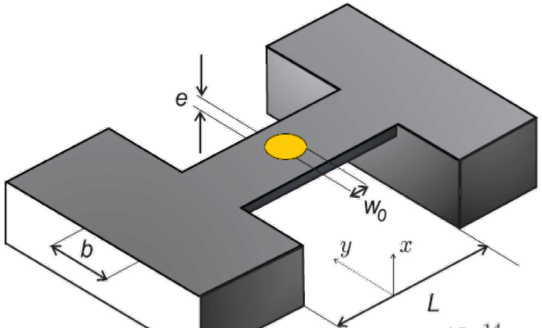


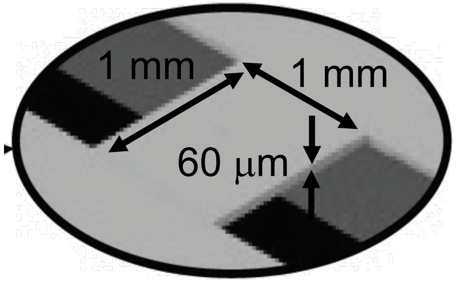
- negative work performed on resonator i.e. energy loss of mechanical system
- “Optomechanical cooling”

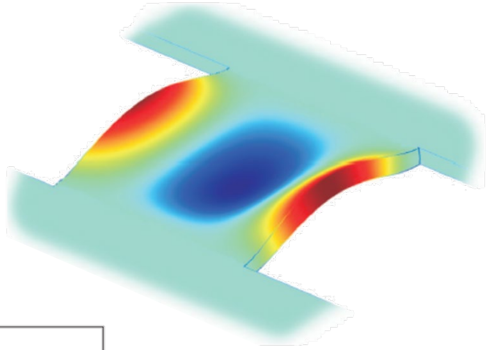
# Radiation-pressure induced cavity optomechanics

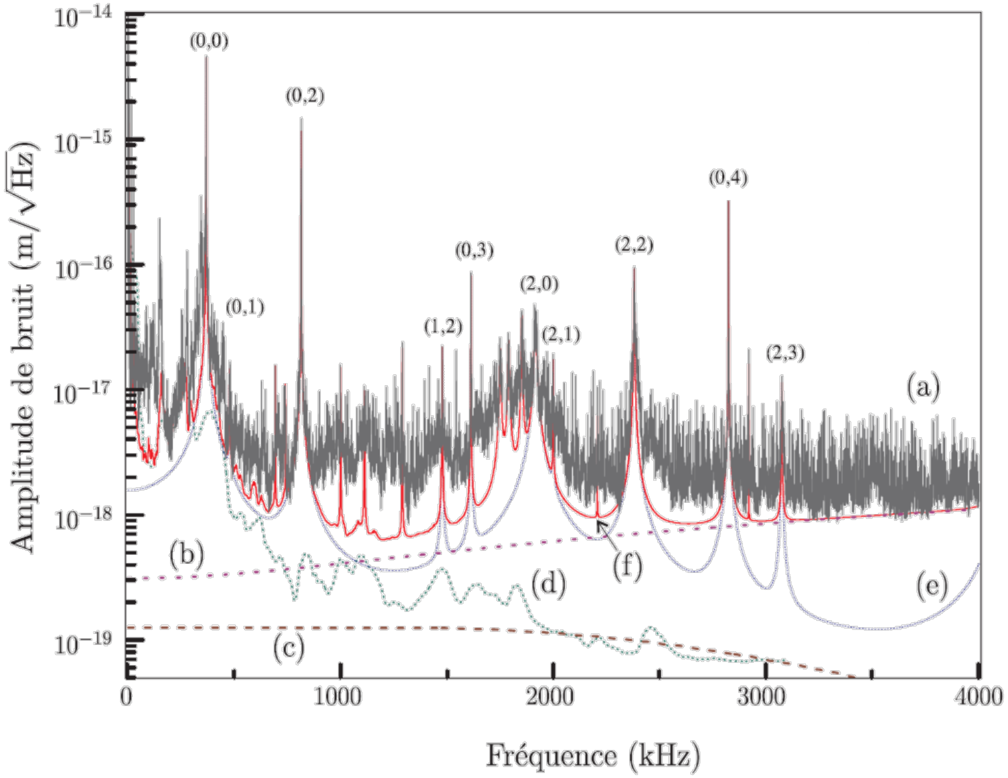
Fabry-Pérot cavity w/ moveable micromirror from the Heidmann group (LKB)

Doubly clamped beam with high quality dielectric coating as micromirror:







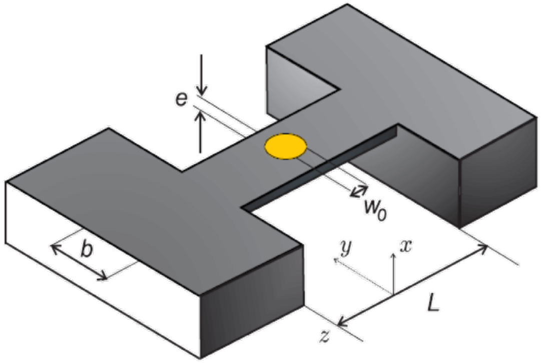


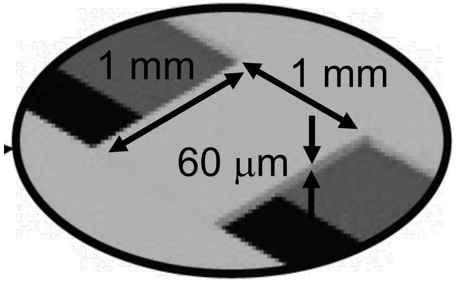
*O. Arcizet,*
*PhD thesis,*
*(Heidmann group)*

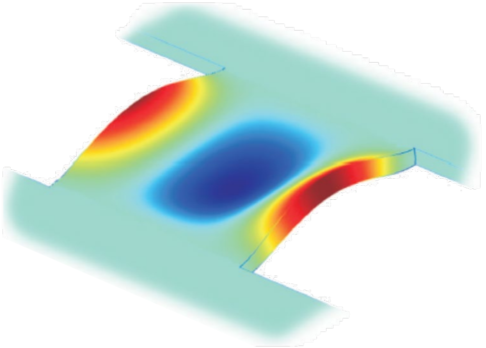
# Radiation-pressure induced cavity optomechanics

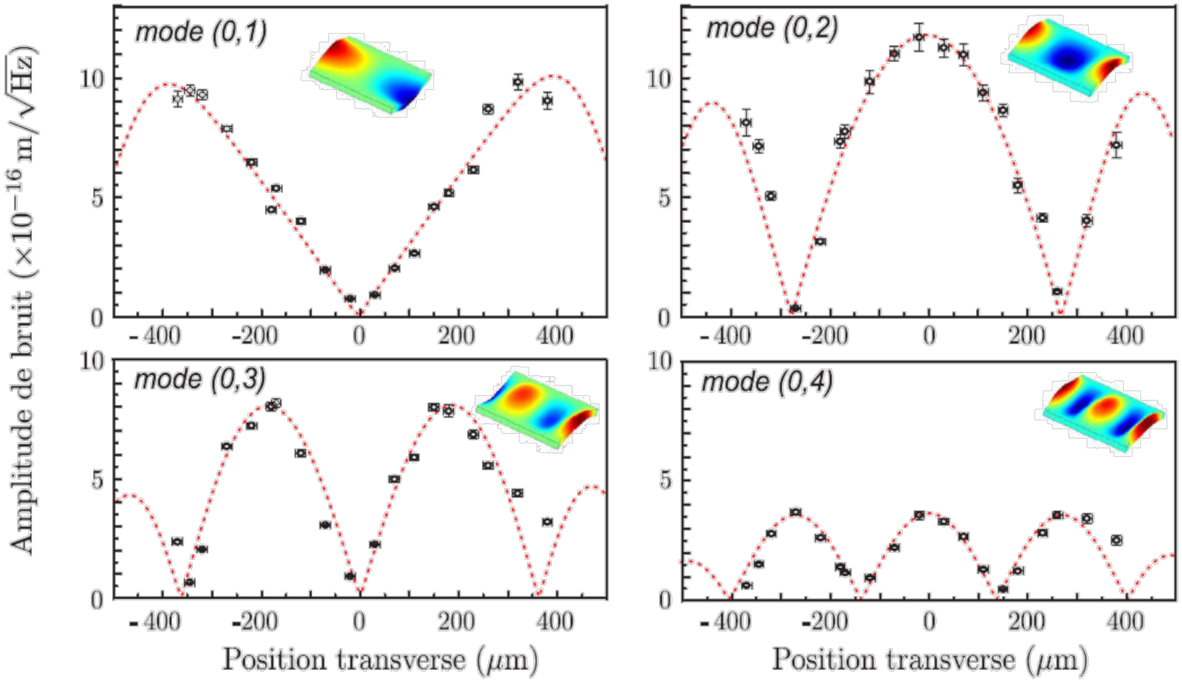
Fabry-Pérot cavity w/ moveable micromirror from the Heidmann group (LKB)

Doubly clamped beam with high quality dielectric coating as micromirror:









$\Omega_{0,2}/2\pi$ = 814 kHz

$m_{eff}$ = 190 µg

$\Gamma_m$ = 81 Hz

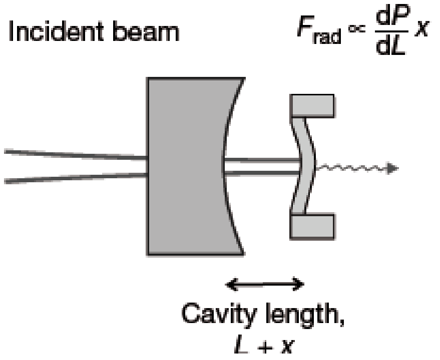
$Q_m$ = 10,000

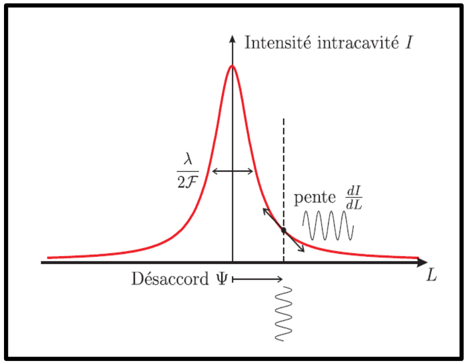
*O. Arcizet,*
*PhD thesis,*
*(Heidmann group)*

# Radiation-pressure induced cavity optomechanics

## Fabry-Pérot cavity w/ moveable micromirror from the Heidmann group (LKB)

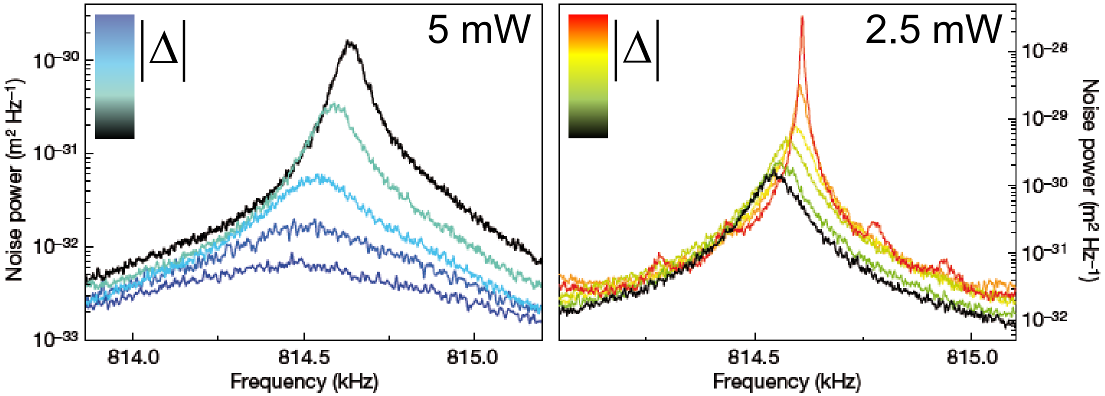
High finesse cavity (F = 30,000), i.e. need to consider $\tau \neq$ const

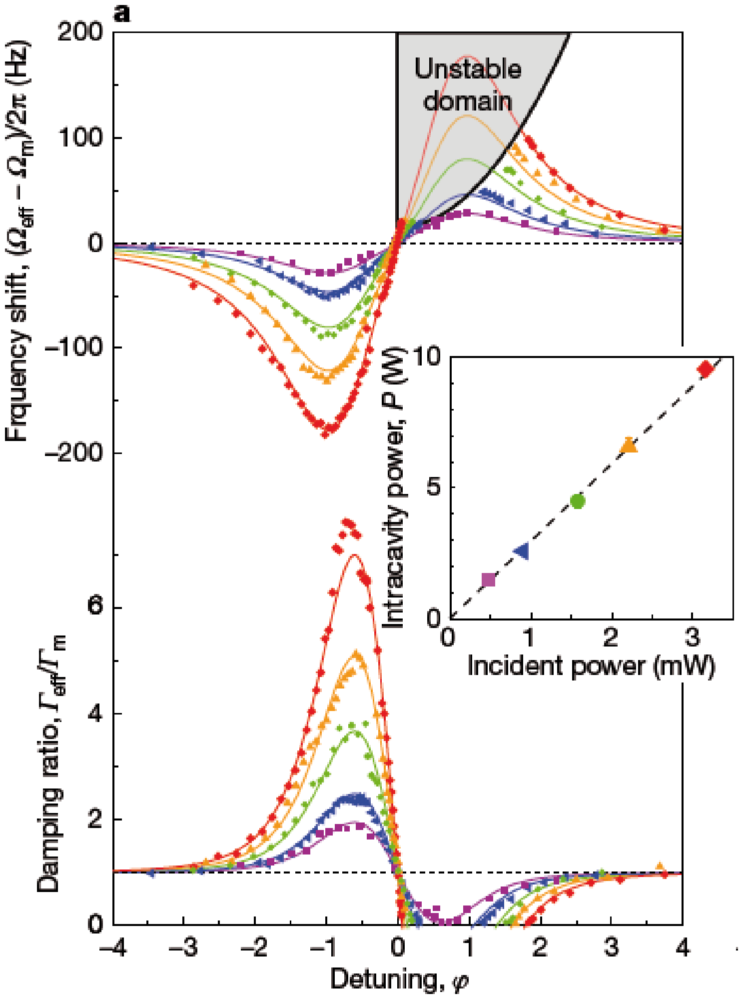




Laser detuning $\Delta = \omega_L - \omega_C$ :

“red detuning“: $\Delta < 0$ “blue detuning“: $\Delta > 0$





*O. Arcizet et al., Nature 444, 71 (2006)*

# The generic model of optomechanical backaction

## Input-output theory in the classical regime

- Optical spring effect:

$$\Omega_{\text{eff}} = \Omega_m + \Omega_{\text{cOM}} \text{ w/ } \Omega_{\text{cOM}} = g^2 \left[ \frac{\Delta + \Omega_m}{(\Delta + \Omega_m)^2 + (\kappa/2)^2} + \frac{\Delta - \Omega_m}{(\Delta - \Omega_m)^2 + (\kappa/2)^2} \right] \text{ for } \Delta = \omega - \omega_{\text{cav}}$$

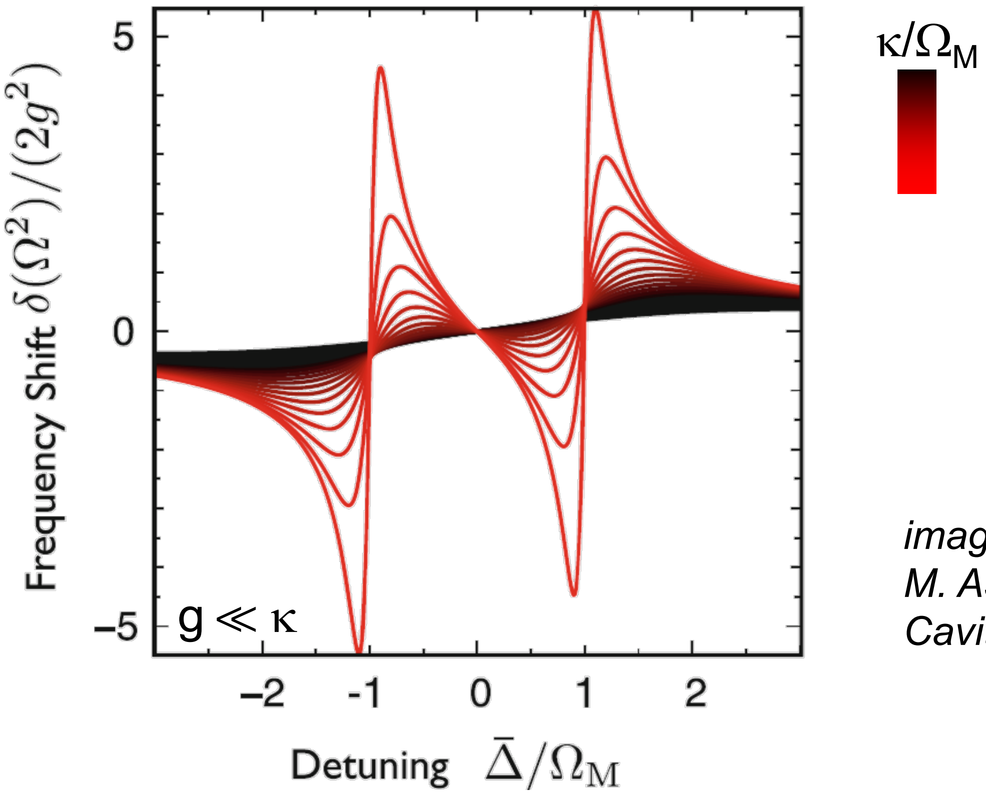


*image:*
*M. Aspelmeyer, T. J. Kippenberg, F. Marquardt, Cavity Optomechanics, Springer (2014)*

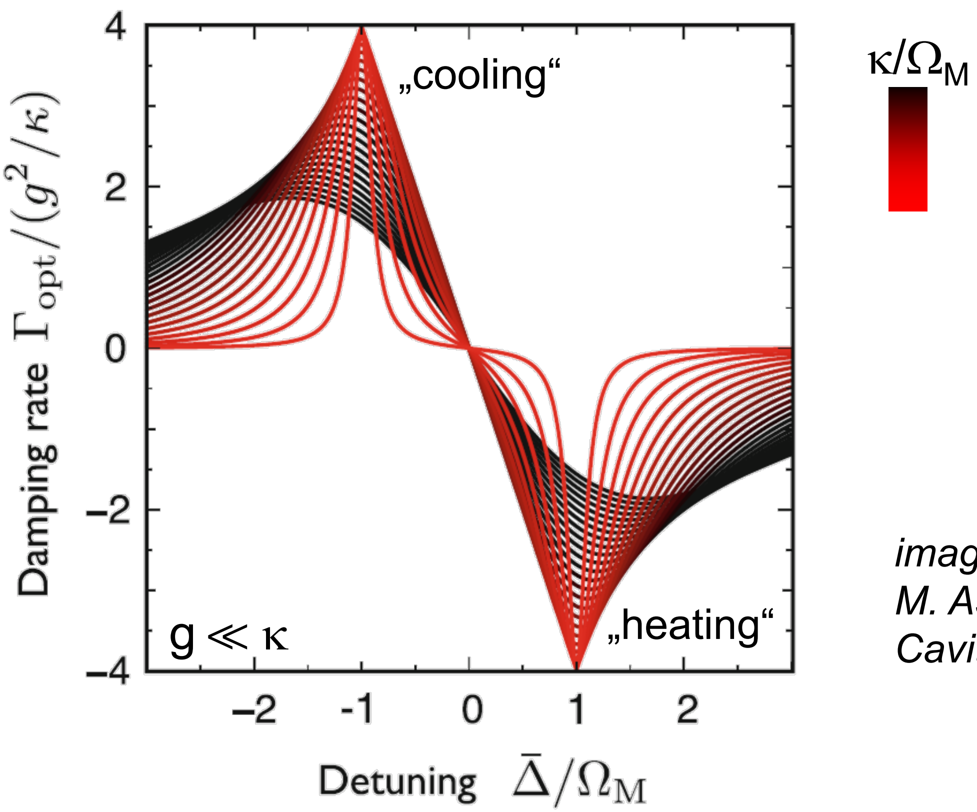
*M. Aspelmeyer, T. J. Kippenberg, F. Marquardt, Rev. Mod. Phys. 86, 1391 (2014)*

# The generic model of optomechanical backaction

## Input-output theory in the classical regime

- Optomechanical damping:

$$\Gamma_{\mathrm{eff}} = \Gamma_{\mathrm{m}} + \Gamma_{\mathrm{cOM}} \quad \text{w/} \quad \Gamma_{\mathrm{cOM}} = \mathrm{g}^2 \left[ \frac{\kappa}{\left(\Delta+\Omega_{\mathrm{m}}\right)^2 + \left(\kappa/2\right)^2} - \frac{\kappa}{\left(\Delta-\Omega_{\mathrm{m}}\right)^2 + \left(\kappa/2\right)^2} \right] \quad \text{for } \Delta = \omega - \omega_{\mathrm{cav}}$$



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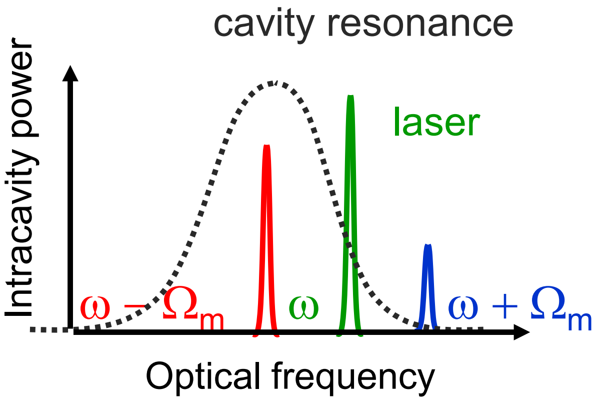
*M. Aspelmeyer, T. J. Kippenberg, F. Marquardt, Cavity Optomechanics, Springer (2014)*

*M. Aspelmeyer, T. J. Kippenberg, F. Marquardt, Rev. Mod. Phys. 86, 1391 (2014)*

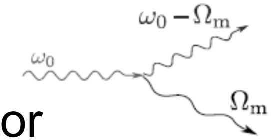
# Dynamical backaction in the sideband resolved regime

## The scattering picture of cavity optomechanics

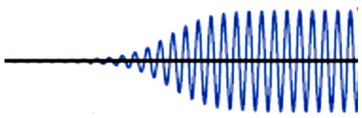
### Blue detuned laser



- Stokes processes dominate, i.e. emission of phonons into resonator

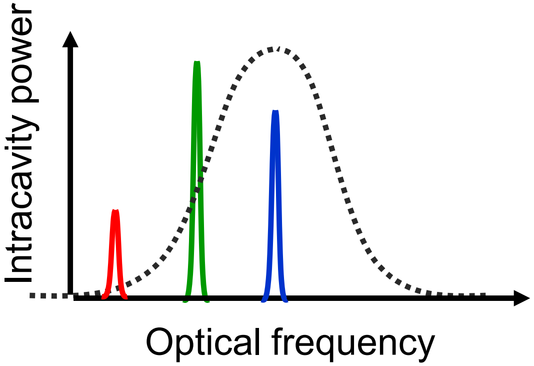


- Optomechanical pumping

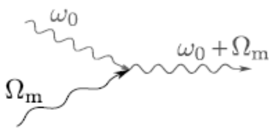


Resolved sideband regime: $\Omega_m \gtrsim \kappa$

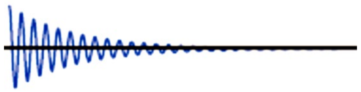
### Red detuned laser



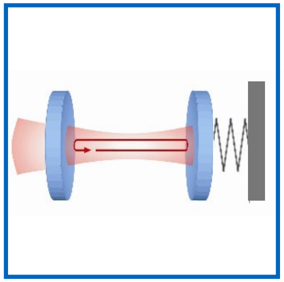
- Anti-Stokes processes dominate, i.e. absorption of resonator phonons



- Optomechanical cooling



# PART 2: CAVITY OPTOMECHANICS



1. An introduction to cavity optomechanics:
   Radiation-pressure induced dynamical backation

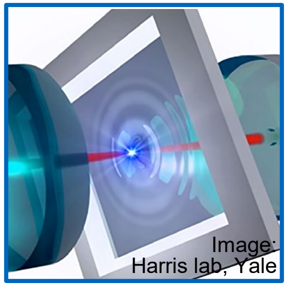
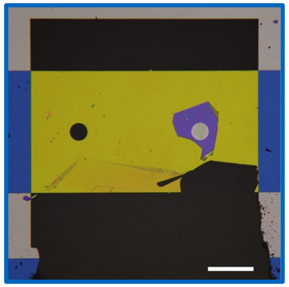


Image:
Harris lab, Yale

2. The membrane-in-the-middle configuration:
   A vibrating membrane inside a Fabry-Pérot cavity



3. Cavity optomechanics with van der Waals materials:
   Radiation pressure backaction on a flake of hBN

# The original membrane-in-the-middle system

## with a commercial SiN membrane from the Harris group (Yale University)

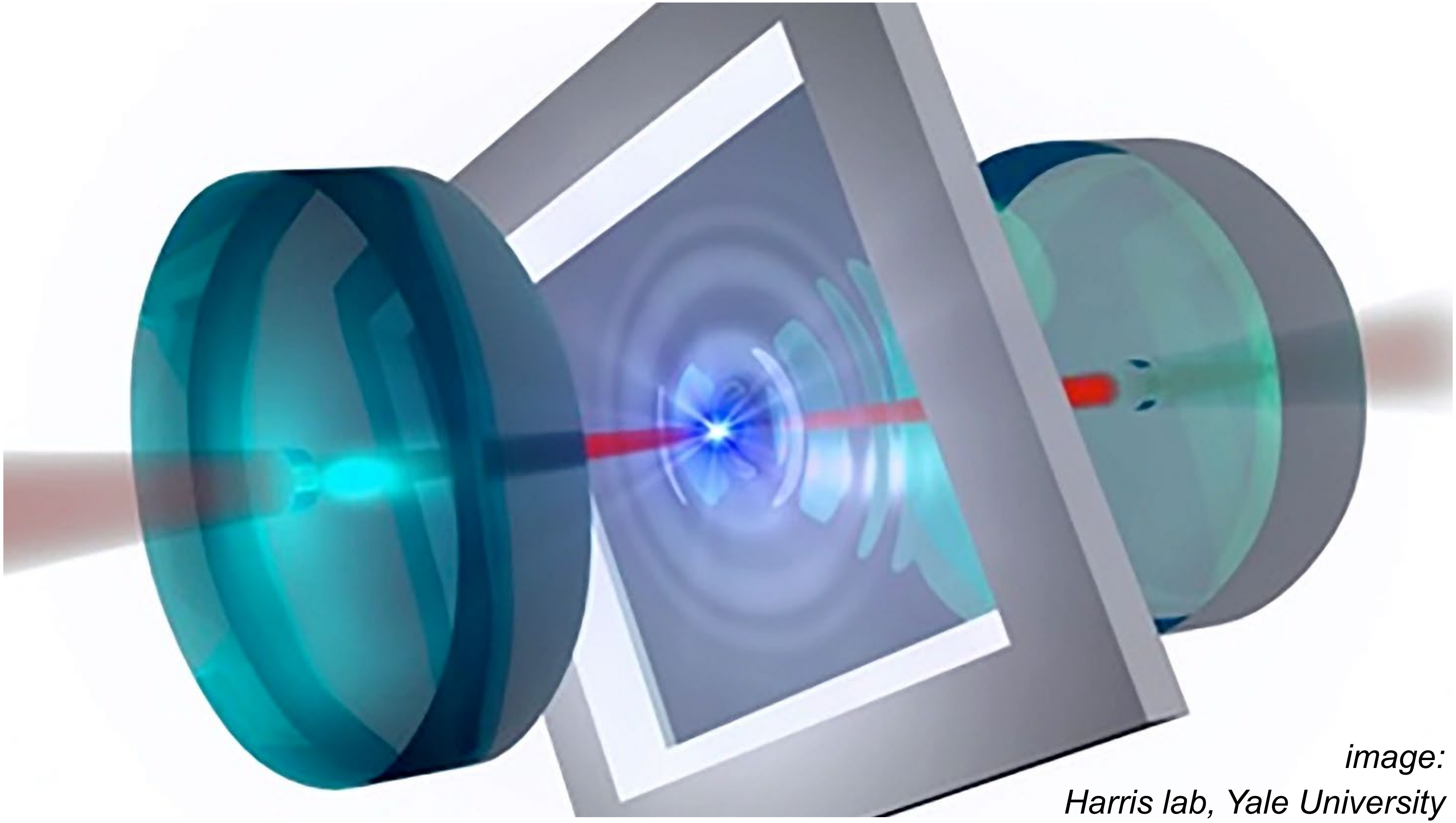
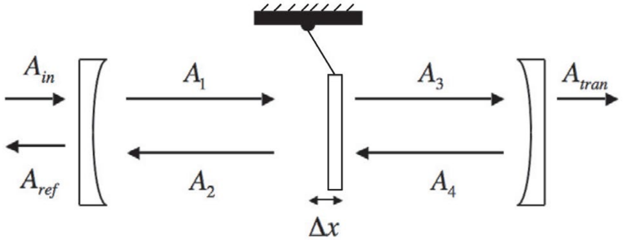


*image:*
*Harris lab, Yale University*

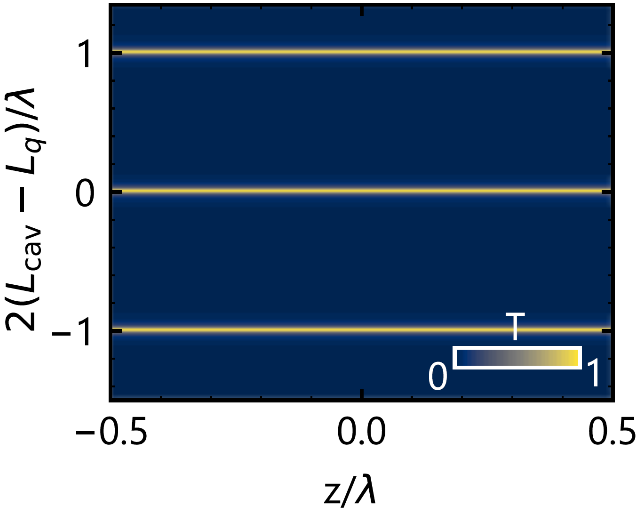
see Thompson et al., Nature 452, 72 (2008) and Jayich et al., New J. Phys. 10, 095008 (2008)

# Cavity transmission as a function of membrane position

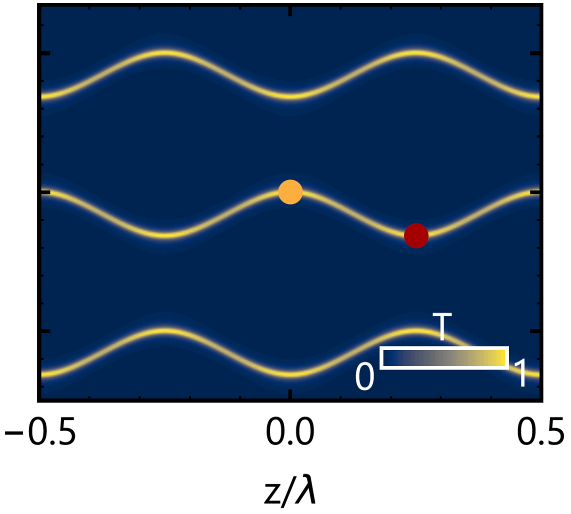
## Linear and quadratic optomechanical coupling



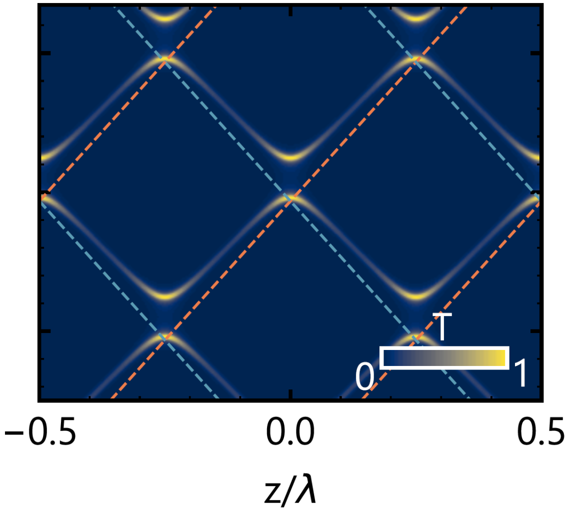
No membrane:



Weakly reflective:

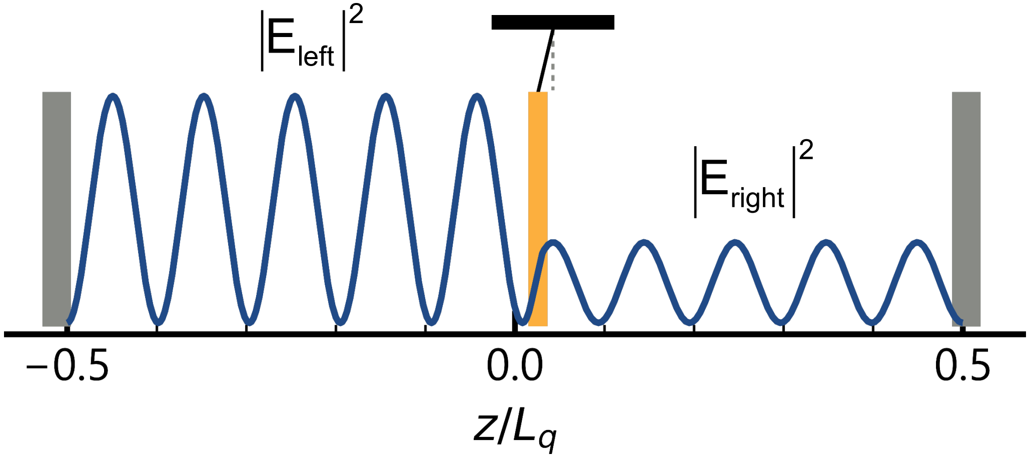


Stongly reflective:

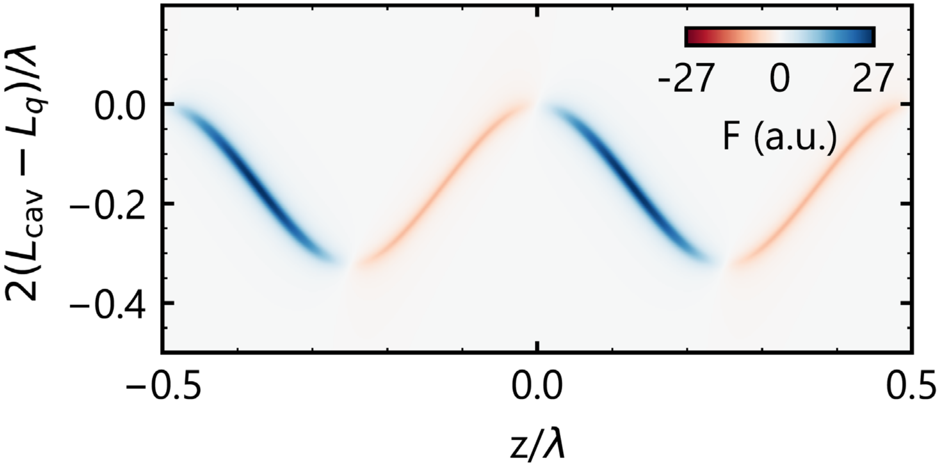


# Radiation pressure force

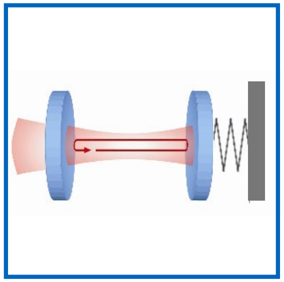
arises from different field intensities in the two sub-cavities



$$F_{\mathrm{rad}} \sim |E_{\mathrm{left}}|^2 - |E_{\mathrm{right}}|^2$$



# PART 2: CAVITY OPTOMECHANICS



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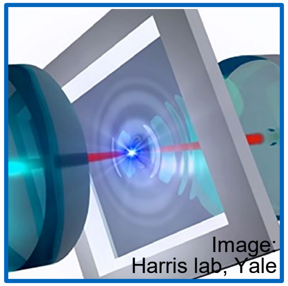

Image: Harris lab, Yale

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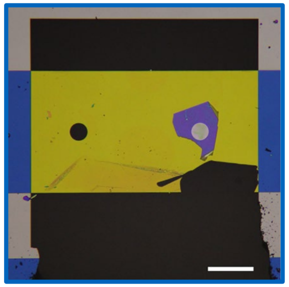
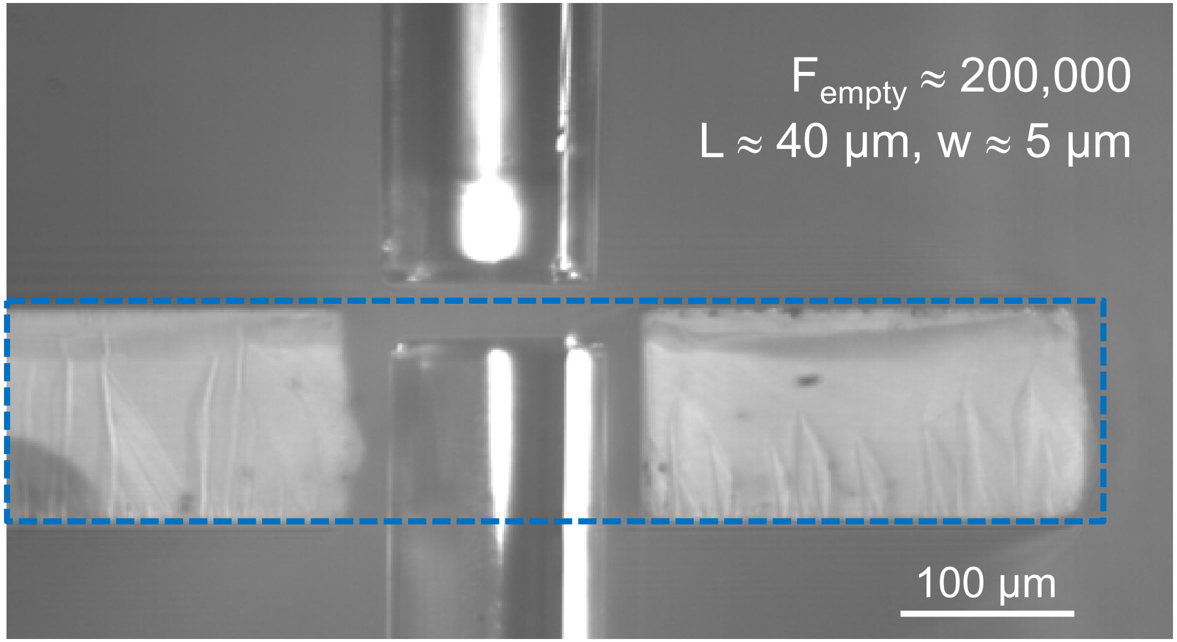


3. Cavity optomechanics with van der Waals materials:
   Radiation pressure backaction on a flake of hBN

# Miniaturizing the membrane-in-the-middle approach

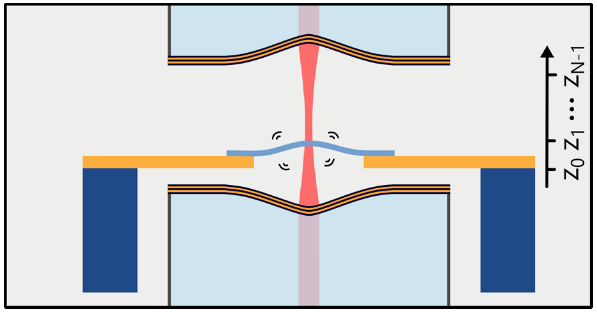
## Optical detection of nanoscale resonators requires a small mode volume

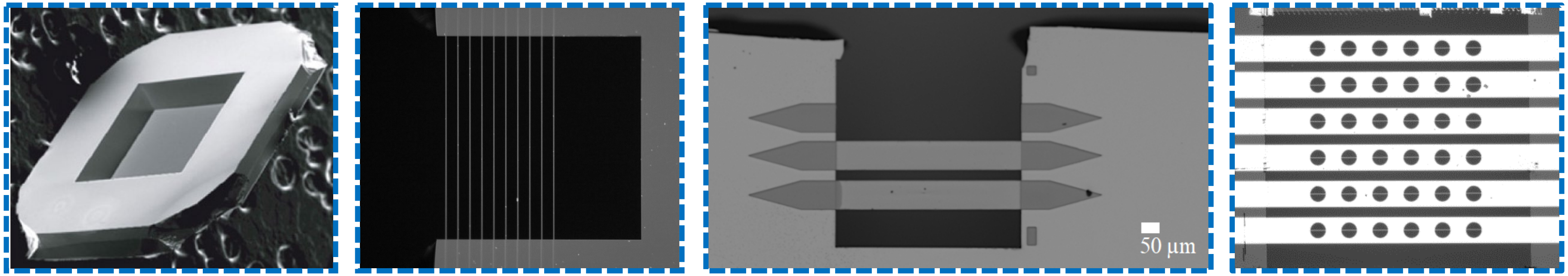
High finesse fiber-based Fabry-Pérot cavity:



$F_{empty} \approx 200{,}000$

$L \approx 40\ \mu m$, $w \approx 5\ \mu m$

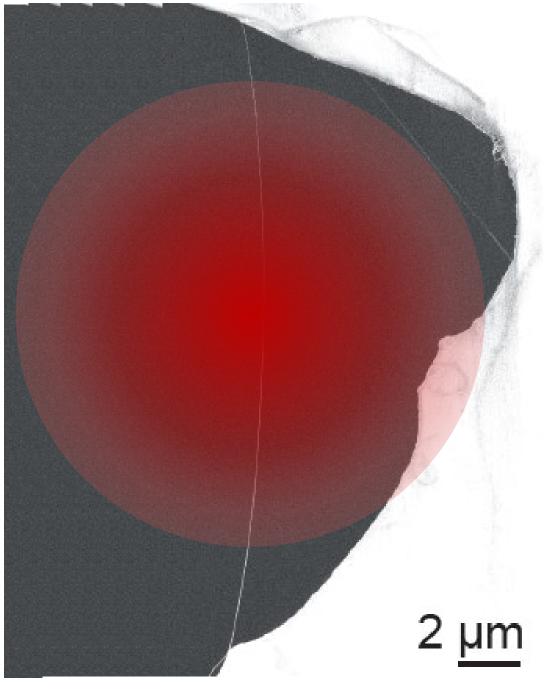


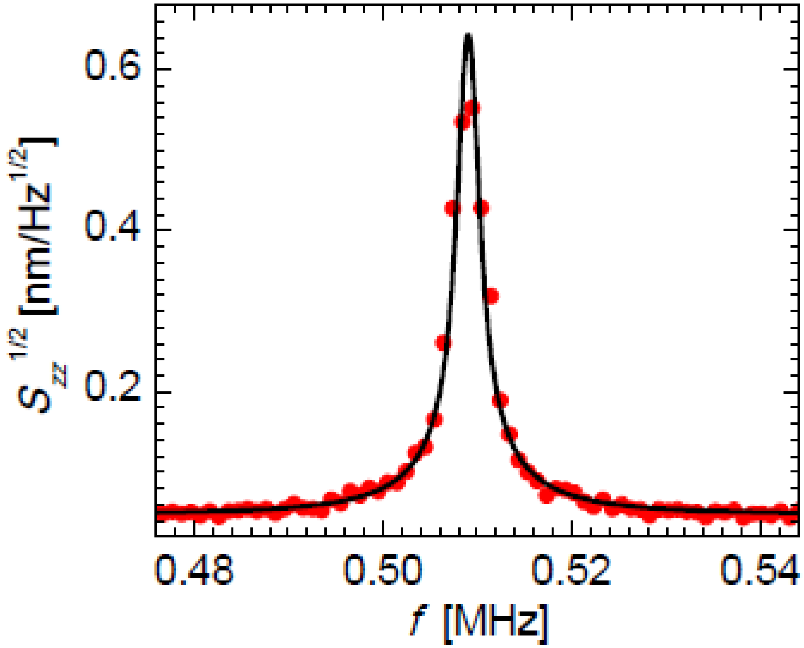


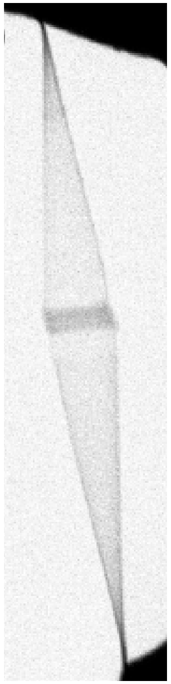
see Rochau, Sanchez Arribas, Brieussel, Stapfner, Hunger, Weig, Phys. Rev. Appl. 16, 014013 (2021)

# Carbon nanotubes (CNT) as optomechanical systems

## Optical detection of Brownian motion of a single CNT







TEM imaging & diffraction:

- diameter < 10 nm
- bundle/multiwall with approx. 6 shells

Thermomechanical fluctuations:

$f_0 = 0.51$ MHz
Q = 300

SEM verification:

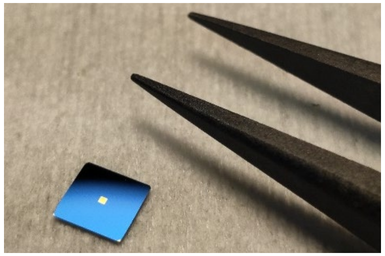
$f_0 = 0.518$ MHz
Q = 250±50

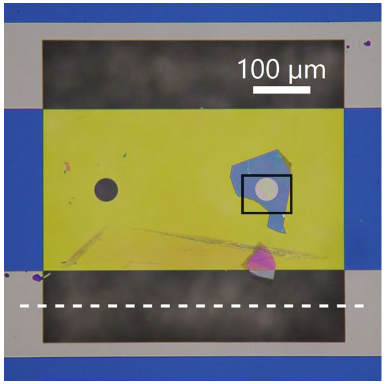
Stapfner, Ost, Hunger, Reichel, Favero, Weig, Appl. Phys. Lett., 102, 151910 (2013)

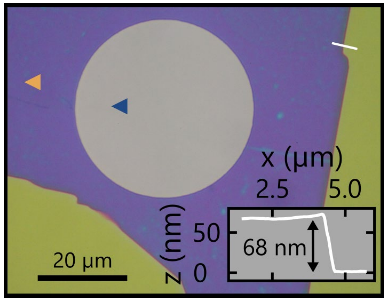
see also: Moser et al., Nature Nano 8, 493 (2013) for electrical detection of CNT Brownian motion

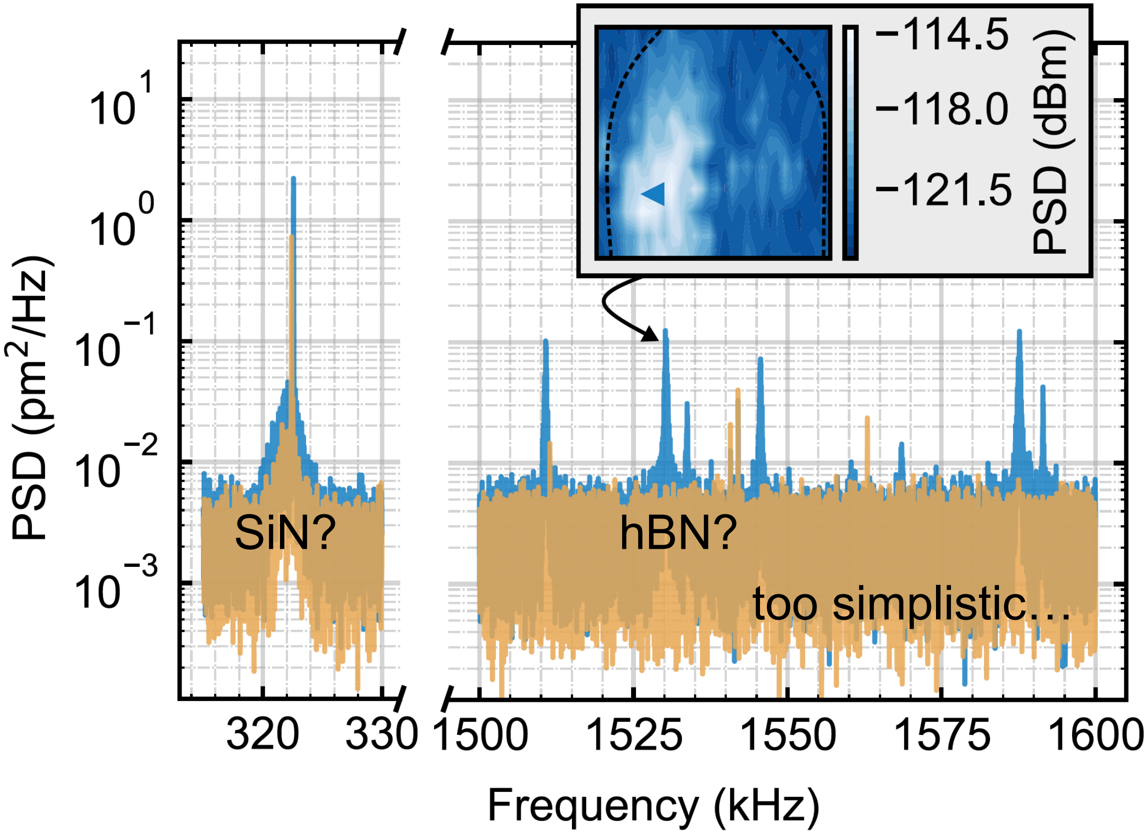
# A hBN drumhead on a hole in a SiN membrane stripe

## Characterization of hybridized mechanical modes in Michelson interferometer









Sánchez Arribas, Taniguchi, Watanabe, Weig, arXiv:2302.04291

see Jaeger et al., Nano Lett. 23, 2016 (2023) for a thorough study of mode hybridization

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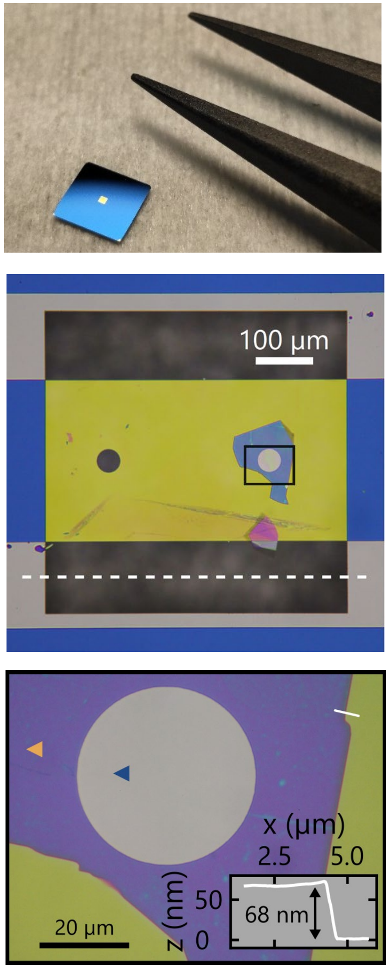

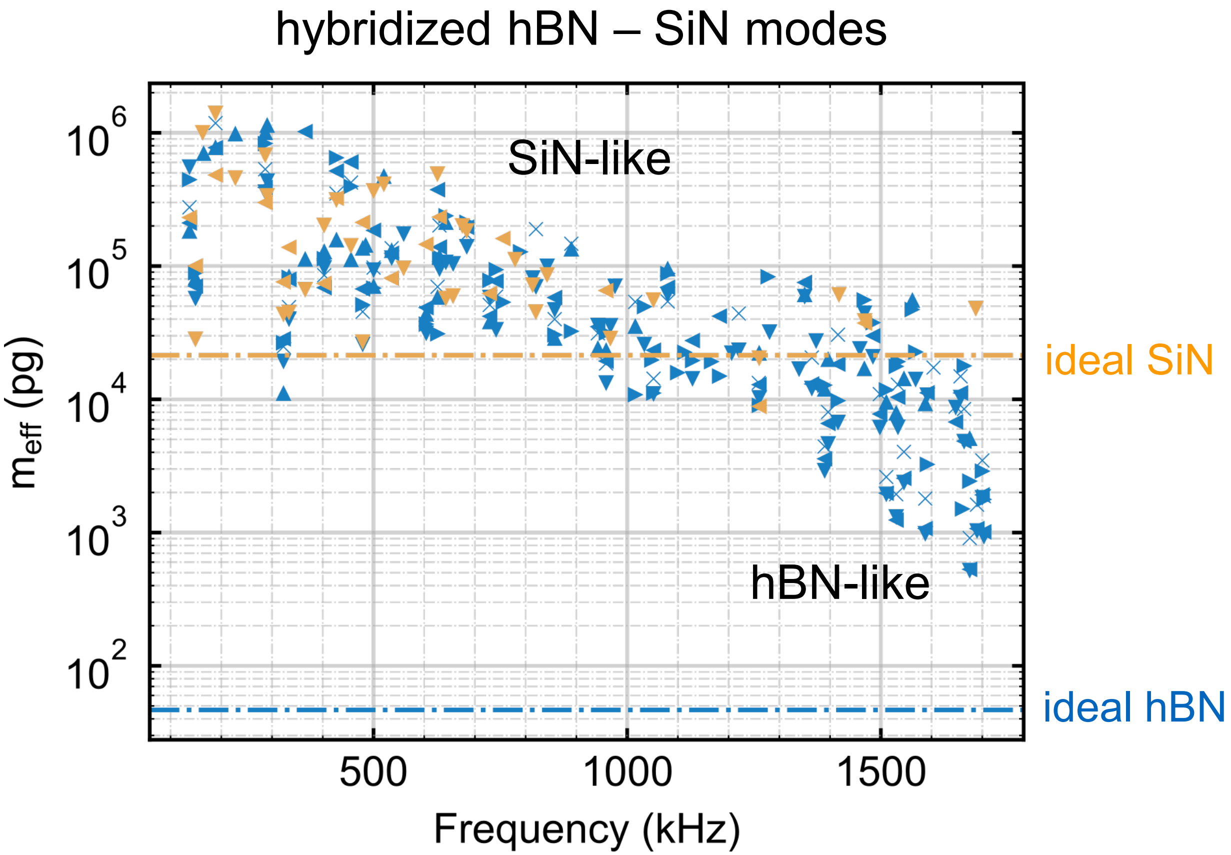

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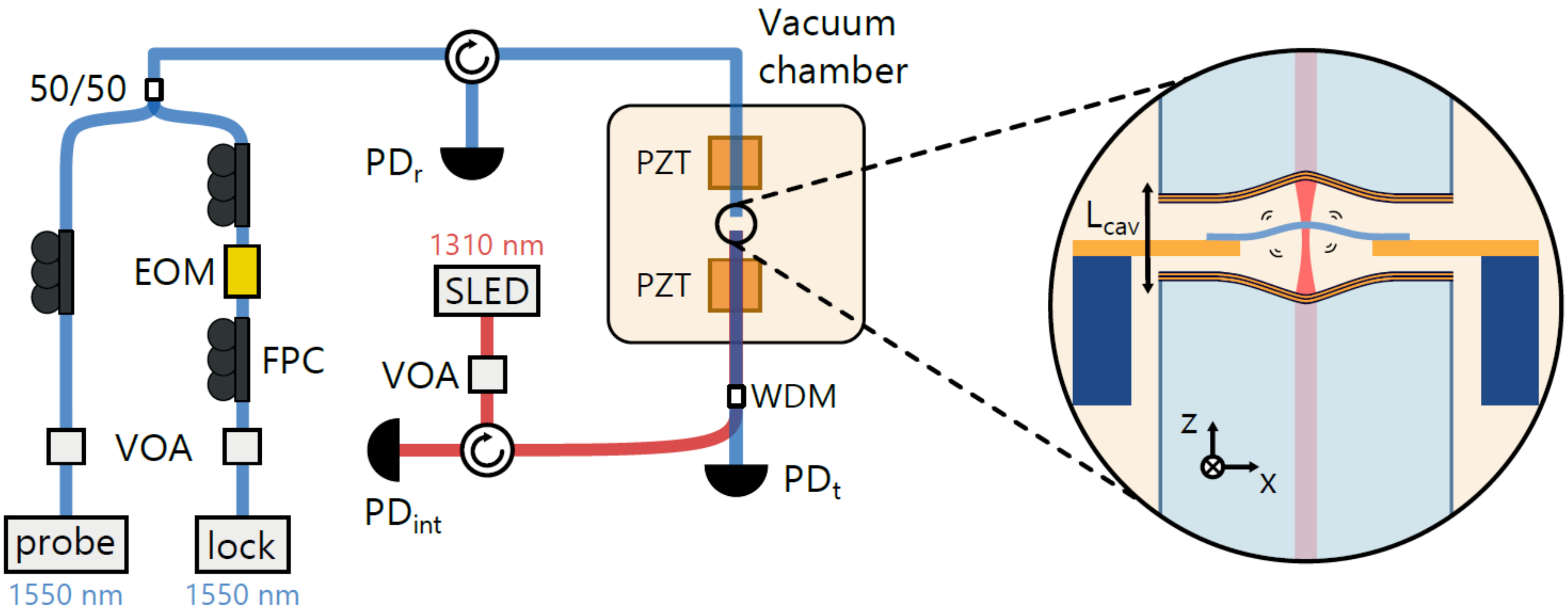
# Dynamical backaction in the membrane-in-the-middle system

## Lock cavity, measure backaction of second, variable-wavelength probe laser



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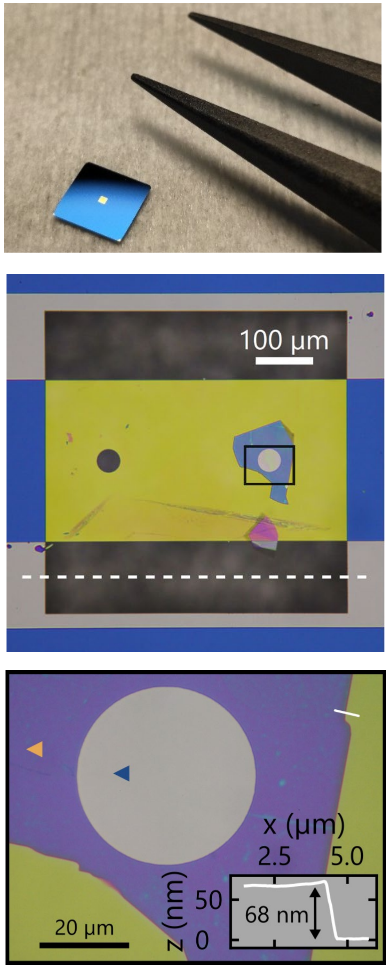
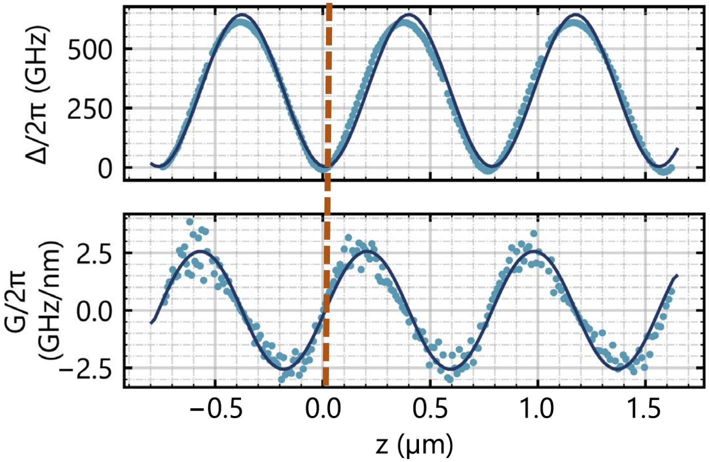
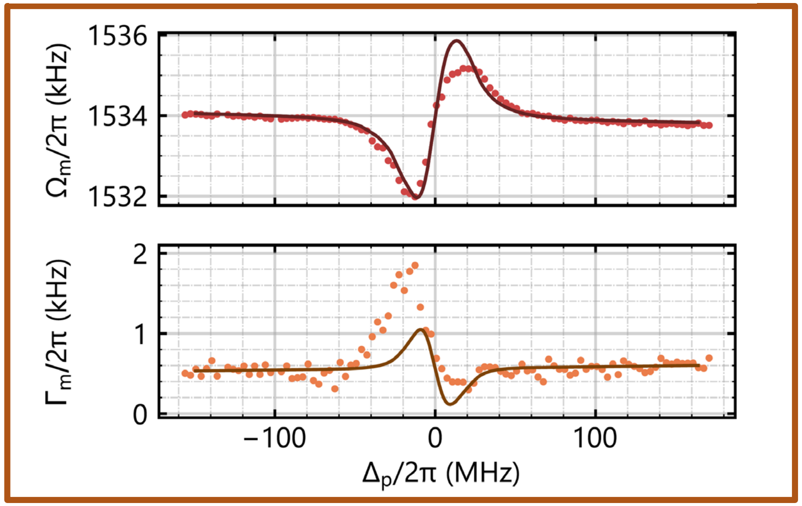
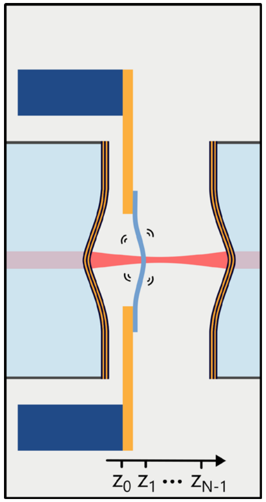
# Radiation pressure backaction on a hBN drumhead

## A first step towards hybrid optomechanics with 2D materials









$g_0/2\pi \approx 1$ kHz

optical spring effect

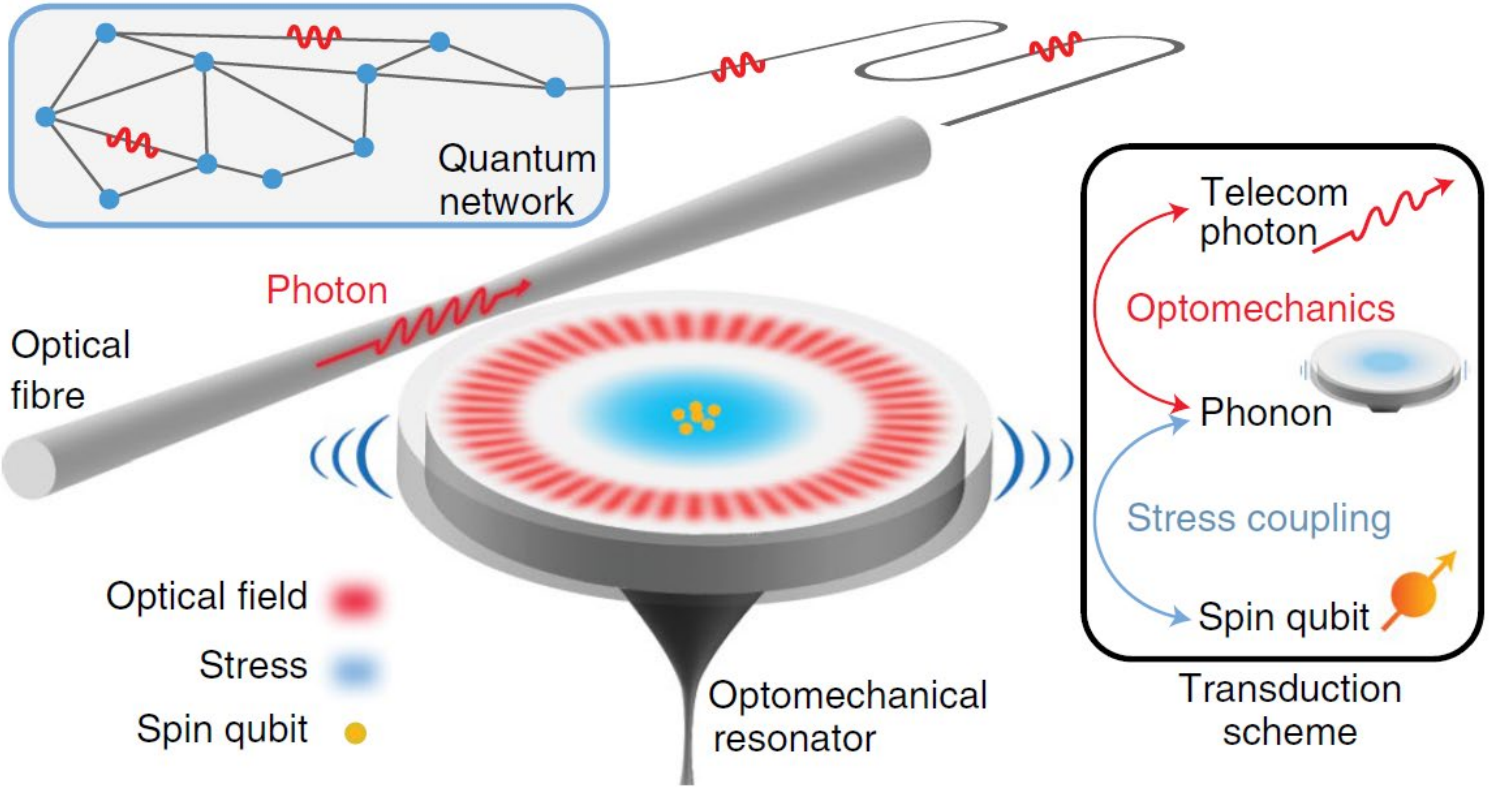
cooling / heating

Sánchez Arribas, Taniguchi, Watanabe, Weig, arXiv:2302.04291

see Zoepfl et al. Phys. Rev. Lett. 130, 033601 (2023) for similar nonlinear behavior

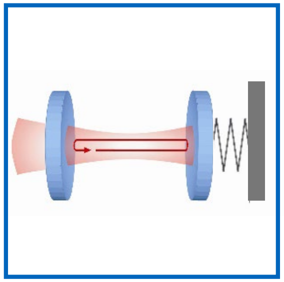
# Hybrid optomechanical systems w/ van der Waals materials

combining cavity optomechanics with spin/charge defect



from: Shandilya et al., Nature Physics 17,1420 (2021)

# PART 2: CAVITY OPTOMECHANICS



1. An introduction to cavity optomechanics:
   Radiation-pressure induced dynamical backation

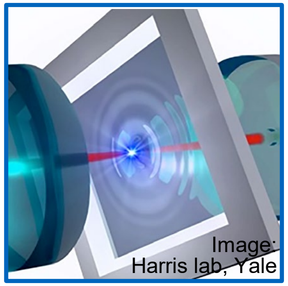
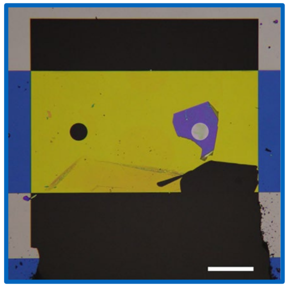


Image:
Harris lab, Yale

2. The membrane-in-the-middle configuration:
   A vibrating membrane inside a Fabry-Pérot cavity



3. Cavity optomechanics with van der Waals materials:
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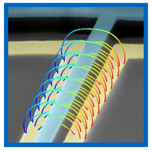
# SUMMARY

## PART 1: (HIGH Q) NANOMECHANICAL SYSTEMS



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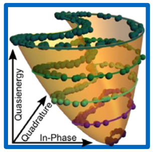

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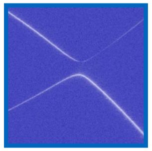

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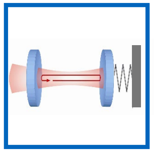

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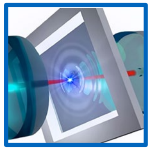

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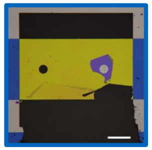

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