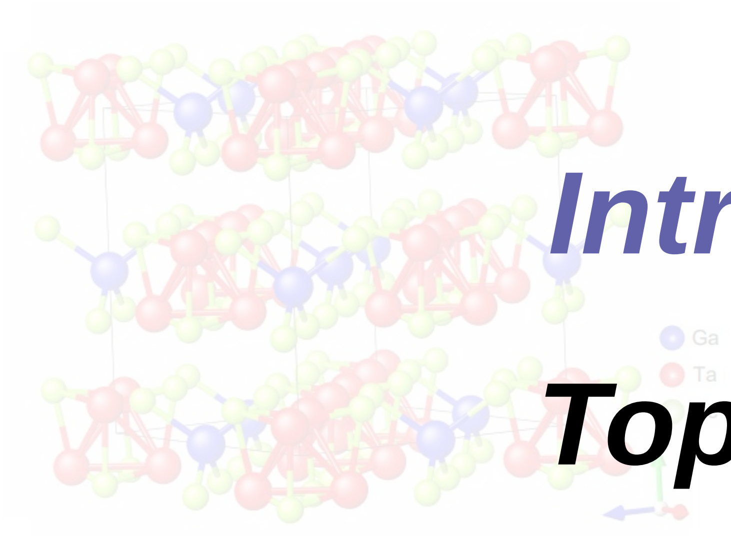
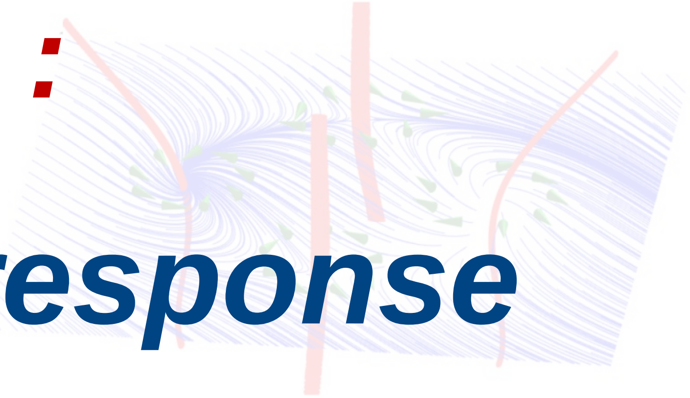


Introductory course of

Topological materials :

Topological materials and response



중시계 여름학교
2024. 05. 25.

박문집
한양대학교 물리학과

한양대학교
HANYANG UNIVERSITY

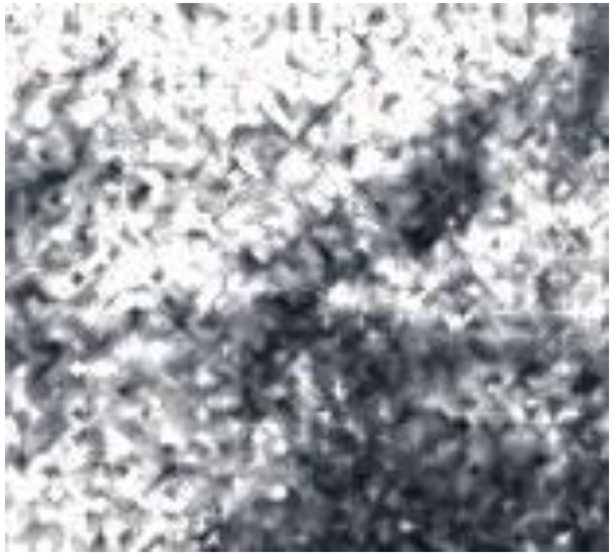
More is different

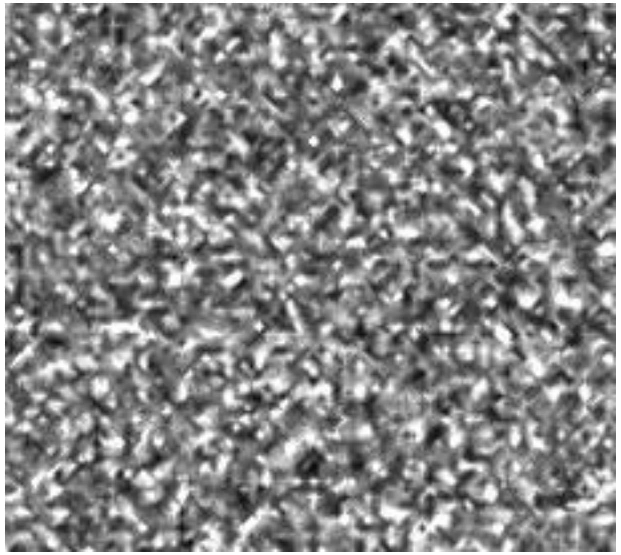
Philip W. Anderson

More is different

- Everything is made up of the same atoms and electrons **at short length scales.**

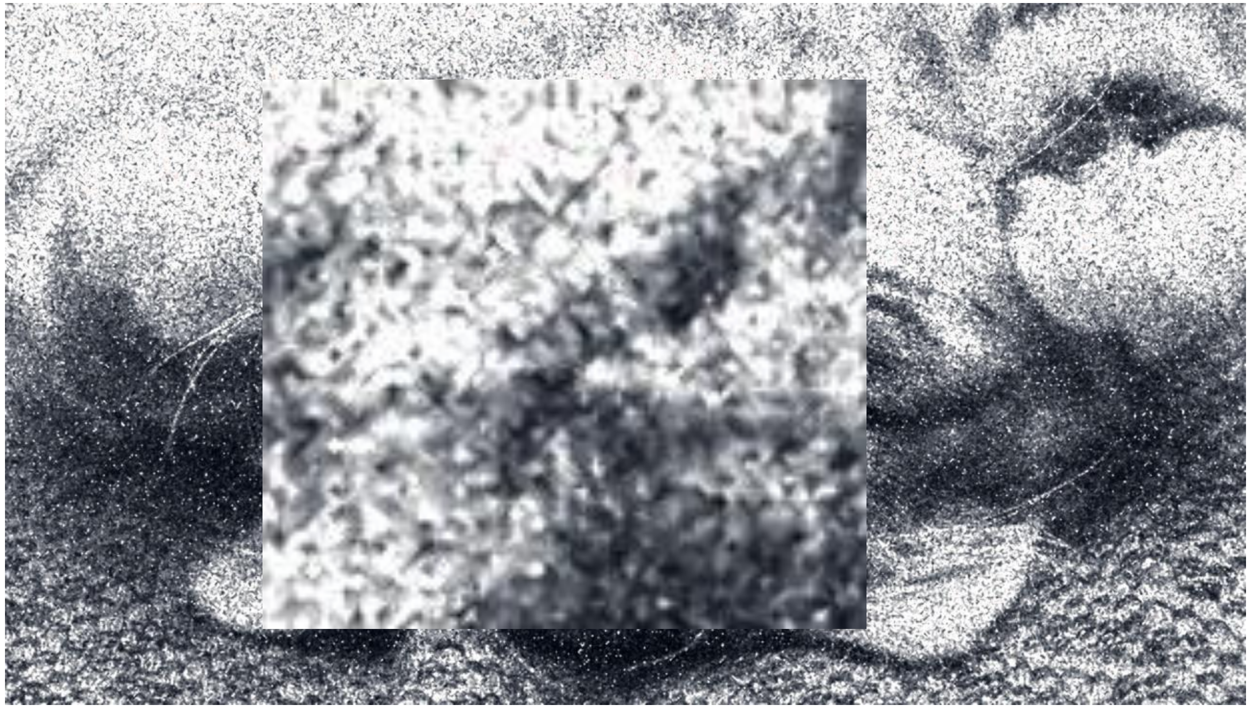
- Different correlations and patterns can emerge **at longer length scales.**

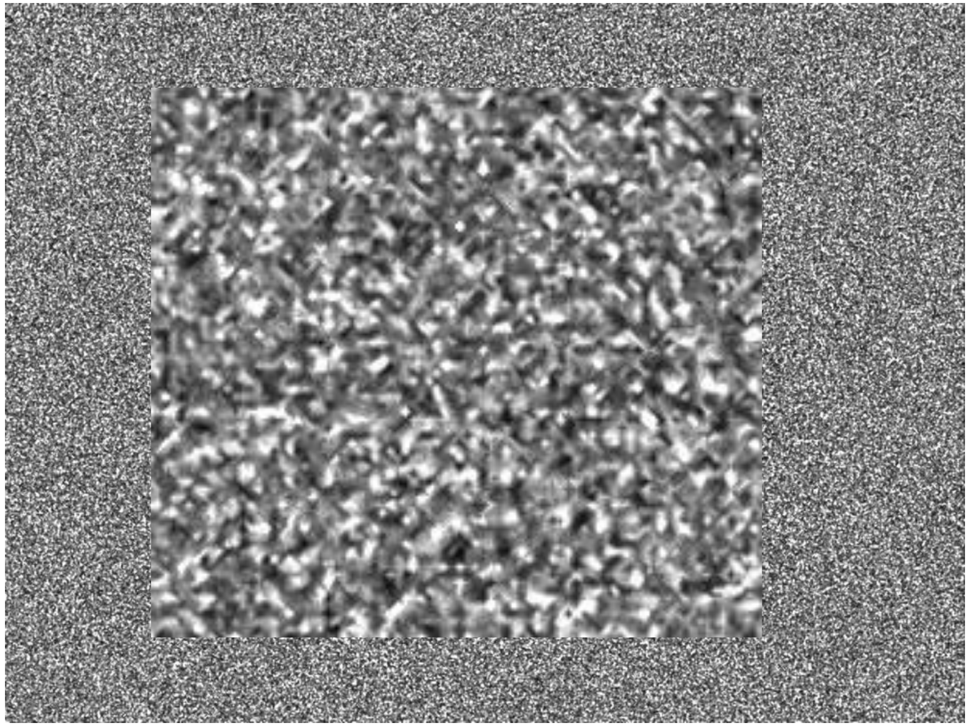




More is different

- Everything is made up of the same atoms and electrons **<u>at short length scales.</u>**
- Different correlations and patterns can emerge **<u>at longer length scales.</u>**

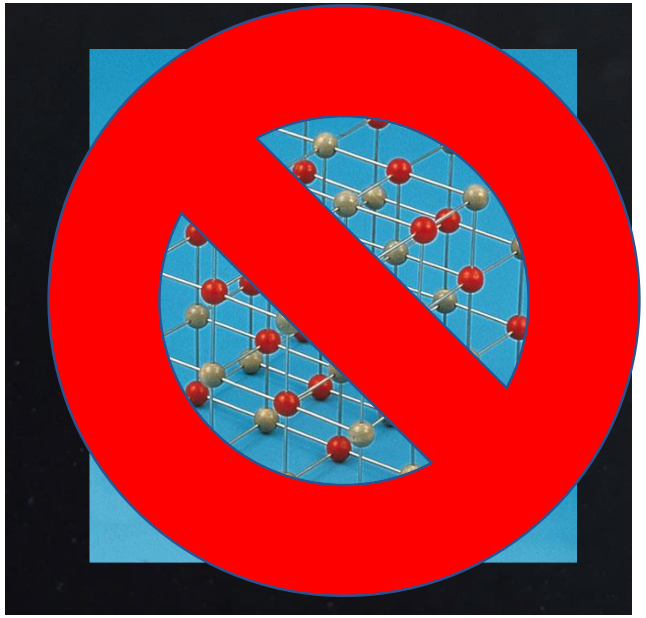


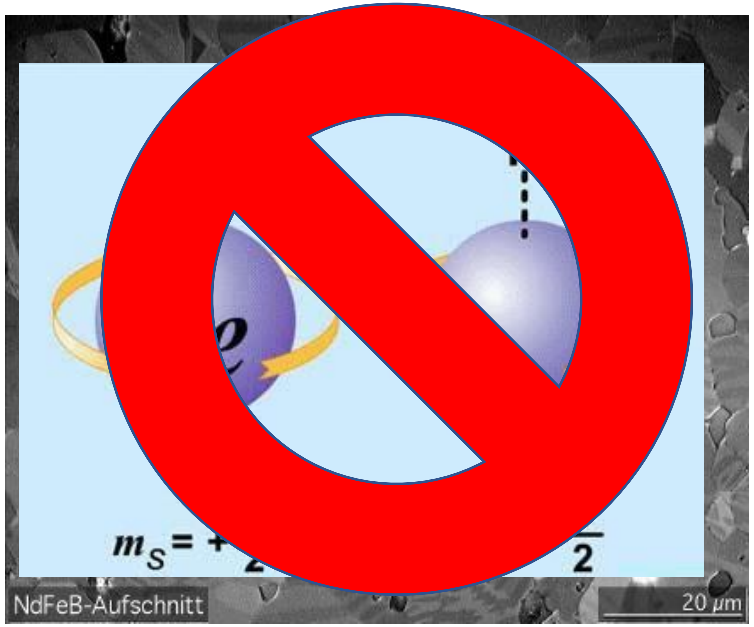


Physics of local correlations

Non-local

Solid state lattice	**Magnetic Domains**	**Self-organization**

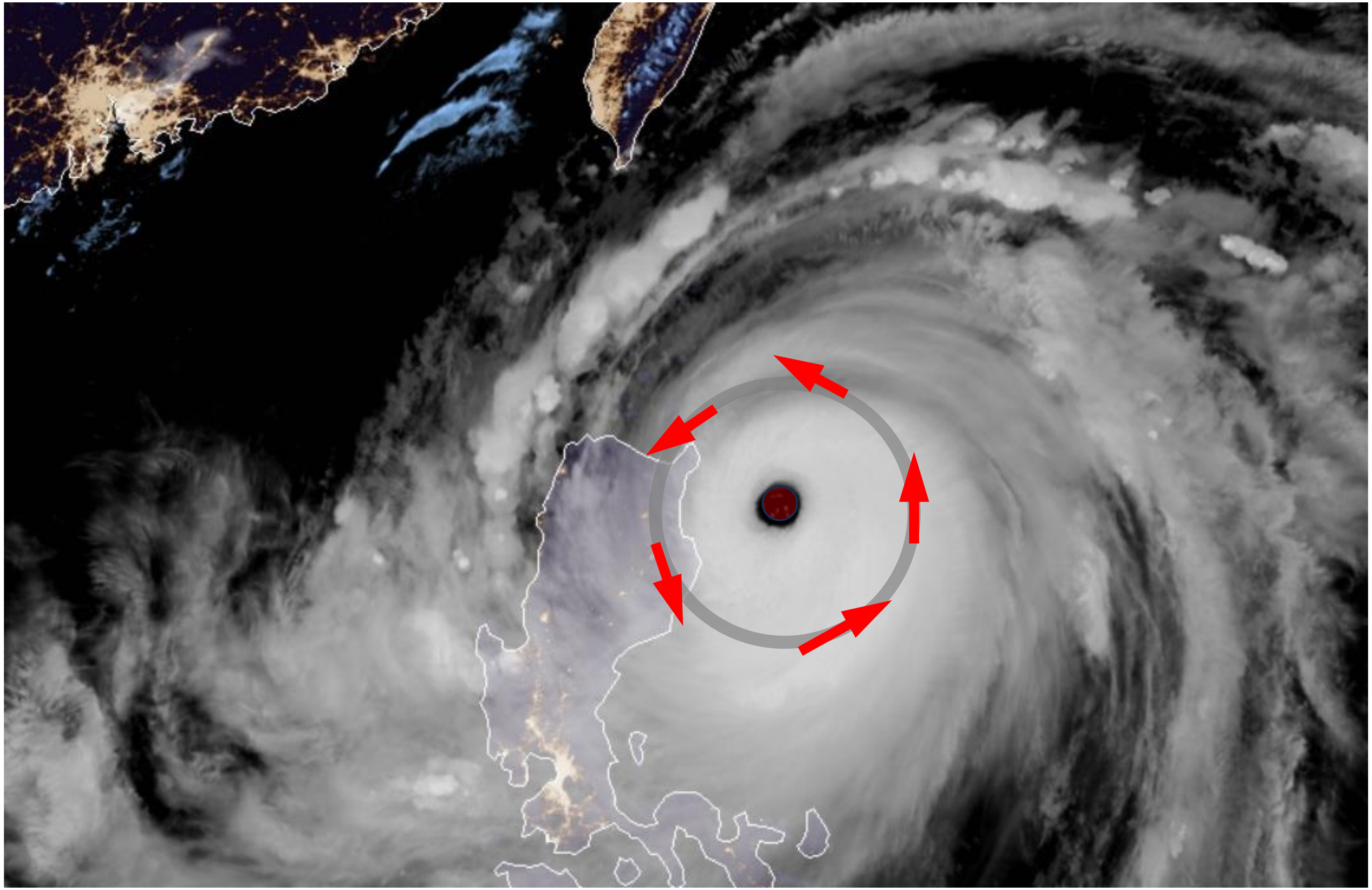




Paradigm of Landau theory:

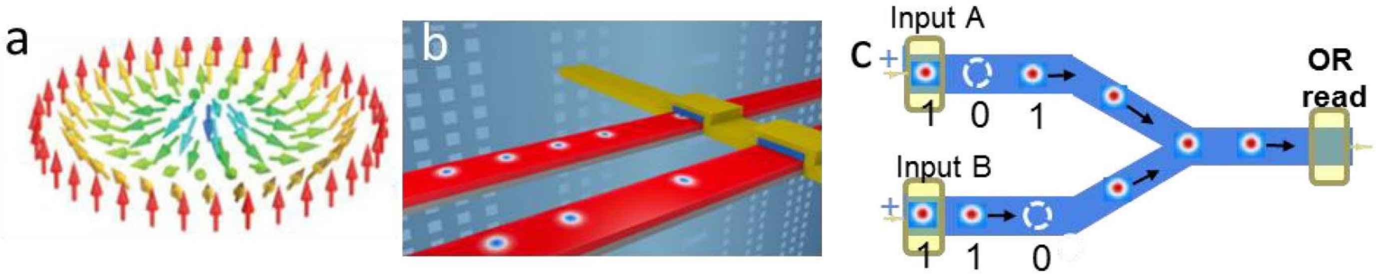
Patterns/Correlations of local order parameters determines **the phases of matters.**

Vortex

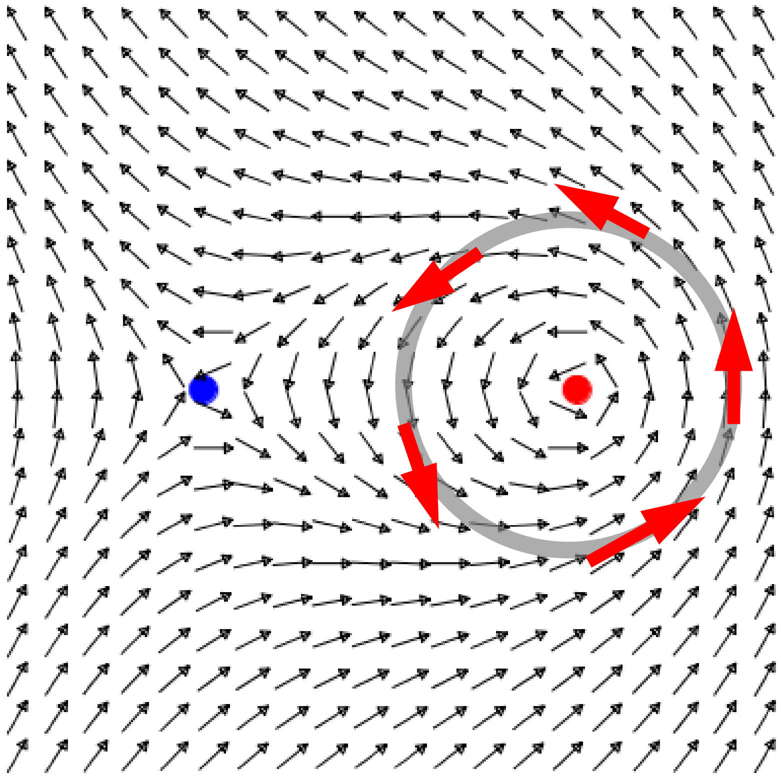


Vortex

➢ **<u>스커미온 스핀트로닉스</u>**



➢ **<u>Vortex in spin model</u>**



$$H = -\sum_{\langle ij \rangle} \cos\left(\theta_i - \theta_j\right)$$

What is topological insulator?

Phenomenological level

Topological boundary mode (Bulk-Boundary correspondence)

Theoretical level

Topological invariant
(Chern insulator, Z2 insulator)

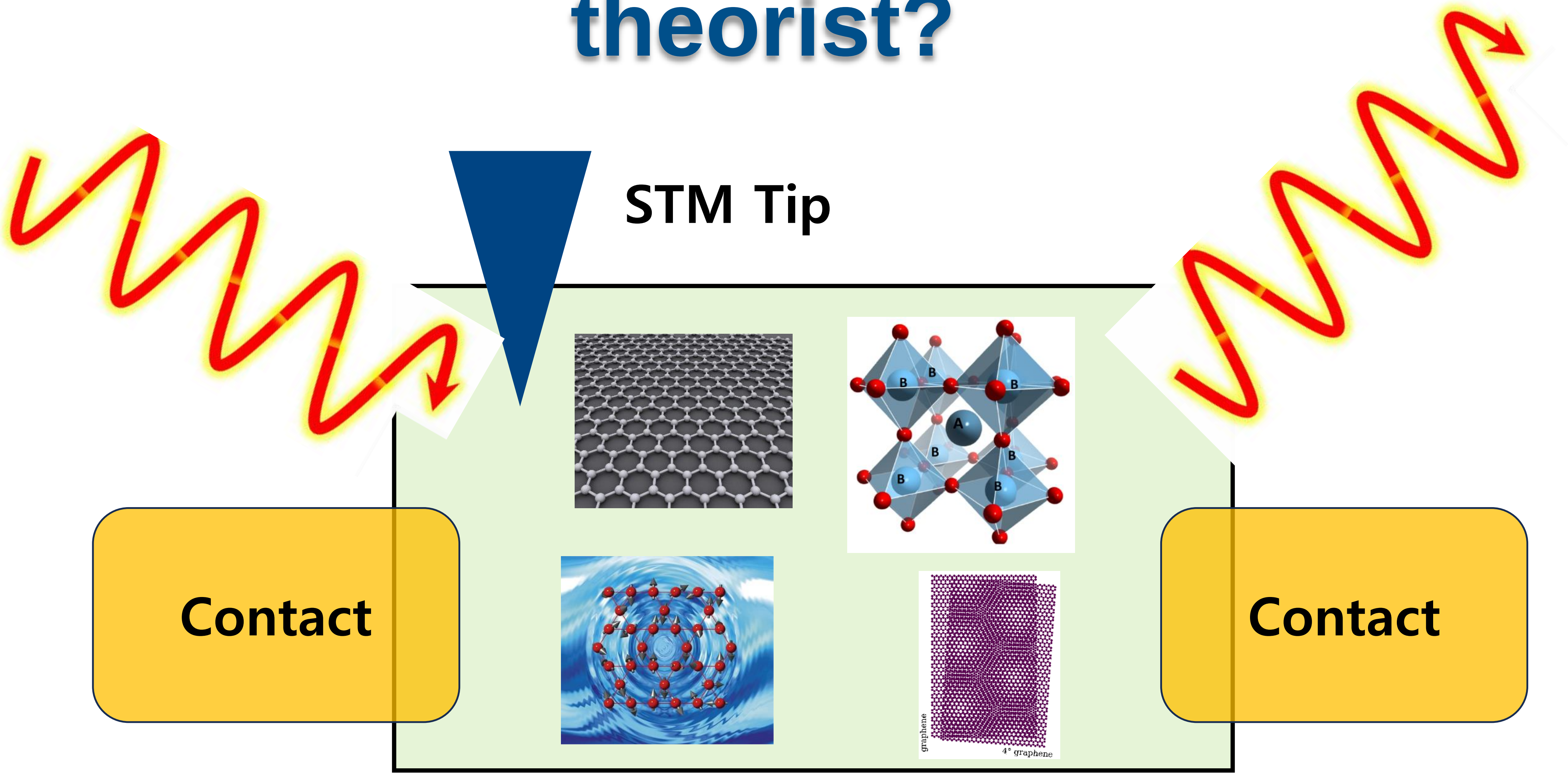
Material level

Graphene, HgTe, Bi2Se3

Experimental level

Quantum Hall effect, Chiral anomaly, Axion E&M

What is response to theorist?



Outline

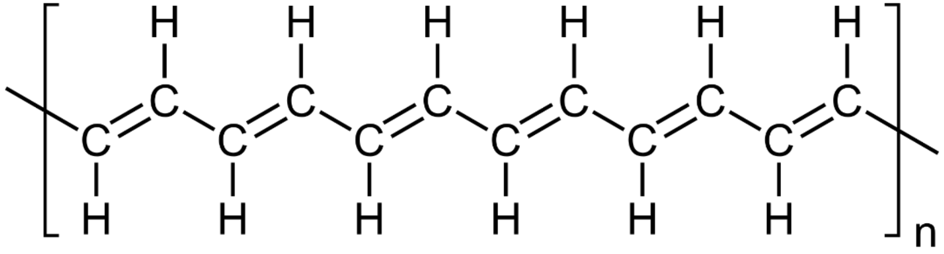
- <u>1D TI</u>
- <u>2D TI</u>
- <u>3D TI</u>
- <u>2D TM</u>
- <u>3D TM</u>

1D TI

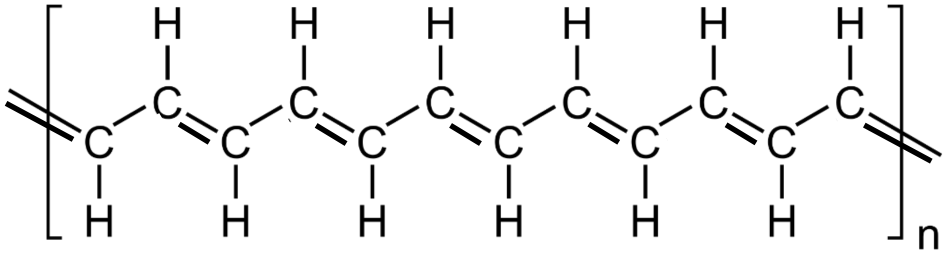
위상학적 결합

➢ Chemist's view

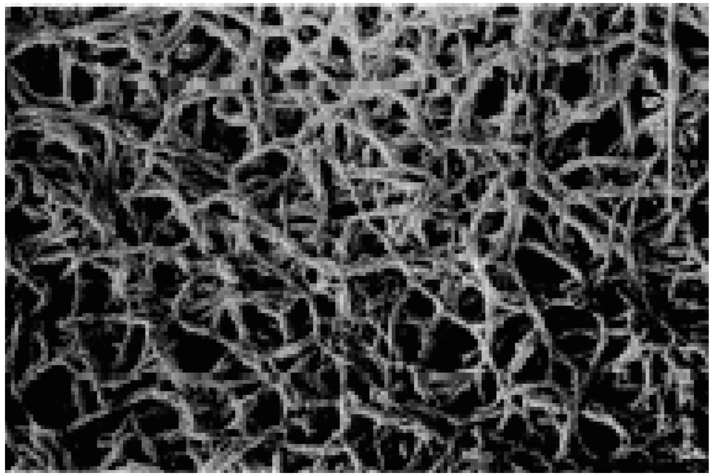
Case I :

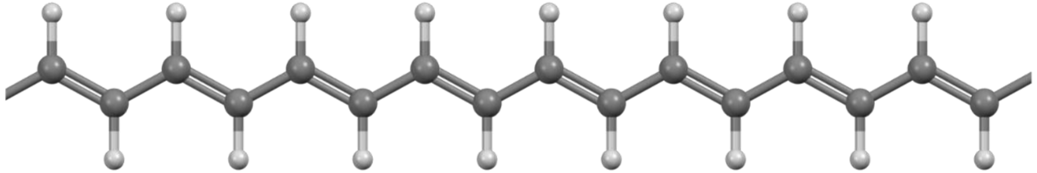


Case II :



➢ 현미경으로 본 이미지

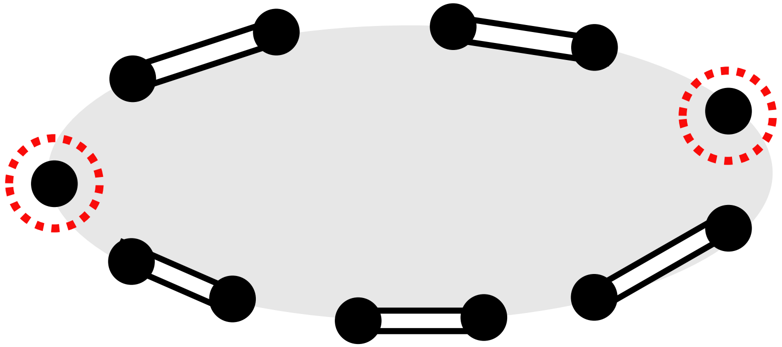




➢ Physicist's view

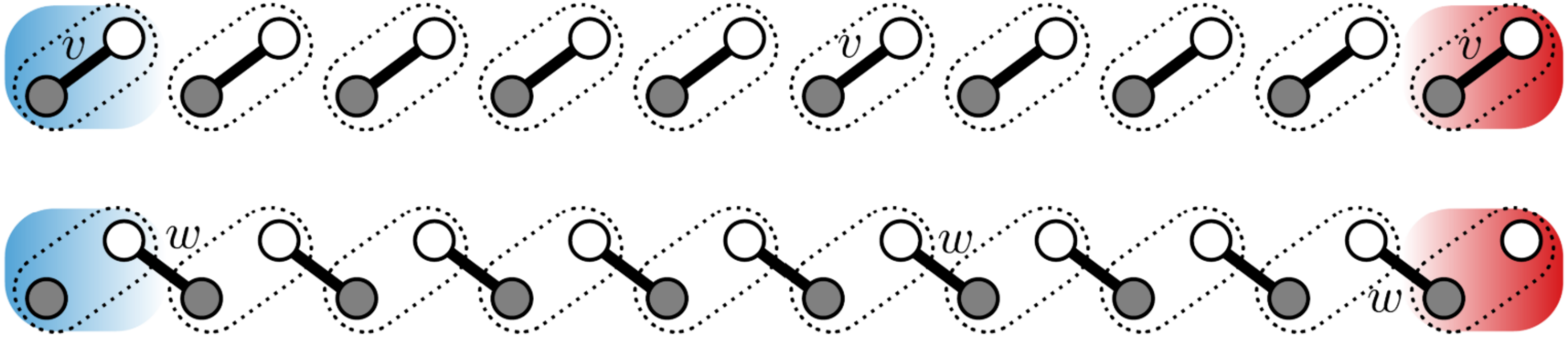
Case I :

Case II :

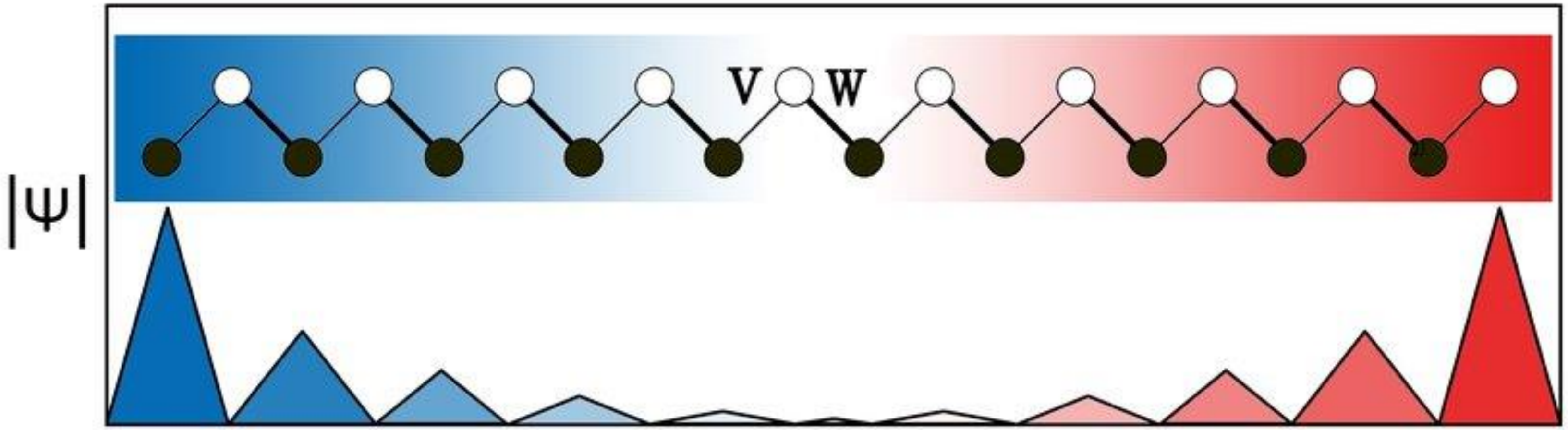


Su-Schrieffer-Heeger model

- **Real space coupling patterns**



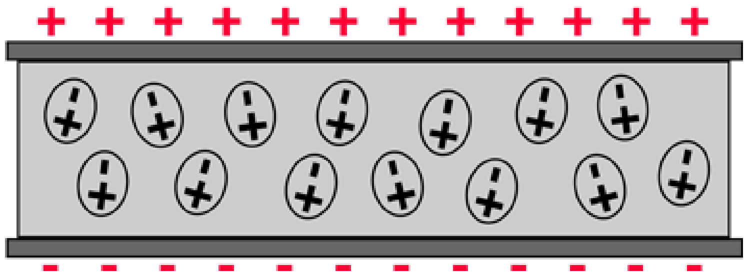
- **Topological edge state**



전하 편극밀도와 베리 위상

➢ 전하 편극밀도

Classical polarization



$$\boldsymbol{P} = \sum_{\boldsymbol{i}} q_i \boldsymbol{r}_{\boldsymbol{i}}$$

Quantum polarization

$$\boldsymbol{P} = \frac{e}{(2\pi)^3} \oint d\boldsymbol{r} \langle \psi | \boldsymbol{r} | \psi \rangle$$

$$= \frac{e}{(2\pi)^3} \oint d\boldsymbol{k} \langle \psi | \boldsymbol{\nabla}_{\boldsymbol{k}} | \psi \rangle$$

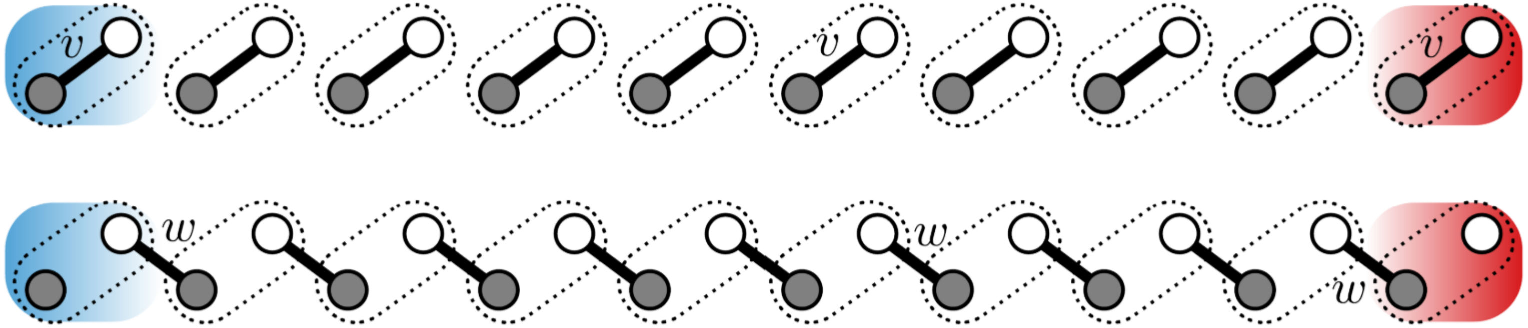
$\boldsymbol{A} = \langle \psi | \boldsymbol{\nabla}_{\boldsymbol{k}} | \psi \rangle$

베리 위상

$$\boldsymbol{k} = -\boldsymbol{i} \frac{\partial}{\partial \boldsymbol{x}}, \qquad \boldsymbol{x} = \boldsymbol{i} \frac{\partial}{\partial \boldsymbol{k}}$$

양자물질의 편극밀도는 베리위상과 같다.

SSH 모델의 베리위상



➢ 실공간 해밀토니안

$$H = v\sum_{m=1}^{N}\left(|m,A\rangle\langle m,B| + \text{h.c.}\right) + w\sum_{m=1}^{N-1}\left(|m+1,A\rangle\langle m,B| + \text{h.c.}\right)$$

$\nu = 0 \text{ if } v > w,\ \nu = 1 \text{ if } v < w$

➢ 운동량-공간 해밀토니안

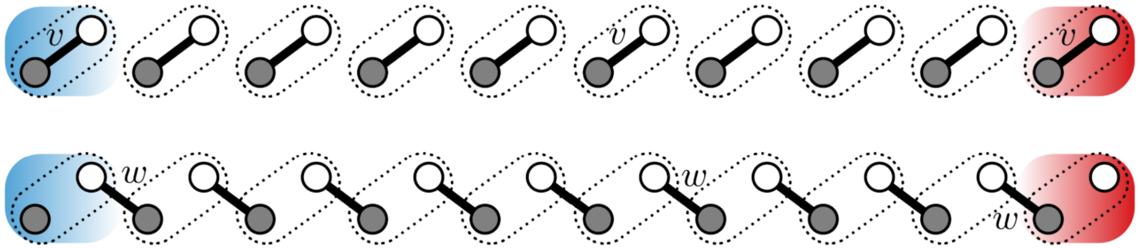
$$H(k) = \begin{pmatrix} 0 & v + we^{-ik} \\ v + we^{ik} & 0 \end{pmatrix}$$

$$H(k) = d_x(k)\sigma_x + d_y(k)\sigma_y + d_z(k)\sigma_z$$

$$d_x(k) = v + w\cos k,\ d_y(k) = w\sin k$$

SSH model은 위상학적 감김수(전하편극밀도)가 양자화 되어있다.

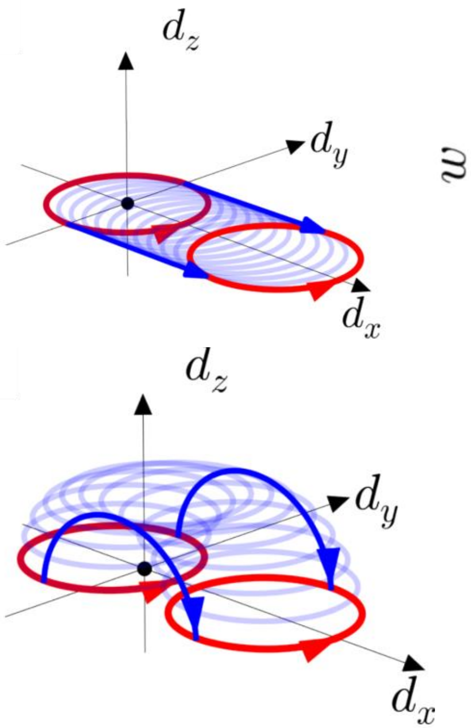
SSH 모델의 베리위상

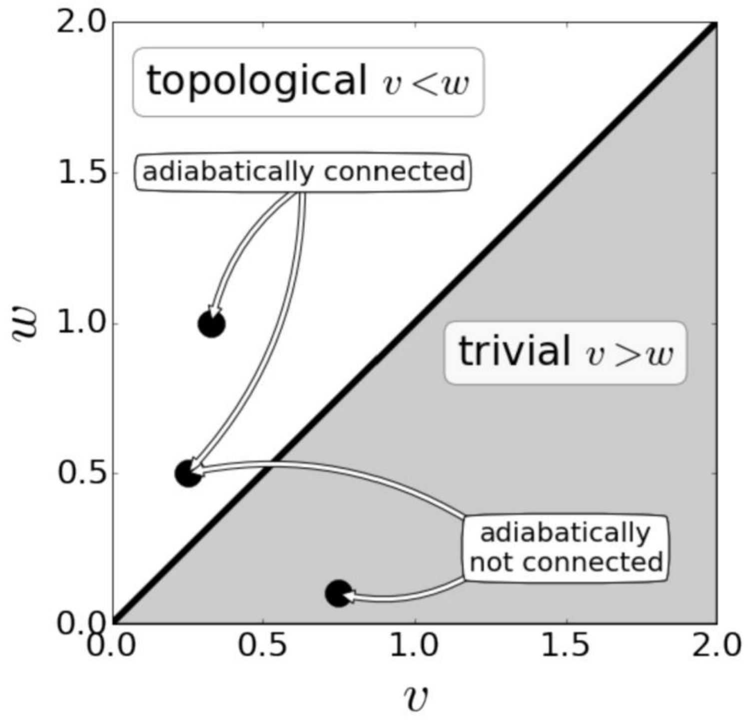


➢ <u>운동량-공간 해밀토니안</u>

$$H(k) = d_x(k)\sigma_x + d_y(k)\sigma_y + d_z(k)\sigma_z$$

$$d_x(k) = v + w\cos k, d_y(k) = w\sin k$$





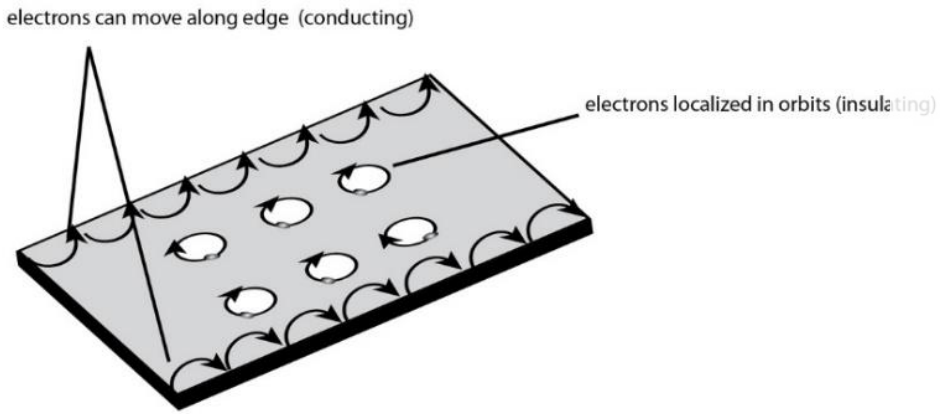
1. 위상학적 상전이는 밴드갭 닫힘(금속)을 동반한다.

2. 대칭성(대칭성 깨짐)은 종종 위상학적 상을 보호(파괴)한다.

2D IQHE

자기장하에서 2차원 전자의 운동

Classical cyclotron orbit



$$\frac{mv^2}{R} = \frac{qvB}{c}$$

원심력 로렌츠힘

$$R = \frac{mvc}{qB}$$

$$\omega_c = \frac{eB}{mc}$$

Quantum cyclotron orbit (Landau level)

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} - \frac{e\vec{A}}{c}\right)^2 \qquad H = \frac{p_x^2}{2m} + \frac{1}{2}\frac{B^2e^2}{m}\left(x - \frac{p_y}{Be}\right)^2$$

$$\vec{A} = Bx\hat{j}$$

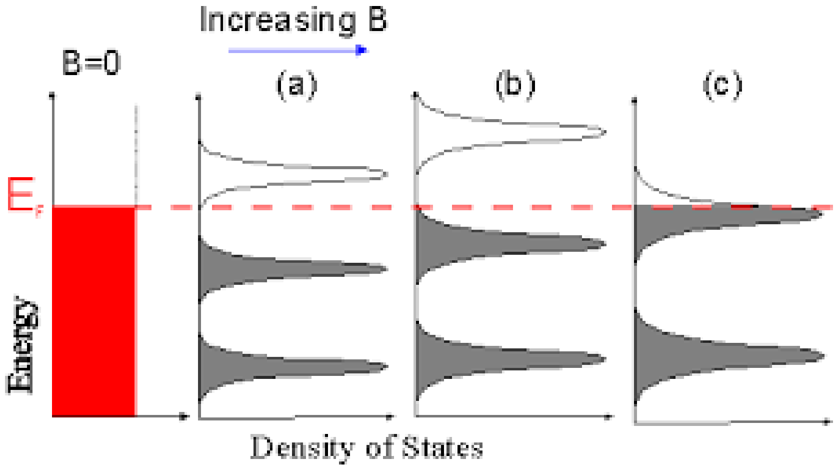
자기장내에서 2차원 전자 에너지는 양자화 되어 있다.

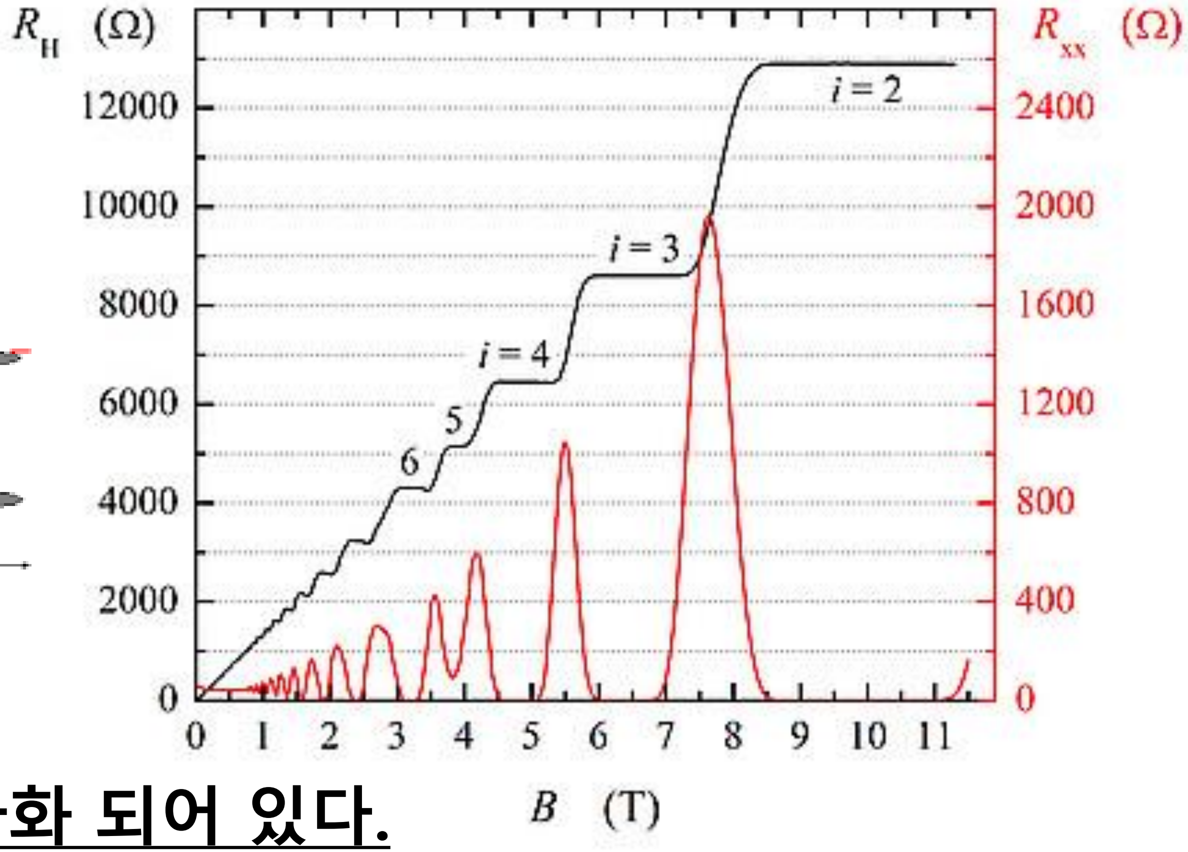
정수 양자홀 효과

- ➢ **Quantized Hall conductance**
- ➢ **Vanishing Longitudinal conductance**

클라우스 폰 클리칭

출생	1943년 6월 28일(80세) 나치 독일 (오늘날 폴란드) 비엘코폴스카주 시로다비엘코폴스카(폴란드어: Środa Wielkopolska)
국적	독일
주요 업적	양자 홀 효과
수상	노벨 물리학상 (1985)
분야	물리학





양자물질의 홀전도도는 양자화 되어 있다.

Quantum anomalous Hall effect (QAHE)

- **TKNN formula :**

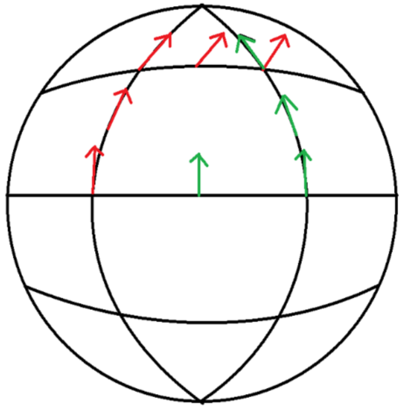
$$\sigma_{xy} = \frac{e^2}{h}\int_{\mathrm{BZ}} \frac{d^2k}{(2\pi)^2}\, f(\mathbf{k})\,\Omega(\mathbf{k})$$

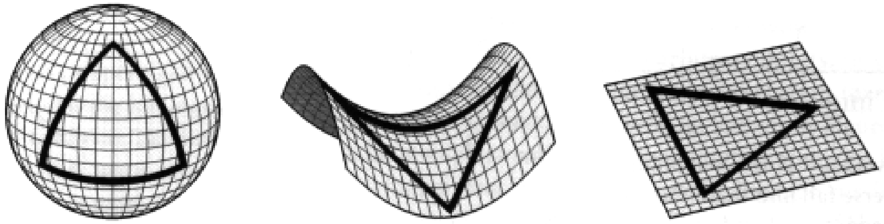
$$\Omega(\mathbf{k}) = \langle u(\mathbf{k})|\partial_x\partial_y|u(\mathbf{k})\rangle$$

Berry curvature

Gauss-Bonnet theorem :

$$\int_M K\,dA = 2\pi(2-2g)$$



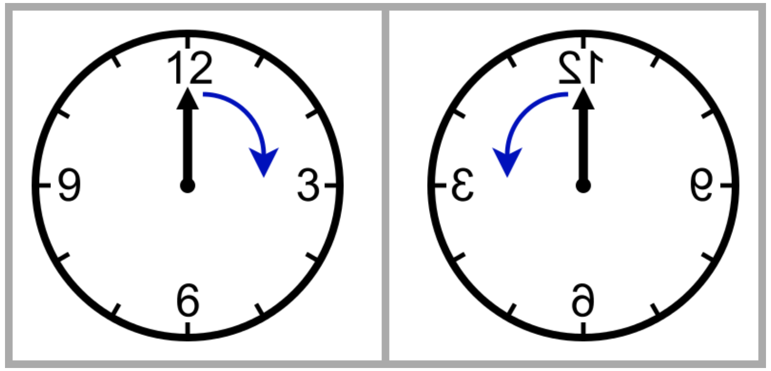


Positive Curvature   Negative Curvature   Flat Curvature

홀전도도는 파동함수의 위상학적 불변량으로 결정된다. (자기장이 필수는 아니다.)

시간-반전 대칭성

➢ Time-reversal symmetry:



$$Q \rightarrow Q \qquad E \rightarrow \mathrm{E}$$

$$I = \frac{\partial Q}{\partial t} \rightarrow -I \qquad B \rightarrow -B$$

➢ 왼손잡이 입자, 오른손잡이 입자:

Left-moving fermion
(negative group velocity)

Right-moving fermion
(positive group velocity)

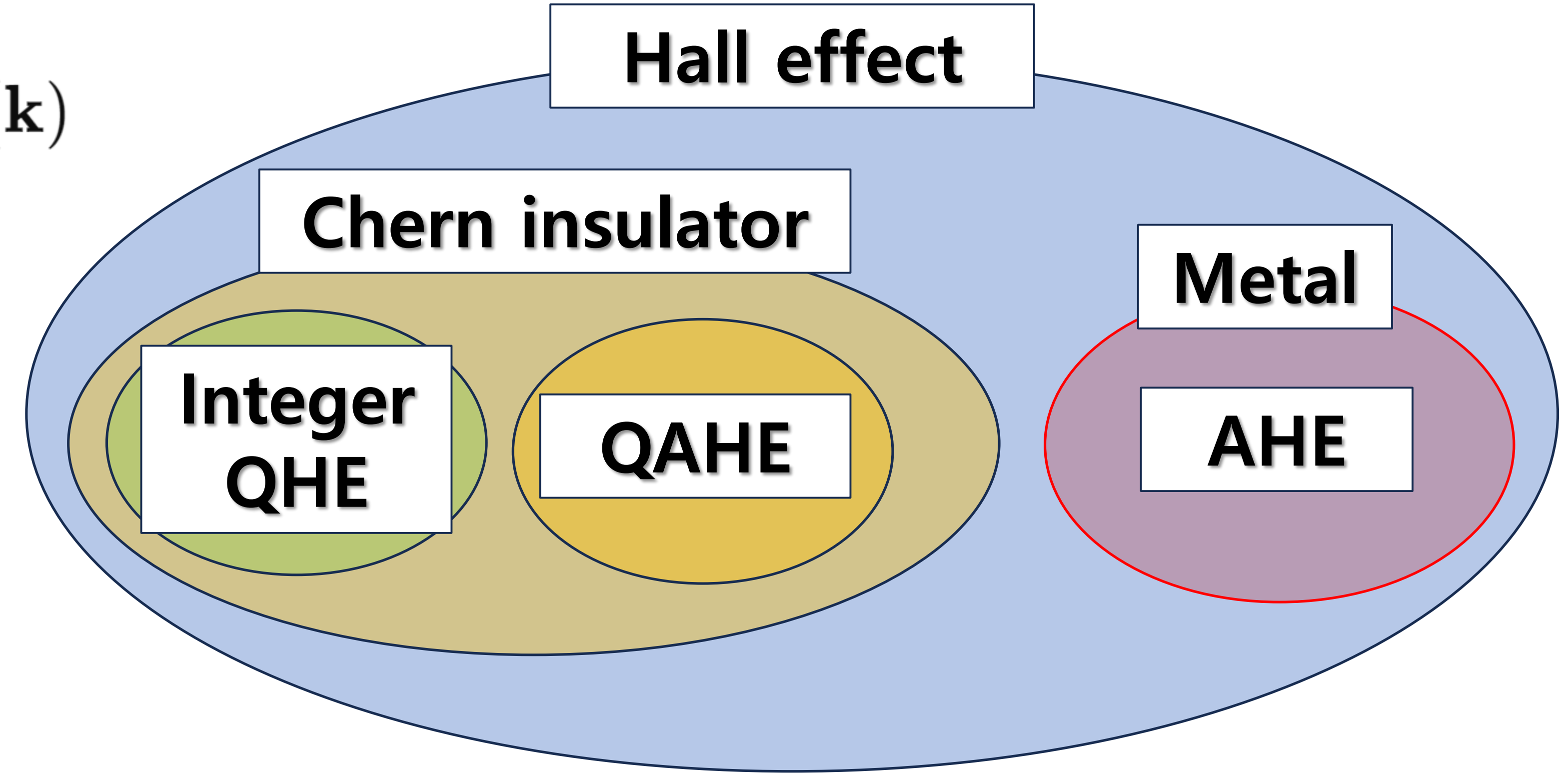
Quantum anomalous Hall effect (QAHE)

➢ **TKNN formula :**

$$\sigma_{xy} = \frac{e^2}{h}\int_{\mathrm{BZ}} \frac{d^2k}{(2\pi)^2}\, f(\mathbf{k})\,\Omega(\mathbf{k})$$

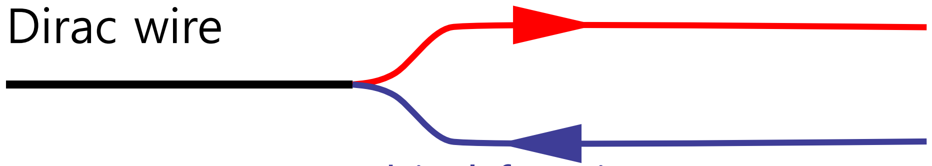
$$\Omega(\mathbf{k}) = \langle u(\mathbf{k})|\partial_x\partial_y|u(\mathbf{k})\rangle$$

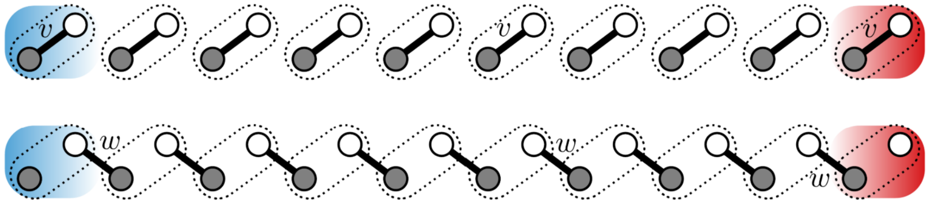
Berry curvature



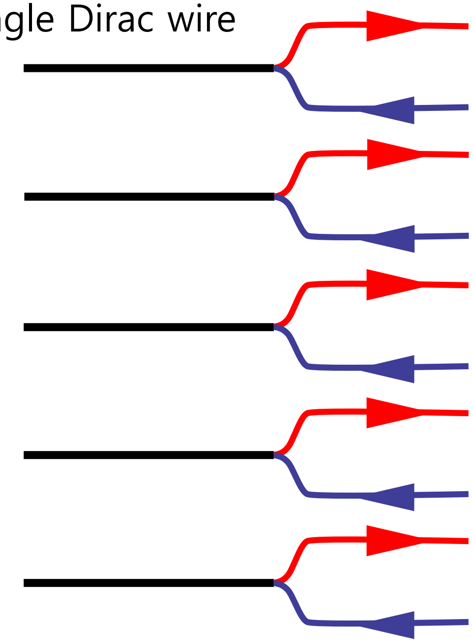
홀전도도는 파동함수의 위상학적 불변량으로 결정된다.

Coupled wire construction





- **실공간 와이어 다발**



- Trivial insulator

- Chern insulator

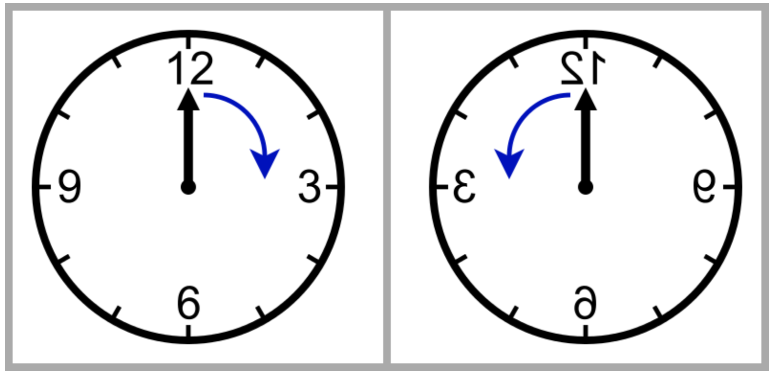
양자홀 효과는 시간반전대칭성 깨짐의 패턴이다.

양자홀 효과는 왼손지기 입자와 오른손지기 입자의 공간분리이다.

2D QSHE

양자 스핀 홀 효과

시간 반전 대칭성 깨짐은 양자홀 효과에 필수적인가? 일단 yes!

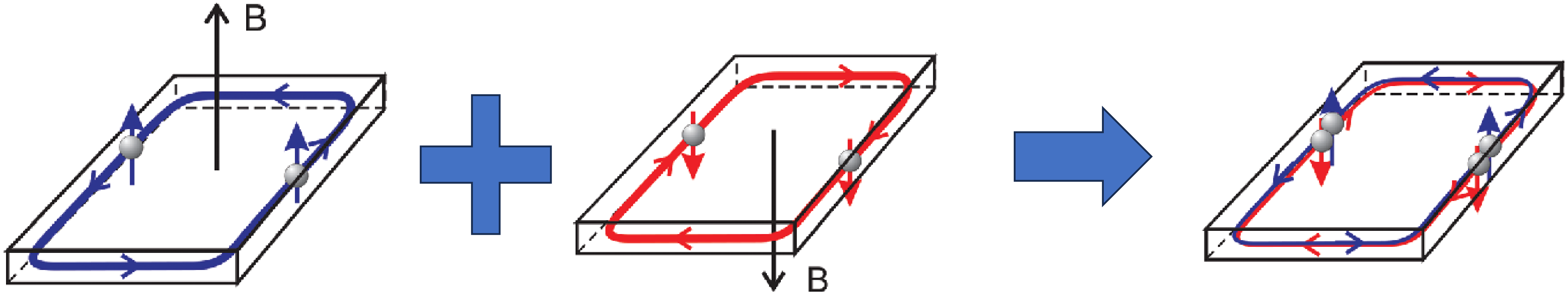


$$Q \rightarrow Q \qquad E \rightarrow E$$

$$I = \frac{\partial Q}{\partial t} \rightarrow -I \qquad B \rightarrow -B$$

고려하지 않은 변수가 있나? 답 : 스핀

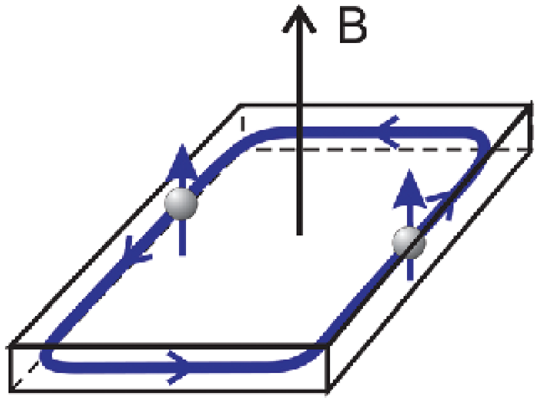
$$\boldsymbol{S} \rightarrow -\boldsymbol{S}$$



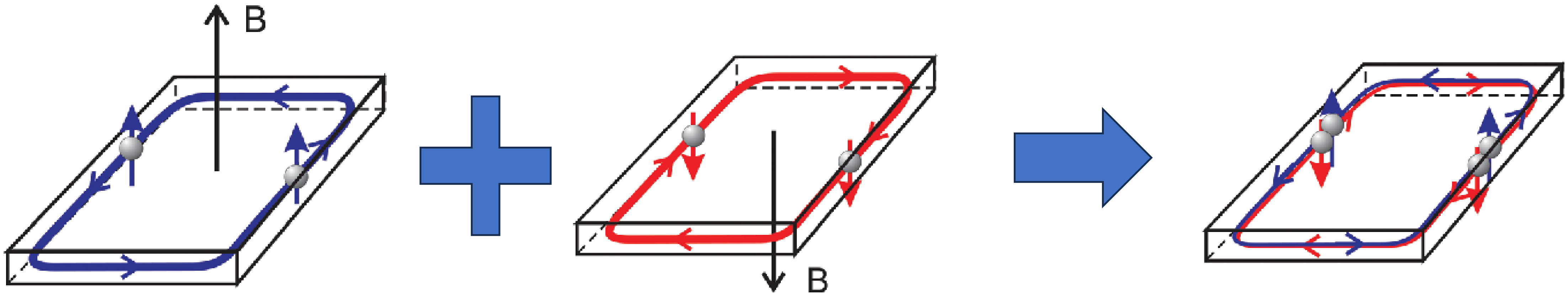
양자 스핀 홀 효과는 홀전도도는 0이지만 스핀 홀 전도도가 양자화된다.

Kane-Mele model

- **외부** 자기장 $(\boldsymbol{L} \cdot \boldsymbol{B})$

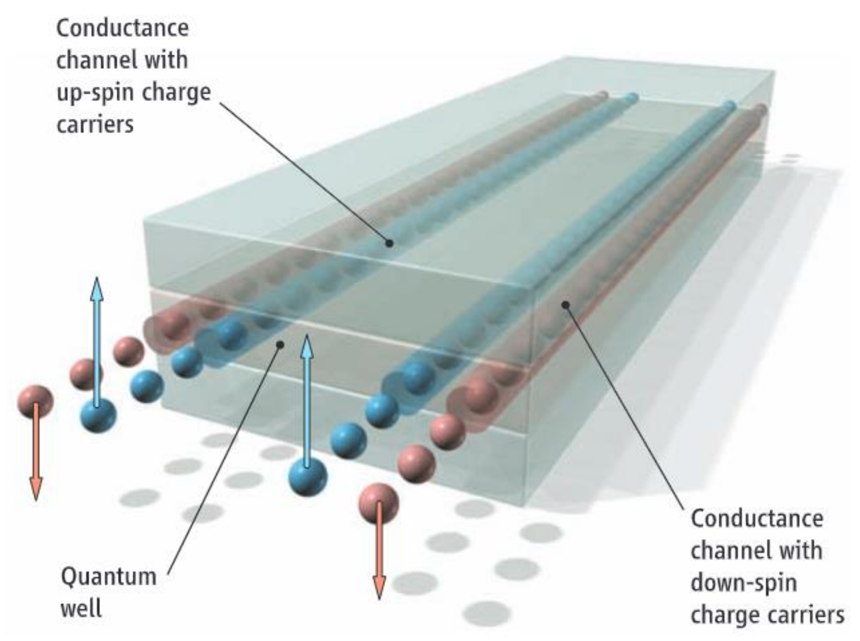


- **스핀-궤도 결합** $(\boldsymbol{L} \cdot \boldsymbol{S})$



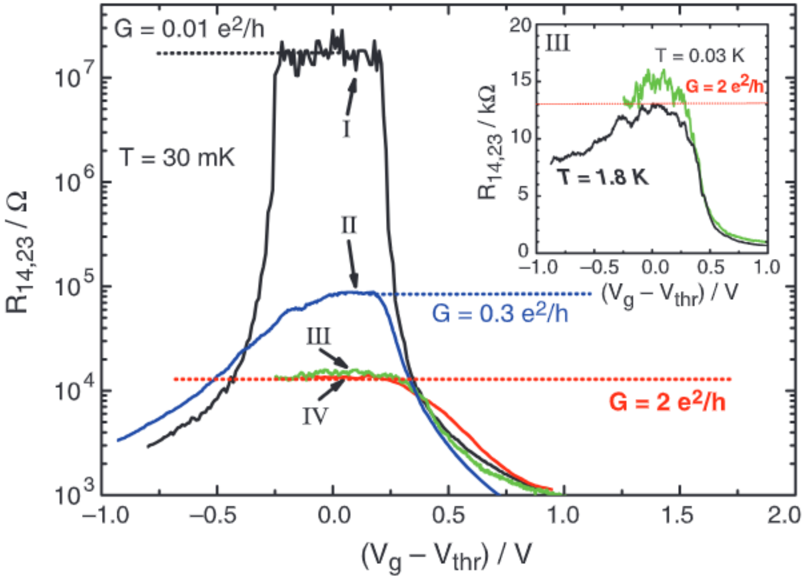
<ins>**스핀-궤도 결합은 자기장보다 매우 세기가 강하다. -> 상온 양자홀효과**</ins>

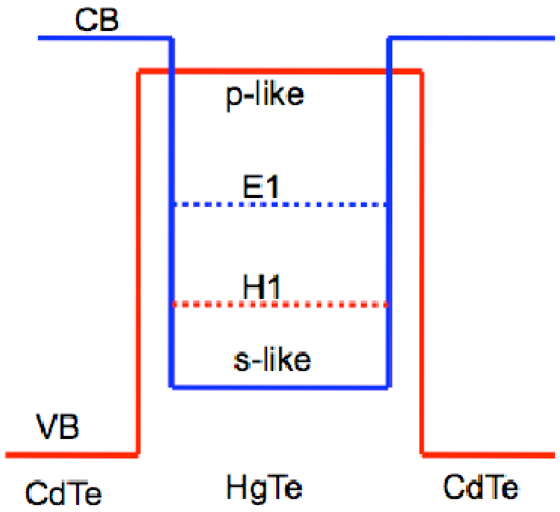
HgTe quantum well



Schematic of the spin-polarized edge channels in a quantum spin Hall insulator.

➢ **Bernevig-Hughes Zhang model**

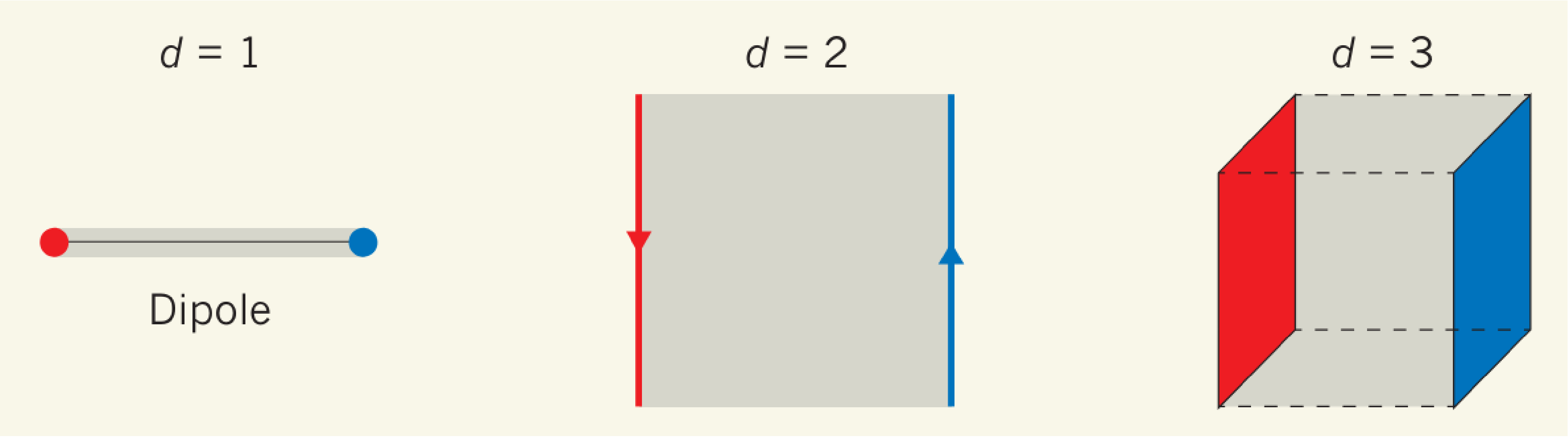




3D TI

3차원 위상 절연체

시간-반전 대칭 3차원 위상 절연체는 존재할까?



양자화된 편극밀도
(전하의 위치)

양자화된 전도도
(전하의 흐름)

양자화된 ??
(전하의 가속운동)

빛-물질 상호작용

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

$\Leftrightarrow$

$$\mathcal{L} = E^2 - B^2$$

Axion Electromagnetism

➢ **Axion E&M**

$$\mathcal{L} = E^2 - B^2 + \frac{\theta(x,t)}{2\pi} E \cdot B$$

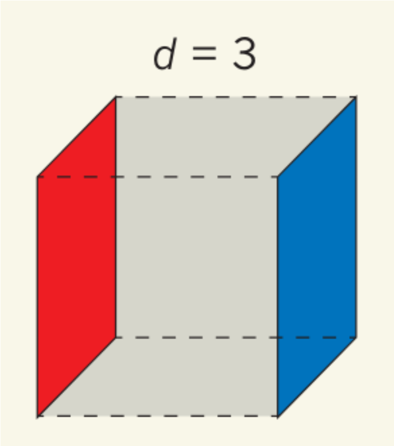
가상의 입자 Axion을 가정하자

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

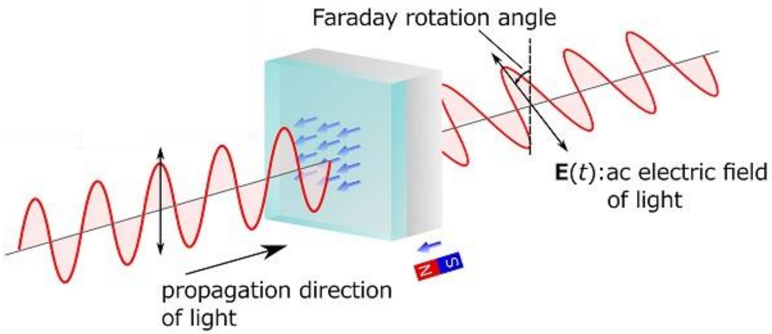
가우스 법칙, 암페어 법칙 변환

$$\boxed{\begin{aligned} \nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} - 2c\alpha \nabla\left(\frac{\theta}{2\pi}\right) \cdot \mathbf{B}. \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} + \frac{2\alpha}{c} \nabla\left(\frac{\theta}{2\pi}\right) \times \mathbf{E}. \end{aligned}}$$

$$\boldsymbol{D} = \epsilon \boldsymbol{E} - \frac{\alpha\Theta}{\pi}\boldsymbol{B},$$
$$\boldsymbol{H} = \boldsymbol{B} + \frac{\alpha\Theta}{\pi}\boldsymbol{E}.$$



양자화된 ??
(전하의 가속운동)

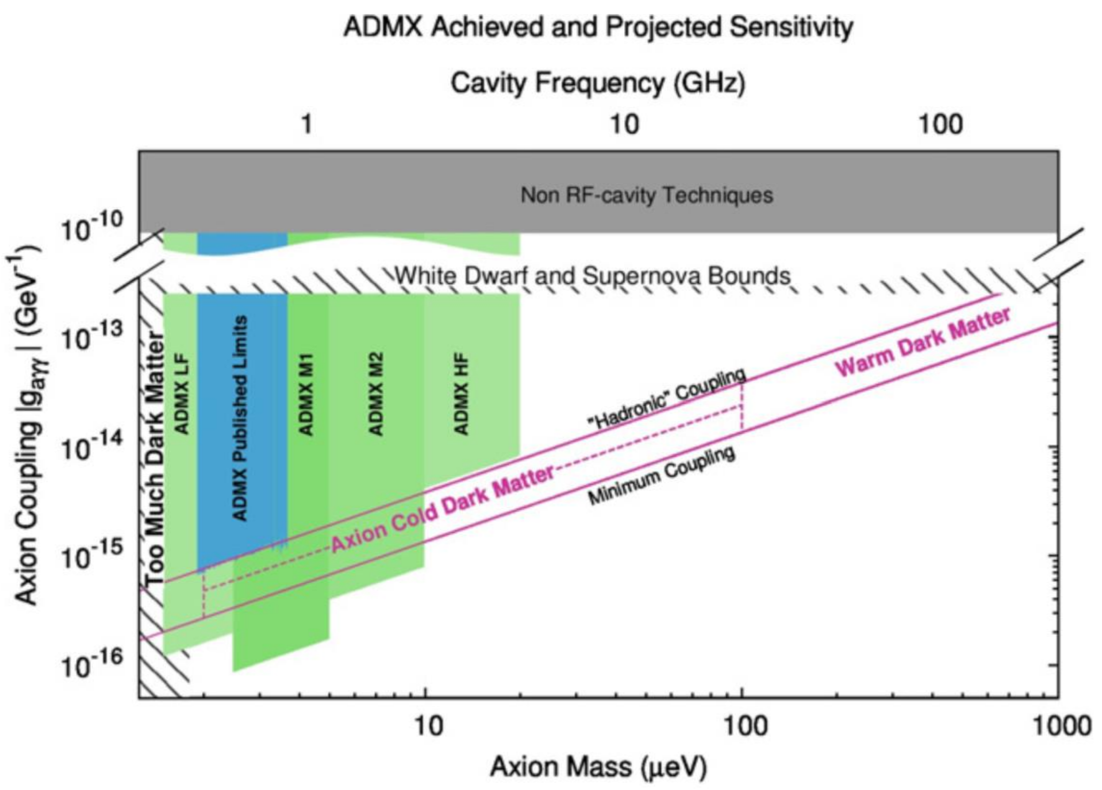


<u>**엑시온의 존재를 페러데이, 커 회전으로 검출할 수 있다.**</u>

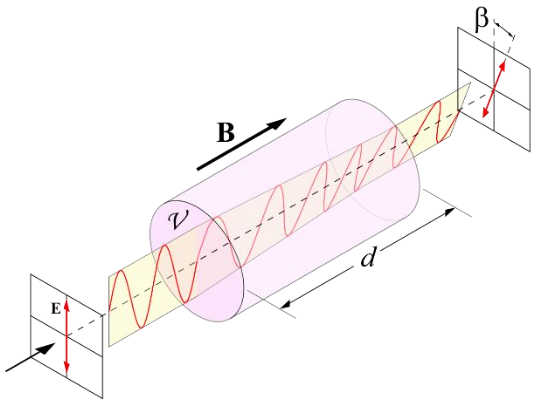
Axion E&M

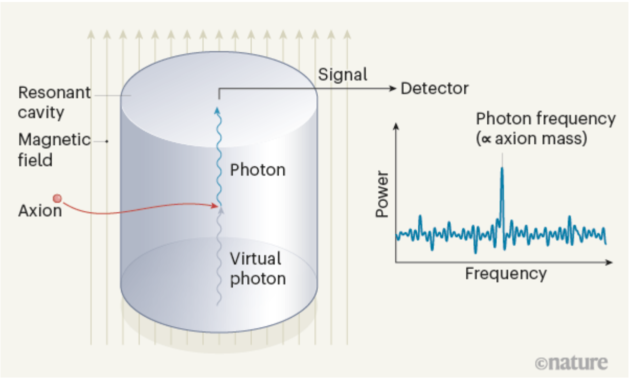
➢ **암흑물질 후보로써의 엑시온**

$$\mathcal{L} = E^2 - B^2 + \frac{\theta(x,t)}{2\pi} E \cdot B$$



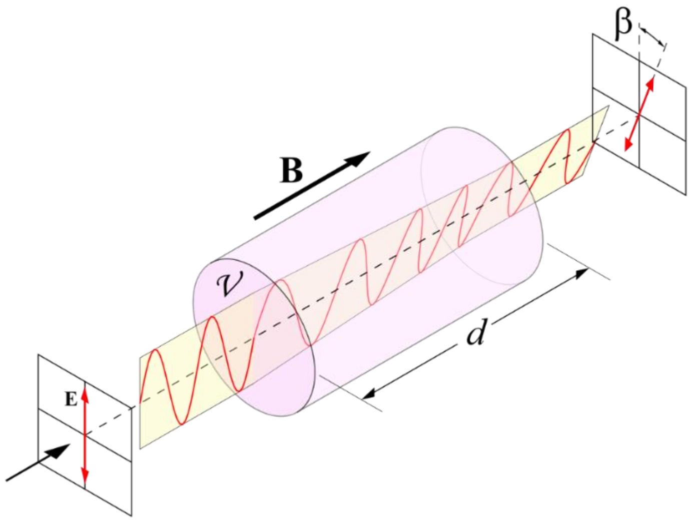
➢ **엑시온-암흑물질 검출기**

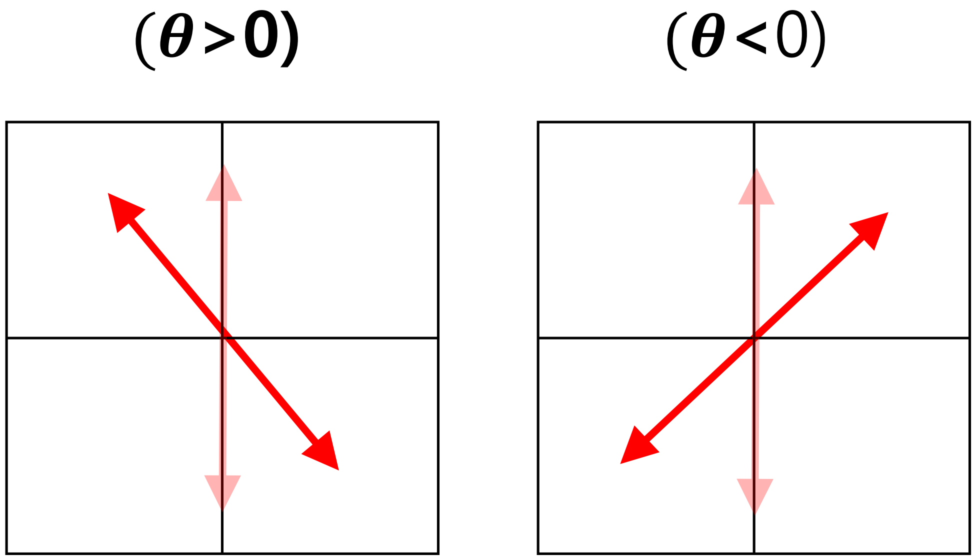




3차원 위상 절연체

➢ **패러데이 회전과 Axion-field 와의 관계**





➢ **물질내부의 Axion-field**

$$\tan(\phi_F) = \frac{2\alpha}{1+n}(N_t + \frac{1}{2} + N_b + \frac{1}{2})$$

$$\tan(\phi_K) = \frac{4n\alpha}{n^2-1}(N_t + \frac{1}{2} + N_b + \frac{1}{2})$$

$\boldsymbol{\theta} = 0$ ⇒ 시간반전 대칭 일반절연체

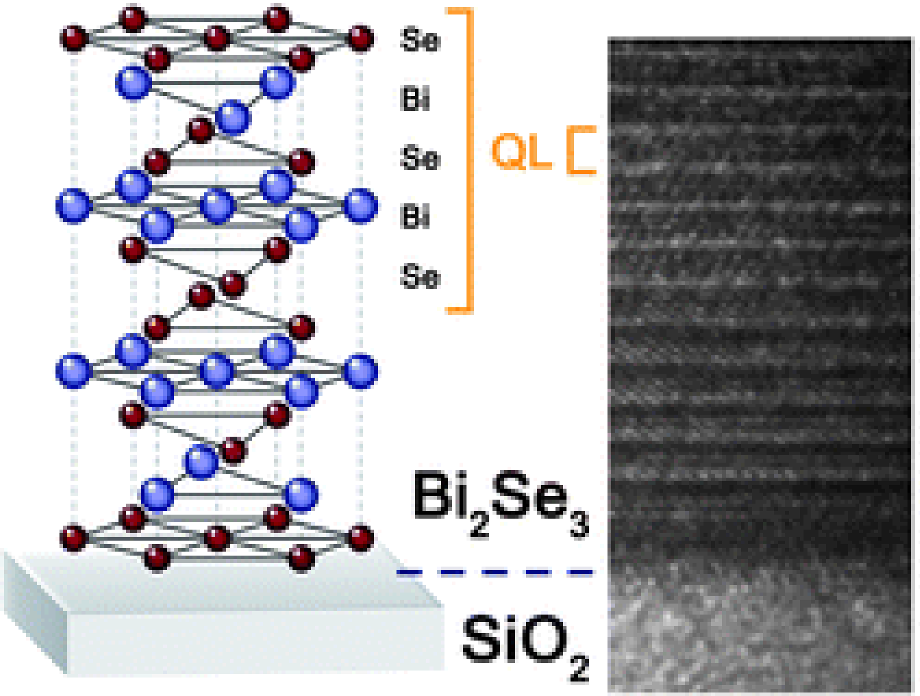
$\boldsymbol{\theta}$ = Arbitrary const ⇒ 시간반전 깨짐 자성절연체

$\boldsymbol{\theta} = \boldsymbol{\pi}$ ⇒ 시간반전 대칭 위상절연체

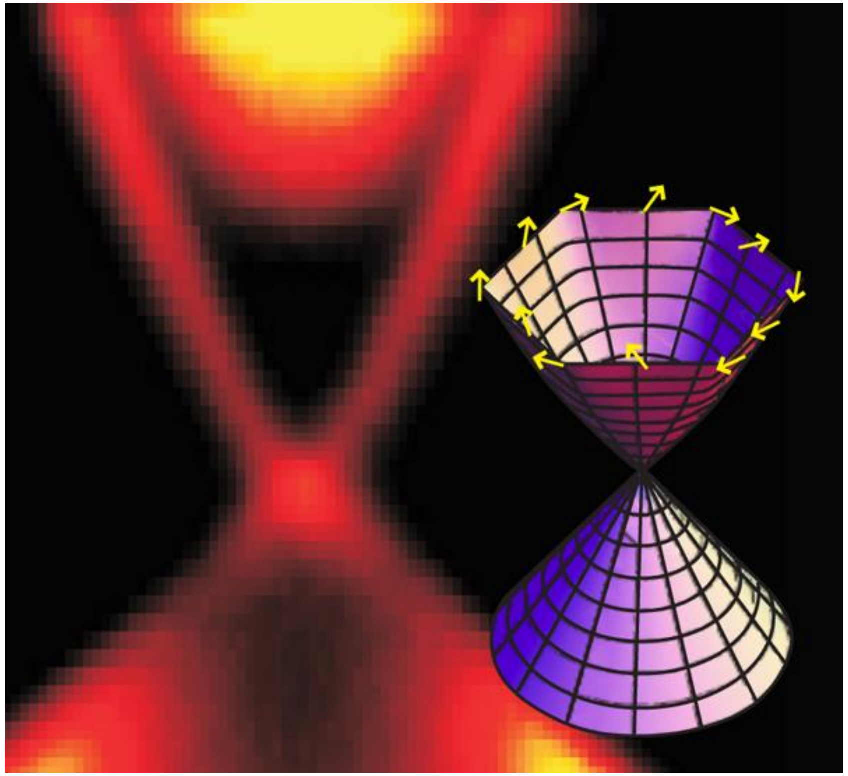
3차원 위상 절연체는 Axion field가 최대화 되는 물질이다 $(\boldsymbol{\theta} = \boldsymbol{\pi})$

3차원 위상 절연체

➢ Bi_2Se_3, Bi_2Te_3 (강한-스핀궤도 결합)

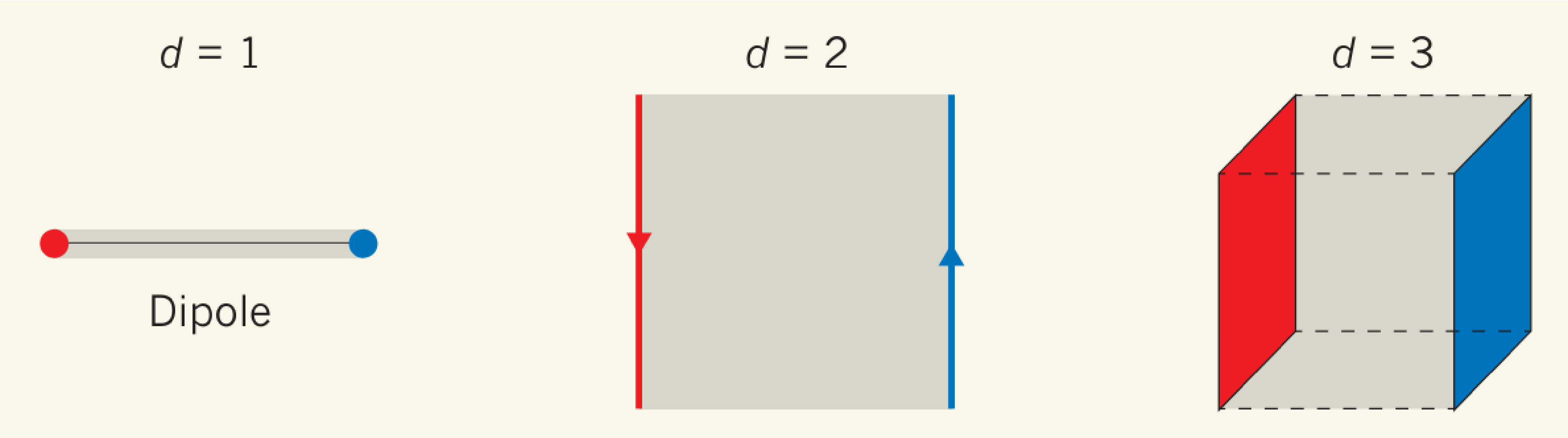


➢ 위상학적 겉면 상태 (디락콘)

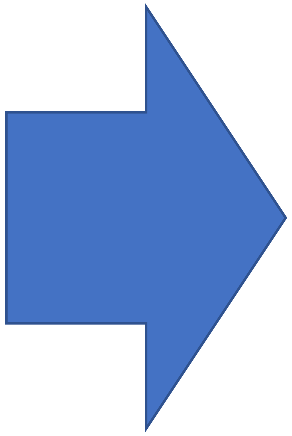


<u>3차원 위상 절연체는 한 개의 디락 콘을 가지는 겉면상태가 있다.</u>

Summary



- **1D TI**
- **2D TI**
- **3D TI**



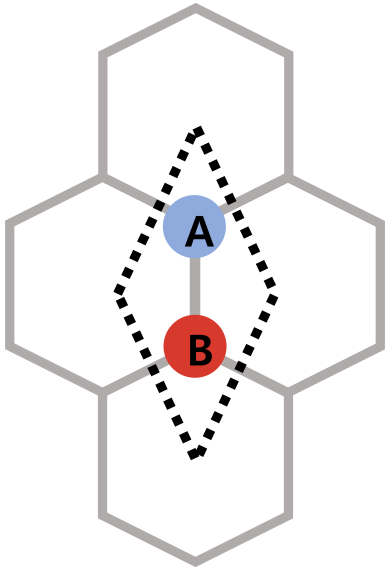
- <u>양자화된 편극밀도</u>
- <u>양자화된 홀전도도</u>
- <u>양자화된 자기광 패러데이/커 효과</u>

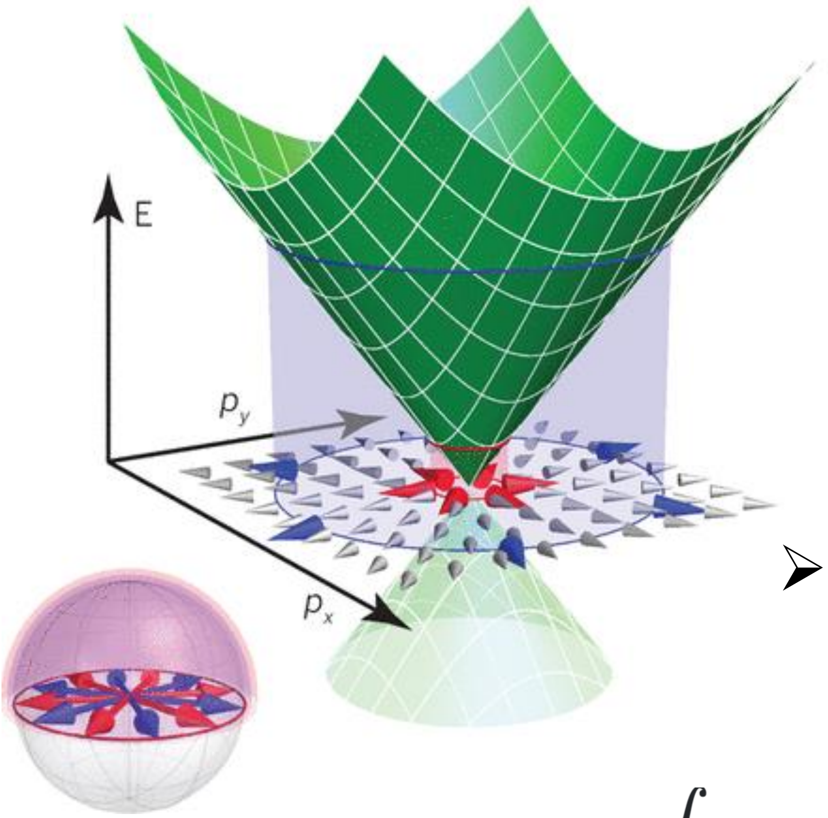
위상학적 준금속

Berry Phase of Graphene

- **Berry phase of graphene**

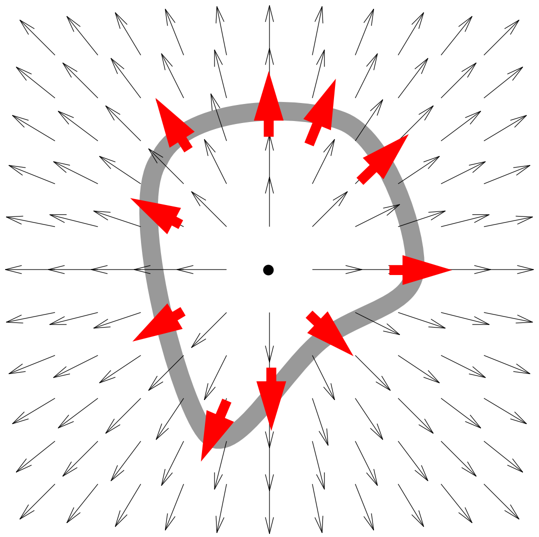
Pseudo-spin



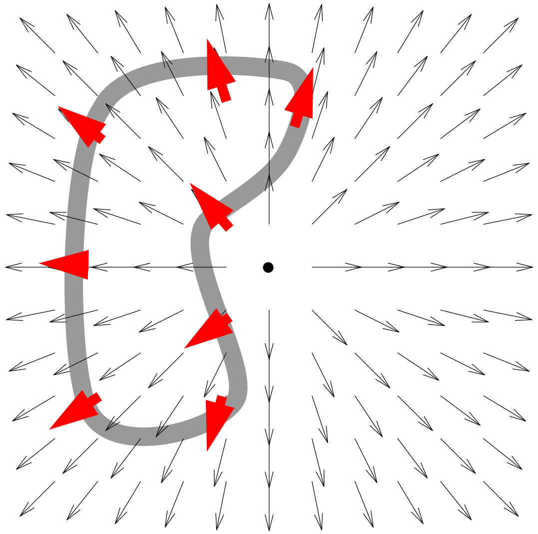


$$\gamma_n(t) = i\int_C \langle \psi_n | \nabla_R \psi_n \rangle \cdot d\vec{R}$$

- **π –Berry phase**



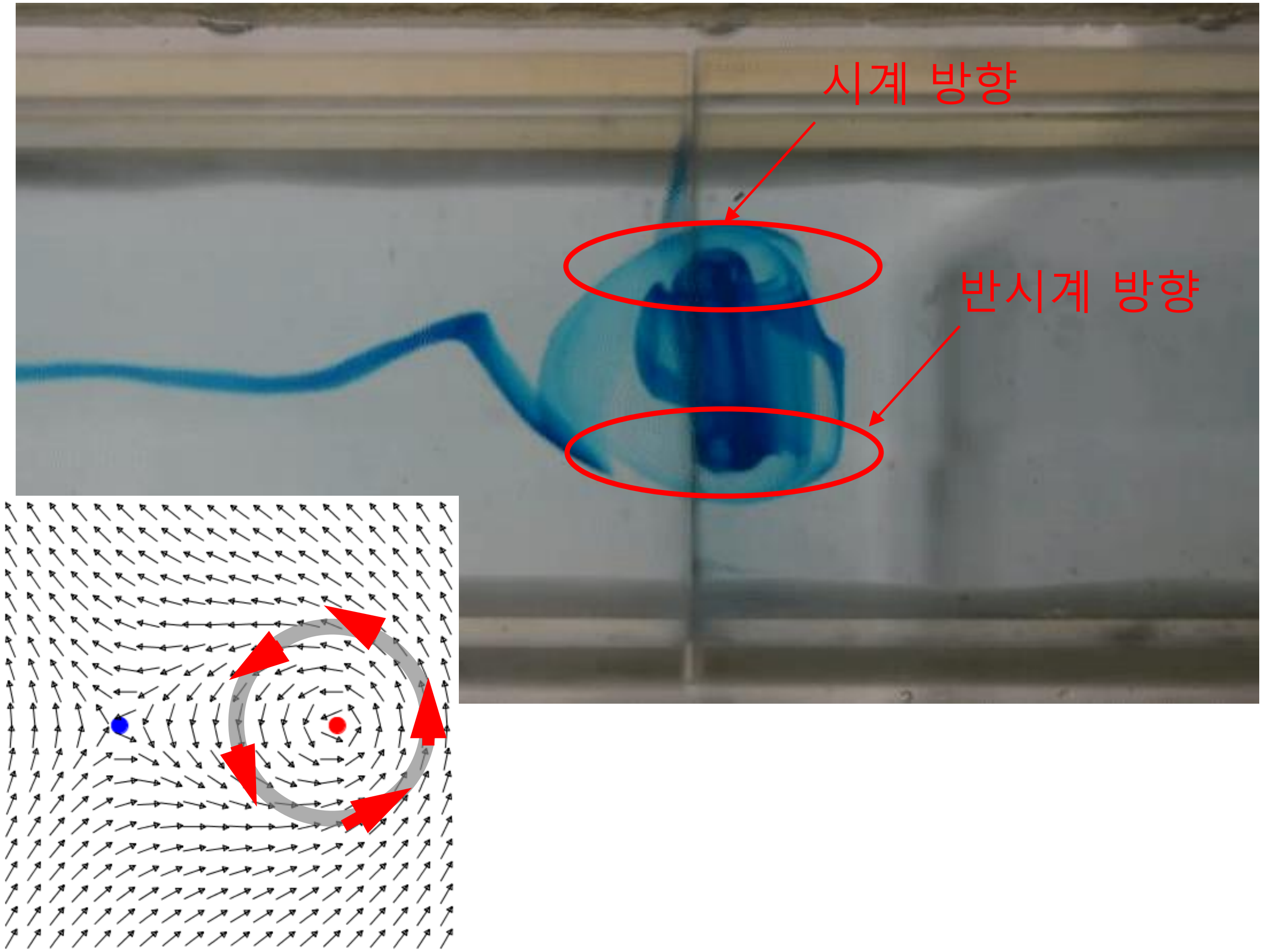
- **0–Berry phase**



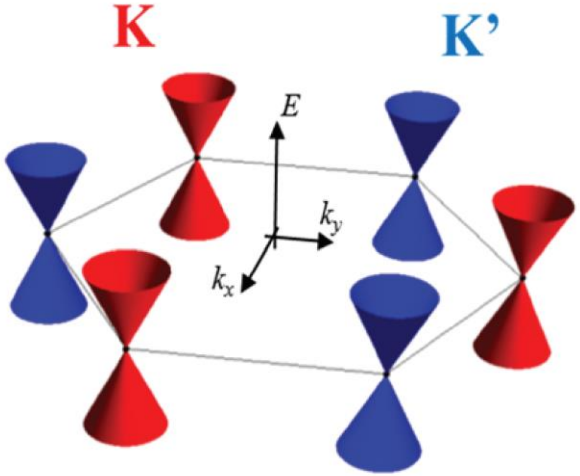
디락 콘을 가지는 겉면상태가 있다.

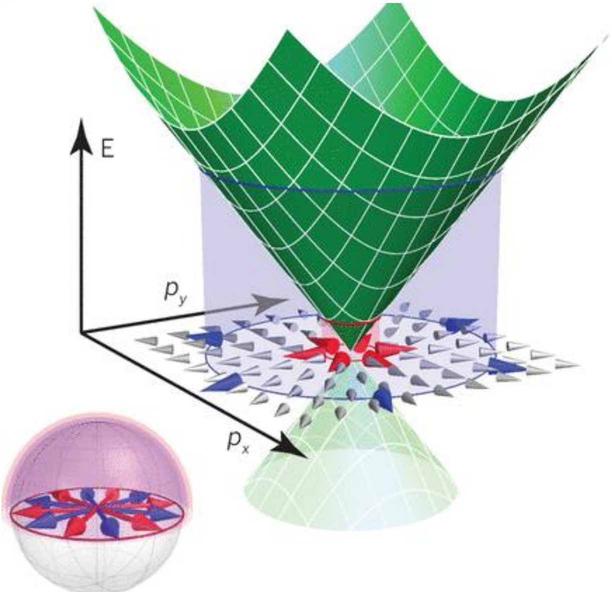
Vortex

- **Vortex as a pair**



- **Dirac cone as vortex core**

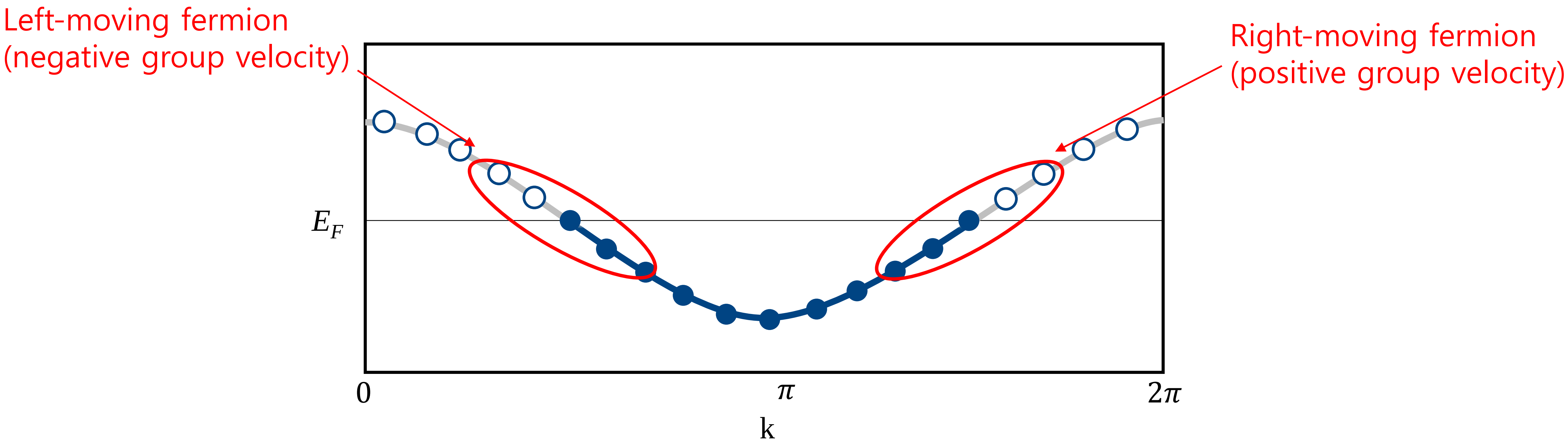




그래핀의 디락콘은 항상 짝수개로 존재한다.

페르미온 더블링 문제

➢ **Nielsen Ninomiya 노고 정리 (fermion doubling) :** In odd-space dimension, any lattice regularized theory must have even-numbers of chiral fermions.



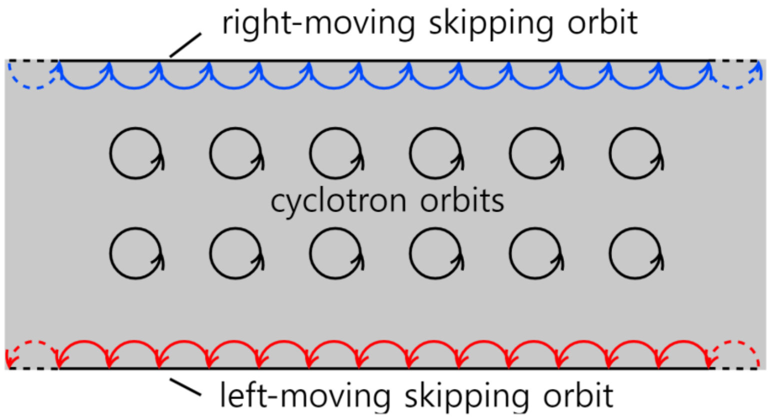
Anything goes up must goes down!

Violation of NN theorem

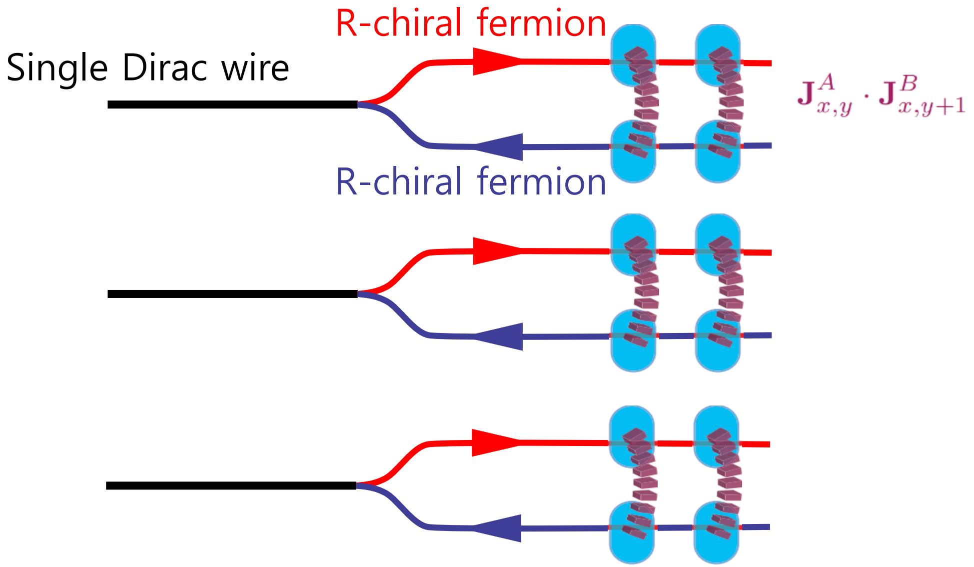
So far.. Known way of avoiding NN theorem is to intoduce..

- Non-locality
- Non-Hermiticity
- Embeding on higher-dimensions

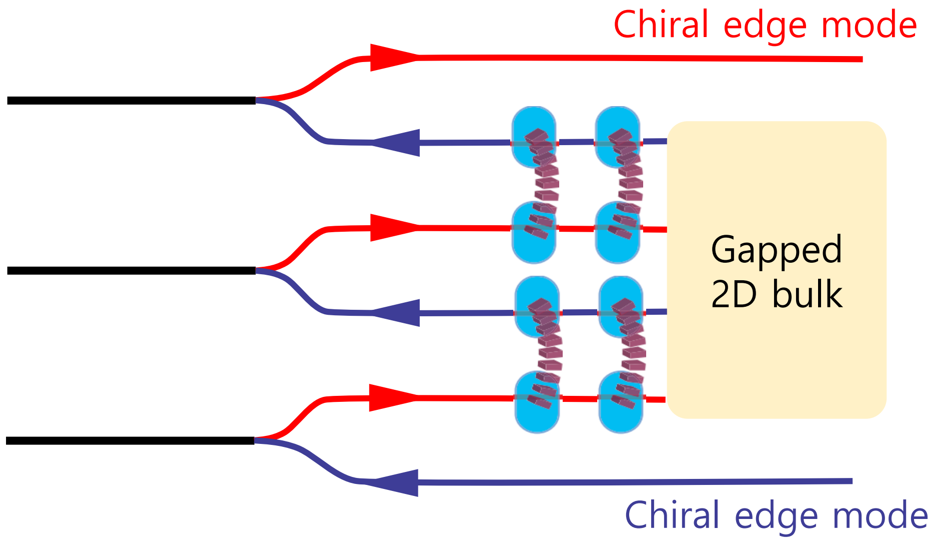
  (Topological insulators)



- **Intra-wire coupling (trivial)**

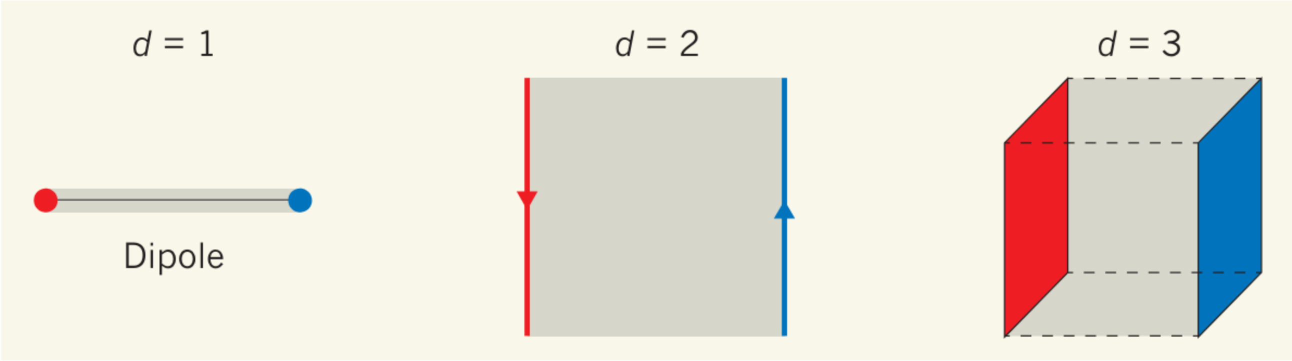


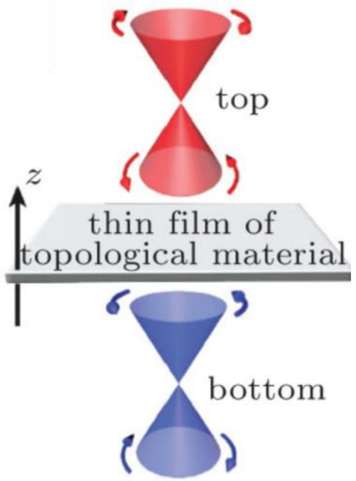
- **Inter-wire coupling (Non-trivial)**



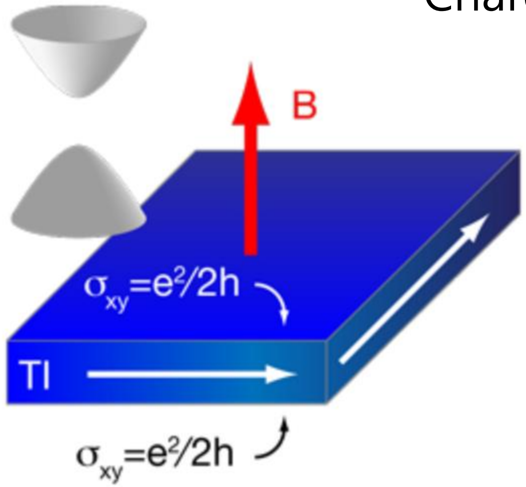
분수화된 양자홀 효과

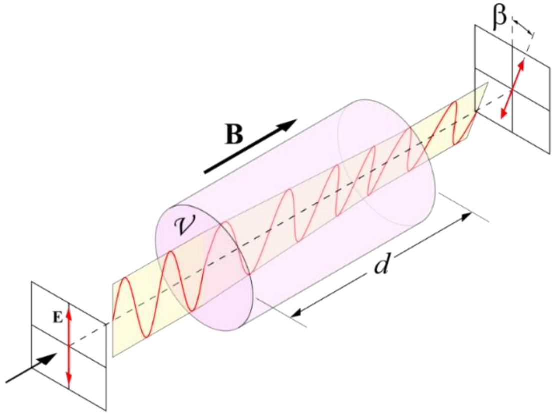
Fractionalization in physical degree of freedom





Charge의 분수화 Chirality의 분수화 Helicity의 분수화





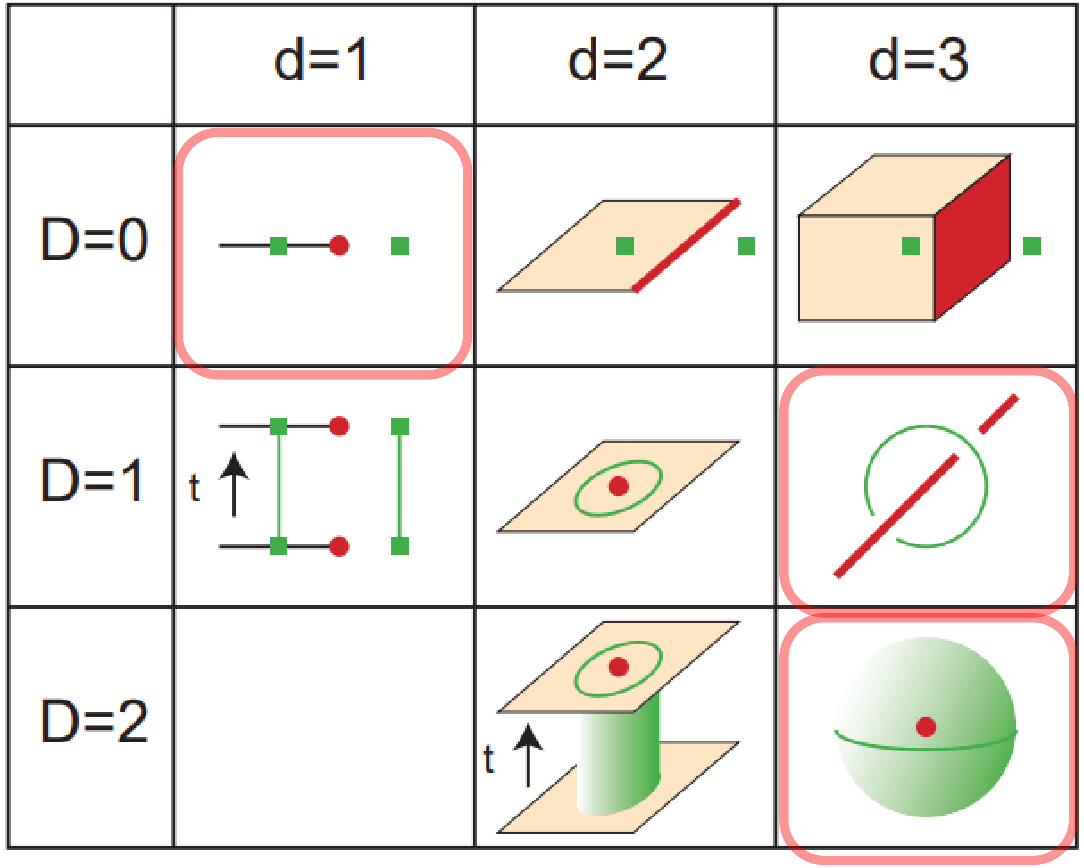
$$\tan(\phi_F) = \frac{2\alpha}{1+n}(N_t + \frac{1}{2} + N_b + \frac{1}{2})$$

3D TI의 한 면의 한 개의 디락콘은 분수화된 양자홀 효과를 만든다.

위상학적 물질의 분류

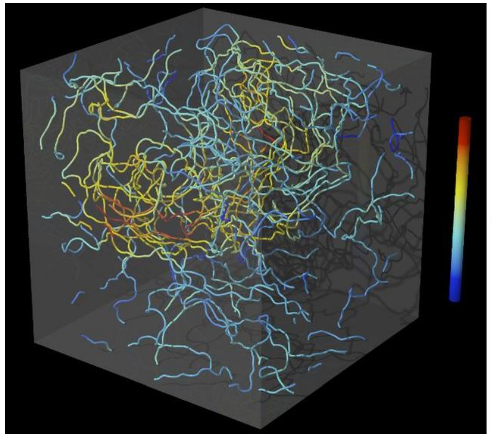
General considerations

- Generalization to various dimensions



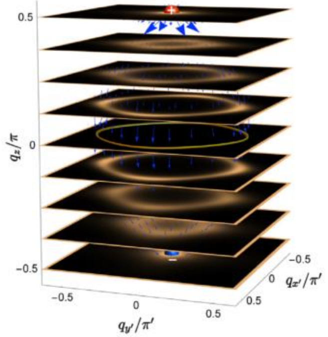
Teo-Kane

Quantum turbulence



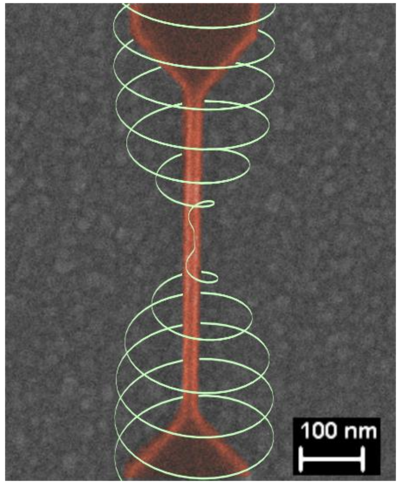
Baggaley group

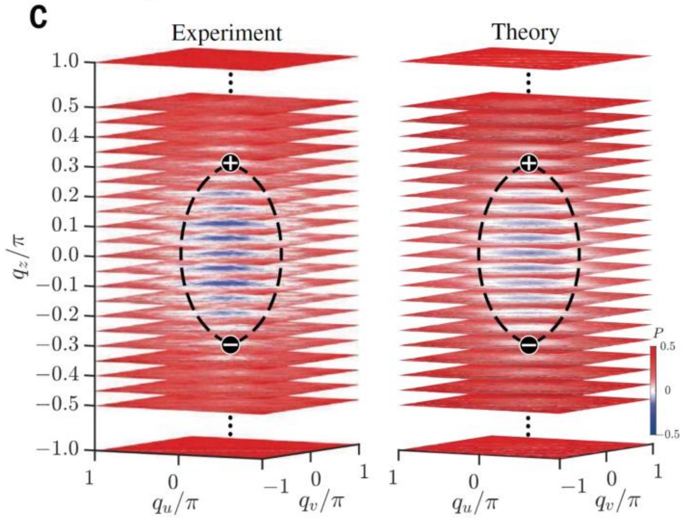
Weyl fermion



Liu,Pan group

SC phase slip

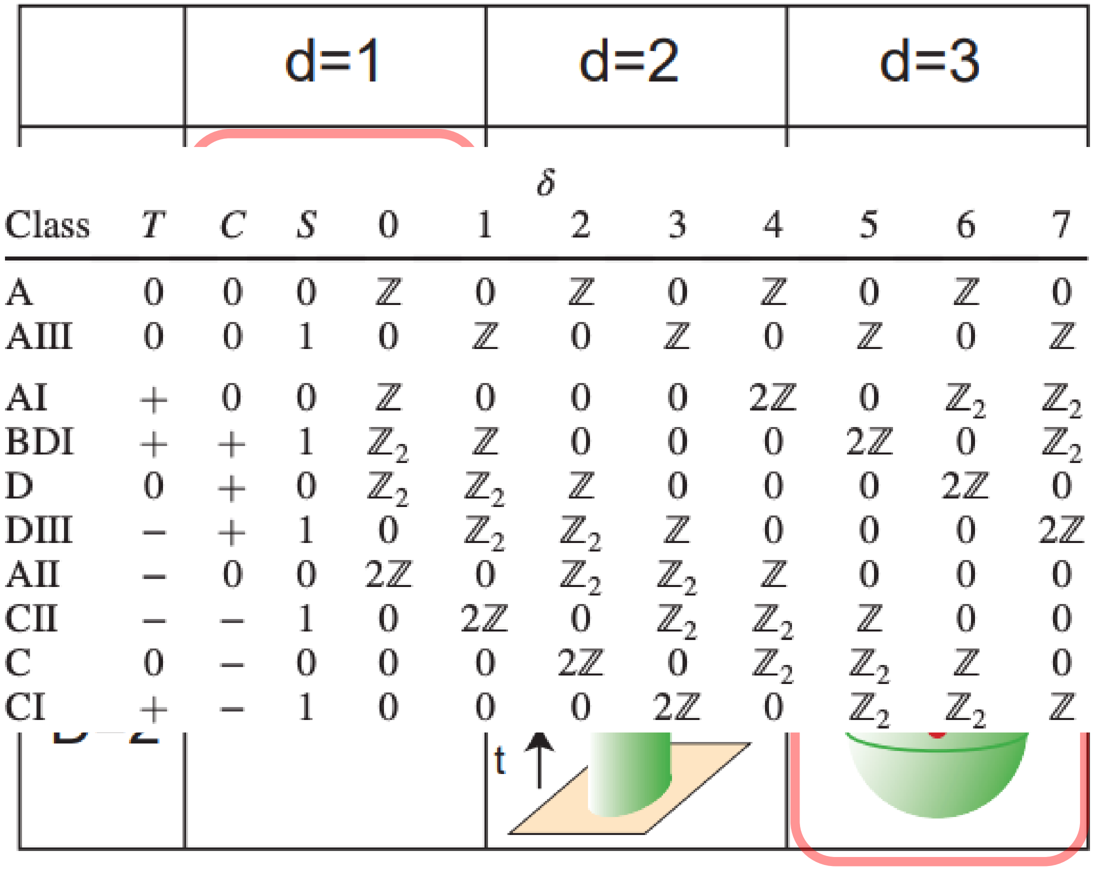




Johannes Rotzinger group

General considerations

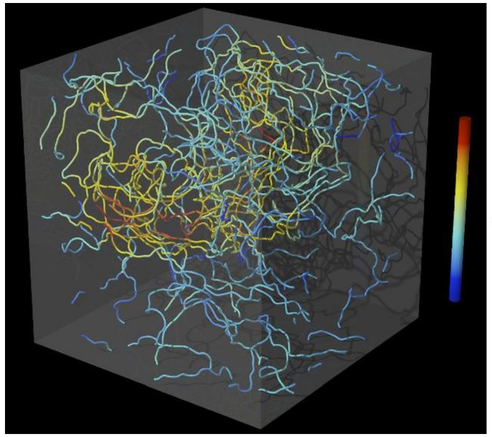
➢ Generalization to various dimensions



Class	T	C	S	δ 0	1	2	3	4	5	6	7
A	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AIII	0	0	1	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AI	+	0	0	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
BDI	+	+	1	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$
D	0	+	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$	0
DIII	−	+	1	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$2\mathbb{Z}$
AII	−	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
CII	−	−	1	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
C	0	−	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
CI	+	−	1	0	0	0	$2\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

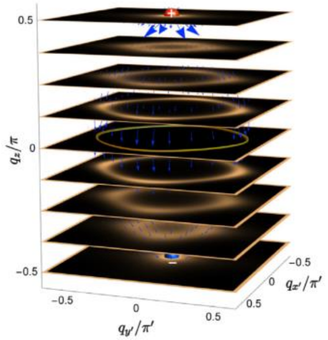
Teo-Kane

Quantum turbulence



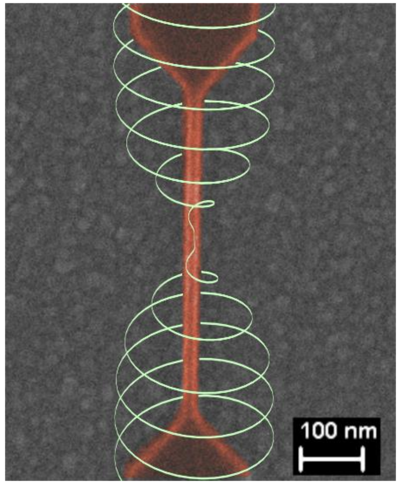
Baggaley group

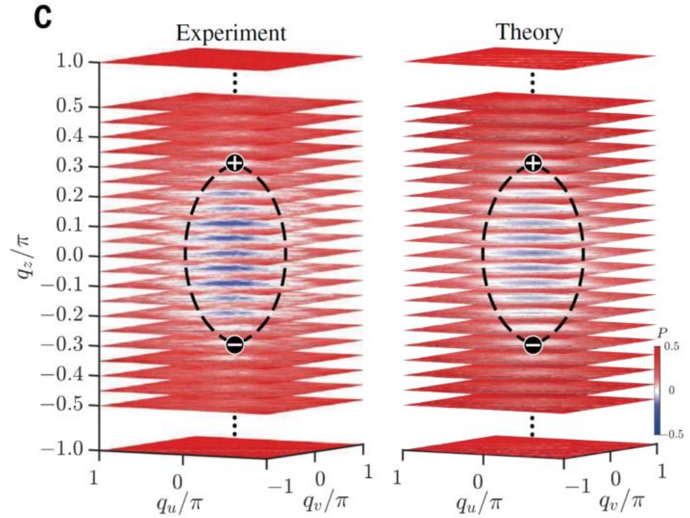
Weyl fermion



Liu,Pan group

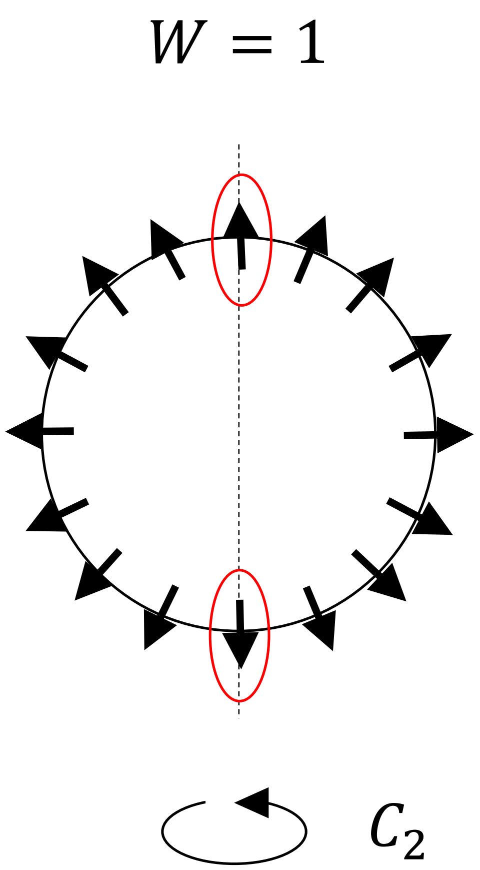
SC phase slip

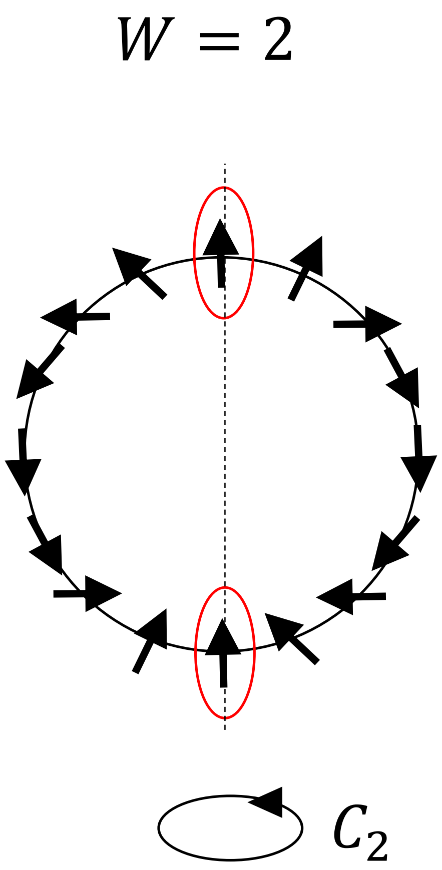




Johannes Rotzinger group

Symmetry indicator





<u>공간 대칭성이 있다면 한 개의 점만으로 위상이 결정된다.</u>

Summary

- **1D TI**
- **2D TI**
- **3D TI**
- **TSM**

- <u>양자화된 편극밀도</u>
- <u>양자화된 홀전도도</u>
- <u>양자화된 자기광 패러데이/커 효과</u>
- <u>양자 아노말리</u>

Out of 26938 stoichiometric materials, 3307 topological insulators and 4078 topological semimetals and 0 fragile phases were found . For these 7385 materials the electronic bands structure – in-