위상 물질과 양자 수송

중시계 여름학교
2024. 05. 22.

박문집
한양대학교 물리학과

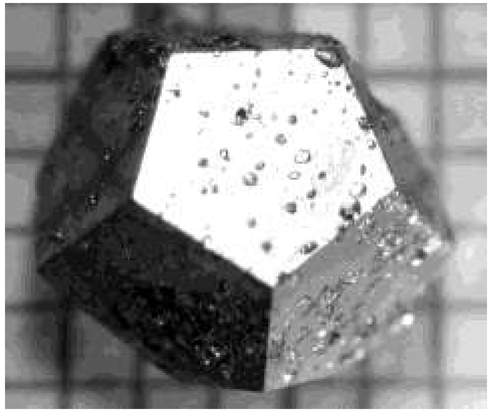
한양대학교
HANYANG UNIVERSITY

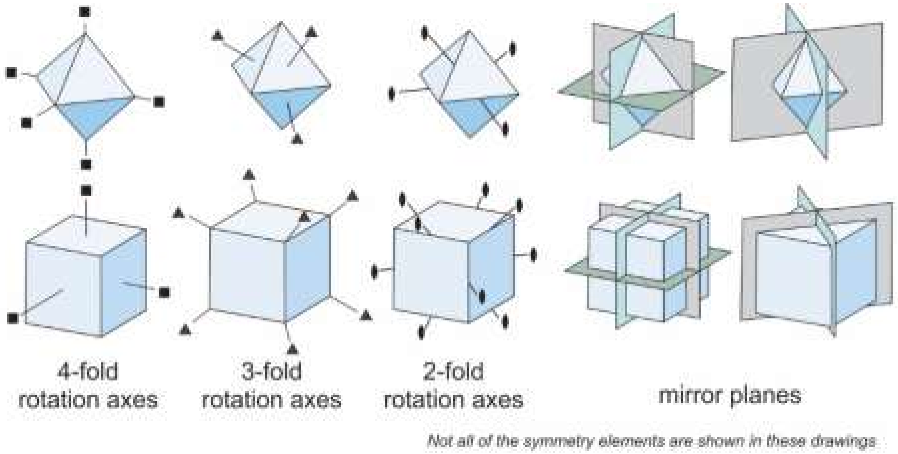

NETFLIX
더 글로리
지난 줄거리

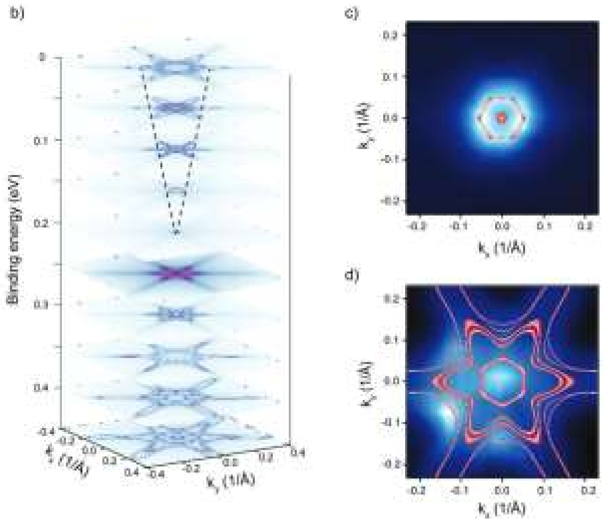
Symmetry classifications of matter

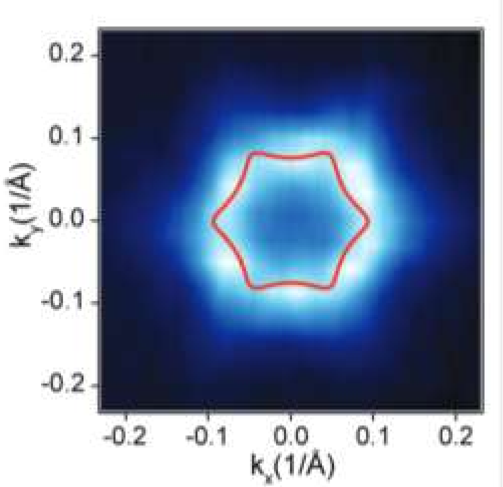
Symmetry classifications of matter

➢ <u>**Symmetries in crystals :**</u>







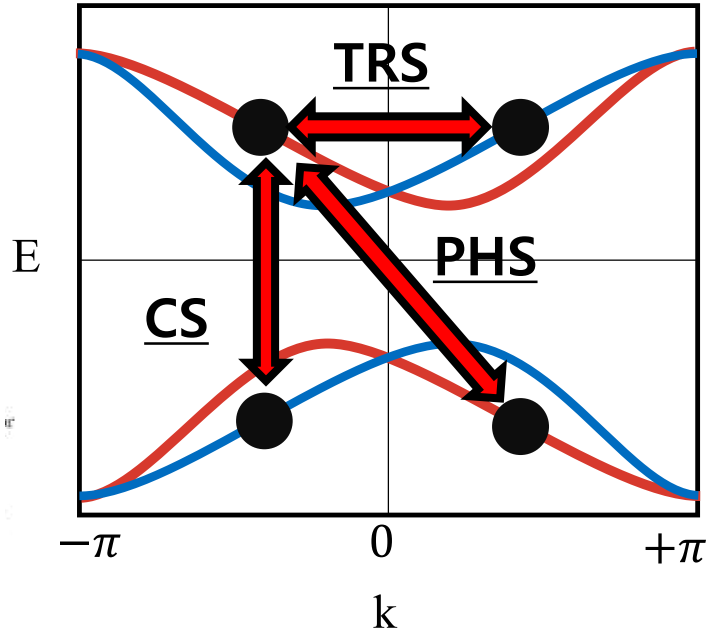


Mn doped Bi2Te3
Q. Li, Advanced materials (2022)

Anti-unitary symmetry

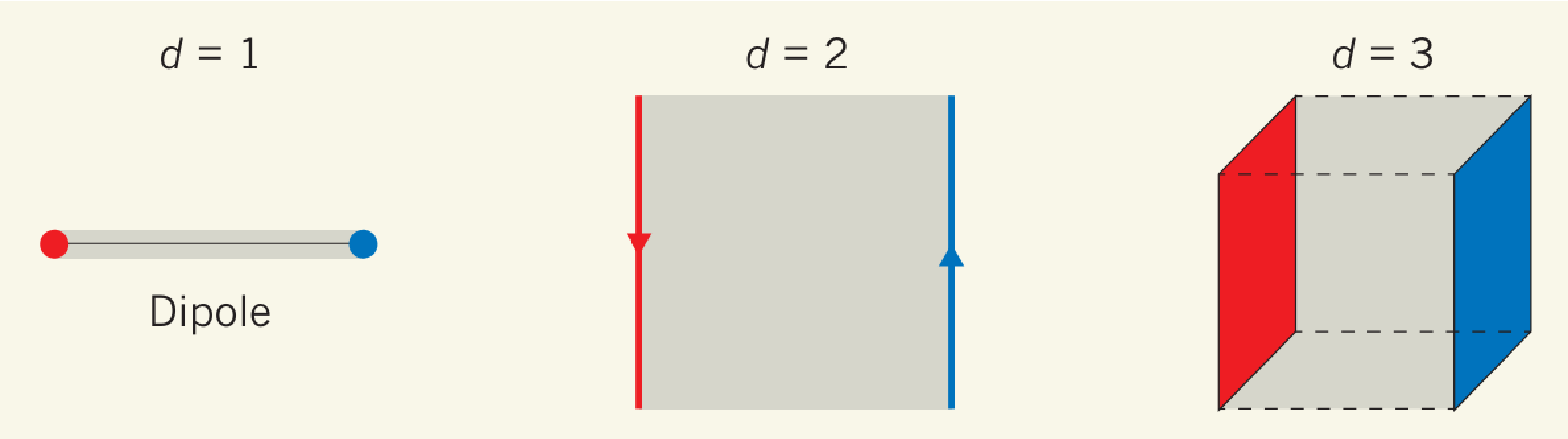
Anti unitary symmetry :

$$\phi \to -\phi \quad (\Psi = e^{i\phi})$$



10-fold classification Altland-Zirnbauer classification.

지난 줄거리



- **1D TI**
- **2D TI**
- **3D TI**

- 양자화된 편극밀도
- 양자화된 홀전도도
- 양자화된 자기광 패러데이/커 효과

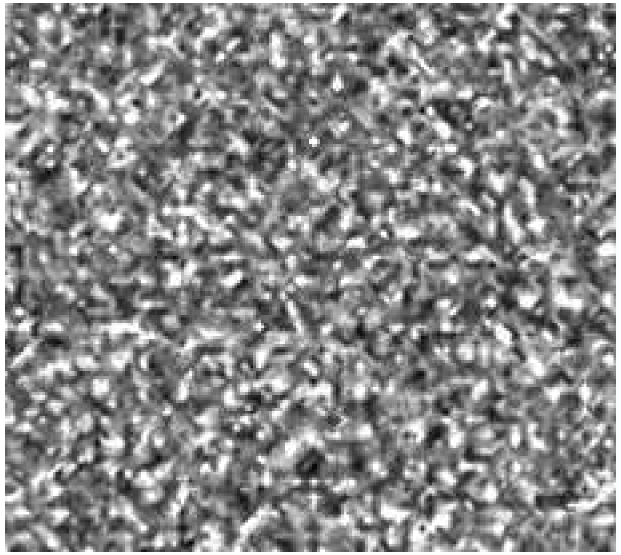
Outlook

- 일반적인 양자 수송
- 위상학적인 양자 수송 (양자 홀 효과)
- 그래핀의 양자 수송
- 실습

Topology
as
Vortex

More is different

- Everything is made up of the same atoms and electrons **at short length scales.**
- Different correlations and patterns can emerge **at longer length scales.**



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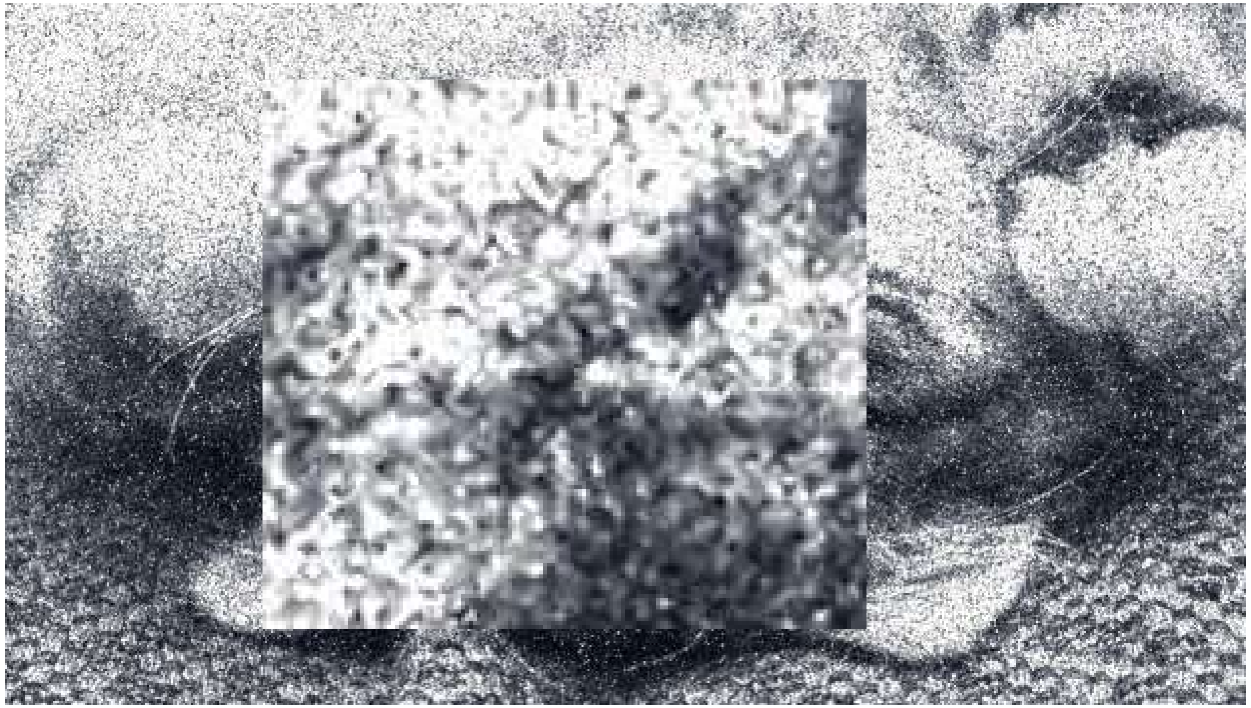
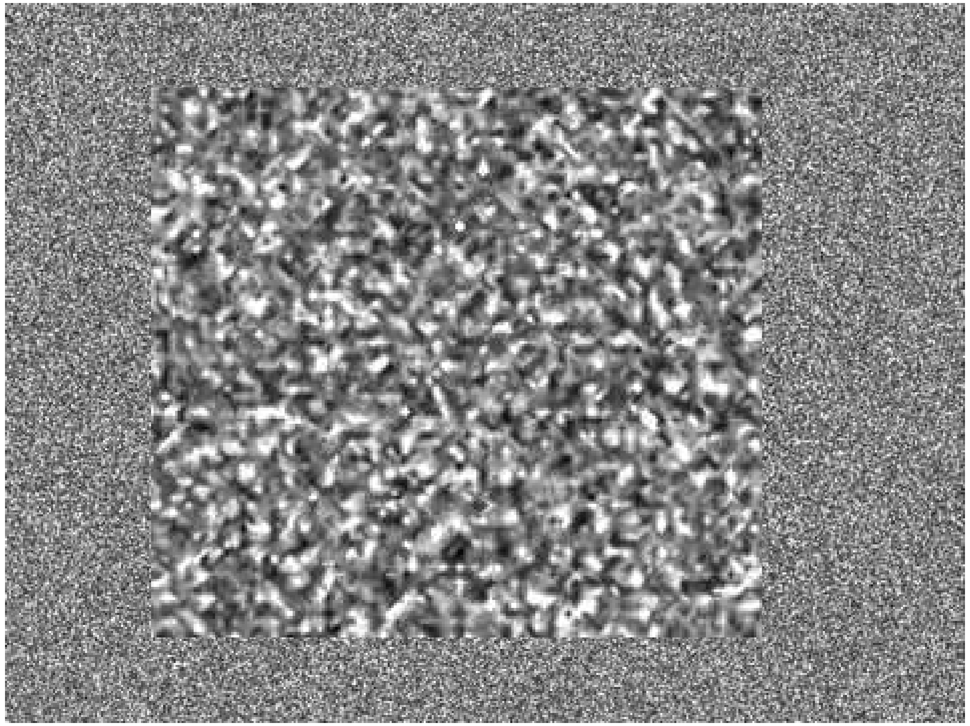




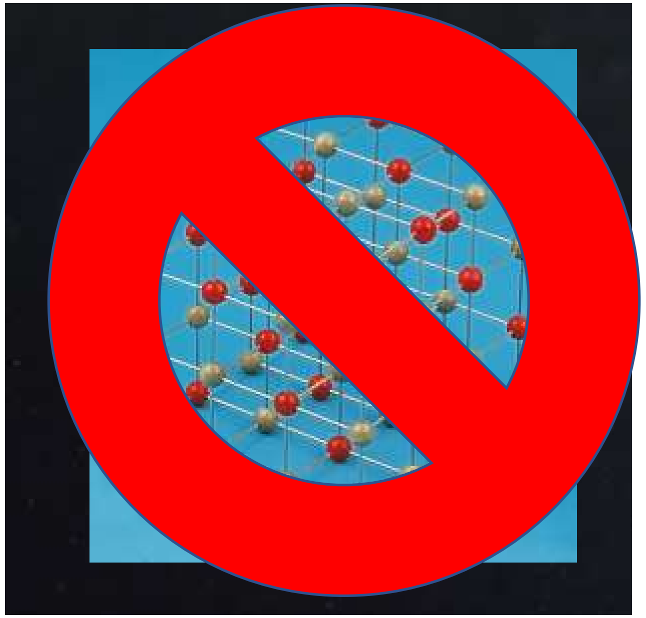
Physics of ~~local~~ correlations

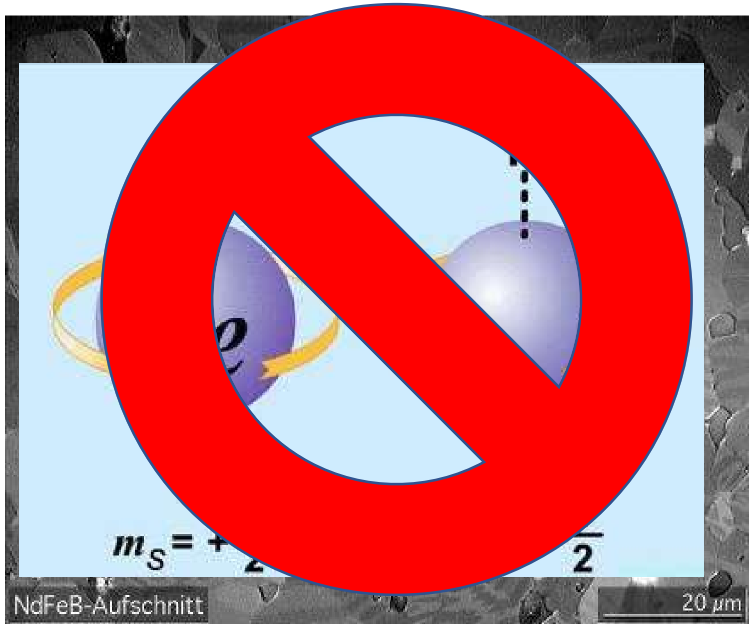
Non-local

Solid state lattice

Magnetic Domains

Self-organization

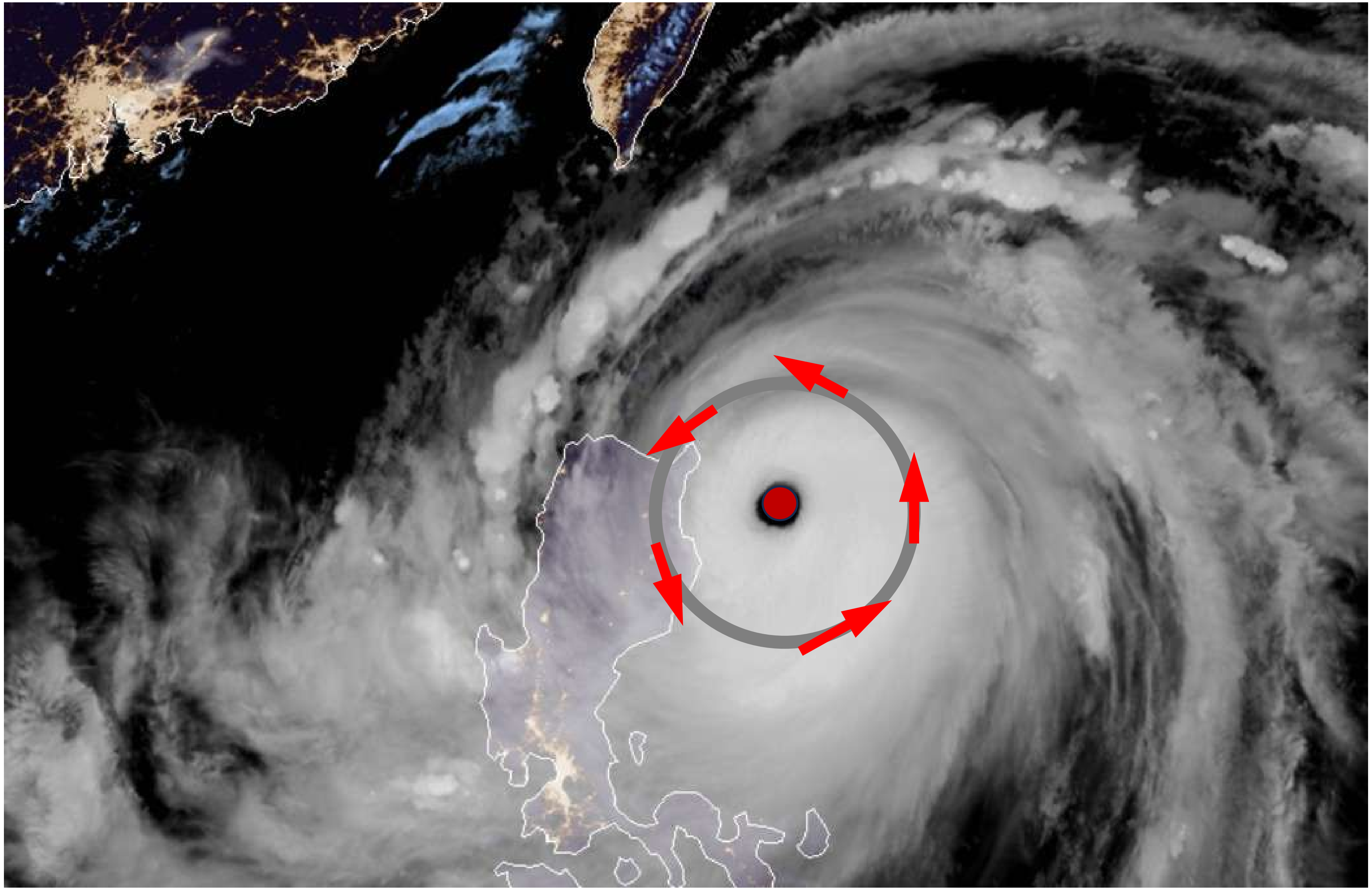




Paradigm of Landau theory:

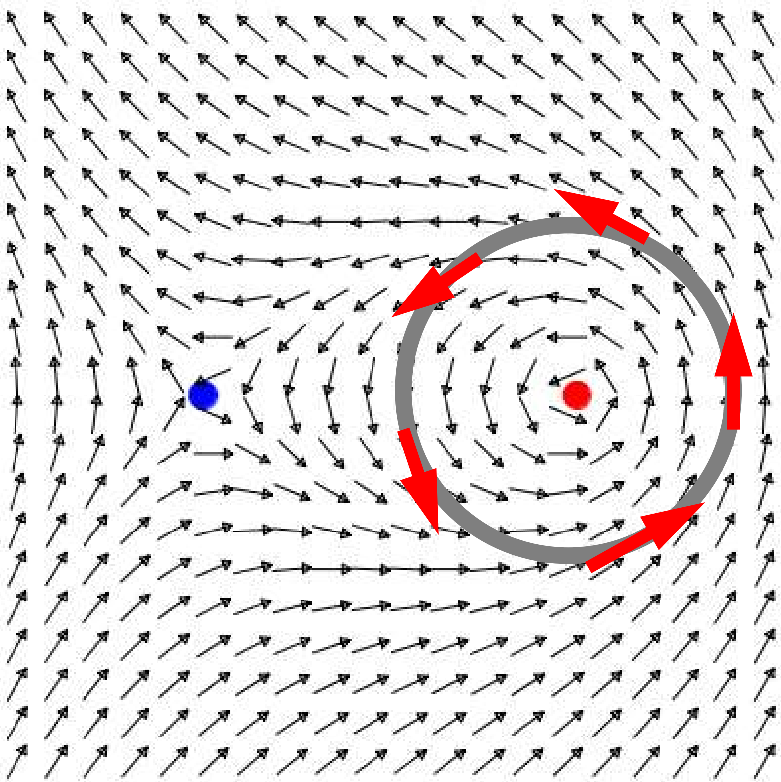
Patterns/Correlations of local order parameters determines **the phases of matters.**

Vortex



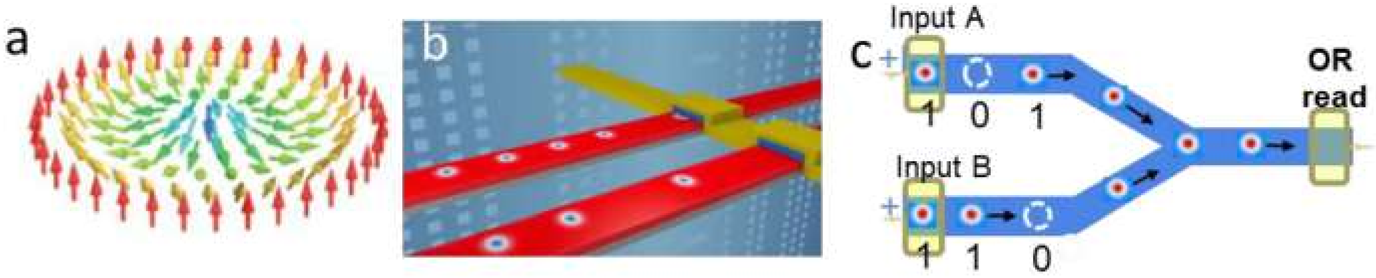
Vortex

➢ **Vortex in spin model**



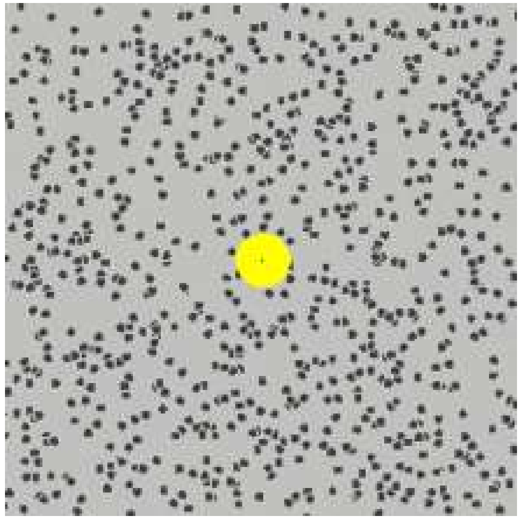
$$H = -\sum_{\langle ij \rangle} \cos(\theta_i - \theta_j)$$

➢ **스커미온 스핀트로닉스**



Conventional
Quantum transport

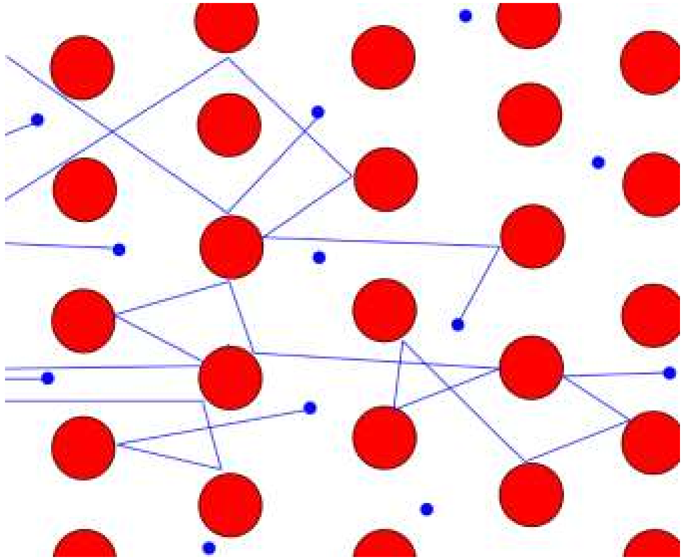
꽃가루의 확산



- 1827년 식물학자 - R. Brown이 꽃가루 입자의 난류 관찰
- 열적 요동의 가시화 - 아인슈타인이(1905) 미시적 분자의 존재를 정량적으로 증명
- 확산운동의 예시 - $\langle r \rangle = Dt^2$

$$D = \frac{k_B T}{\gamma}$$

전자의 확산



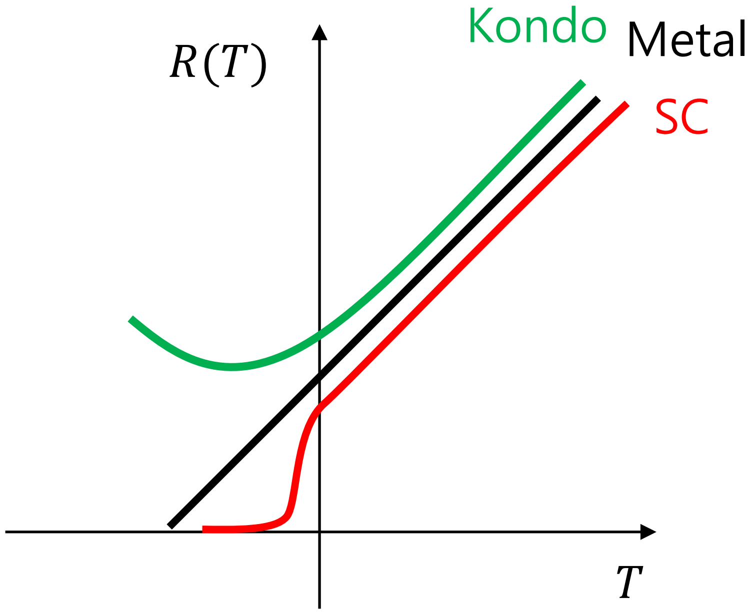
- Drude가 금속을 ‘자유전자 기체’로 모델링

→ 옴의 법칙과 Wiedemann-Franz 법칙의 미시적 기원 제시

- 산란 시간 τ 도입 – 전기전도도 $\sigma = ne^2\tau/m$,

- 고전적 픽처의 한계 – 낮은 온도에서 전자 기체의 양자성 필요

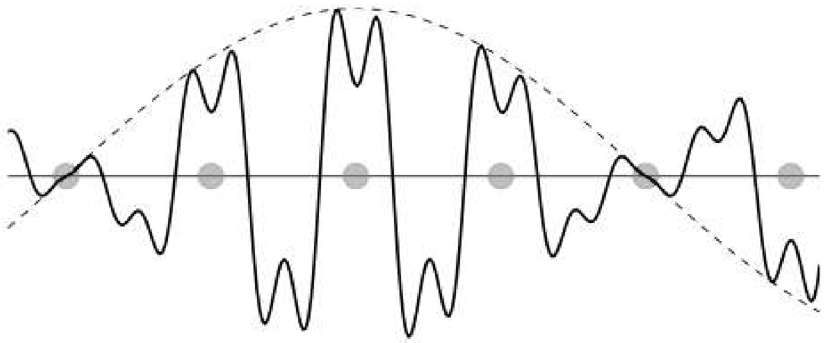
고전적 수송의 한계



- Drude 모델에 따르면 0K에서 유한한 저항이 발생함
- 그러나 완벽하게 깨끗한 금속은 저항이 0임

$$R(T) = R_0(1 + \alpha T)$$

파동의 확산

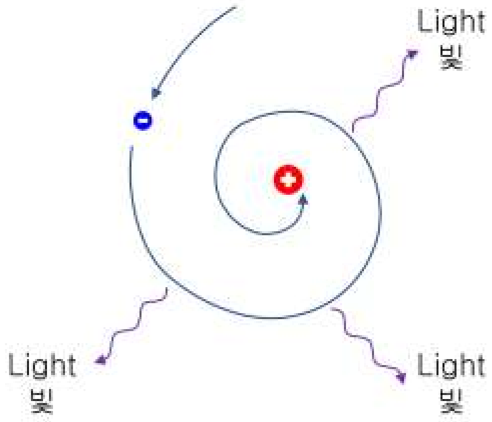


• Bloch 정리 – 주기적인 퍼텐셜에서 전자는 안정적인 상태로 존재함.
→ 저항이 0이 되는 사실을 설명

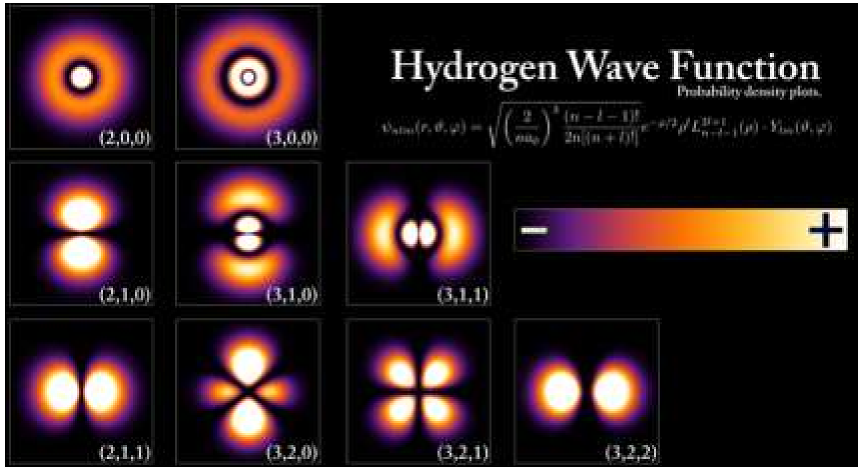
• 크리스탈 운동량 k의 보존 – 밴드 이론의 기초

수송이론의 수소모델

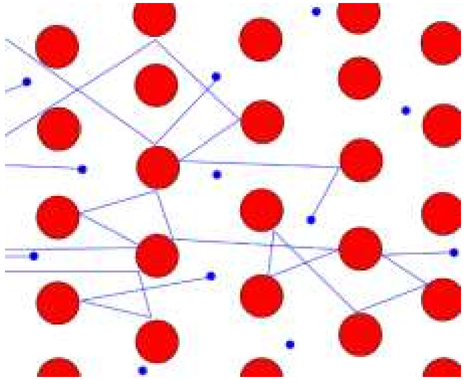
러더포드 원자모형



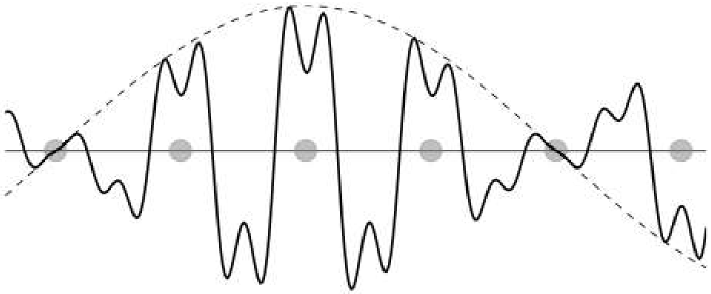
양자 수소 모형



Drude 모델



Bloch 모델



안정된 수송 궤도 -> Bloch 이론은 수송이론의 수소모형!

양자화된 전도도

$$\frac{dn}{d\epsilon} = \frac{2}{hv}$$

$$j = -ev(\mu_1 - \mu_2)\frac{dn}{d\epsilon}$$

$$V = -\frac{(\mu_1 - \mu_2)}{e}$$

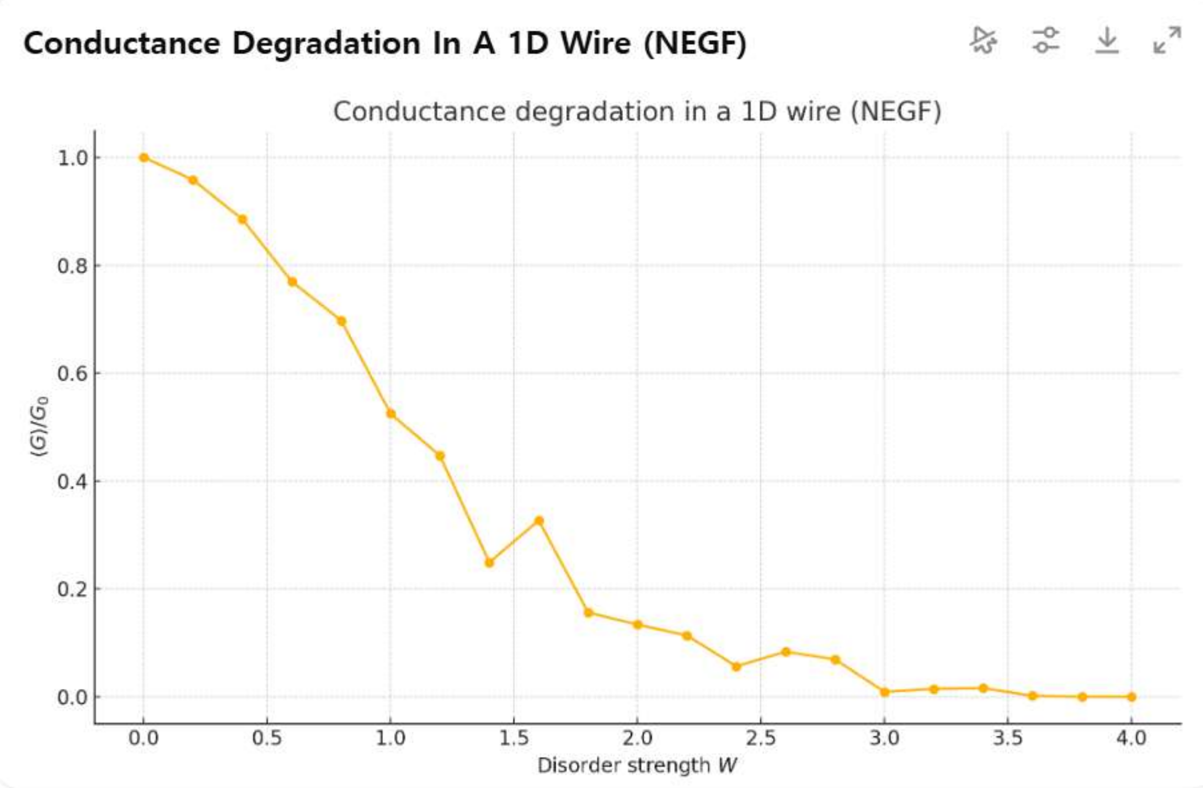
$$G_0 = \frac{I}{V} = \frac{j}{V} = \frac{2e^2}{h}$$

양자 전도는 채널당 $G_0 = \frac{2e^2}{h}$로 양자화 됨!

불순물이 존재하는 양자수송

- ChatGPT를 이용한 MATLAB코드 (렉처노트 참고)

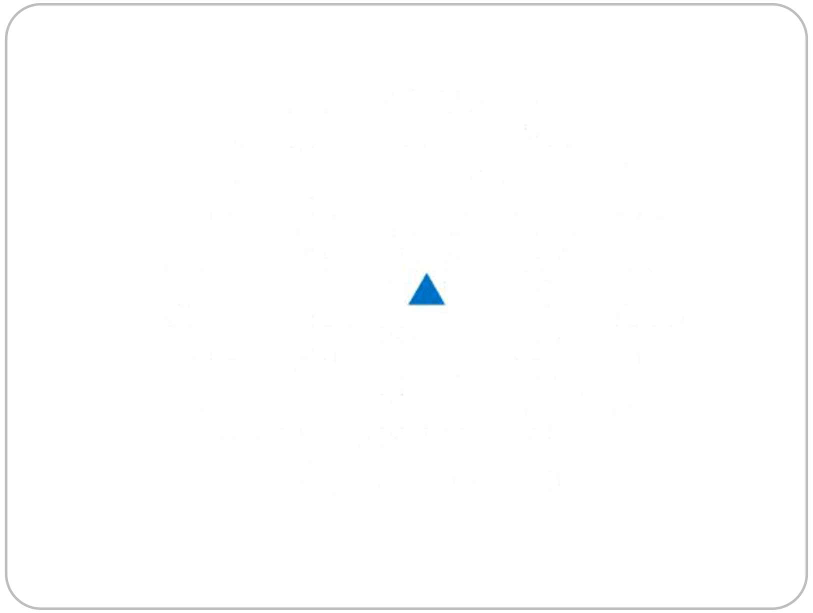
NEGF로 1D WIRE의 CONDUCTANCE를 계산하는데 디스오더를 넣어가면서 CONDUCTANCE QUANTUM 에서부터 DEGRADE되는걸 보여줘

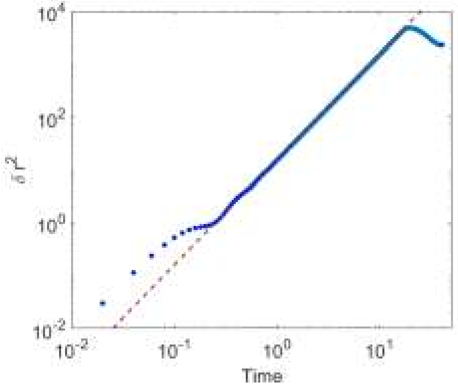


파동의 확산 수송

무산란 수송(ballistic transport)

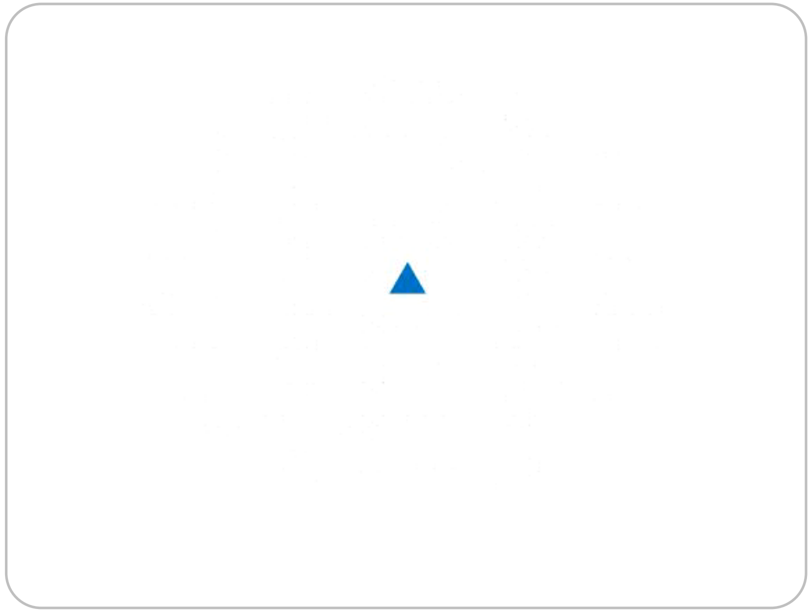
$\delta r^2(t) \sim vt^2$

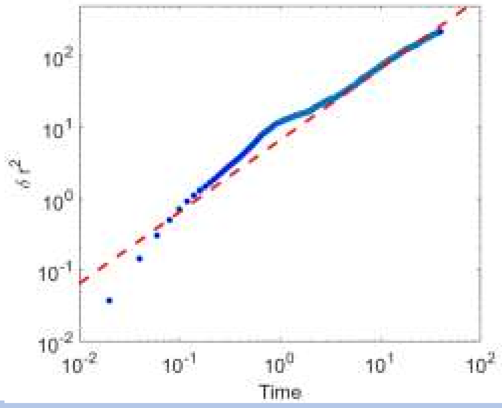




산란 수송(diffusive transport)

$\delta r^2(t) \sim Dt$





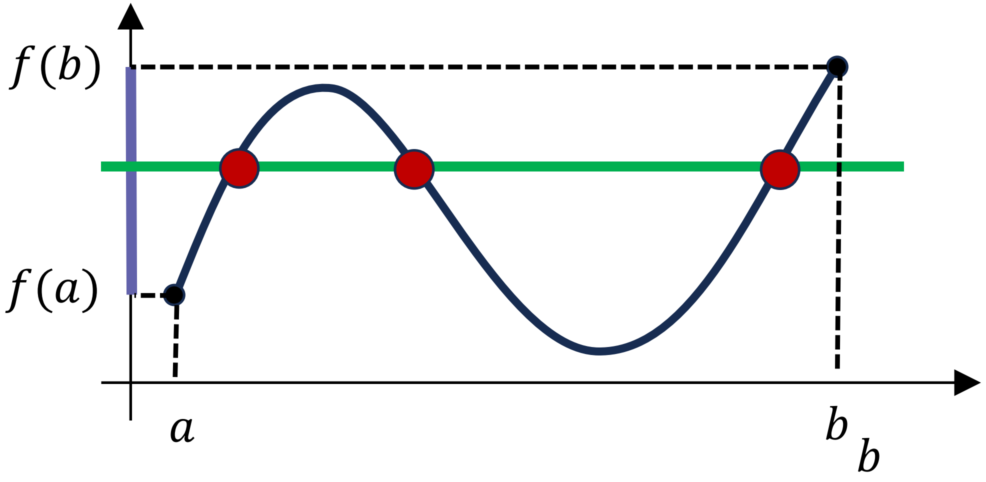
페르미온 더블링 문제

닐슨–니노미야 정리(Nielsen–Ninomiya no-go theorem)

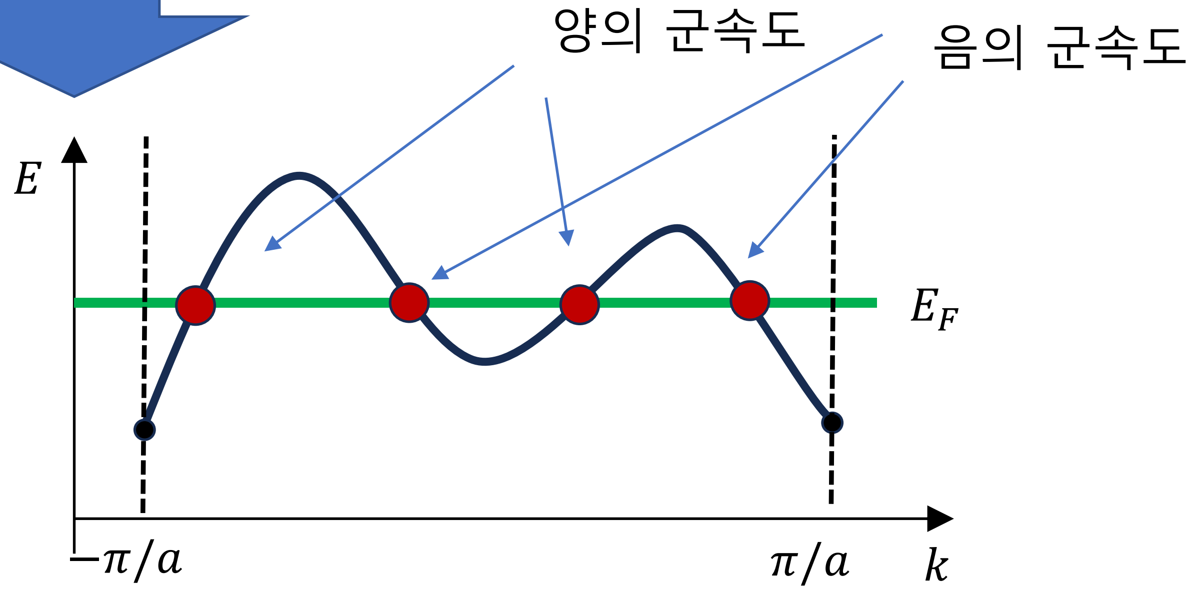
	내용
정리 한 줄 요약	유한한 격자(lattice)에서 국소적(local)·전이대칭(translational)·에르미트(Hermitian)한 디랙 연산자를 쓰면, chirality(손지향성)가 한쪽으로만 치우친 단일 Weyl 페르미온은 구현 불가능하다. 즉 Weyl 노드는 항상 짝수(동일 수의 왼·오른 chirality)로 나타난다.

페르미온 더블링 문제

중간값정리



NN 정리



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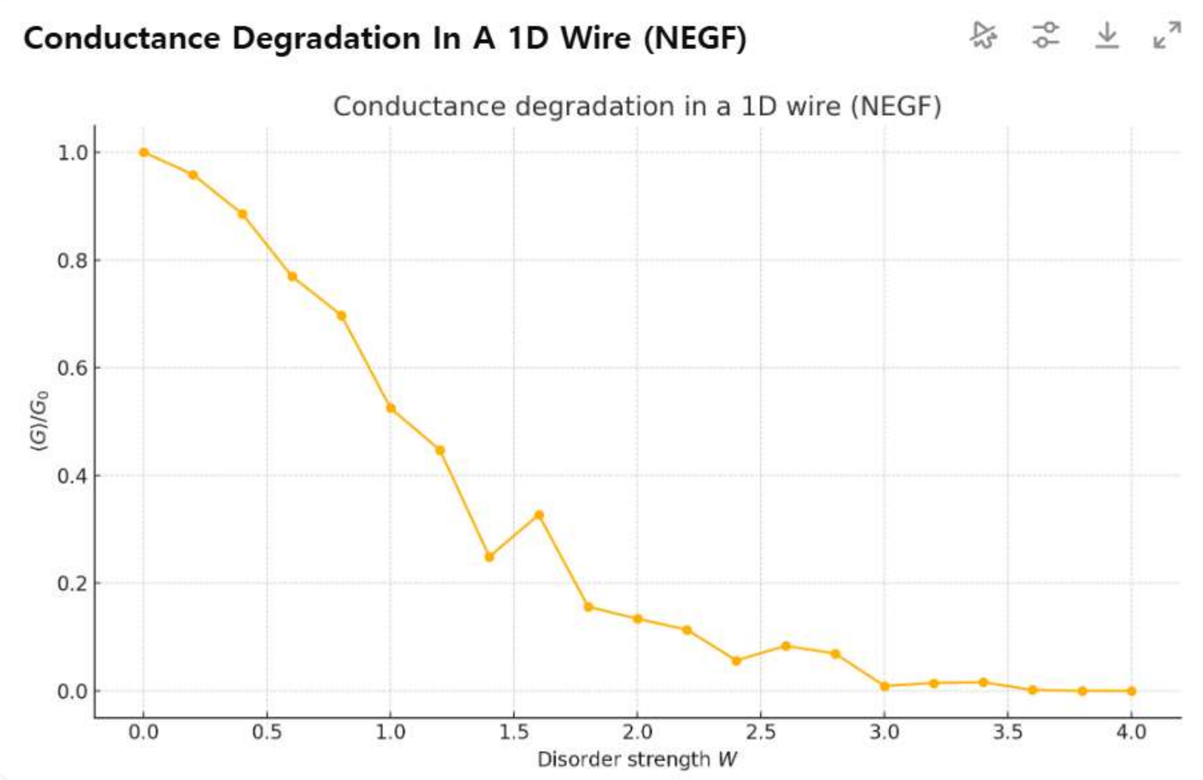
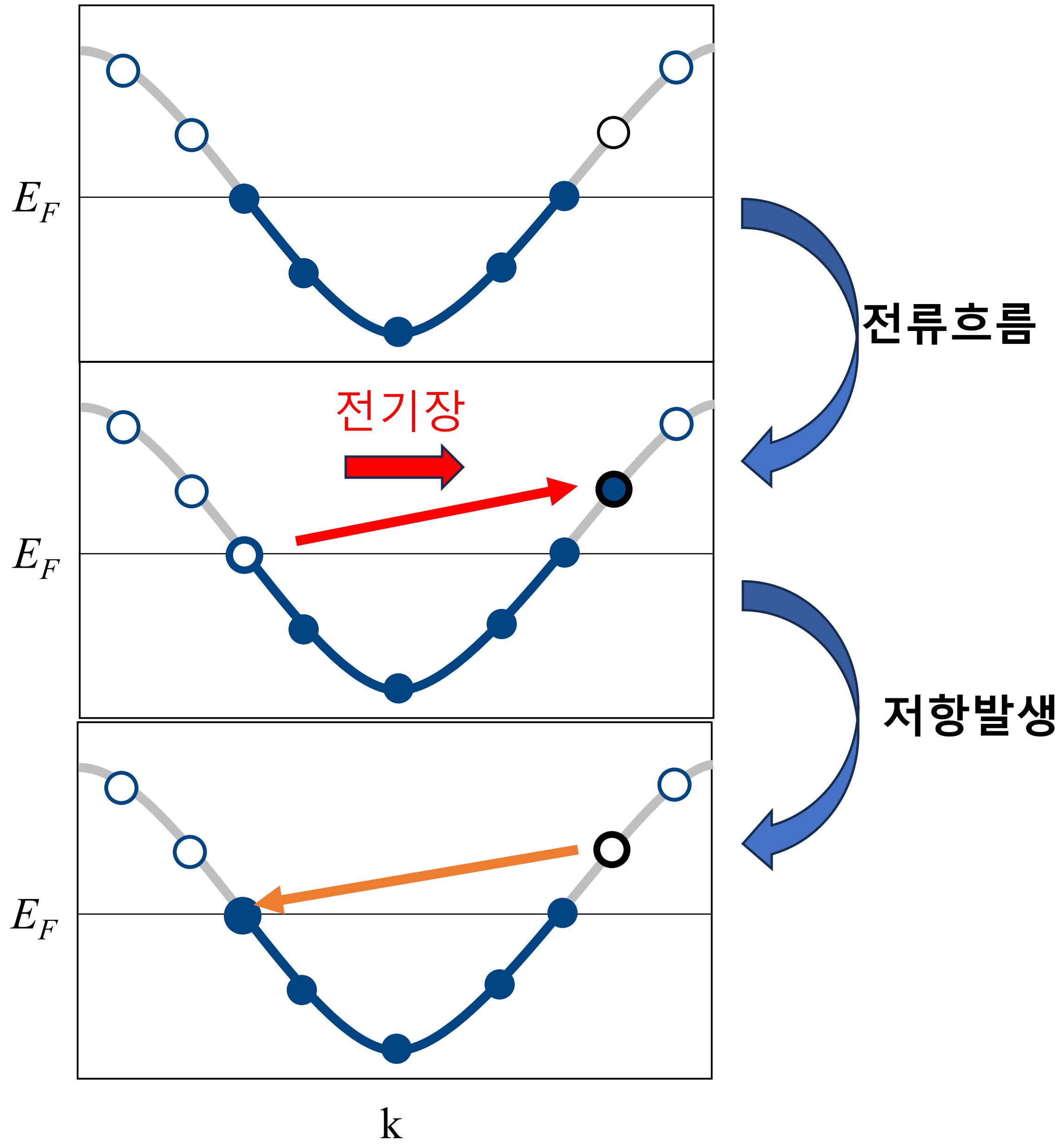
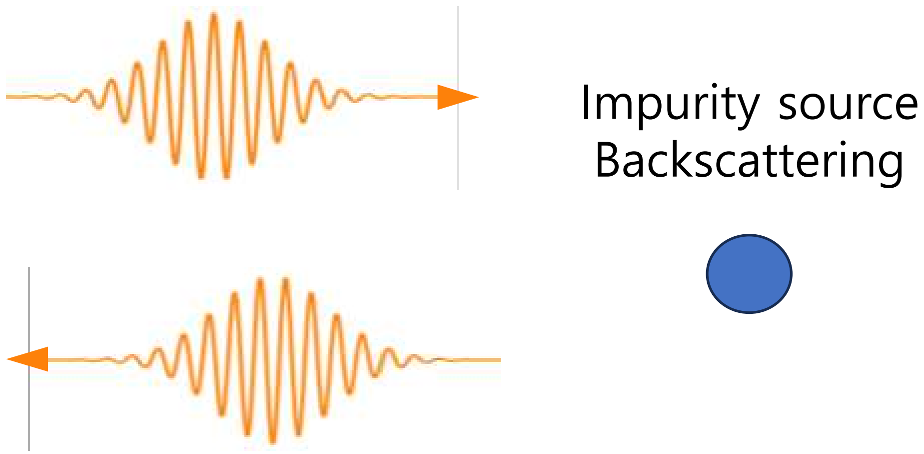
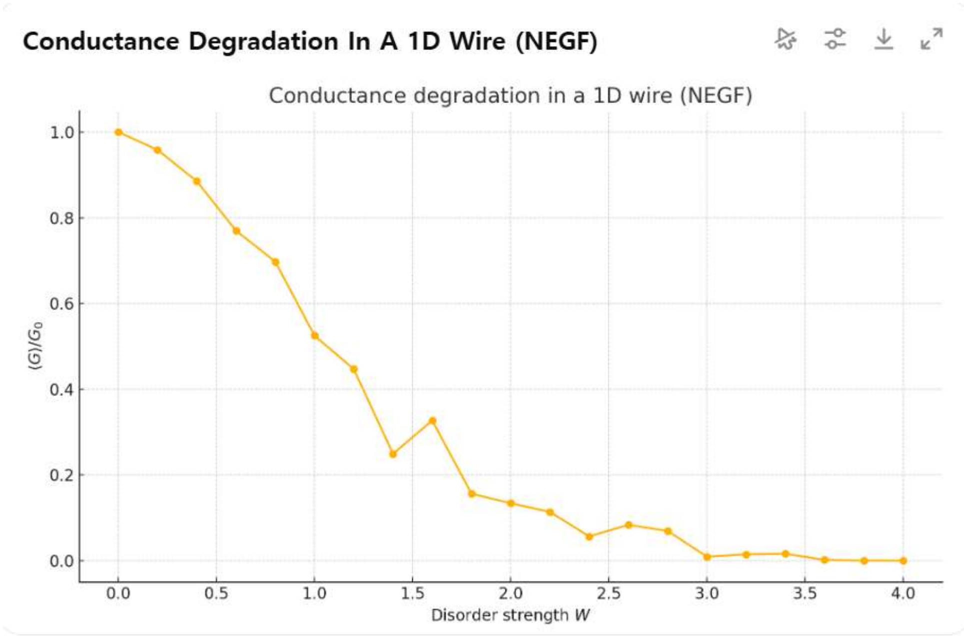


불순물이 존재하는 양자수송







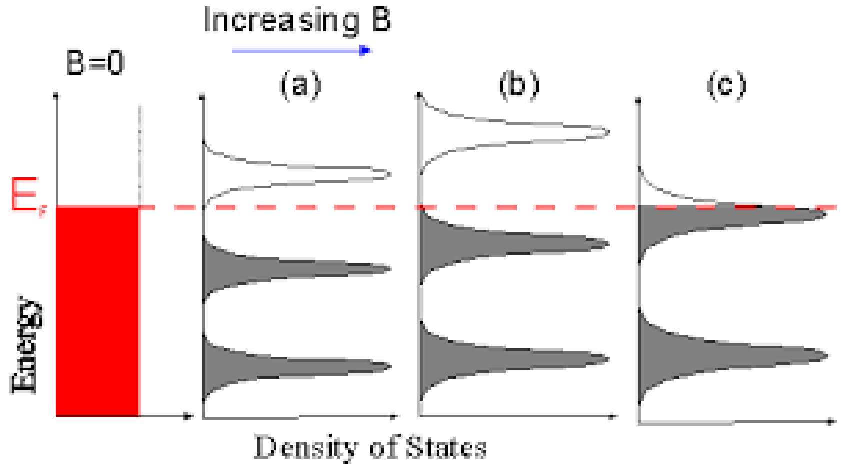
Topological
Quantum transport

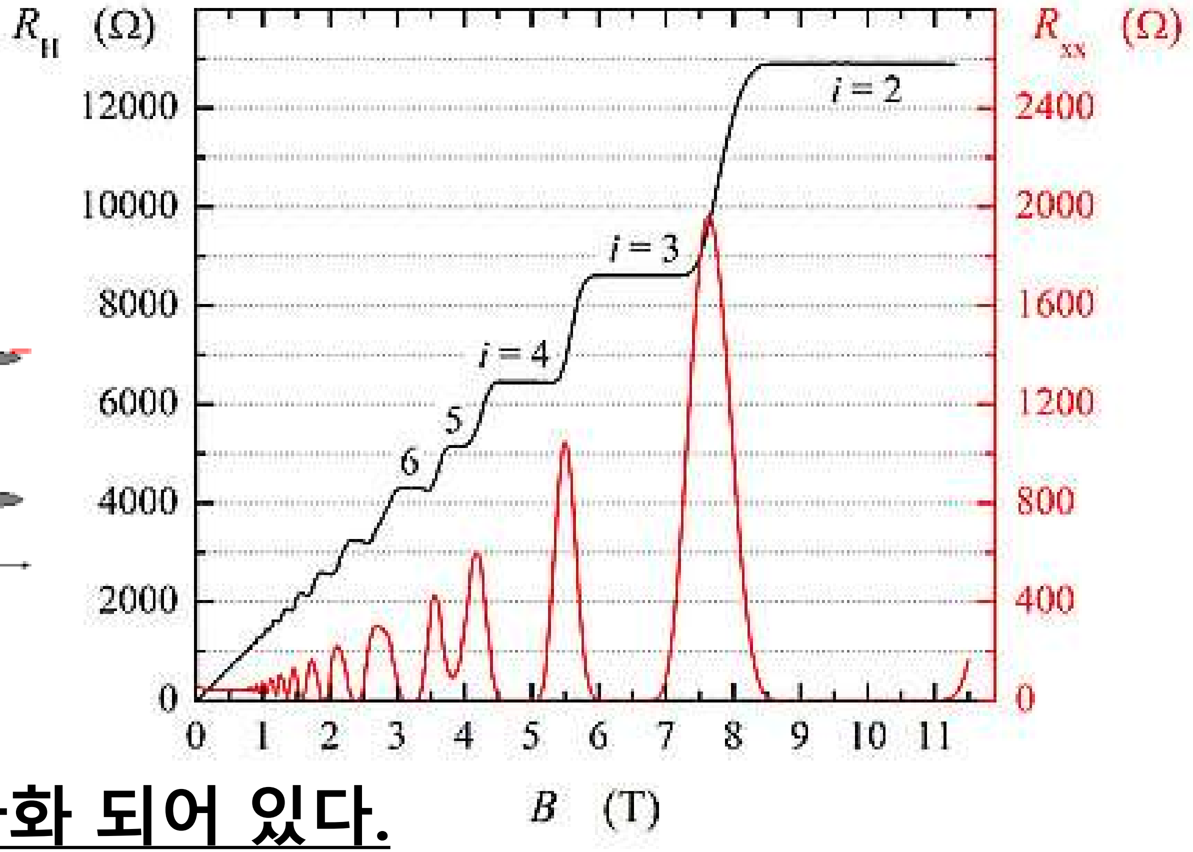
정수 양자홀 효과

"**New method** for **high-accuracy** determination of

the fine-structure constant based on **quantized**

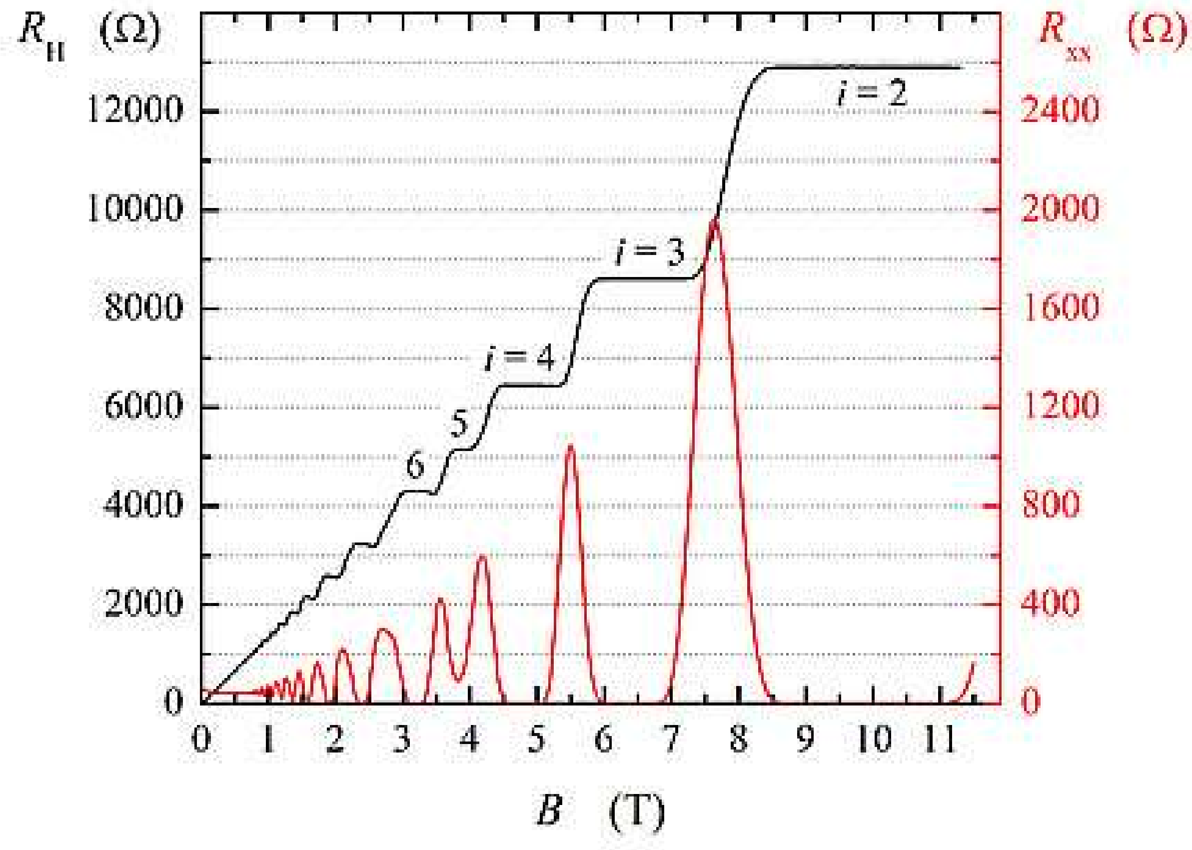
Hall resistance."





<u>**양자물질의 홀전도도는 양자화 되어 있다.**</u>

정수 양자홀 효과



- ChatGPT를 이용한 MATLAB코드 (렉처노트 참고)

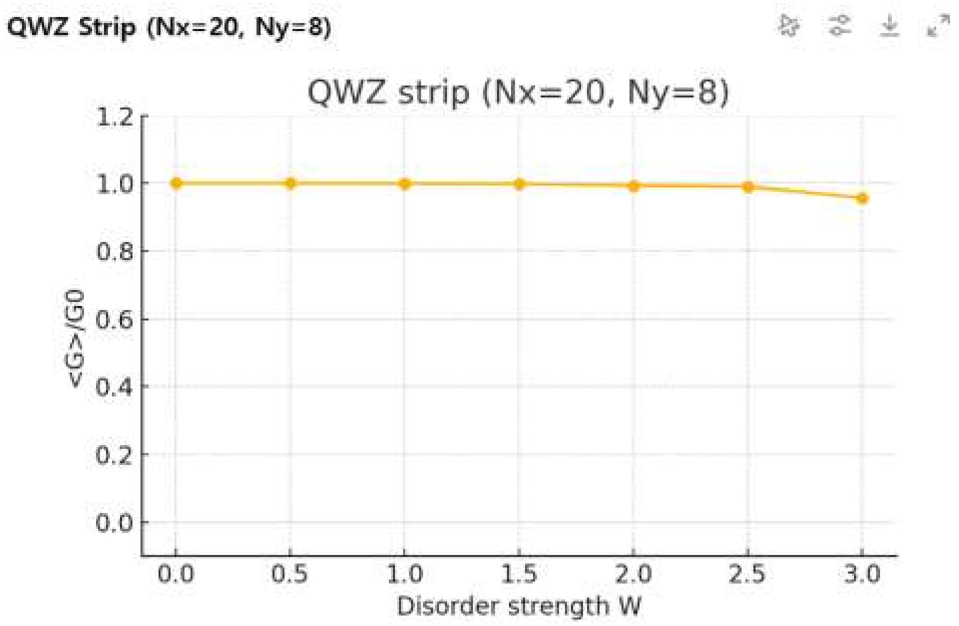
2차원 QI-WU-ZHANG 모델에 대해 똑같이할수있어 ?

너가 돌려봐

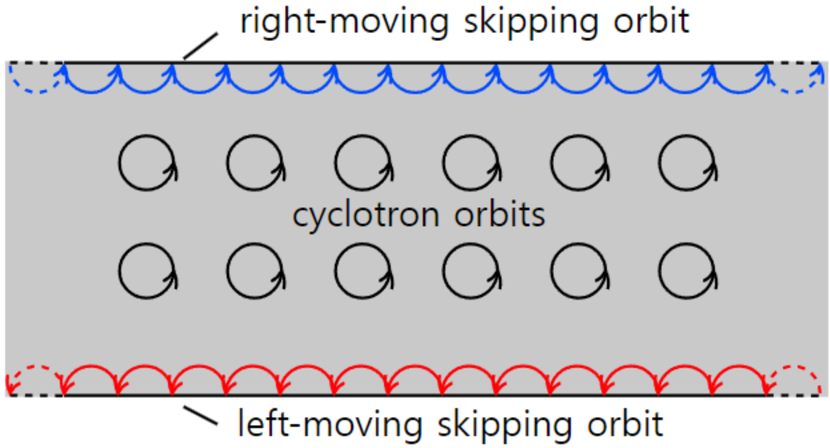
[redacted]

robust edege 모드에 의한 conductance protection이 안보이는데

계산결과 보여줘 그렇게되게해서



NN 정리의 실패



	Band gap
Edge(Left-mover)	closed
Bulk (insulator)	opened
Edge(right-mover)	closed

닐슨–니노미야 정리(Nielsen–Ninomiya no-go theorem)

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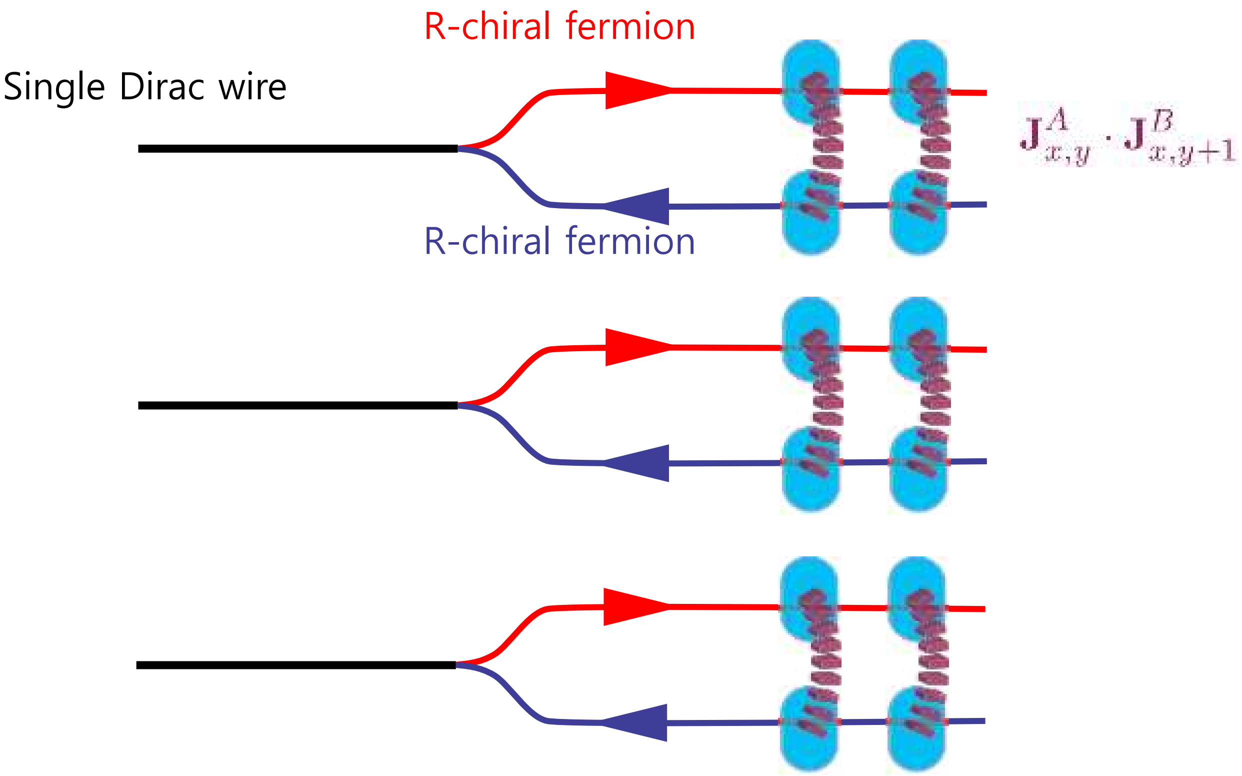
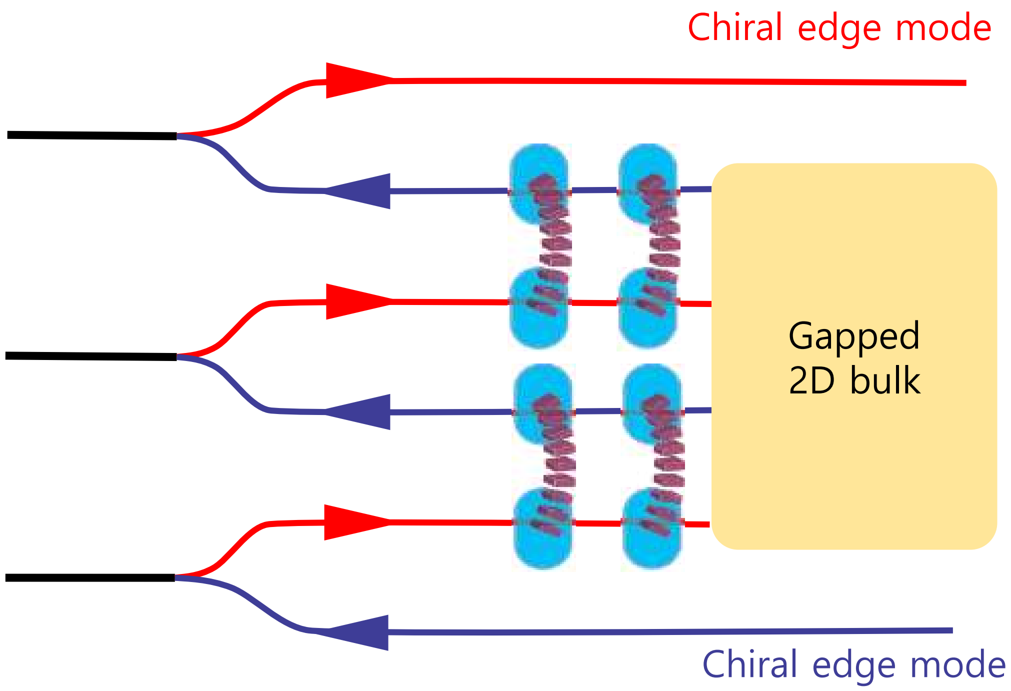
- ❖ Non-locality
- ❖ Non-Hermiticity
- ❖ Embeding on higher-dimensions (Topological insulators)

NN 정리의 실패

➢ **일반 부도체**

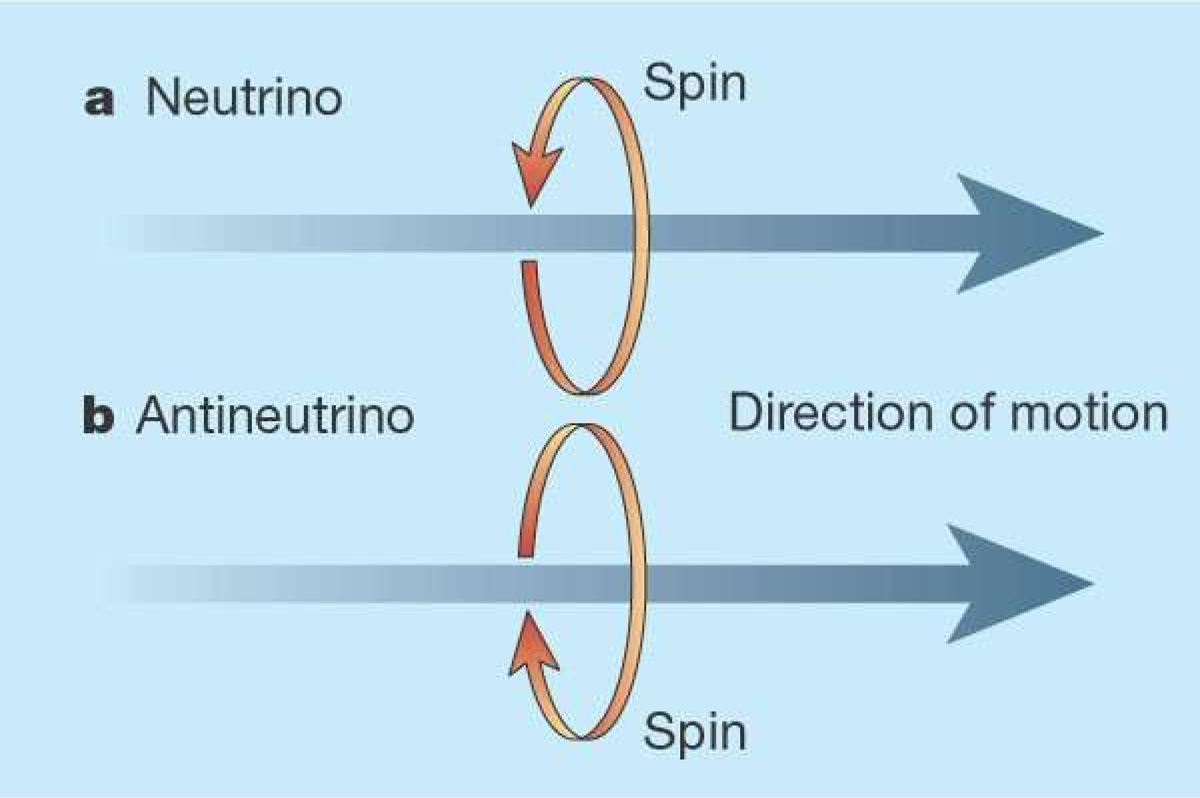


➢ **양자 홀 부도체**



Graphene
as
Topological material

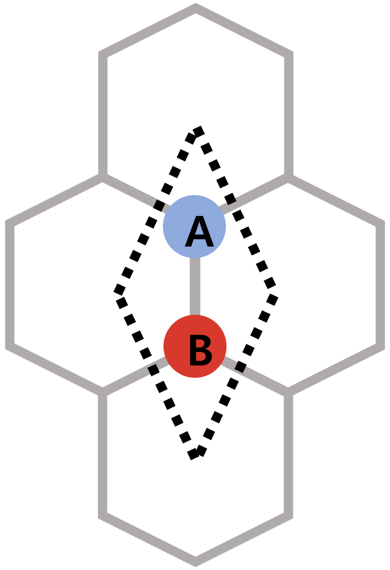
손지기 성질

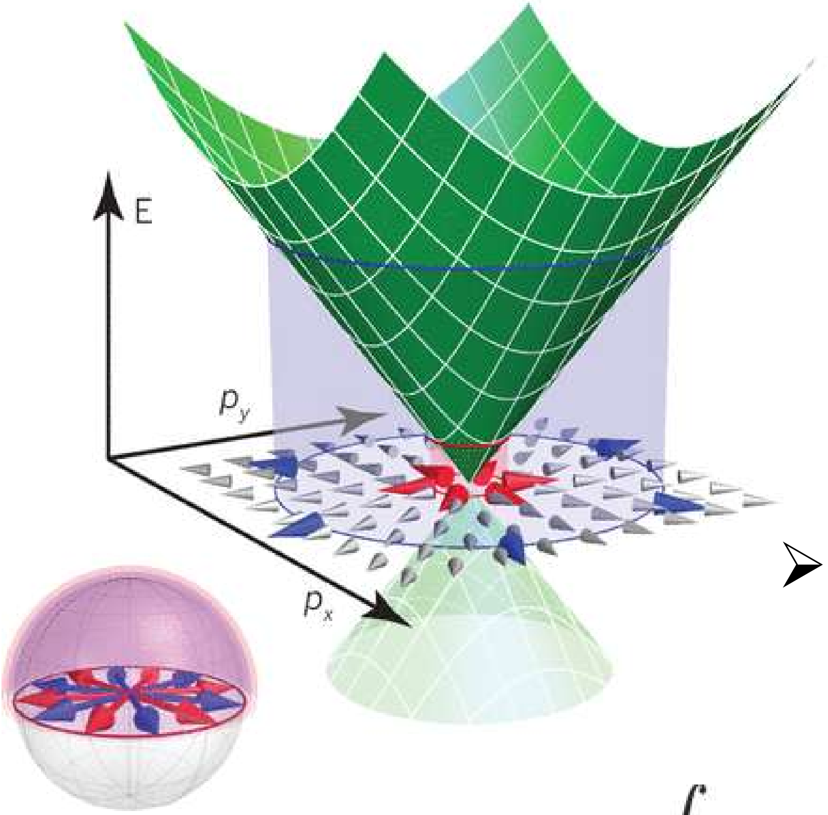


슈도스핀-모멘텀 라킹

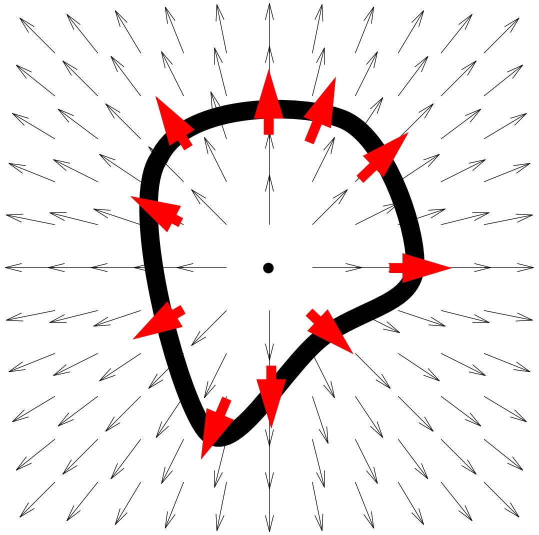
- **Berry phase of graphene**

Pseudo-spin

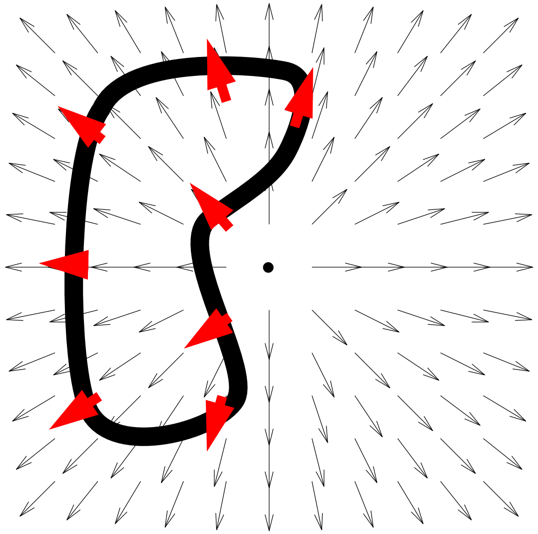




- **π –Berry phase**



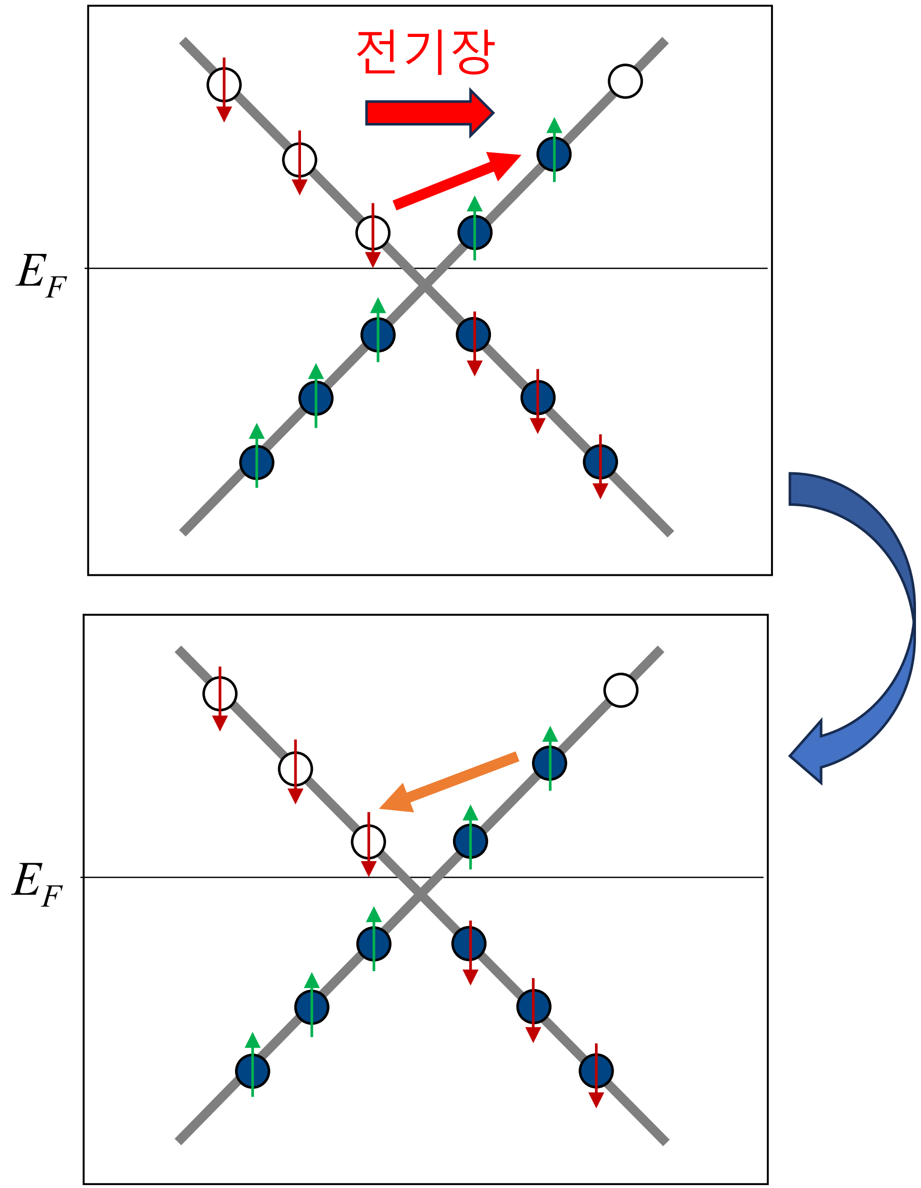
- **0–Berry phase**



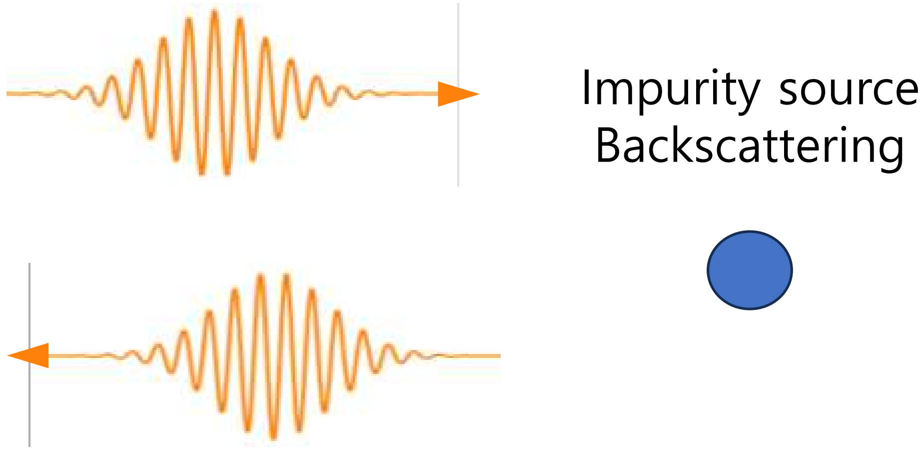
$$\gamma_n(t) = i \int_C \langle \psi_n | \nabla_R \psi_n \rangle \cdot d\vec{R}$$

디락 콘은 운동 방향과 슈도스핀이 라킹되어 있다.

그래핀의 양자수송



저항발생
(잉?)

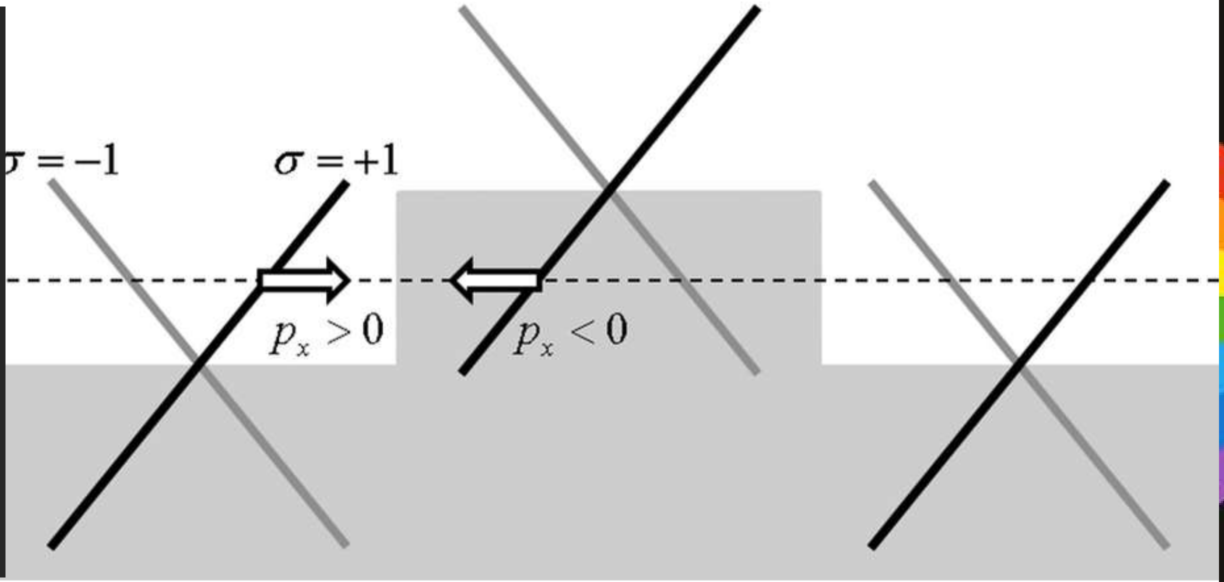


Backscattering이 일어나려면 운동방향과 스핀이 뒤집혀야함!

스핀을 뒤집을 수 없다면 Backscattering이 안일어남!

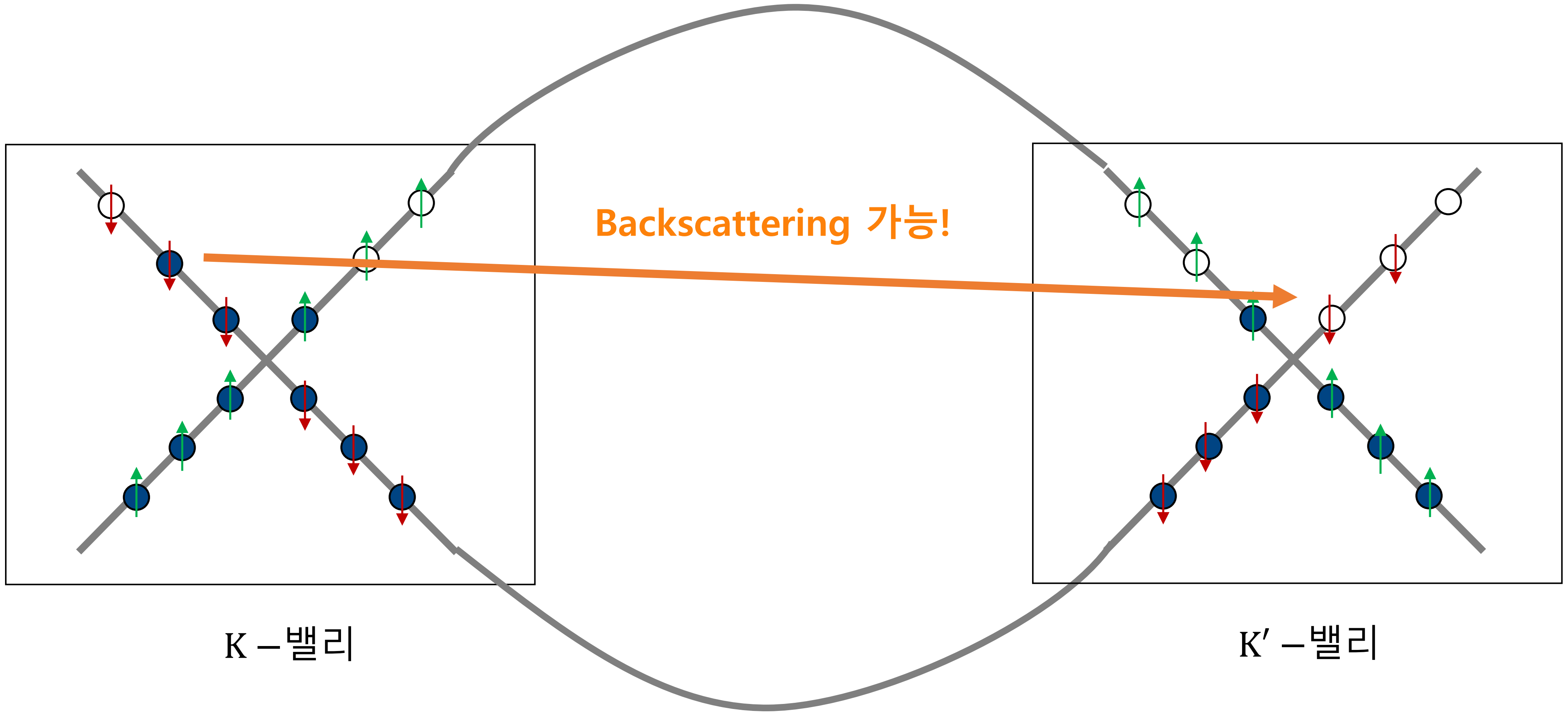
클라인 터널링

디락콘은 완벽한 트랜스미션(T=1 R=0)을 가진다



그래핀... 완벽한가?

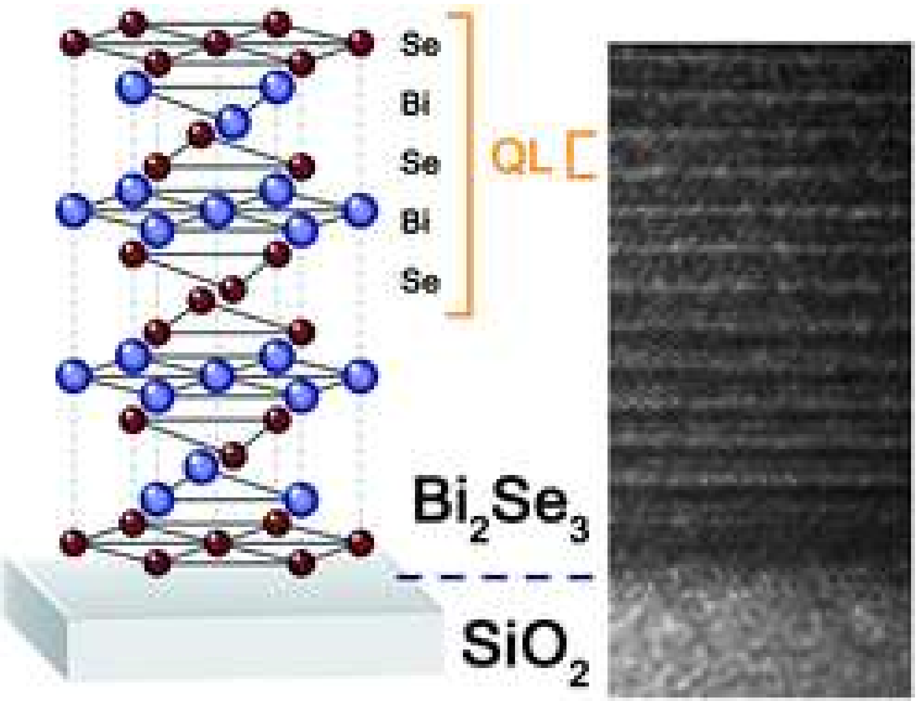
그래핀의 페르미온 더블링



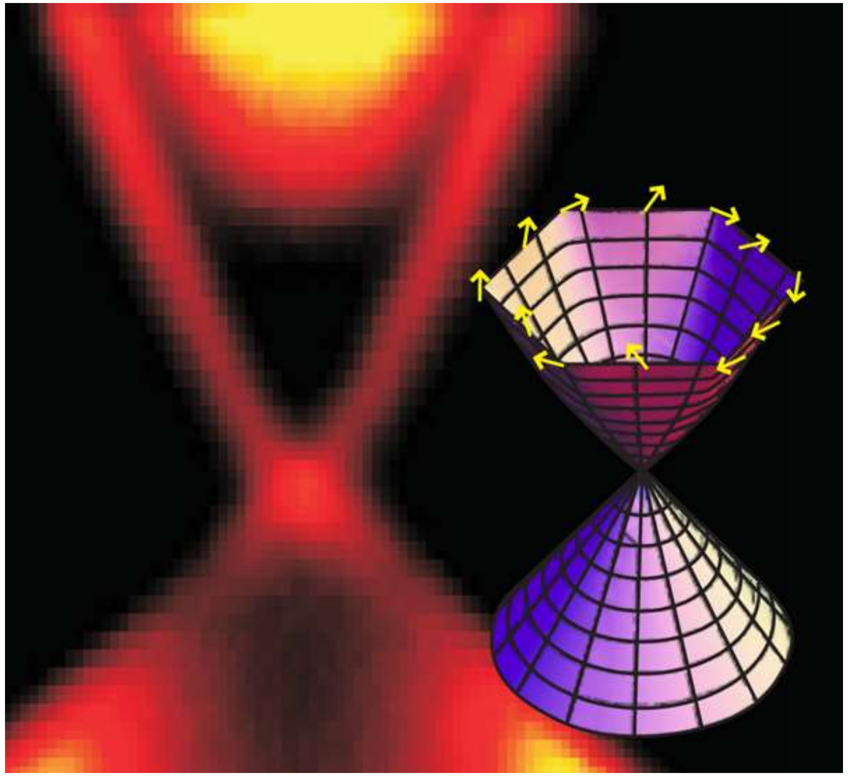
$\lambda = \frac{2\pi}{k}$ => Δk가 크다는것 => Short-length backscattering은 가능!

3차원 위상 절연체

- Bi$_2$Se$_3$, Bi$_2$Te$_3$ (강한-스핀궤도 결합)



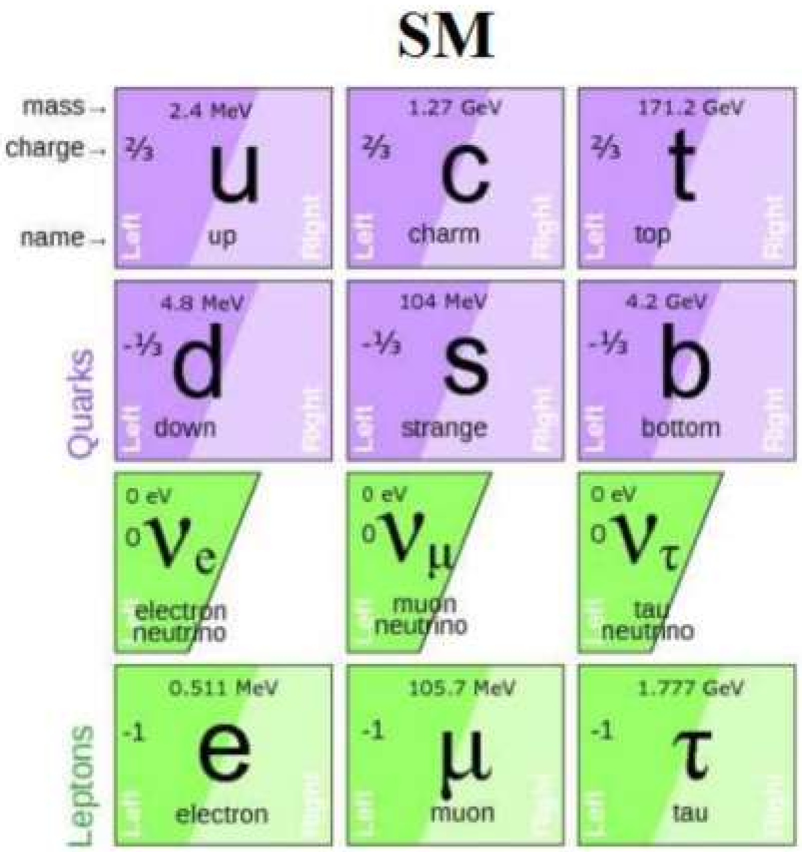
- 위상학적 겉면 상태 (디락콘)



<u>3차원 위상 절연체는 한 개의 디락 콘을 가지는 겉면상태가 있다.</u>

손지기 성질

그렇다면 우리 우주는..?

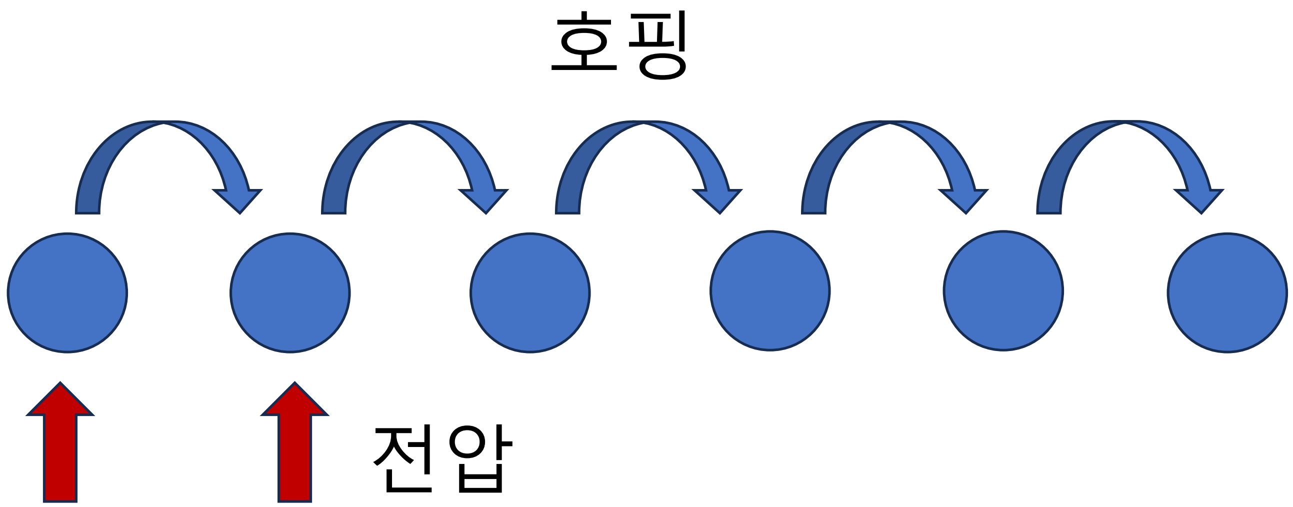


우주는 위상절연체의 겉면일지도..?

실습

실습- 1차원 양자수송

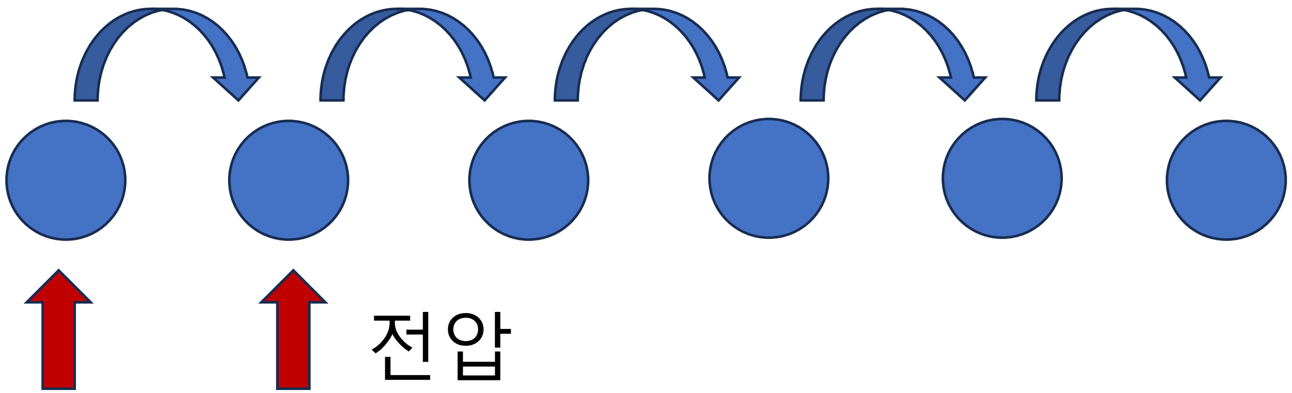
```
clear all
%%%%% 샘플 준비 %%%%%
L=800;
h=zeros(L);
for i=1:(L-1)
h(i,i+1)=1;
h(i+1,i)=1;
end
h(420,420)=5; % impurity
%%%% 전압 준비 %%%%%
V=zeros(L);
for i=1:400
V(i,i)=1;
end
```




```
>> h(1:5,1:5)

ans =

     0     1     0     0     0
     1     0     1     0     0
     0     1     0     1     0
     0     0     1     0     1
     0     0     0     1     0
```

```
>> V(399:403,399:403)

ans =

     1     0     0     0     0
     0     1     0     0     0
     0     0     0     0     0
     0     0     0     0     0
     0     0     0     0     0
```

실습- 1차원 양자수송



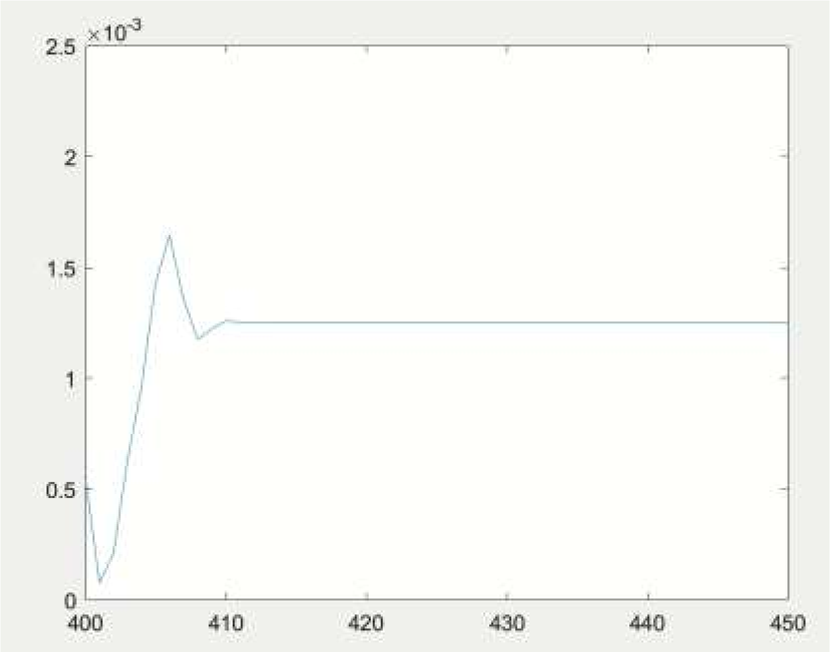

```
%%%%%%%%%%%%%%%%%%
psi=1/sqrt(L)*ones(L,1);

dt=0.2; tott=200;

for t=1:tott
    psi=expm(-1i*(h+V)*dt)*psi;
    psi=psi/norm(psi);

    n(:,t)=abs(psi).^2;

    pause(0.1)
    plot(n(:,t));
    axis([400 450 0 2/L])
end

plot(sum(n(1:400,:)))
% plot(-sum(n(1:400,1:end-1))+sum(n(1:400,2:end)))
```

$$\psi(x) = \frac{1}{\sqrt{L}}$$

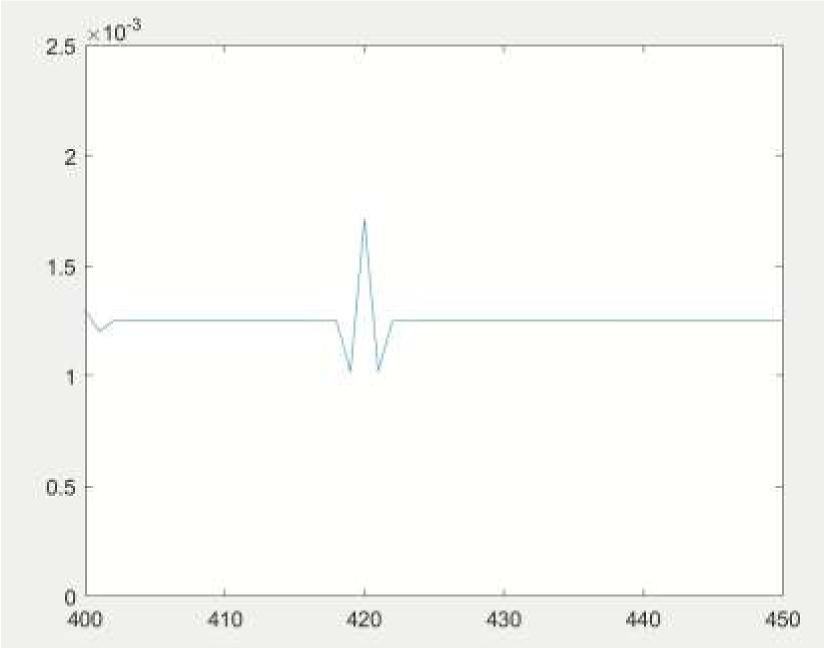
$$\psi(x,t) = e^{-iHt}\psi(x,t=0)$$

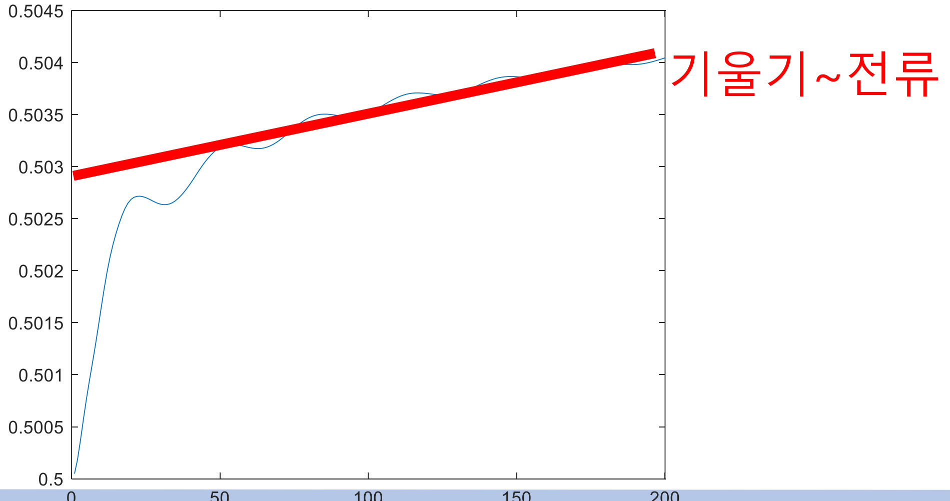
실습- 1차원 양자수송

노 스캐터링



임퓨리티 스캐터링



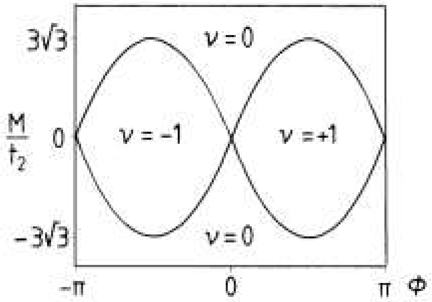


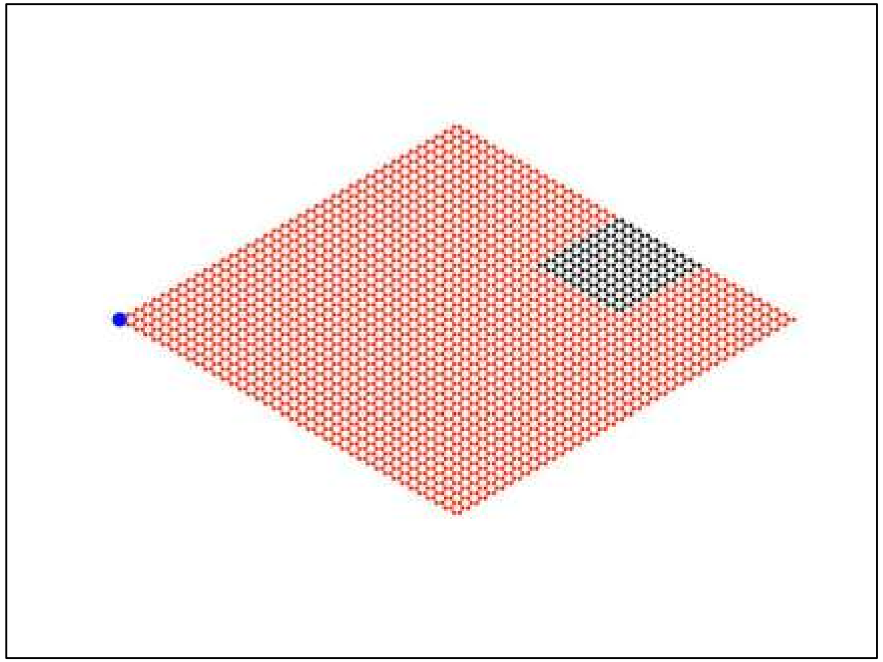
실습- 2차원 양자홀 수송

Time evolution of Hamiltonian : chiral edge mode

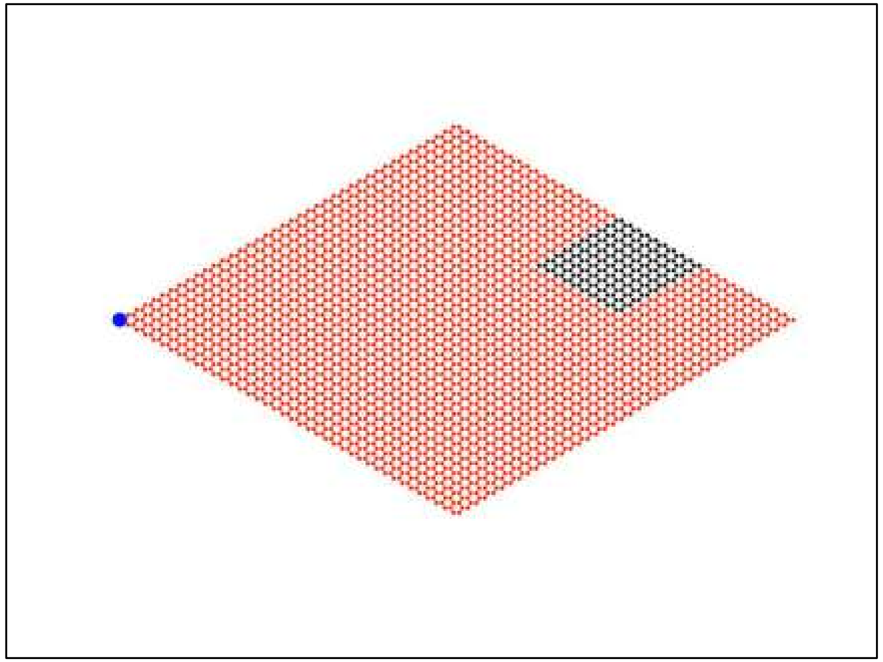
Direction → Chern number

$$|\psi(t)\rangle = e^{-i\hat{H}t/\hbar}|\psi(0)\rangle$$





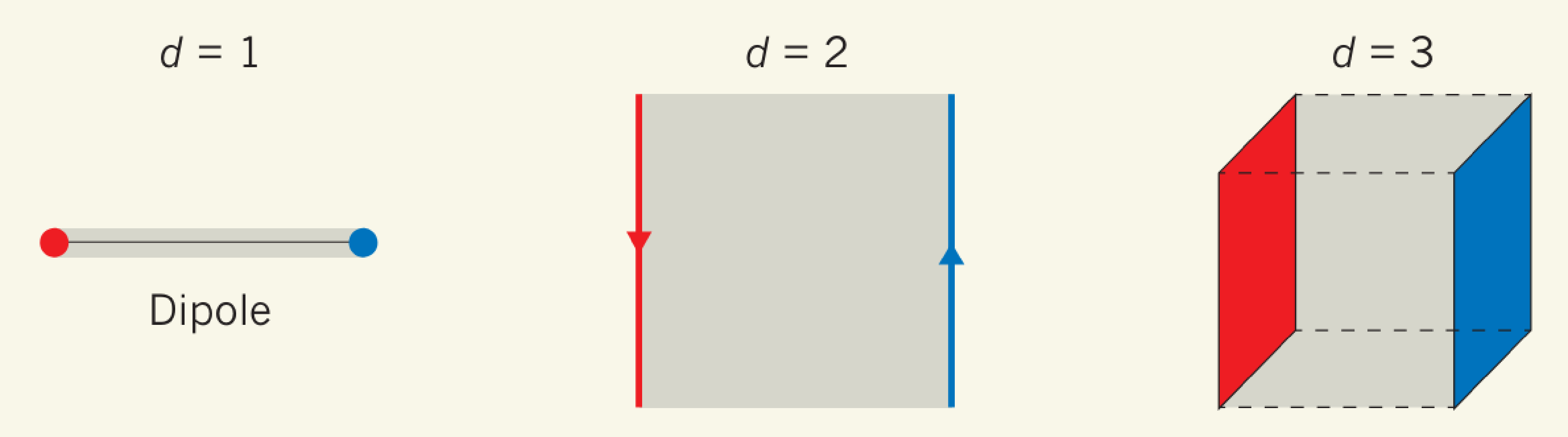
$\Phi = \pi/2$

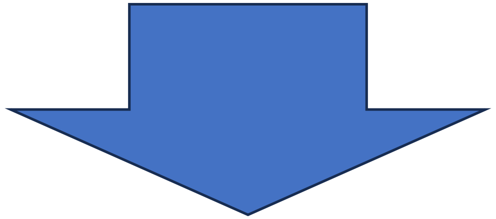


$\Phi = -\pi/2$

Summary

위상 절연체 : 금속 경계면을 갖는 물질





(페르미온 더블링을 위배하는) 금속 경계면을 갖는 물질

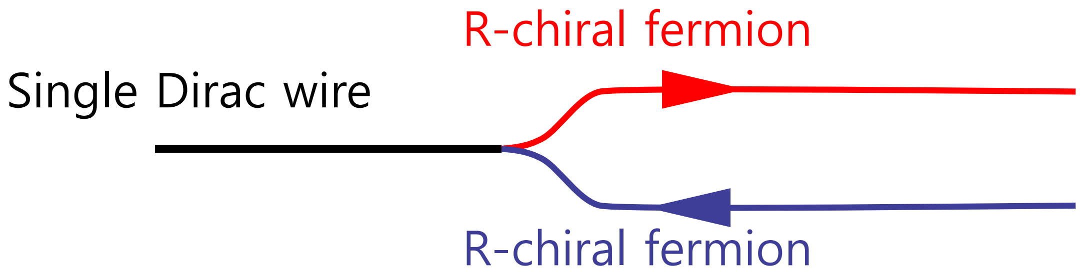
Outlook

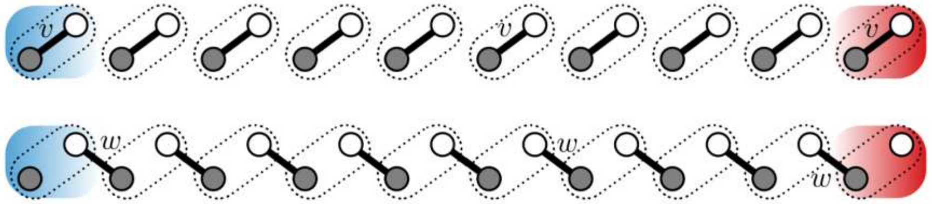
- 1D QHE
- 2D Graphene
- 3D Weyl semimetal

- Robust edge mode
- Robust transmission
- Quantum anomaly

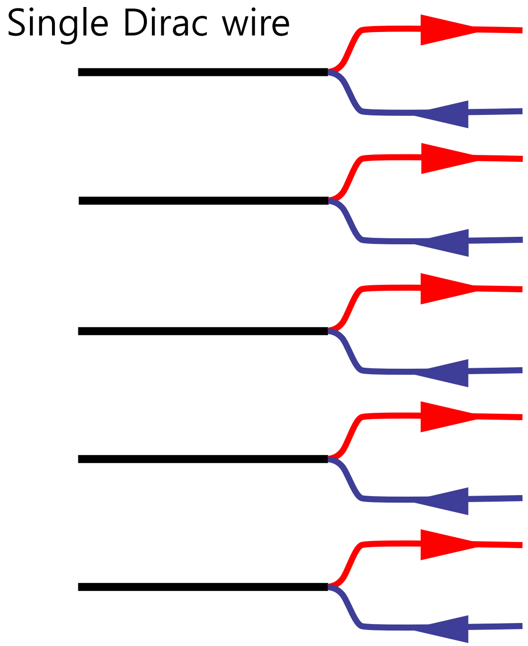
Out of 26938 stoichiometric materials, 3307 topological insulators and 4078 topological semimet-
als and 0 fragile phases were found . For these 7385 materials the electronic bands structure – in-

Coupled wire construction



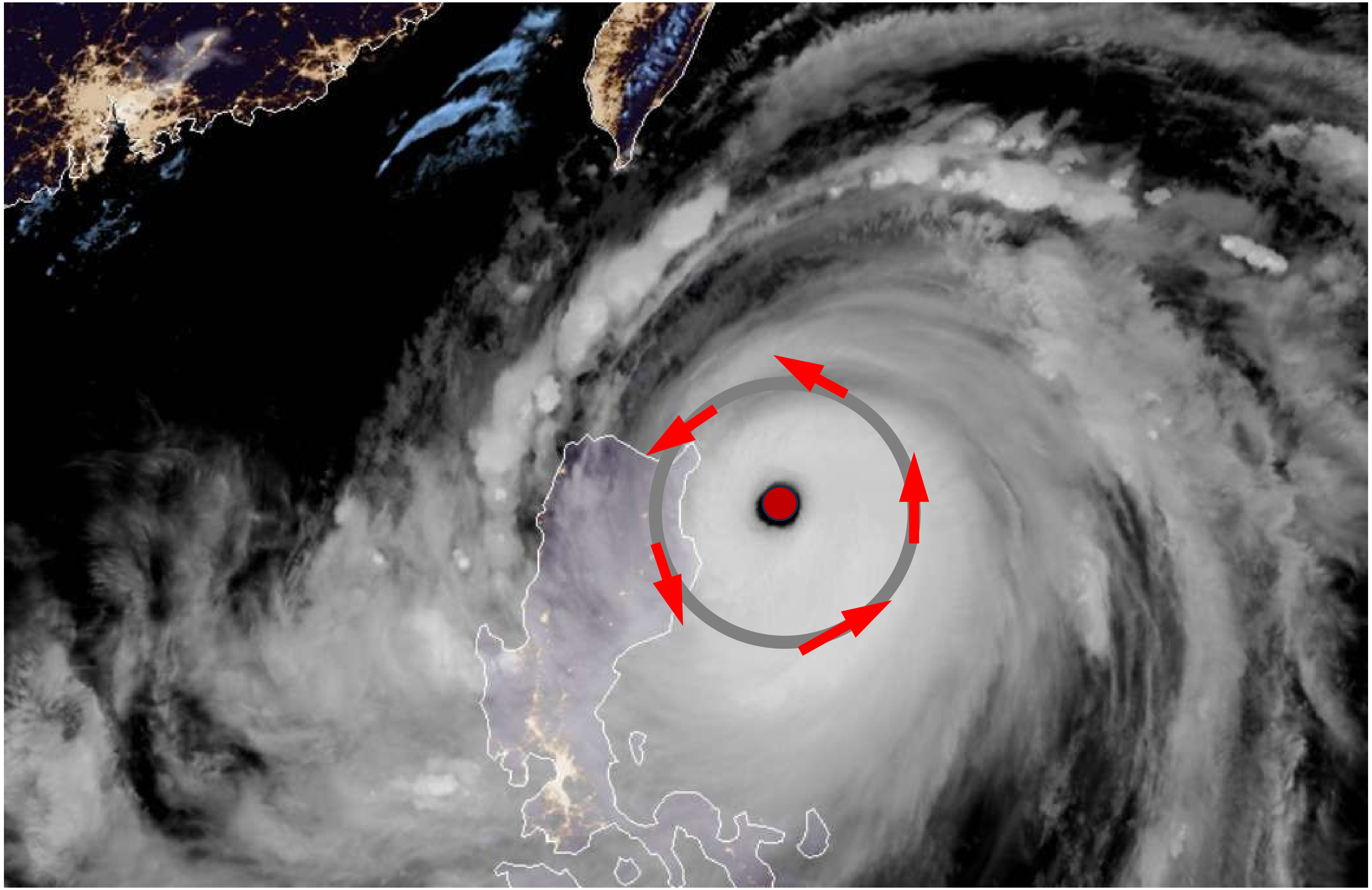


- **<u>실공간 와이어 다발</u>**



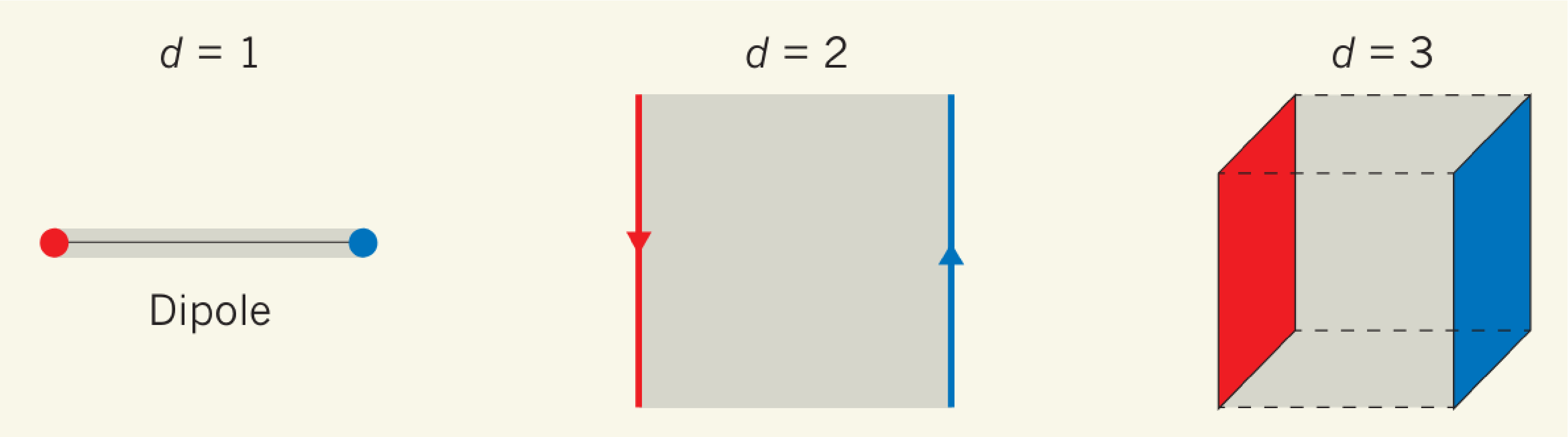
3D TI

Vortex



3차원 위상 절연체

시간-반전 대칭 3차원 위상 절연체는 존재할까?



Axion Electromagnetism

- **<u>Axion E&M</u>**

$$\mathcal{L} = E^2 - B^2 + \frac{\theta(x,t)}{2\pi} E \cdot B$$

가상의 입자 Axion을 가정하자

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \end{aligned}$$

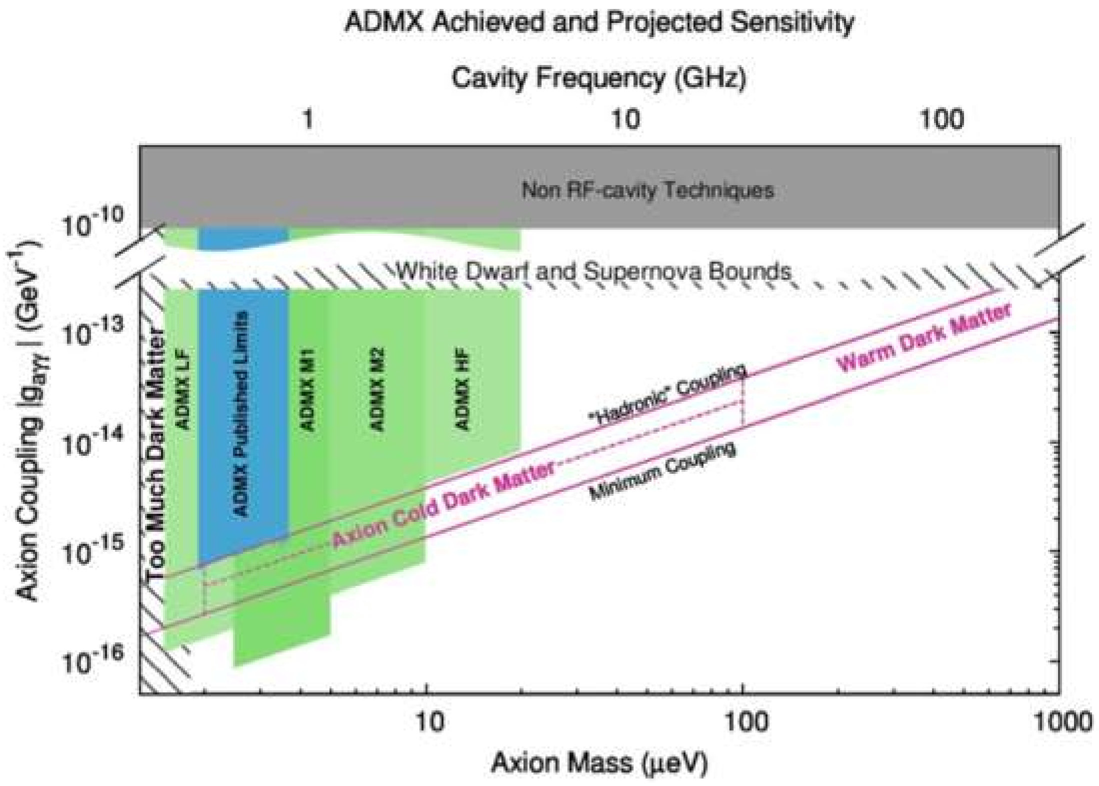
Chiral (ABJ) anomaly

- **Anomaly** = Breaking of underlying symmetry at quantum level
  = Consequence of violation of NN theorem

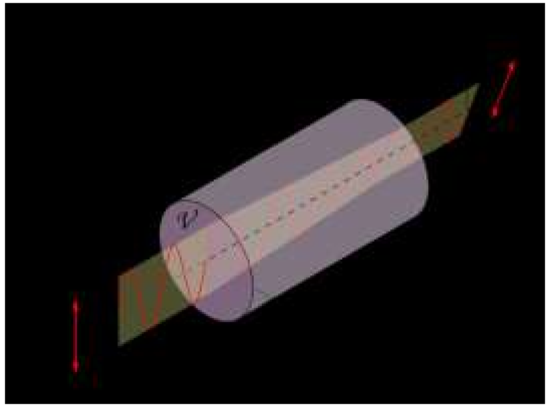
Axion E&M

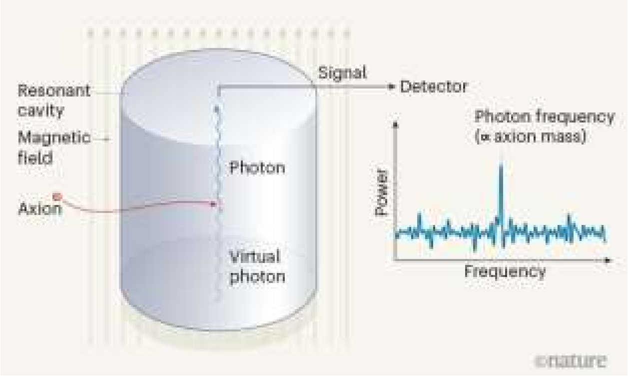
➢ **암흑물질 후보로써의 엑시온**

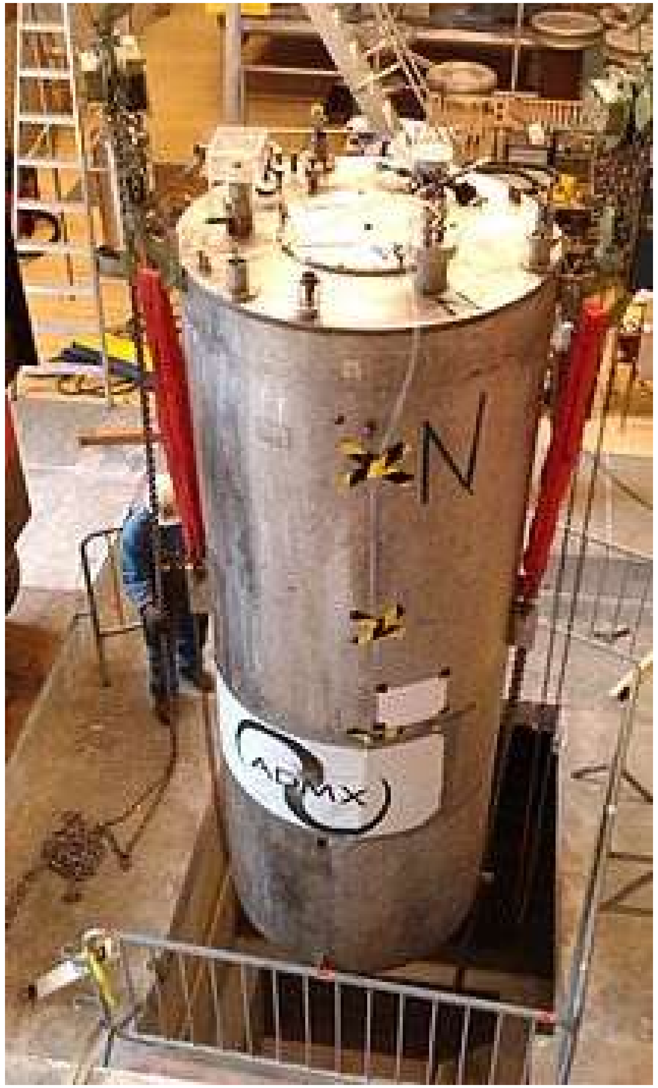
$$\mathcal{L} = E^2 - B^2 + \frac{\theta(x,t)}{2\pi} E \cdot B$$



➢ **엑시온-암흑물질 검출기**

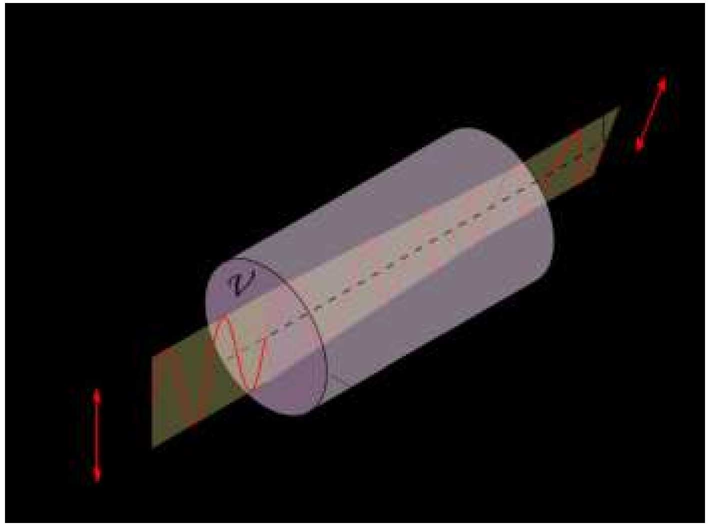




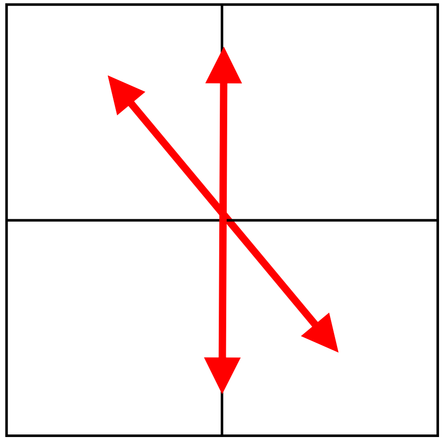


3차원 위상 절연체

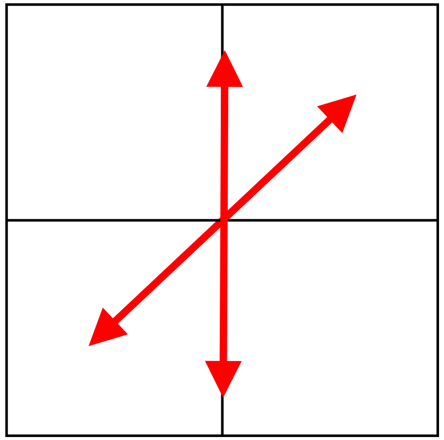
➢ <u>**패러데이 회전과 Axion-field 와의 관계**</u>



$(\boldsymbol{\theta} > \mathbf{0})$



$(\boldsymbol{\theta} < 0)$

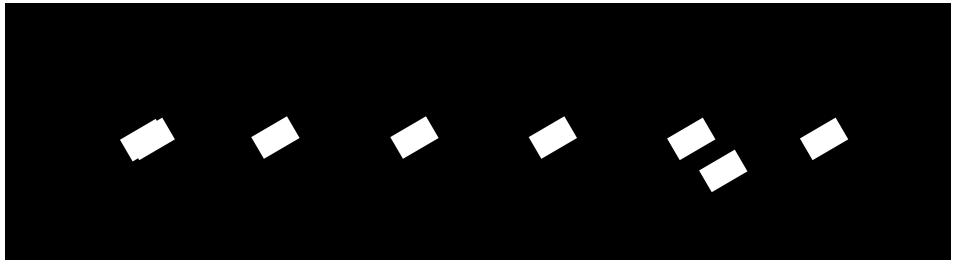


위상학적 결합

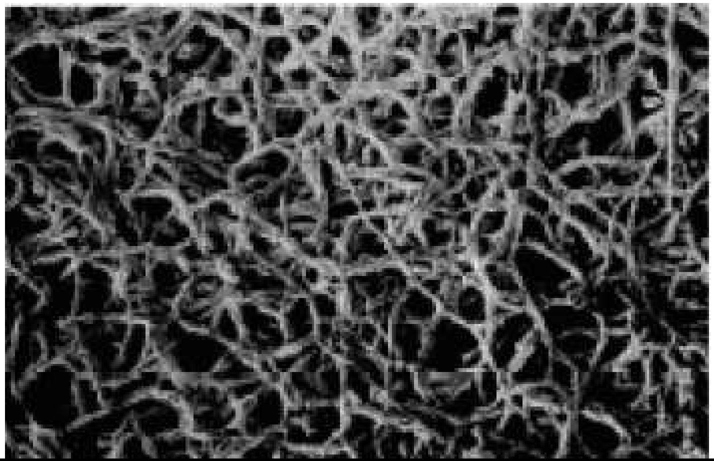
➢ Chemist's view

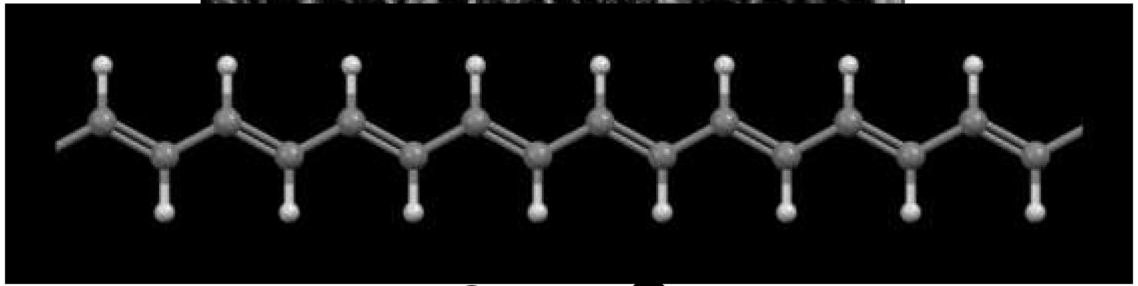
Case I :

Case II :



➢ 현미경으로 본 이미지

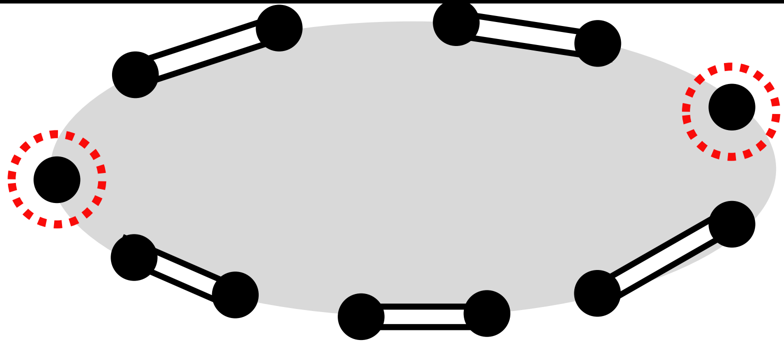




➢ Physicist's view

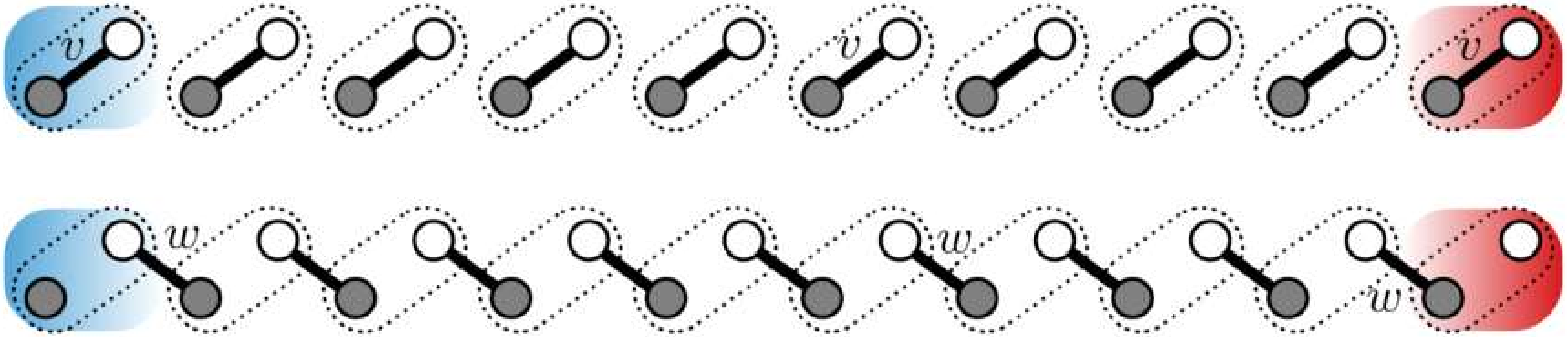
Case I :

Case II :

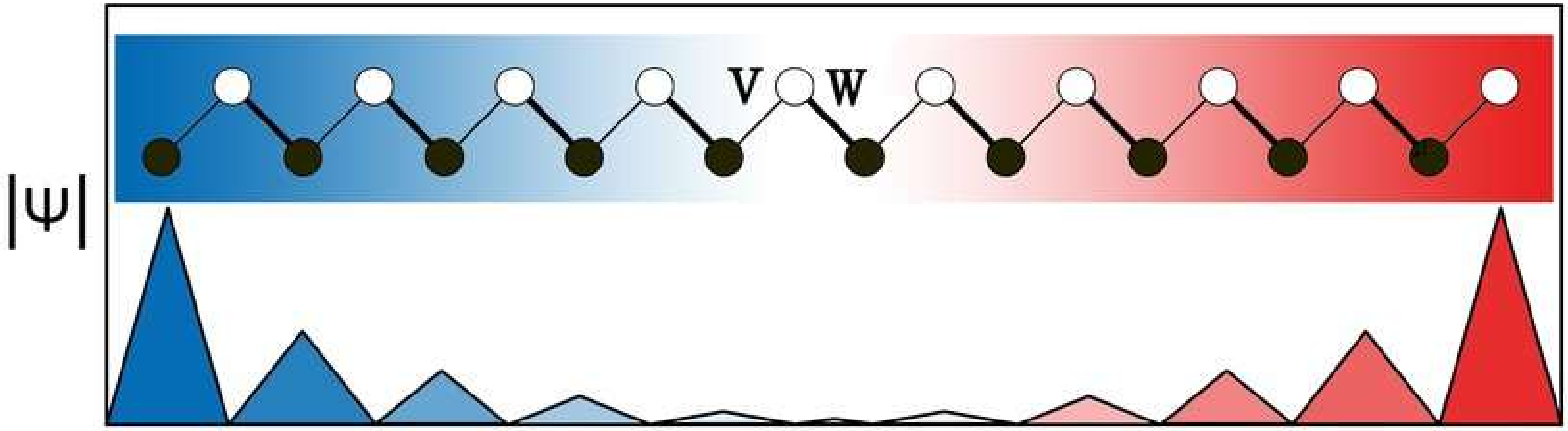


Su-Schrieffer-Heeger model

- **Real space coupling patterns**



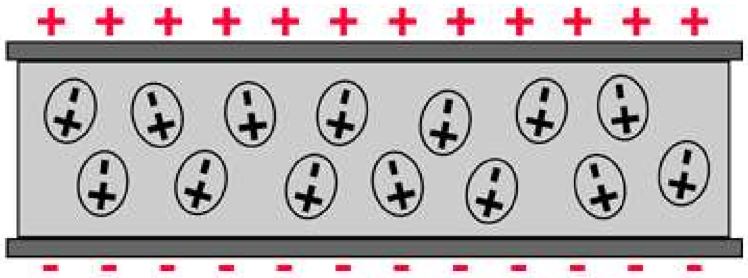
- **Topological edge state**



전하 편극밀도와 베리 위상

➢ **전하 편극밀도**

Classical polarization

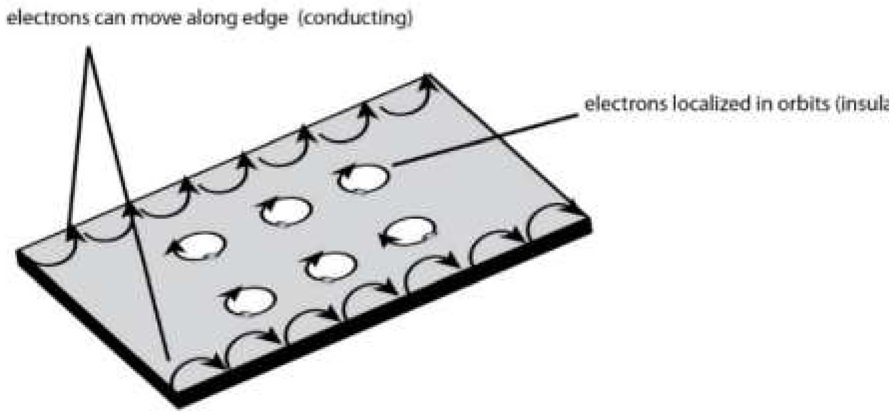


$$\boldsymbol{P} = \sum_{\boldsymbol{i}} q_i \boldsymbol{r}_{\boldsymbol{i}}$$

양자물질의 편극밀도는 베리위상과 같다.

자기장하에서 2차원 전자의 운동

Classical cyclotron orbit



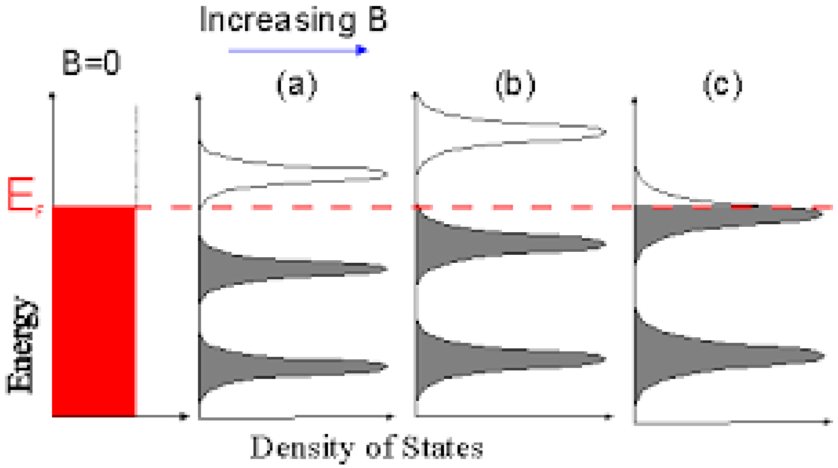
$$\underbrace{\frac{mv^2}{R}}_{\text{원심력}} = \underbrace{\frac{qvB}{c}}_{\text{로렌츠힘}}$$

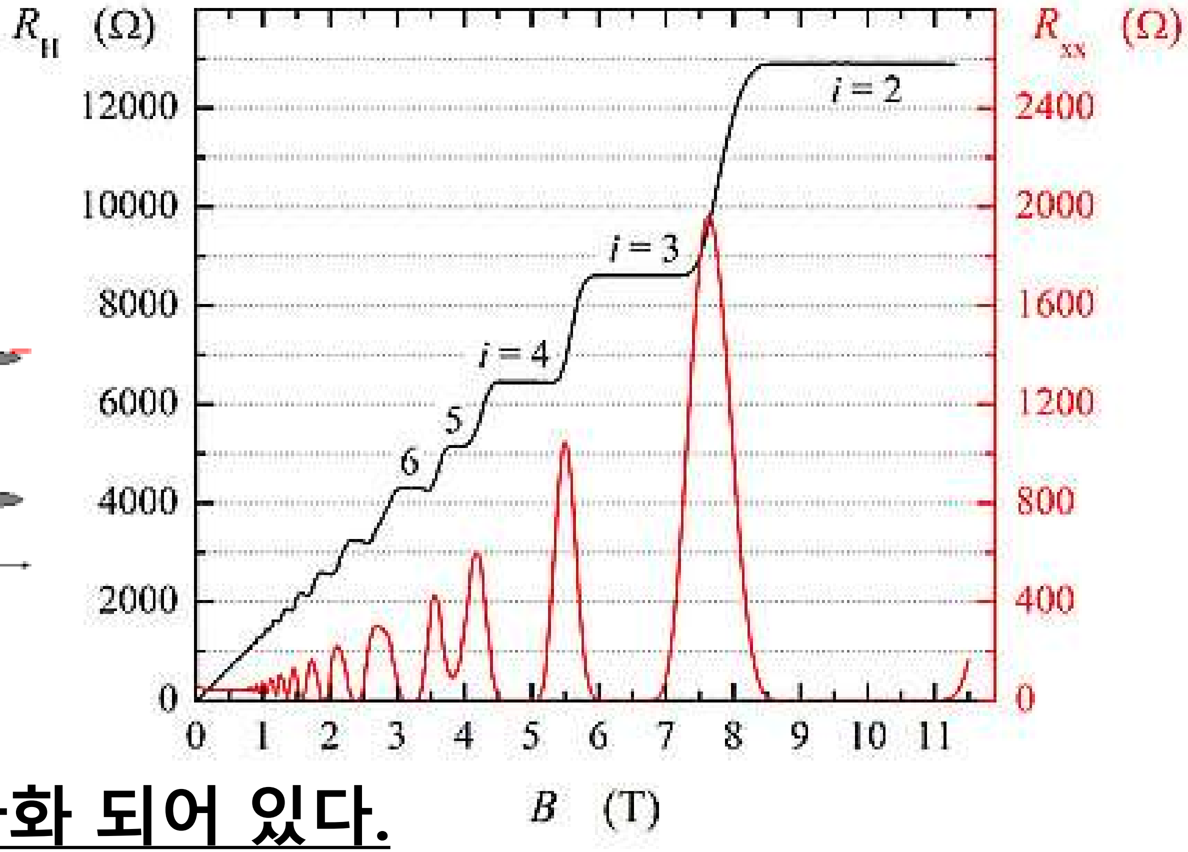
$$R = \frac{mvc}{qB}$$

$$\omega_c = \frac{eB}{mc}$$

정수 양자홀 효과

- ➢ **Quantized Hall conductance**
- ➢ **Vanishing Longitudinal conductance**

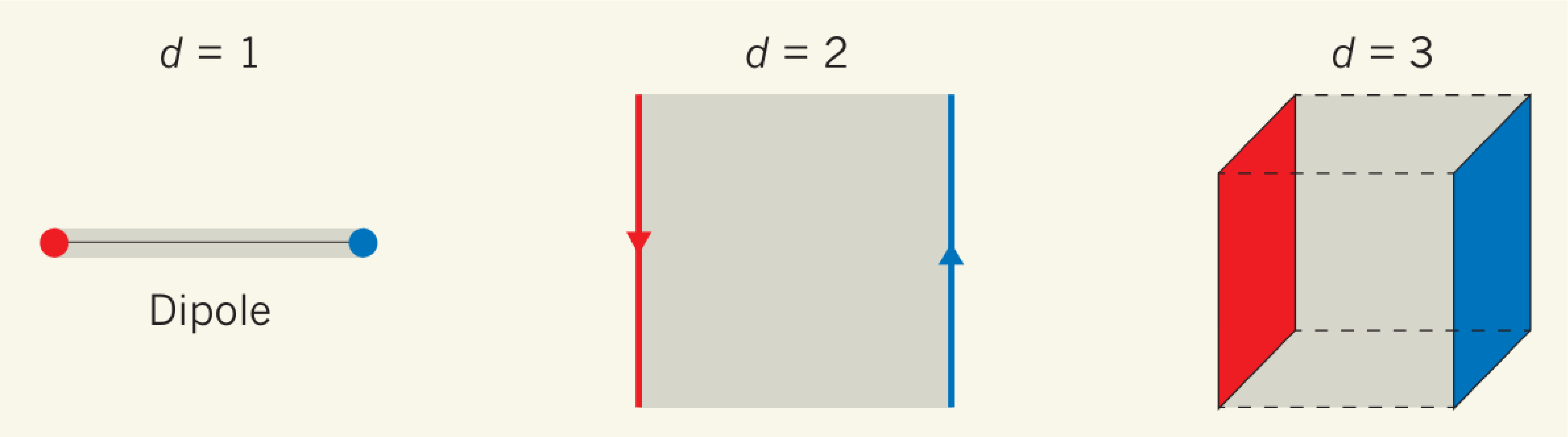




양자물질의 홀전도도는 양자화 되어 있다.

3차원 위상 절연체

시간-반전 대칭 3차원 위상 절연체는 존재할까?



Axion Electromagnetism

- **<u>Axion E&M</u>**

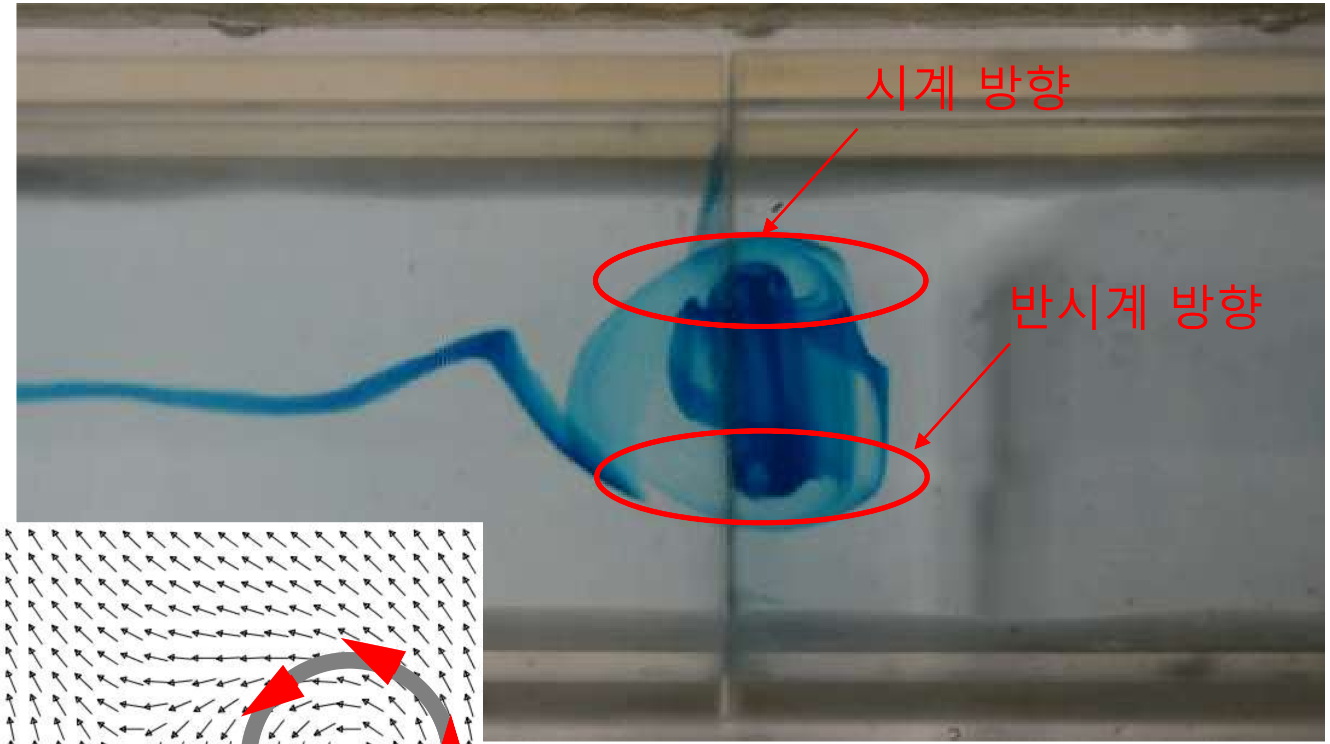
$$\mathcal{L} = E^2 - B^2 + \underline{\frac{\theta(x,t)}{2\pi}} E \cdot B$$

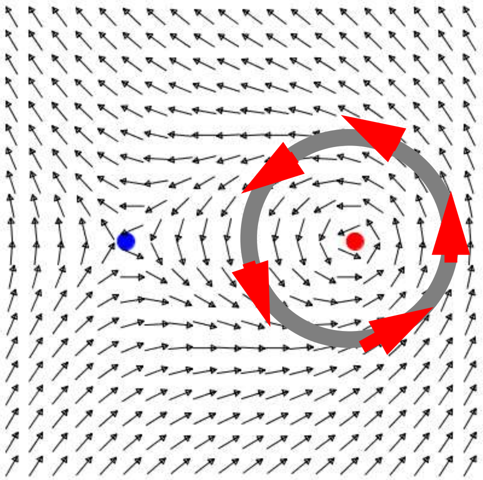
가상의 입자 Axion을 가정하자

$$\begin{aligned}
\nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} &= \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)
\end{aligned}$$

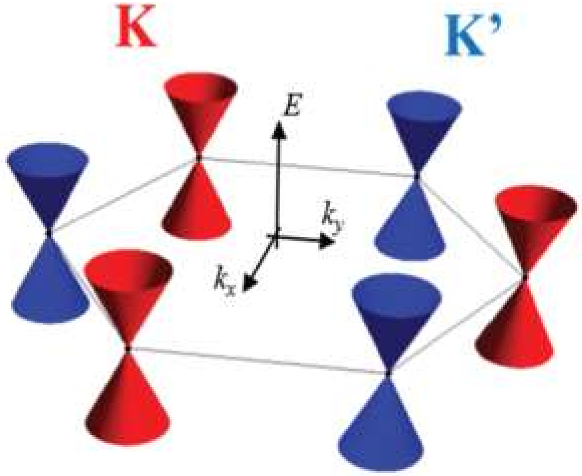
Vortex

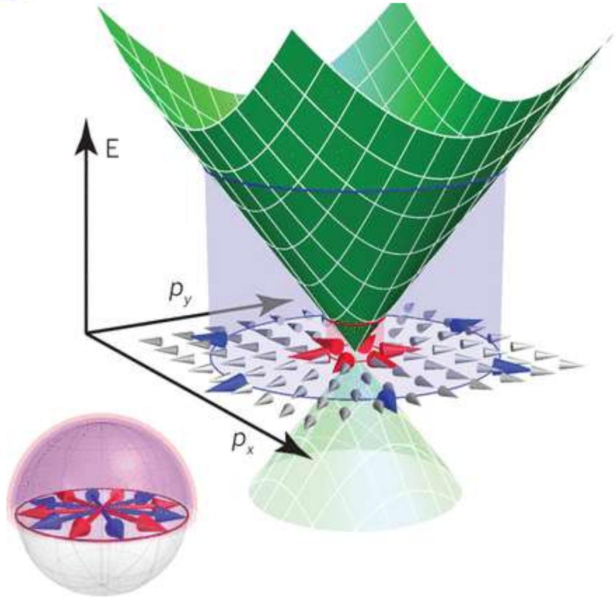
- **Vortex as a pair**





- **Dirac cone as vortex core**

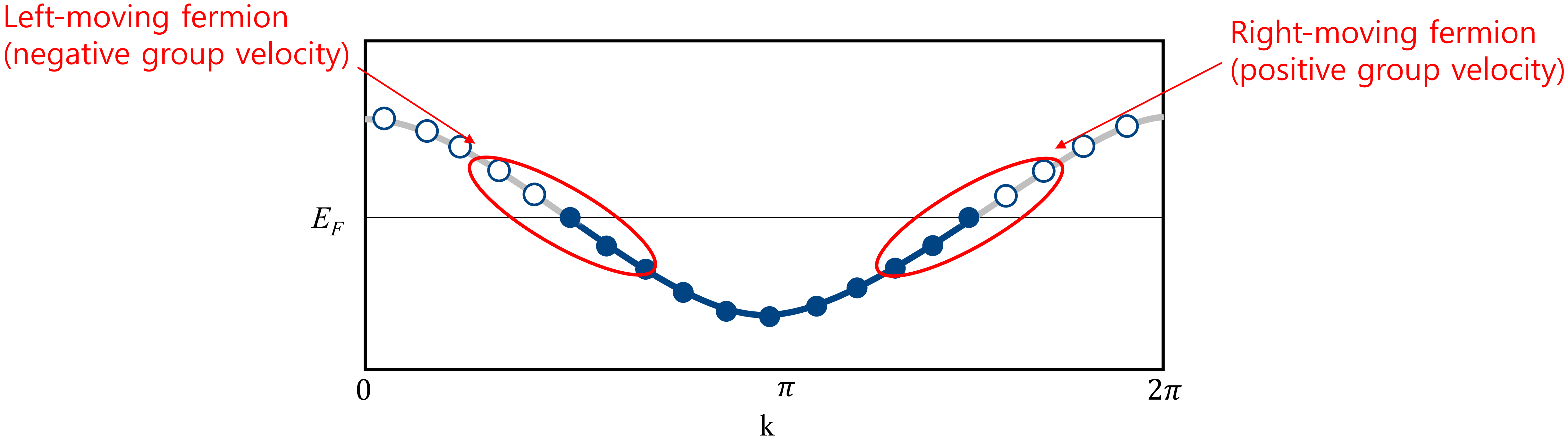




그래핀의 디락콘은 항상 짝수개로 존재한다.

페르미온 더블링 문제

➢ **<u>Nielsen Ninomiya 노고 정리 (fermion doubling) :</u>** In odd-space dimension, any lattice regularized theory must have even-numbers of chiral fermions.



Anything goes up must goes down!

Violation of NN theorem

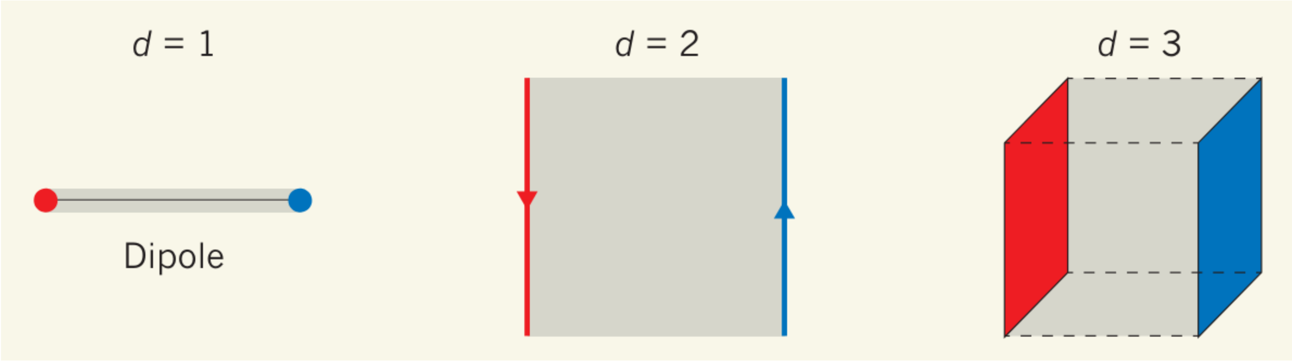
So far.. Known way of avoiding NN theorem is to intoduce..

- Non-locality
- Non-Hermiticity
- Embeding on higher-dimensions

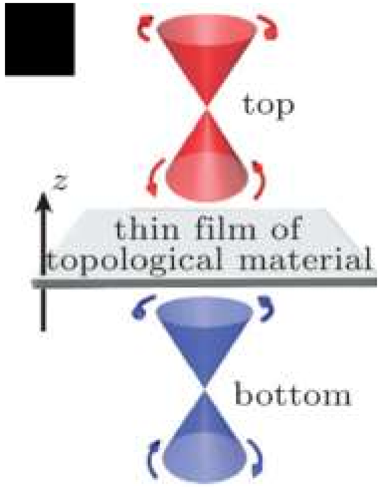
  (Topological insulators)

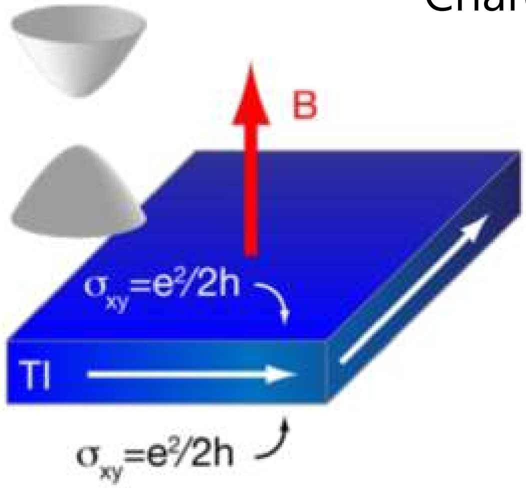
분수화된 양자홀 효과

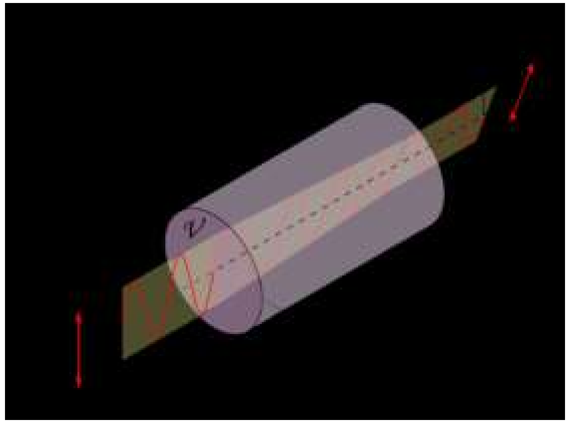
Fractionalization in physical degree of freedom



Charge의 분수화 Chirality의 분수화 Helicity의 분수화







$$\tan(\phi_F) = \frac{2\alpha}{1+n}(N_t + \frac{1}{2} + N_b + \frac{1}{2})$$

3D TI의 한 면의 한 개의 디락콘은 분수화된 양자홀 효과를 만든다.

바일 준금속

PT-Symmetry

- **Kramer's degeneracy** **($T^2=-1$) :** Orthogonal TRS partner exists in spin-half particles.

Weyl semimetal

- In 3D, there is a interesting class of band degeneracy which does not require symmetry

Symmetry of Weyl semimetal

Source of Berry curvature

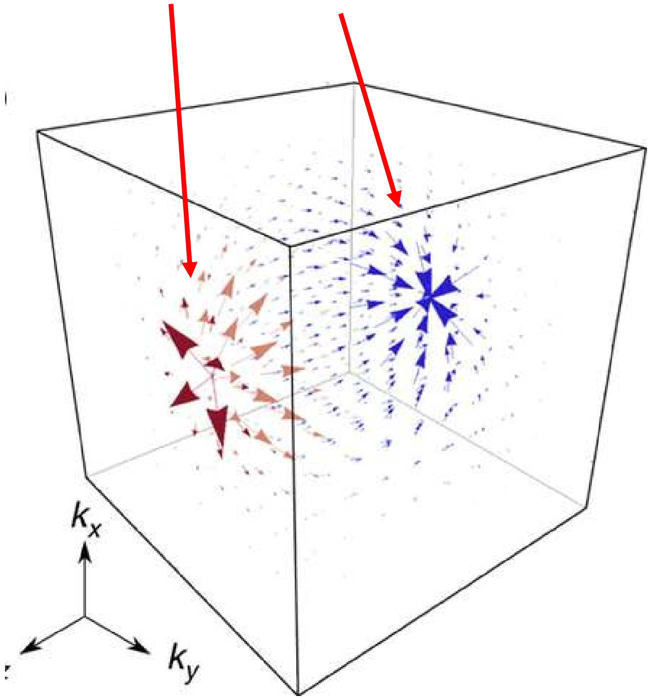
Stability of Weyl fermion can be understood using the tool of topology.

Bulk-boundary correspondence

Quantized Chern number manifests as an interesting bulk boundary correspondence.

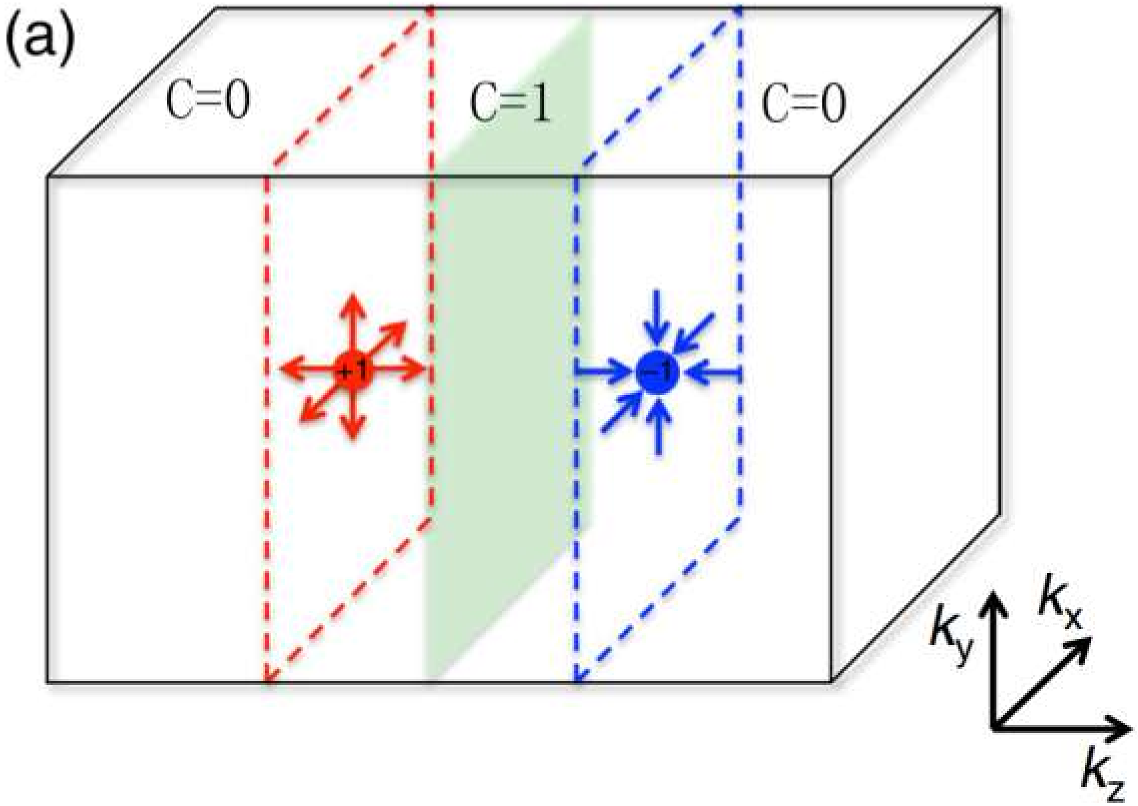
- **Fermi arc**

Monopole charge ± 1



Bulk-boundary correspondence

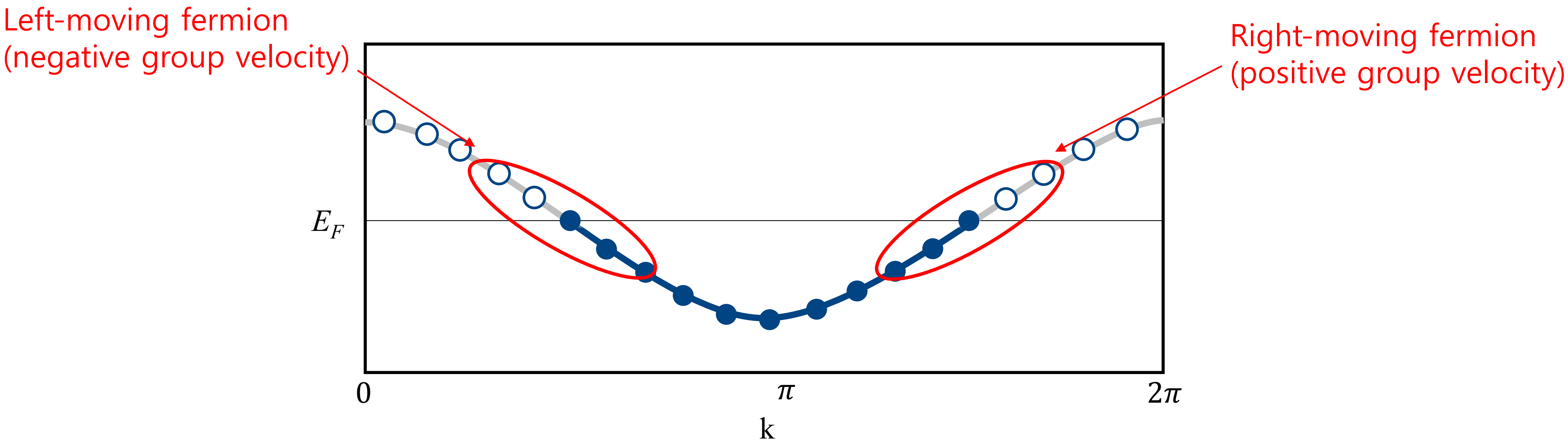
- **Fermi arc as a stack of QHE**



Quantum Anomaly

Fermion doubling

➢ **Nielsen Ninomiya theorem (fermion doubling) :** In odd-space dimension, any lattice regularized theory must have even-numbers of chiral fermions.



Anything goes up must goes down!

Chiral (ABJ) anomaly

- **Anomaly** = Breaking of underlying symmetry at quantum level
  = Consequence of violation of NN theorem

3D Chiral anomaly

- **Fujikawa method** = Source of anomaly is path-integral measure

$$Z[A] = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-\int d^d x\, \bar{\psi} i \not{D} \psi}.$$

3D Chiral anomaly in Lattice

➢ **Zeroth Landau levels**

$$H = (k - A) \cdot \sigma$$

$$= \begin{bmatrix} k_z & k_x - A_x - i(k_y - A_y) \\ k_x - A_x + i(k_y - A_y) & -k_z \end{bmatrix}$$

$$= \begin{bmatrix} k_z & a^\dagger \\ a & -k_z \end{bmatrix}$$

$$\psi = \begin{pmatrix} |0> \\ |1> \end{pmatrix}, E = k_z$$