昆山杜克大学
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Physics of Light-Matter Interaction
Part 2: Cavity magnonics

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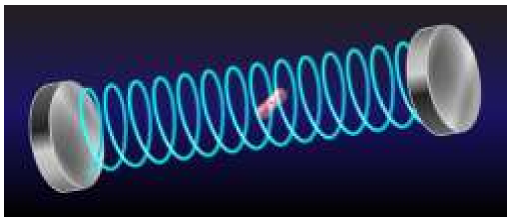
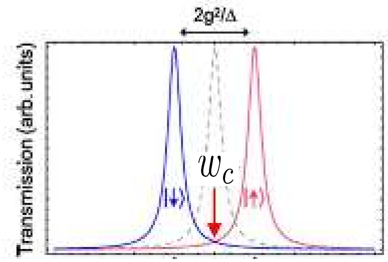
Circuit Quantum Electrodynamics

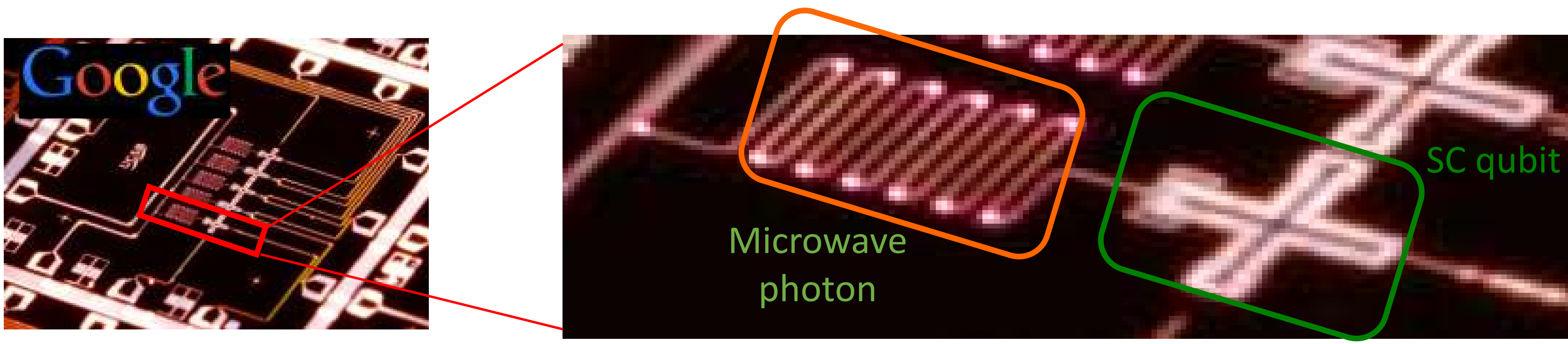
- Quantum light-matter interaction with natural atoms (Cavity QED)





AC stark shift of photon frequency

- Quantum light-matter interaction with superconducting qubit and microwave photons



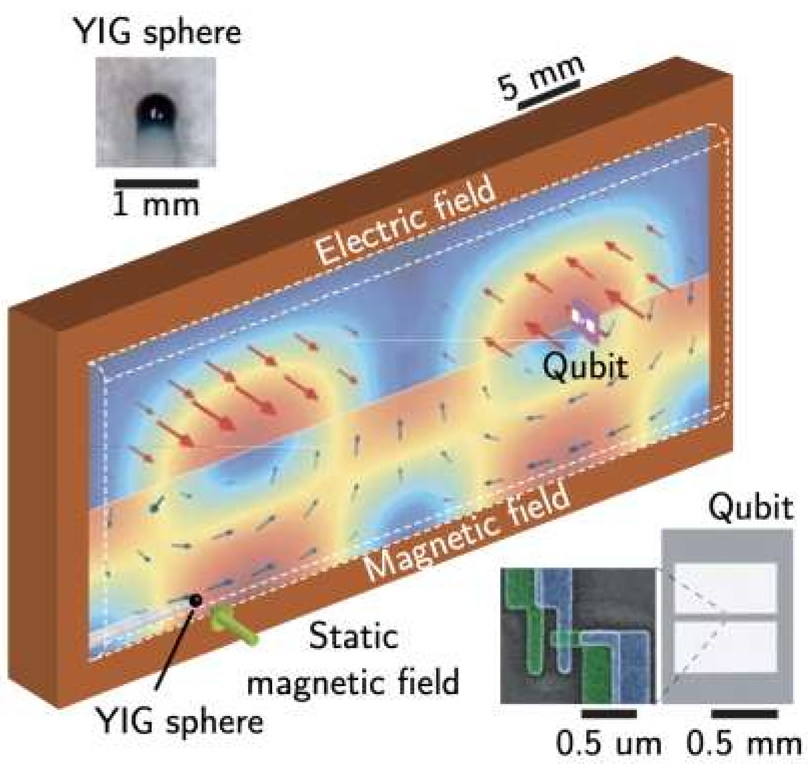
Qubit measurement using AC stark shift of microwave photons!

- Major breakthrough in superconducting quantum computing architecture (Yale, 2004)
- Coherence time : ~1ns (NEC, 1999) to ~0.1ms (IBM)

Cavity magnonics

[redacted]

- Quantum mechanical interaction between magnons, photons, and qubits



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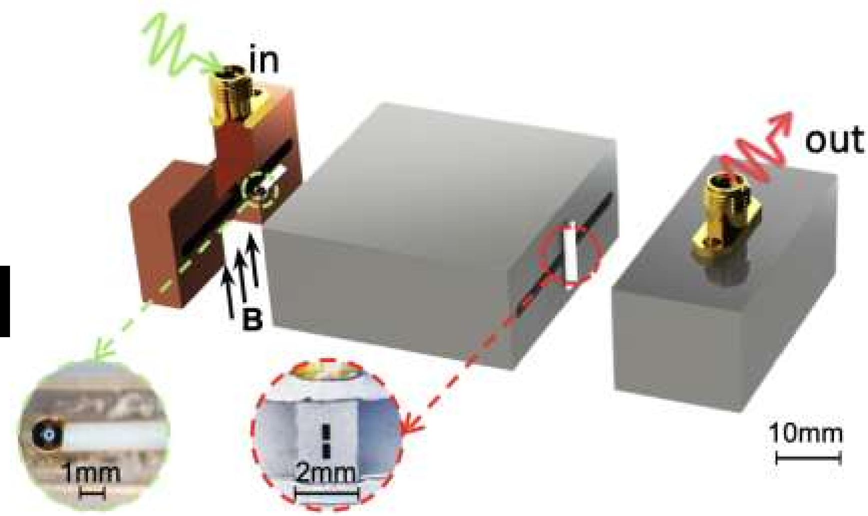


[redacted]

Tabuchi et al Science 2015

Xu et al PRL 2023

Magnons

[redacted]ood Heisenberg interaction & Zeeman field

$$\hat{H} = -\frac{J}{2} \sum_{ij=\mathrm{nn}} \left[\hat{S}_i^+ \hat{S}_j^- + \hat{S}_i^z \hat{S}_j^z \right] + g_{\mathrm{Z}} \mu_{\mathrm{B}} B_0 \sum_i \hat{S}_i^z$$

Holstein-Primakoff transformation (boson operators)

[redacted]

$$\hat{S}_i^+ = \sqrt{2S} \sqrt{1 - \frac{\hat{m}_i^\dagger \hat{m}_i}{2S}} \hat{m}_i, \qquad \hat{S}_i^z = \left(S - \hat{m}_i^\dagger \hat{m}_i \right)$$

[redacted]

In the weak excitation limit, the spin Hamiltonian becomes coupled harmonic oscillators.

$$\sqrt{1 - \hat{m}_i^\dagger \hat{m}_i / 2S} = 1 - \hat{m}_i^\dagger \hat{m}_i / 4S + \ldots$$

Magnons

Magnons:

A collective excitation that spreads the flip of a single electron with angular momentum change hbar over the entire lattice

$$\hat{m}_{\mathbf{k}} = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}_i} e^{-i\mathbf{k}\cdot\mathbf{R}_i} \hat{m}_i$$

[redacted]

Diagonalized spin Hamiltonian in terms of magnons:

[redacted] $$\hat{H}_{\mathrm{sw}} = E_0(B_0) + \sum_{\mathbf{k}} \hbar\omega(\mathbf{k}) \hat{m}^{\dagger}_{\mathbf{k}} \hat{m}_{\mathbf{k}},$$ [redacted]

with a quadratic dispersion for $ka \ll 1$

$$\hbar\omega(\mathbf{k}) \approx g_{\mathrm{Z}}\mu_{\mathrm{B}} B_0 + JSa^2k^2$$

Kittel mode

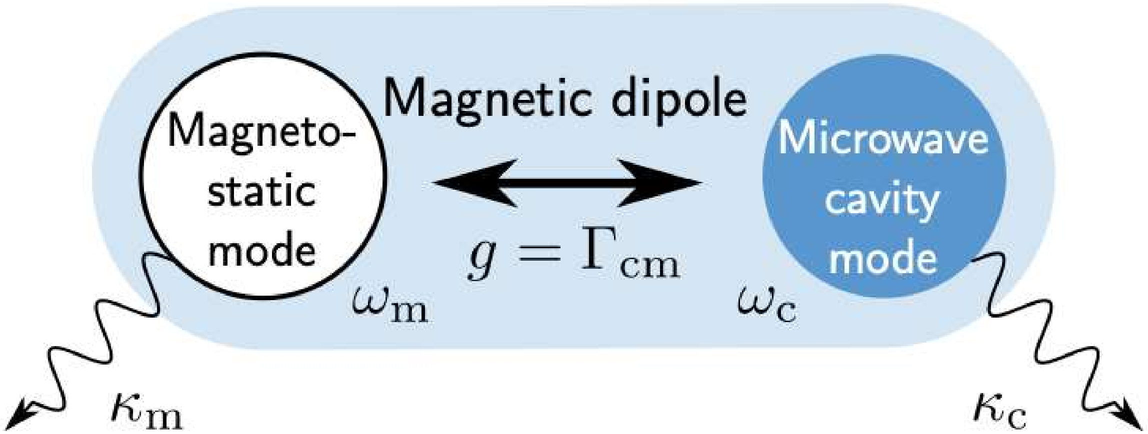
Kittel model: the fundamental mode of magnon (k=0)

We will mainly focus on the kittle mode magnon in today’s talk.

Cavity-magnon interaction

[redacted]ld of photon and spins

[redacted]



[redacted]

$$\hat{H}_{\rm cm} = \hbar \sum_{p\eta} \left(\Gamma_{p\eta} \hat{a}_p \hat{m}_\eta^\dagger + \Gamma_{p\eta}^* \hat{a}_p^\dagger \hat{m}_\eta \right)$$

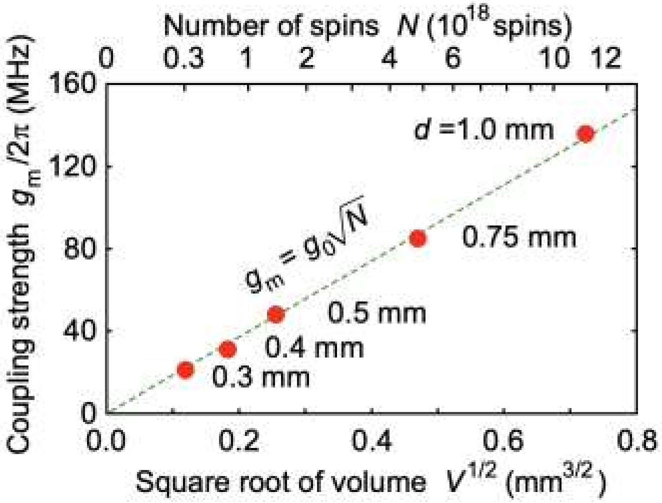
Laschance-Quirion et al Science 2015

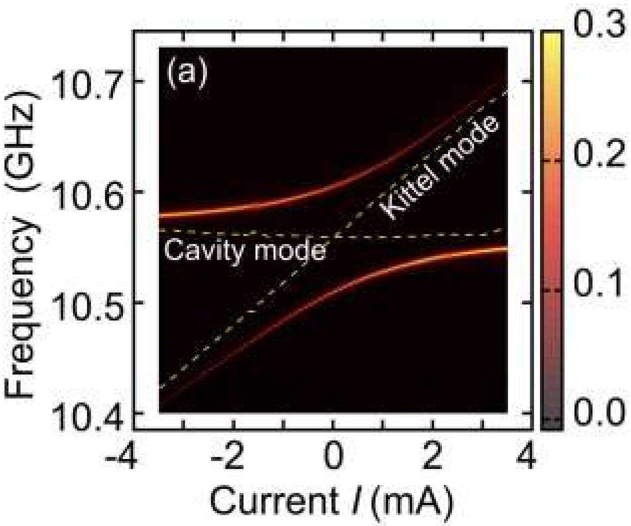
Strong coupling : collective enhancement

[redacted]

Magnetic dipole coupling between the single spin and photon : Weak (< Hz)

Strong coupling is reached by collective enhancement $g_m = g_0\sqrt{N}$





[redacted]

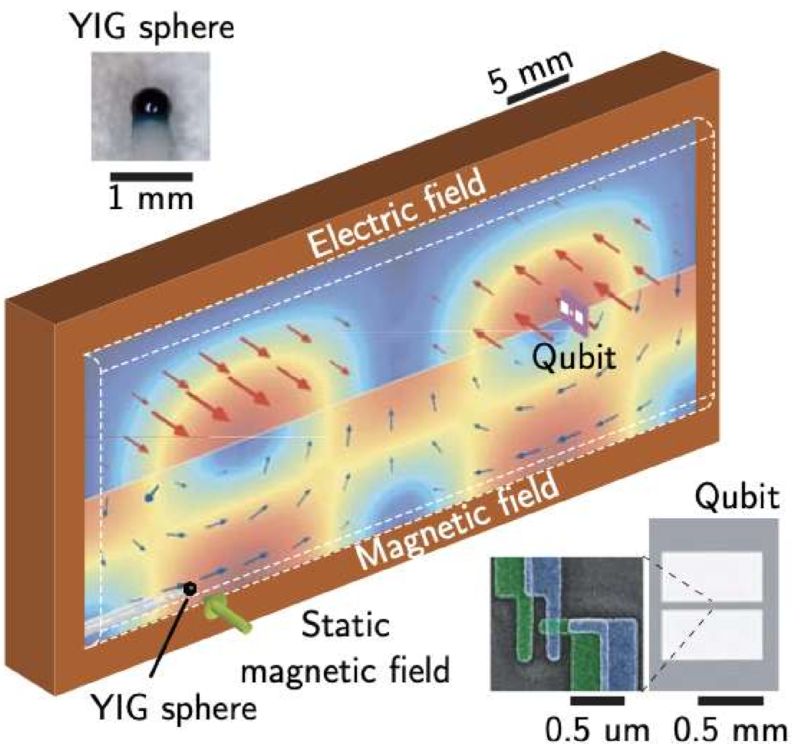
Tabuchi et al PRL 2014

Quantum cavity magnonics

[redacted]

Introduce the transmon!

$$\hat{H}_{\mathrm{q}} = \hbar\left(\omega_{\mathrm{q}} - \frac{K_{\mathrm{q}}}{2}\right)\hat{q}^{\dagger}\hat{q} + \hbar\frac{K_{\mathrm{q}}}{2}\left(\hat{q}^{\dagger}\hat{q}\right)^{2}$$



Tabuchi et al Science 2015

Cavity-qubit interaction

$$\hat{H}_{\text{q-c}} = \hbar g_{\text{q-c}}\left(\hat{a}\hat{q}^{\dagger} + \hat{a}^{\dagger}\hat{q}\right)$$

[redacted]

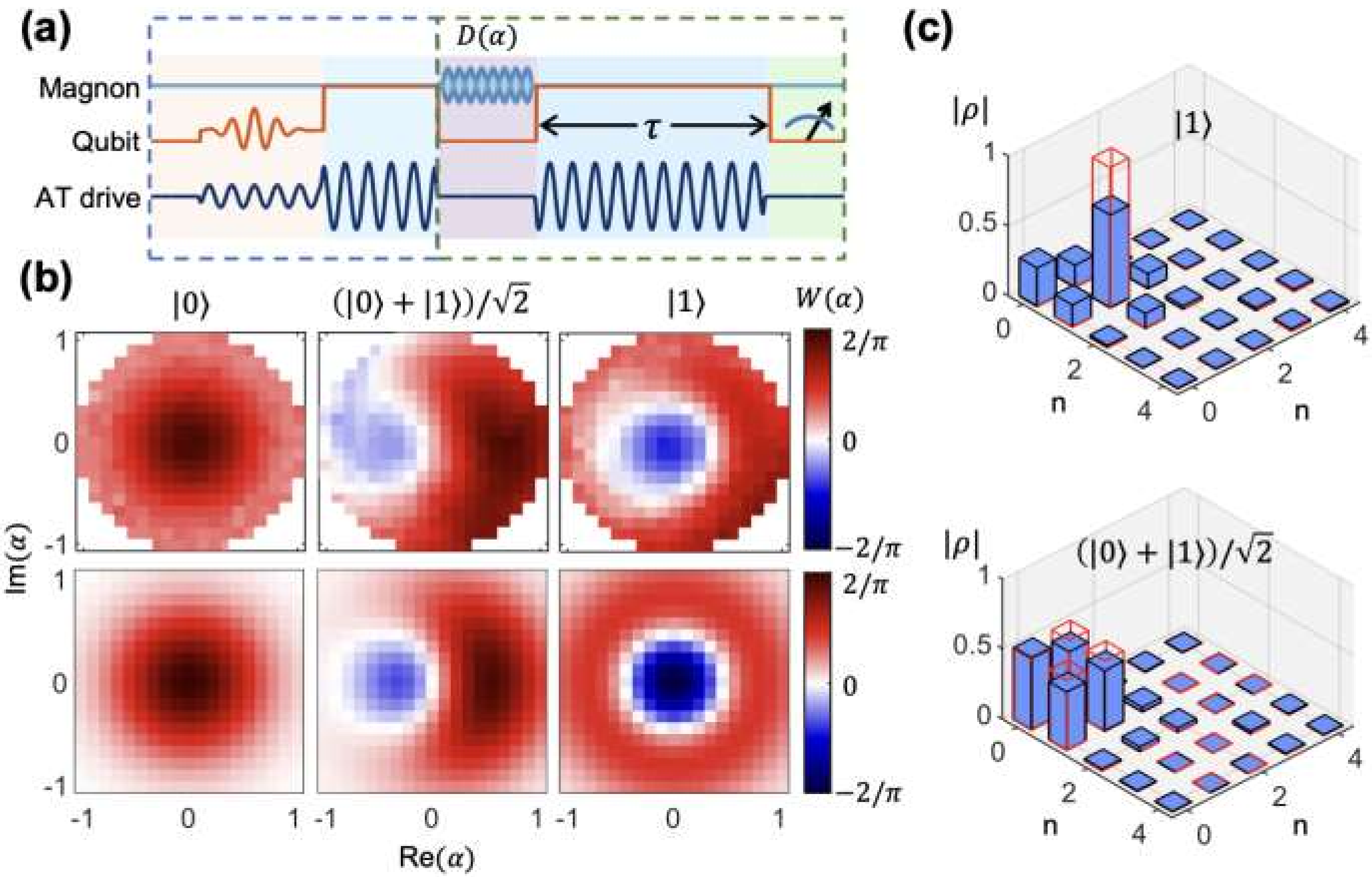
Cavity-induced qubit-magnon interaction

$$\hat{H}^{\text{res.}}_{\text{q-m}} = \hbar g_{\text{q-m}}\left(\hat{q}\hat{m}^{\dagger} + \hat{q}^{\dagger}\hat{m}\right)$$

$$g_{\text{q-m}} \approx \frac{g_{\text{q-c}}g_{\text{m-c}}}{\omega_{\text{q,m}} - \omega_{\text{c}}}$$

Quantum control of single magnon

The non-linearity of the qubit can be used to control the magnon at a single quanta level!



Xu et al PRL 2023

Easy-axis in Ferromagnet

[redacted]

$$\mathcal{H}_{\mathrm{FM}} = -J \sum_{\langle \boldsymbol{r},\boldsymbol{r}' \rangle} \boldsymbol{S}_{\boldsymbol{r}} \cdot \boldsymbol{S}_{\boldsymbol{r}'} - \sum_{\boldsymbol{r}} \left(K \hat{S}_{\boldsymbol{r}}^{z2} + \gamma \mu \boldsymbol{H} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{r}} \right)$$

Ferromagnetic exchange interaction | Easy-axis anisotropy | Zeeman term

- Exchange energy J is the largest energy scale : Ferromagnetic phase

[redacted]

- Due to the anisotropy, the magnetization is along easy-axis direction when H=0.

- Zeeman interaction due to the field, the magnetization tilt away from z-axis and toward y-axis

$$\boldsymbol{H} = H_0 \hat{y}$$

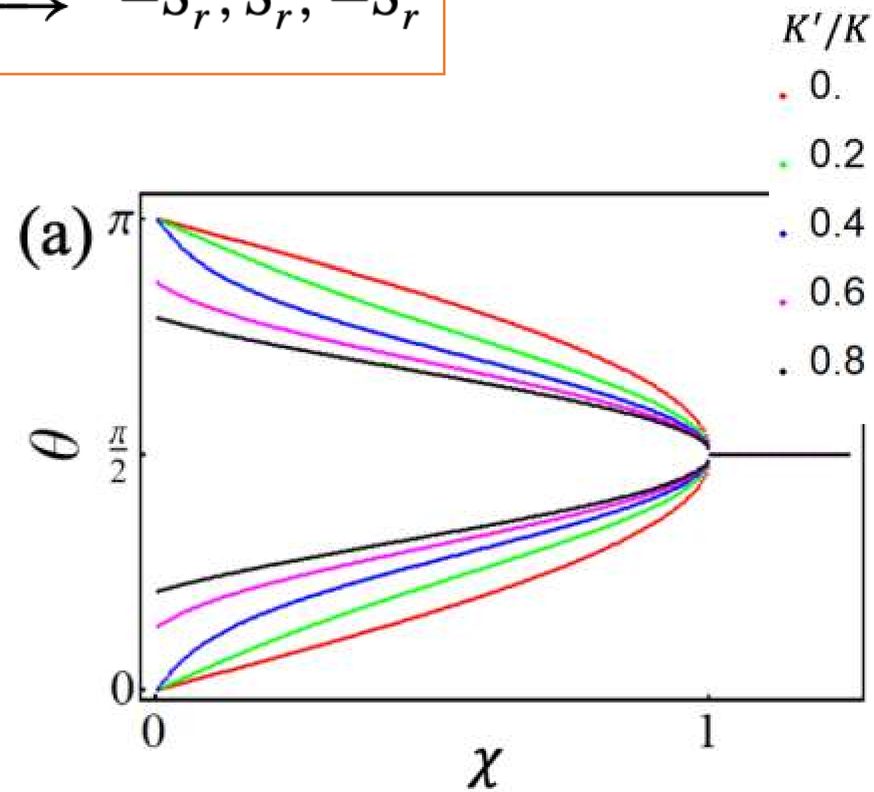
Easy-axis ferromagnet – Phase transition

[redacted] $\hat{\boldsymbol{S}}_{\boldsymbol{r}'} - \sum_{\boldsymbol{r}} (K \hat{S}_{\boldsymbol{r}}^{z2} + \gamma\mu \boldsymbol{H} \cdot \hat{\boldsymbol{S}}_{\boldsymbol{r}})$

- Mirror symmetry: $\hat{S}_r^x, \hat{S}_r^y, \hat{S}_r^z \xrightarrow{\mathcal{M}_y} -\hat{S}_r^x, \hat{S}_r^y, -\hat{S}_r^z$

[redacted] $\chi = |\gamma\mu H_0|/2KS$ [redacted]

$$\hat{\boldsymbol{S}}_{\boldsymbol{r}} = S(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$



[redacted]

- Weak-field limit : broken-symmetry
- Strong-field limit : symmetric phase

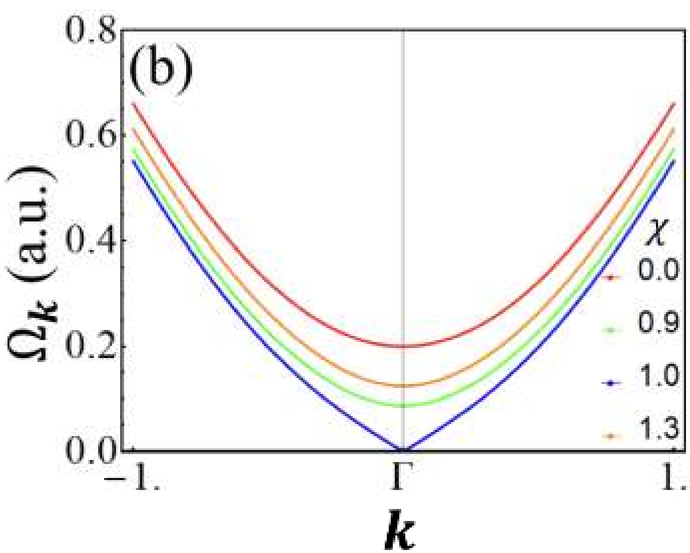
Easy-axis ferromagnet – Critical squeezing

[redacted]se transition?

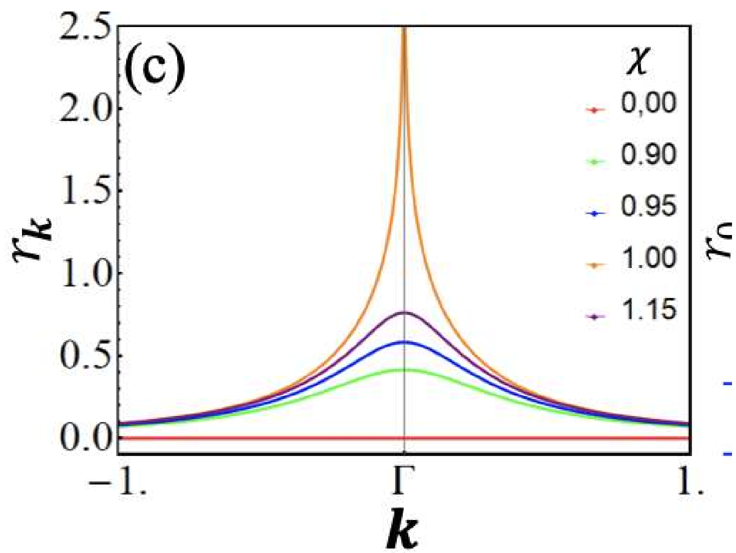
Magnon Hamiltonian: fluctuation around the magnetization

$$-KS\sum_{\boldsymbol{k}}\left[-\frac{\sin^2\theta}{2}(a_{\boldsymbol{k}}-a^\dagger_{-\boldsymbol{k}})(a_{-\boldsymbol{k}}-a^\dagger_{\boldsymbol{k}})-2\cos^2\theta a^\dagger_{\boldsymbol{k}}a_{\boldsymbol{k}}\right]+\gamma' H_0\sin\theta\sum_{\boldsymbol{k}}a^\dagger_{\boldsymbol{k}}a_{\boldsymbol{k}}.$$

Magnon squeezing



Magnon Dispersion

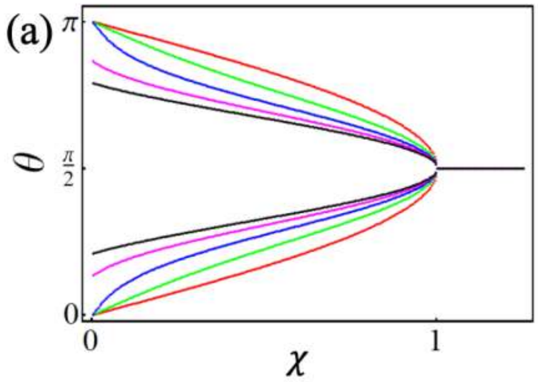


Degree of squeezing

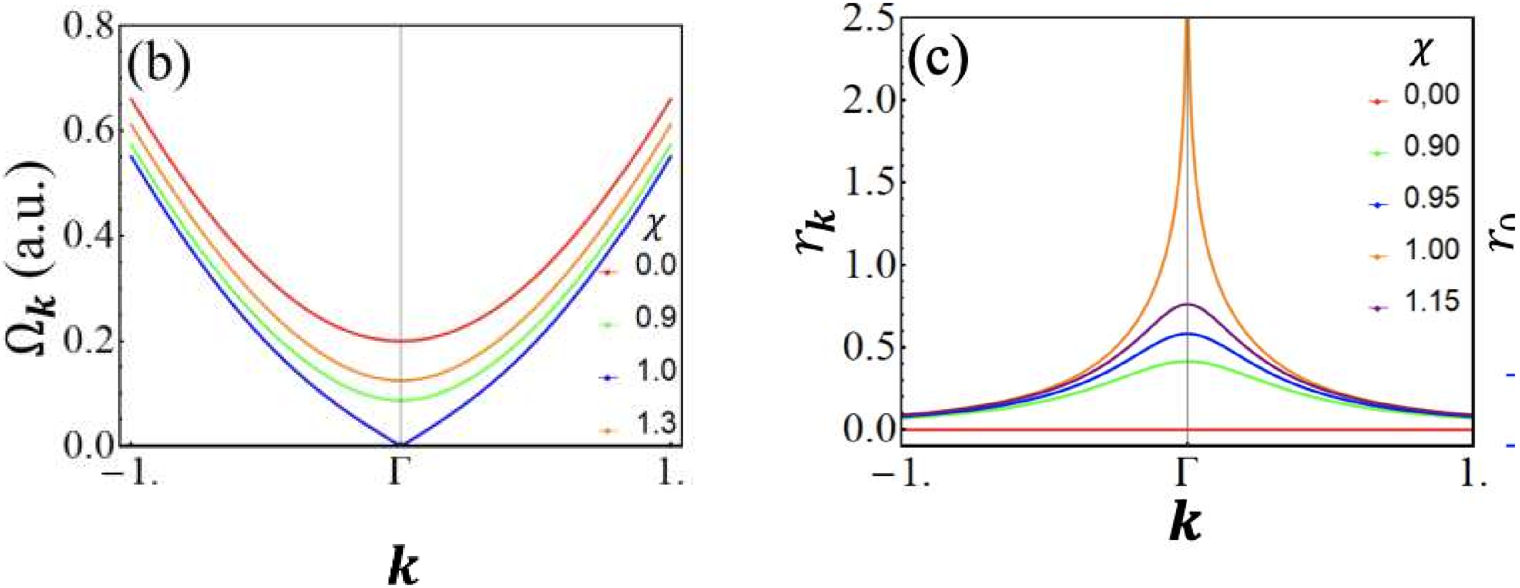
Easy-axis ferromagnet – Critical squeezing

[redacted]

Q: What drives the mechanism of the phase transition?



The softening of the magnon frequency is due to the *diverging* magnon squeezing!

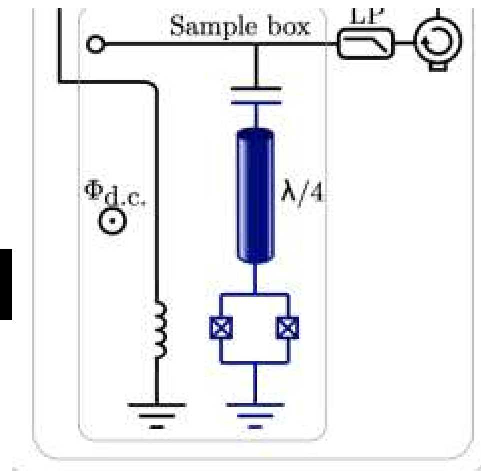


Magnon squeezing as a resource

[redacted]oolbox for many quantum information processing. Eg: Cat-code for QC, quantum parametric amplifiers. $\propto a^2 + a^{\dagger 2}$

Typically, two-photon processes are dynamically generated using non-linear system or parametric driving.

[redacted]



[redacted]

The anisotropy naturally gives the two magnon driving term $\propto a^2 + a^{\dagger 2}$

How can we leverage this in a cavity magnonic system?

Cavity magnonics with Easy-axis ferromagnet

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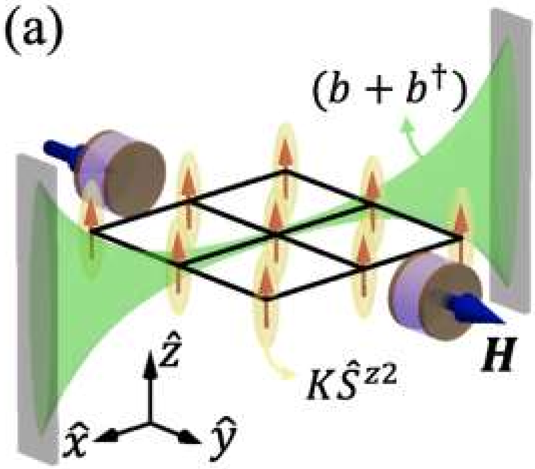


$$\mathcal{H}_{\text{tot}} = \omega b^\dagger b - \frac{g}{\sqrt{N}}\sqrt{\frac{2}{S}}(b+b^\dagger)\sum_{\boldsymbol{r}} \hat{S}^z_{\boldsymbol{r}} + \mathcal{H}_{\text{FM}},$$

$$\mathcal{H}_{\text{FM}} = -J\sum_{\langle \boldsymbol{r},\boldsymbol{r}'\rangle} \hat{\mathbf{S}}_{\boldsymbol{r}} \cdot \hat{\mathbf{S}}_{\boldsymbol{r}'} - \sum_{\boldsymbol{r}} \left(K\hat{S}^{z2}_{\boldsymbol{r}} + \gamma\mu \boldsymbol{H}\cdot\hat{\mathbf{S}}_{\boldsymbol{r}}\right),$$

In the strong field limit, the magnetization will be aligned to the B-field direction.
Therefore, there is no displacement of the cavity due to the magnetization.
(Normal phase)

Cavity magnonics with Easy-axis ferromagnet

[redacted]

Normal phase effective Hamiltonian (for Kittel mode)

$$\mathcal{H} = KS(2\chi - 1)a^\dagger a + \omega b^\dagger b + \boxed{\frac{KS}{2}(a^2 + a^{\dagger 2})} - ig(a - a^\dagger)(b + b^\dagger),$$

Magnon squeezing

Bogoliubov transformation (squeezing transformation)

$$\mathcal{H} = \Omega_0 \tilde{a}^\dagger \tilde{a} + \omega b^\dagger b - i\tilde{g}(\tilde{a} - \tilde{a}^\dagger)(b + b^\dagger)$$

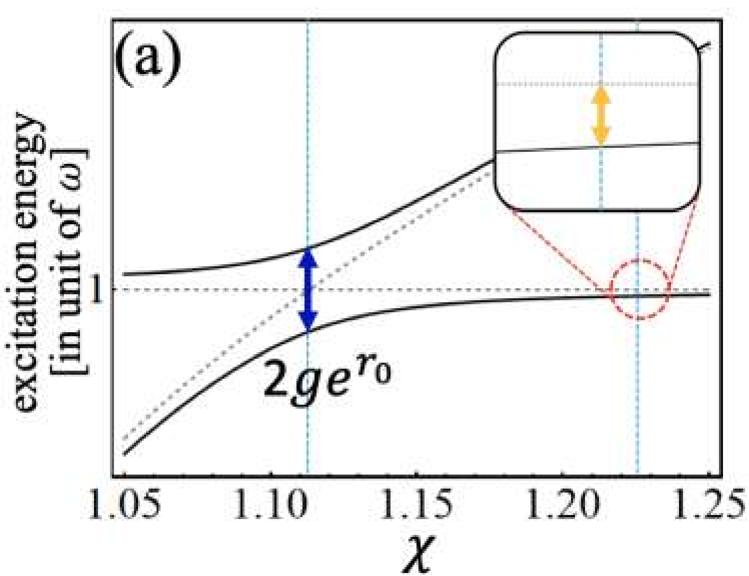
Enhanced coupling between due to the magnon squeezing

$$\tilde{g} = g e^{r_0} \qquad r_0 = -\frac{1}{4}\log(1 - \chi^{-1})$$

$$\chi = |\gamma \mu H_0| / 2KS$$

Enhanced cavity-magnon coupling strength

[redacted] by the gap of the avoided crossings!

[redacted] 

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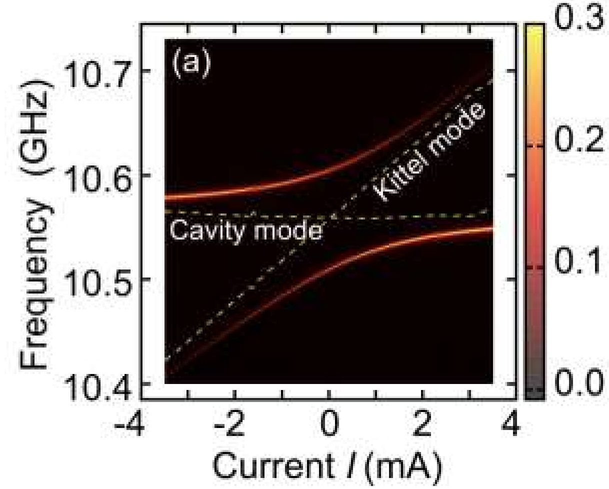
$$\mathcal{H} = \Omega_0 \tilde{a}^\dagger \tilde{a} + \omega b^\dagger b - i\tilde{g}(\tilde{a} - \tilde{a}^\dagger)(b + b^\dagger)$$



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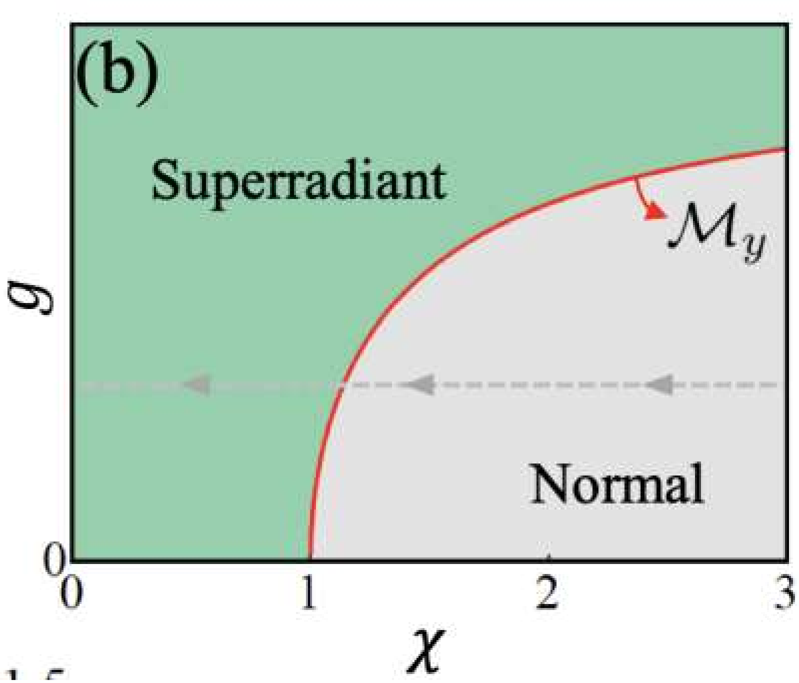
This avoided crossing is a smoking gun experiment showing the quantum nature of magnon due to squeezing.

Ultrastrong coupling and Superradiant PT

[redacted]ic magnetic field can be increased arbitrarily large.

At a threshold value, the diverging magnon squeezing induces a superradiant phase transition breaking a mirror symmetry (Z2)



$$\mathcal{H}_{\rm tot} = \omega b^\dagger b - \frac{g}{\sqrt{N}}\sqrt{\frac{2}{S}}(b+b^\dagger)\sum_{\boldsymbol{r}} \hat{S}^z_{\boldsymbol{r}} + \mathcal{H}_{\rm FM},$$

[redacted]

$$(b, \hat{S}^x_{\boldsymbol{r}}, \hat{S}^y_{\boldsymbol{r}}, \hat{S}^z_{\boldsymbol{r}}) \xrightarrow{\mathcal{M}_y} (-b, -\hat{S}^x_{\boldsymbol{r}}, \hat{S}^y_{\boldsymbol{r}}, -\hat{S}^z_{\boldsymbol{r}})$$

Since the magnetic dipole coupling drives the SPT, it is free of the A^2-term that inhibits SPT.

Superradiant Phase Transition

[redacted]

Diverging photon numbers