중시계 양자역학의 실험적 실현:
빛과 초전도체의 상호작용

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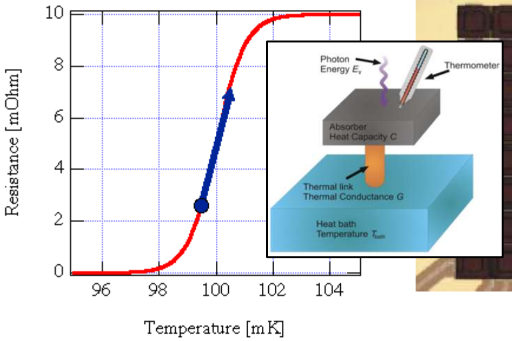
Outline

1. Light-superconductor interaction in mesoscopic system
2. Introduction to Superconductivity
3. Introduction to Josephson Junction
4. Josephson Junction with light
5. SC-light interaction: optical regime
6. SC-light interaction: microwave regime

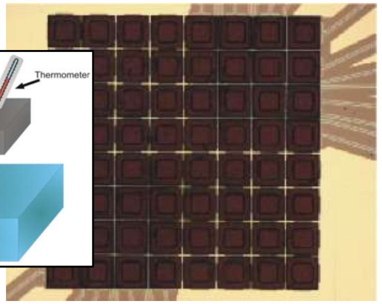
“Light” + “Superconductor”

1. Photon Detector

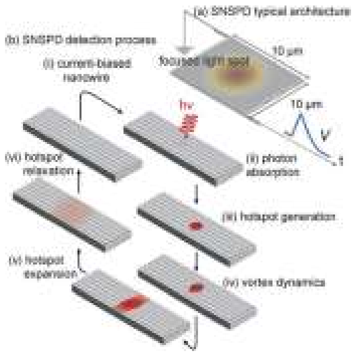
Transition Edge Sensor (TES)



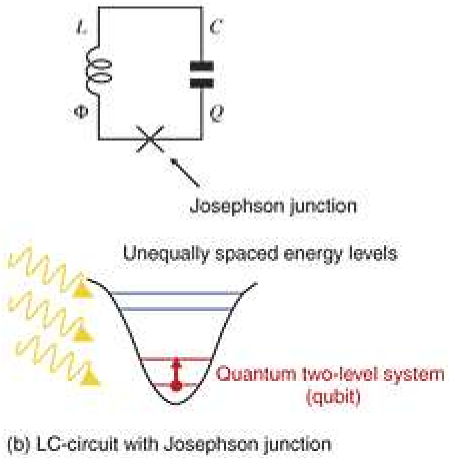
Array of TES

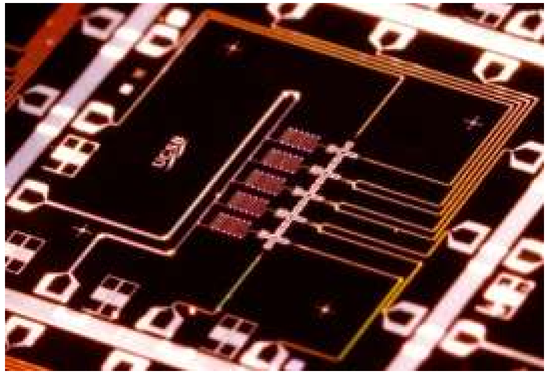


Superconducting nanowire single photon detector (SNSPD)

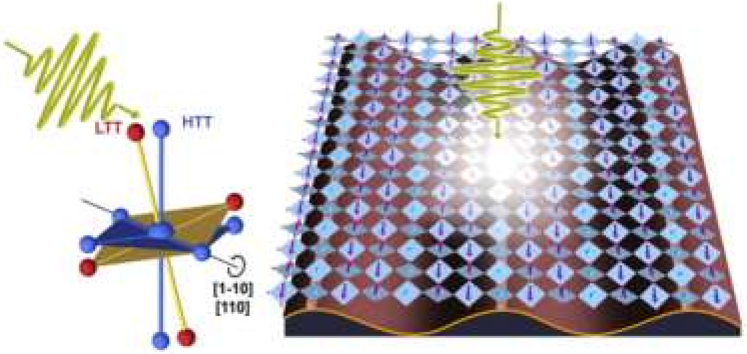


2. Qubit Control/Readout





3. Non-equilibrium QM

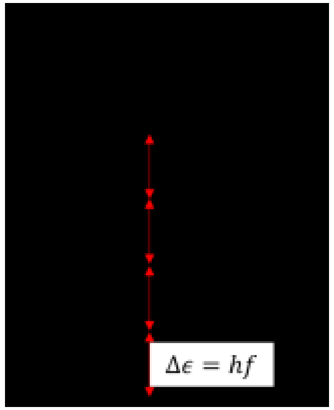


transient high-Tc superconductivity

[Science 331, 189-191 (2011)]

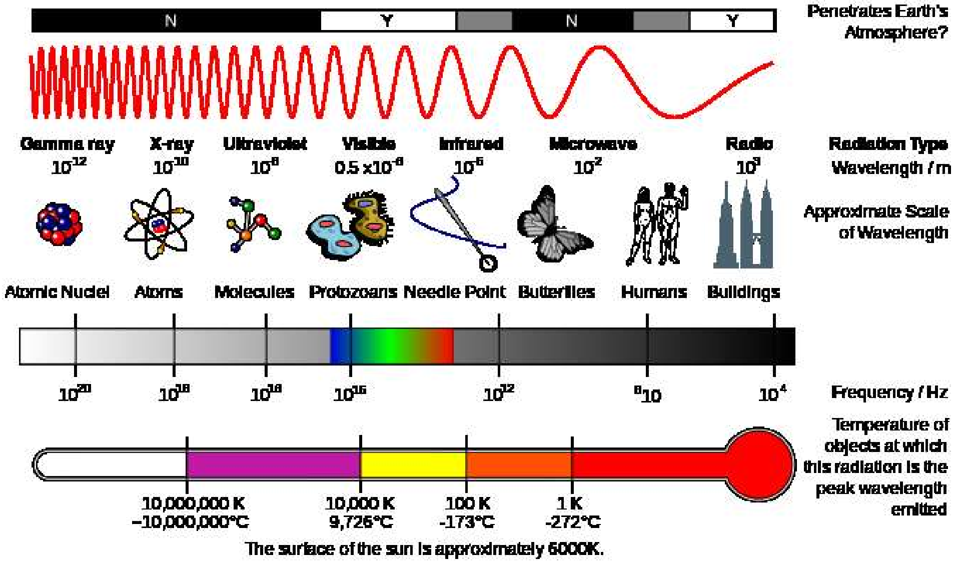
Floquet state

[Science 342, 453-457 (2013),
Nature 603, 421–426 (2022)]



Energy Scale Comparison

- Energy $E = hf(= \hbar\omega) = k_B T$
- Wavelength $\lambda = c/f = hc/E$



- ($E_{photon} > k_B T$)
→ *non-equilibrium* without thermal excitation

항목	에너지 E	파장 λ	온도 T
Optical photon	~1.0–3.0 eV	0.4–1.2 μm	10,000–30,000 K
Infrared photon	~10–200 meV	6–120 μm	100–2,000 K
THz photon	~1–10 meV	30–300 μm	10–100 K
Superconducting gap	~0.1-1 meV (Al, Nb, etc.)		1-10 K
Microwave photon (6 GHz)	~25 μeV	5 cm	0.3 K
Dilution refrigerator temperature	~1 μeV		0.01 K

- ($E_{photon} > \Delta$) photons break Cooper pairs
→ good for *photon sensor*

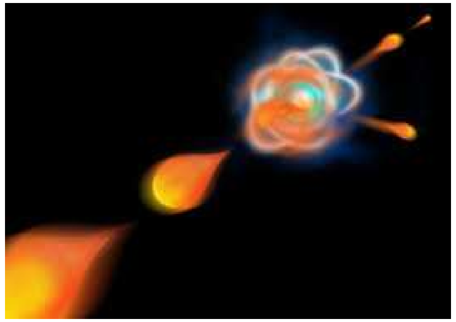
- ($E_{photon} < \Delta$) no effect on SC itself
→ good for *qubit operation*

Mesoscopic System (중시계)

*system size L, coherence length L_ϕ

미시계 (microscopic system)

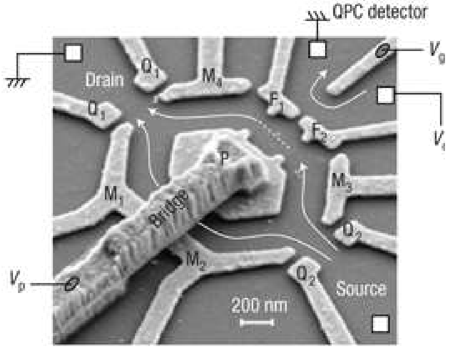
$L \ll L_\phi$



photon, atom, molecule, etc.
$L \sim$ Å

중시계 (mesoscopic system)

$L \lesssim L_\phi$



graphene, 2DEG, nanowires, JJs
$L \sim \mu m$

거시계 (macroscopic system)

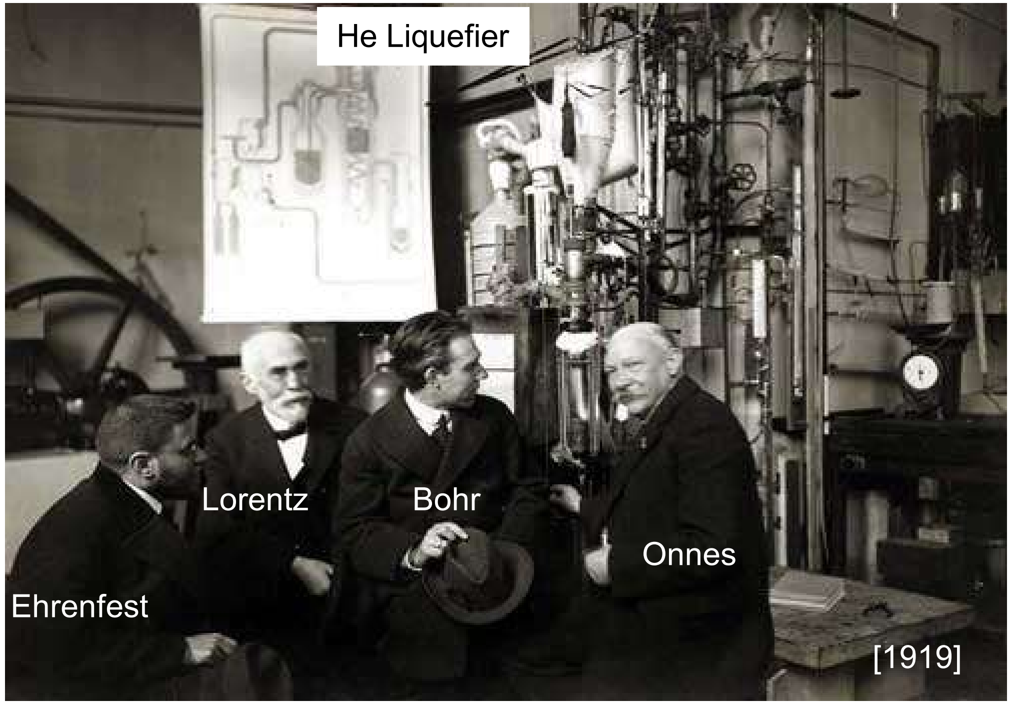
$L \gg L_\phi$

human, cat
$L \sim m$

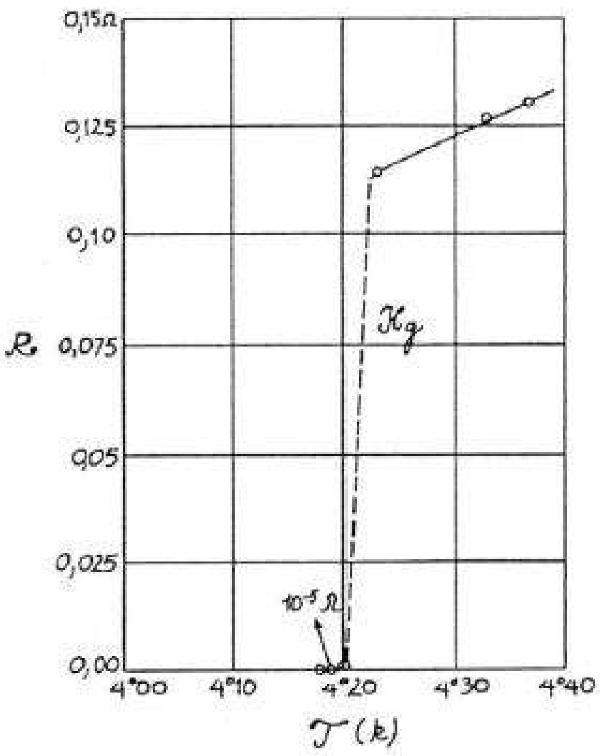
In a mesoscopic system, even a single photon can trigger a *nonequilibrium quantum response*.

Introduction to Superconductivity

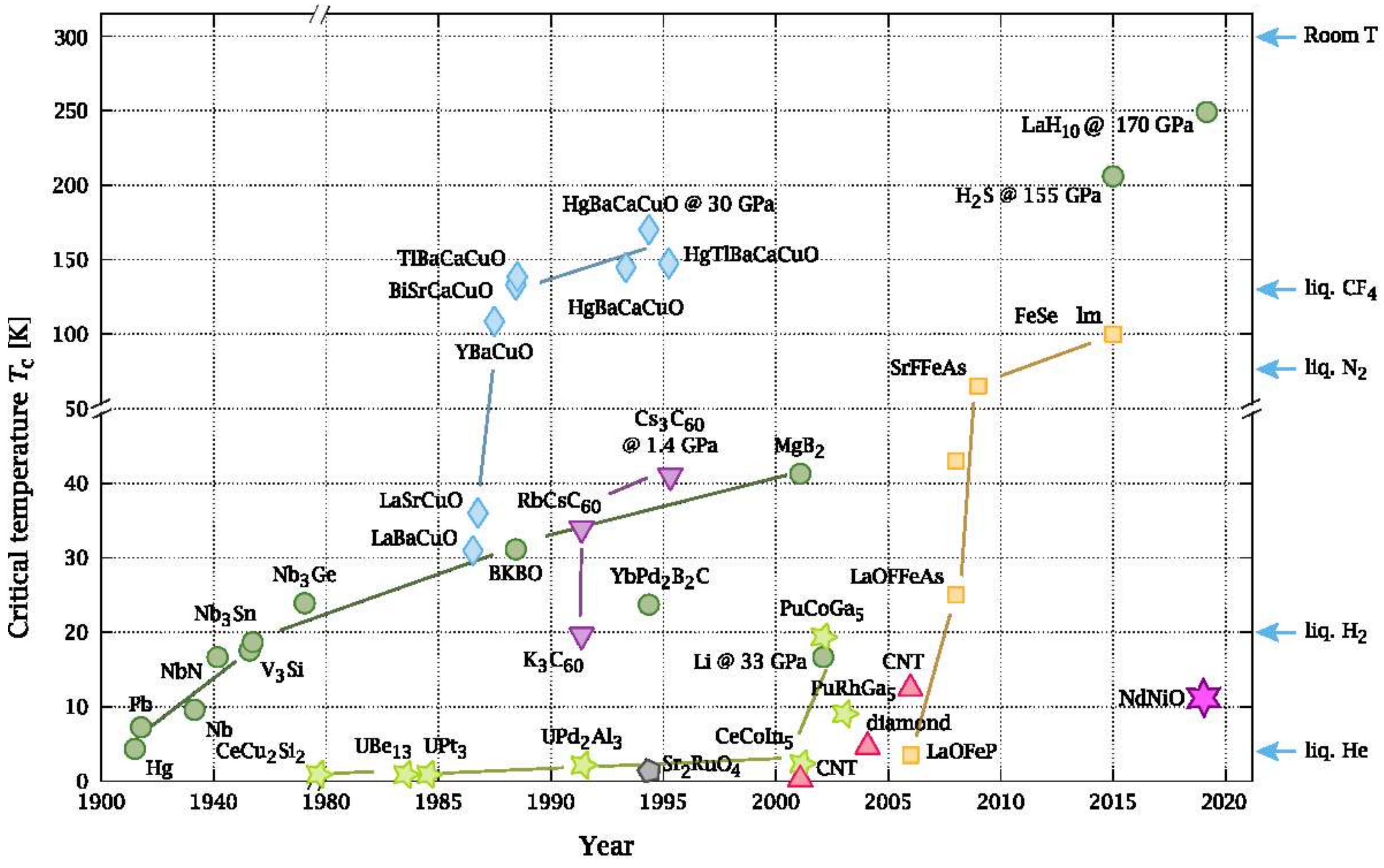
Superconductivity (1911)



Superconductivity of Mercury (1911)



History of Superconductor



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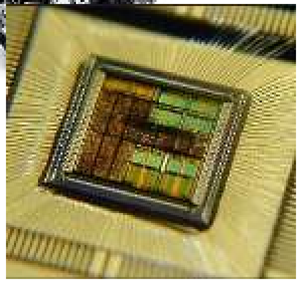
Quantum Electronics

- Study of quantum mechanical behavior of electrons in solids

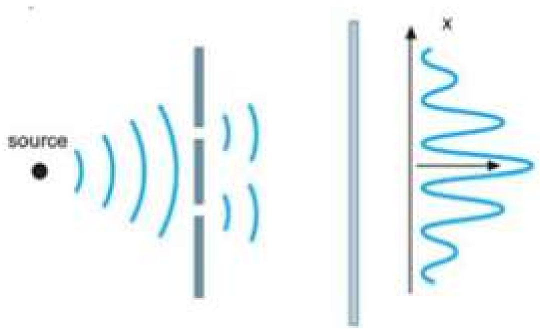
Fundamental Science		Applied Science
Classical Mechanics	→	Mechanical Engineering
Electromagnetics	→	Electrical Engineering
Quantum Mechanics	→	**Quantum Engineering**

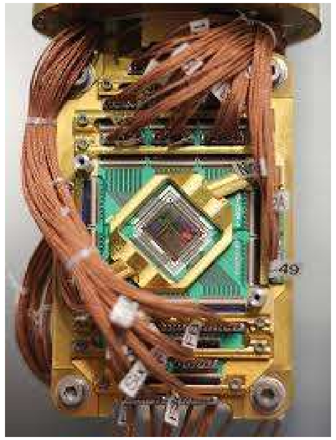
ex.)
Engine



ex.)
CPU

e.g.) quantum coherence,
superconductivity

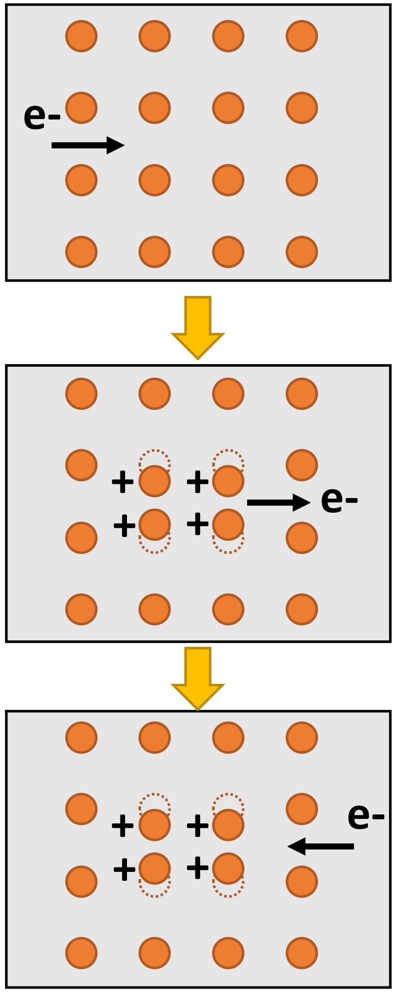


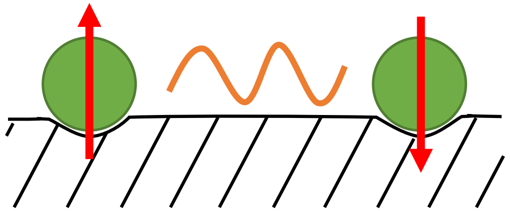


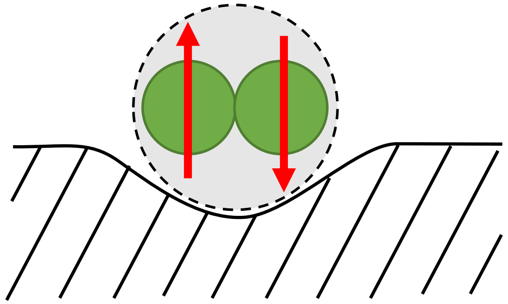
ex.)
Quantum Computers

BCS theory (1957)

Microscopic mechanism – BCS theory (1957)







in BCS superconductor

Spin-singlet (anti-sym.)

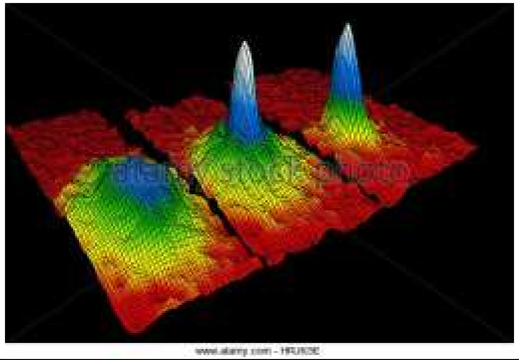
$$|S\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Orbital : *s*-wave (sym.)

Macroscopic quantum phenomena

10^{23} electrons in superconductor form a single quantum state

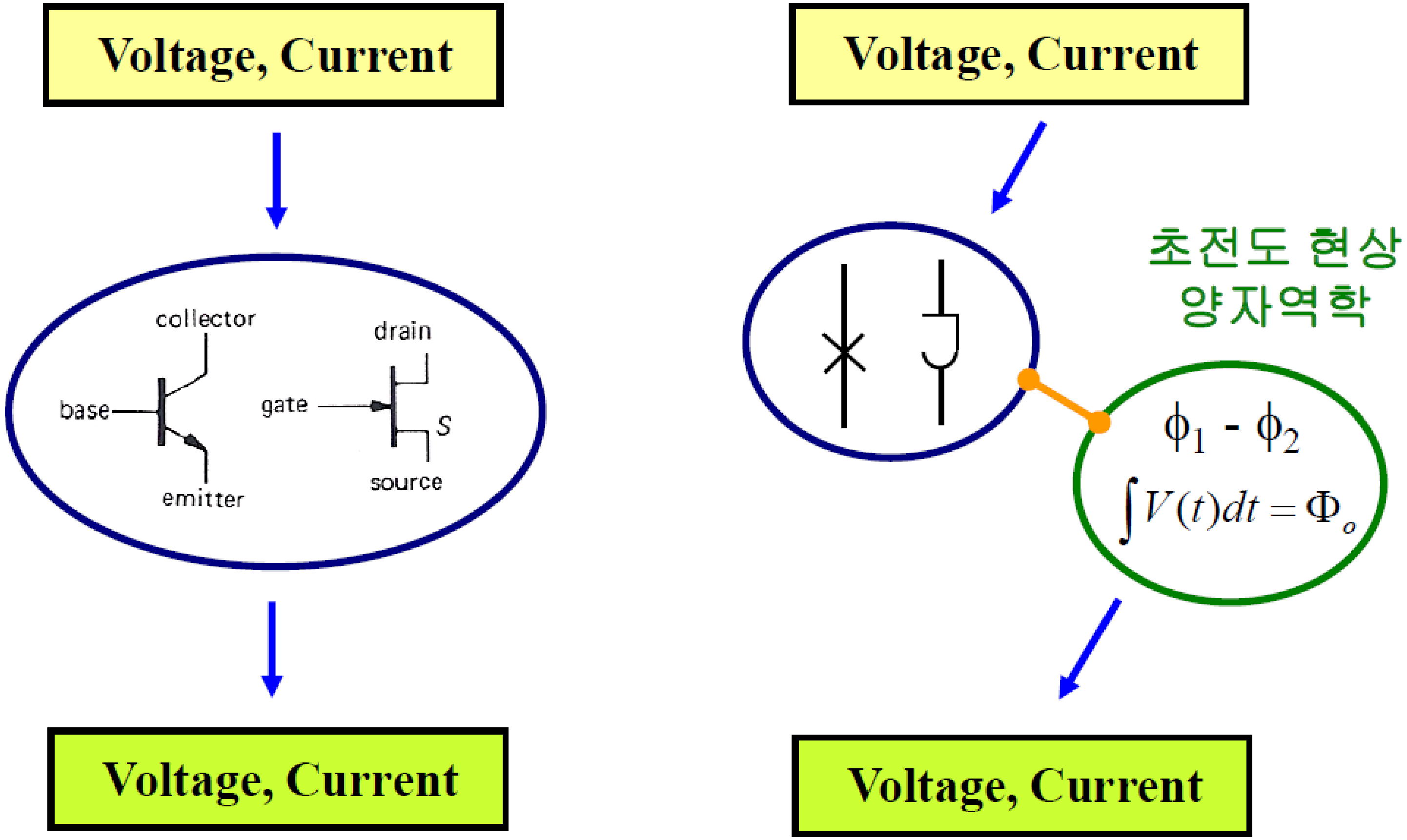
e.x.) BEC condensate



POSTECH

Introduction to Josephson Junction

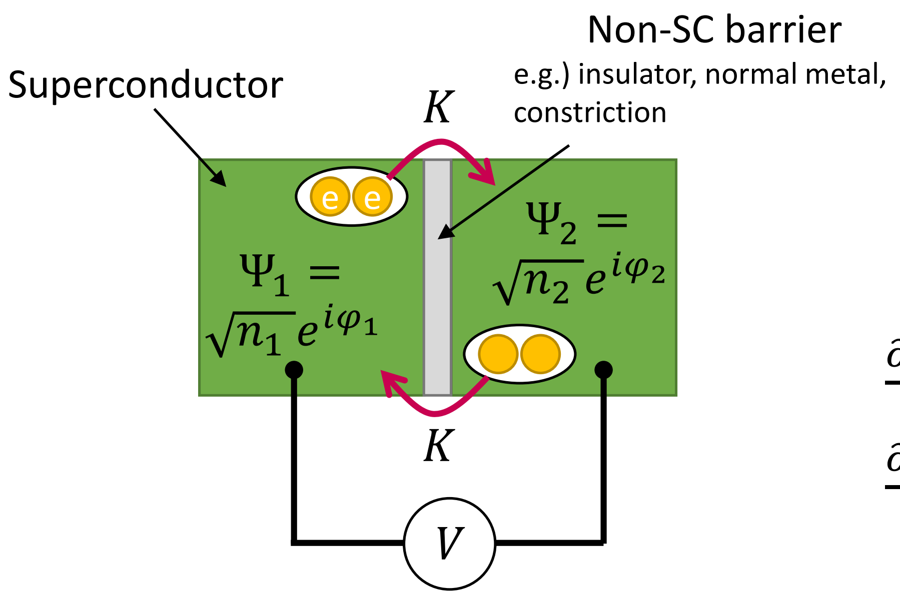
Superconducting Electronic Device



[Slide from Prof. 정연욱]

Tunneling Josephson Junction (JJ)

Predicted by B. D. Josephson in 1962



n: density of Cooper pair
φ: phase of order parameter
K: Coupling parameter

Equation of motion for JJ

$$U_1 - U_2 = qV$$
$$q = 2e$$

$$\begin{cases} i\hbar \dfrac{\partial \Psi_1}{\partial t} = U_1\Psi_1 - K\Psi_2 \\ i\hbar \dfrac{\partial \Psi_2}{\partial t} = U_2\Psi_2 - K\Psi_1 \end{cases}$$

We set $\frac{U_1+U_2}{2} = 0$, then $U_1 = \frac{qV}{2}, U_2 = -\frac{qV}{2}$

$$\frac{\partial \Psi_1}{\partial t} = \frac{1}{2\sqrt{n_1}} e^{i\varphi_1} \frac{dn_1}{dt} + i\sqrt{n_1} e^{i\varphi_1} \frac{d\varphi_1}{dt} = \frac{qV}{2i\hbar}\sqrt{n_1}e^{i\varphi_1} - \frac{K}{i\hbar}\sqrt{n_2}e^{i\varphi_2} \quad \text{- Eq. (1)}$$

$$\frac{\partial \Psi_2}{\partial t} = \frac{1}{2\sqrt{n_2}} e^{i\varphi_2} \frac{dn_2}{dt} + i\sqrt{n_2} e^{i\varphi_2} \frac{d\varphi_2}{dt} = -\frac{qV}{2i\hbar}\sqrt{n_2}e^{i\varphi_2} - \frac{K}{i\hbar}\sqrt{n_1}e^{i\varphi_1} \quad \text{- Eq. (2)}$$

$(1) \times e^{-i\varphi_1}, (2) \times e^{-i\varphi_2}$

Phase difference: $\varphi \equiv \varphi_2 - \varphi_1$

$$\frac{1}{2\sqrt{n_1}}\frac{dn}{dt} + i\sqrt{n_1}\frac{d\varphi_1}{dt} = -i\frac{qV}{2\hbar}\sqrt{n_1} + i\frac{K}{\hbar}\sqrt{n_2}e^{i(\varphi_2-\varphi_1)} \quad \text{- Eq. (1)'}$$

$$\frac{1}{2\sqrt{n_2}}\frac{dn}{dt} + i\sqrt{n_2}\frac{d\varphi_2}{dt} = +i\frac{qV}{2\hbar}\sqrt{n_2} + i\frac{K}{\hbar}\sqrt{n_1}e^{-i(\varphi_2-\varphi_1)} \quad \text{- Eq. (2)'}$$

DC & AC Josephson Relationship

by using $e^{i\varphi} = \cos\varphi + i\sin\varphi$,

$$\frac{1}{2\sqrt{n_1}}\frac{dn_1}{dt} + i\sqrt{n_1}\frac{d\varphi_1}{dt} = -i\frac{qV}{2\hbar}\sqrt{n_1} + i\frac{K}{\hbar}\sqrt{n_2}(\cos\varphi + i\sin\varphi)$$ - Eq. (1)''

$$\frac{1}{2\sqrt{n_2}}\frac{dn_2}{dt} + i\sqrt{n_2}\frac{d\varphi_2}{dt} = +i\frac{qV}{2\hbar}\sqrt{n_2} + i\frac{K}{\hbar}\sqrt{n_1}(\cos\varphi - i\sin\varphi)$$ - Eq. (2)''

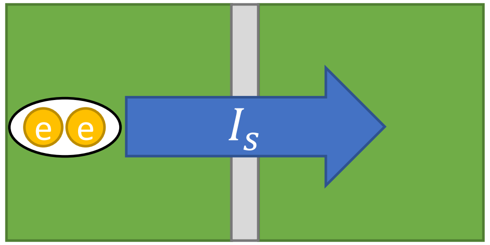
- Real part of Eqs. (1)'' and (2)''

$$\frac{dn_1}{dt} = -2\frac{K}{\hbar}\sqrt{n_1 n_2}\sin\varphi$$

$$\frac{dn_2}{dt} = +2\frac{K}{\hbar}\sqrt{n_1 n_2}\sin\varphi$$

Supercurrent:

$$I_s \propto -\frac{dn_1}{dt} = \frac{dn_2}{dt}$$



DC Josephson relationship

$$I_s = I_c \sin\varphi$$

- Imaginary part of Eqs. (1)'' and (2)''

$$\frac{d\varphi_1}{dt} = -\frac{qV}{2\hbar} + \frac{K}{\hbar}\sqrt{\frac{n_2}{n_1}}\cos\varphi$$

$$\frac{d\varphi_2}{dt} = \frac{qV}{2\hbar} + \frac{K}{\hbar}\sqrt{\frac{n_1}{n_2}}\cos\varphi$$

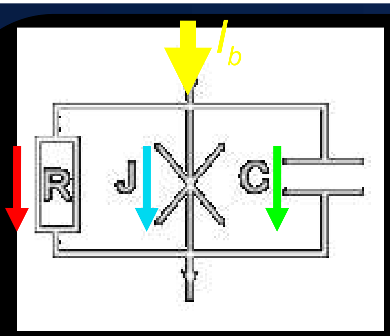
If $n_1 \approx n_2$,

$$\frac{d\varphi}{dt} = \frac{d\varphi_2}{dt} - \frac{d\varphi_1}{dt} = \frac{qV}{\hbar} = \frac{2eV}{\hbar}$$

AC Josephson relationship

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar}V$$

Josephson Junction



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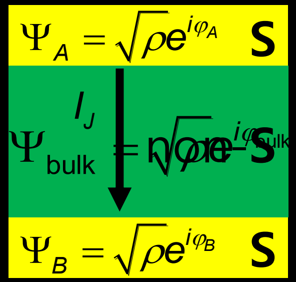

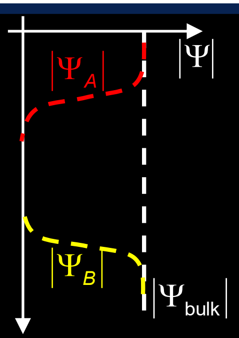

Josephson relations

$I_J = I_{c0}\sin\varphi$, $V = \frac{\quad}{2e}$

Phase difference

$\varphi = \varphi_B - \varphi_A$

Brian Josephson
Nobel prize in 1973

$$I_b = \frac{V}{R} + I_{c0}\sin\varphi + C\frac{dV}{dt}$$

$$m_\varphi \quad \frac{1}{RC} \quad \frac{\partial U(\varphi)}{\partial\varphi} = 0$$

kinetic — damping — potential

Equation of motion of the phase particle in $U(\varphi) = E_{J0}(-\cos\varphi - \gamma\varphi)$

- φ: position of phase particle
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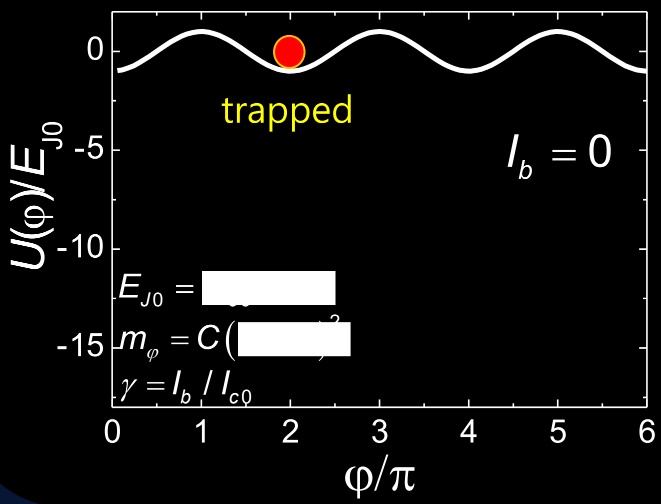
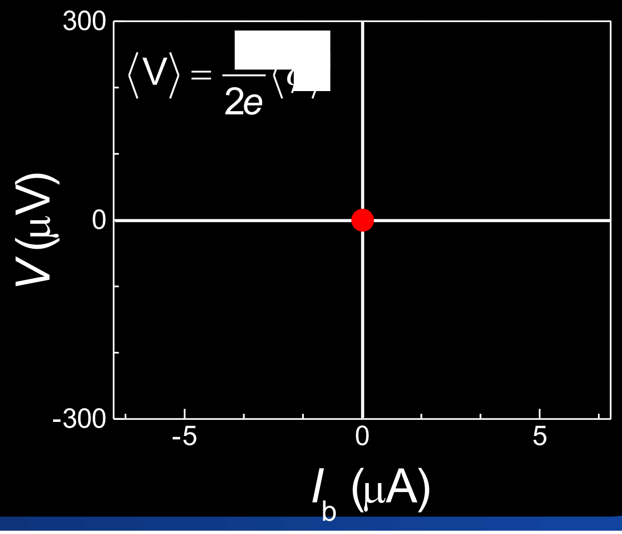
washboard potential



current-voltage characteristic



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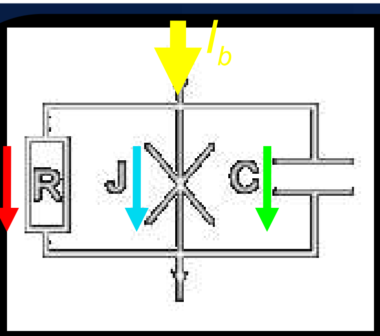

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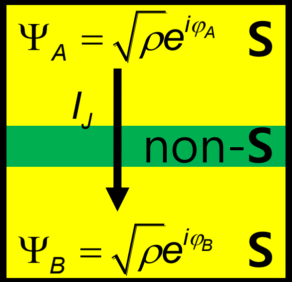

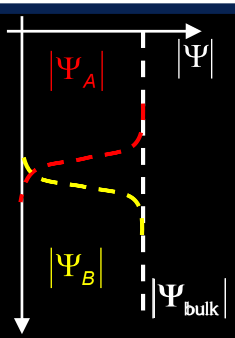

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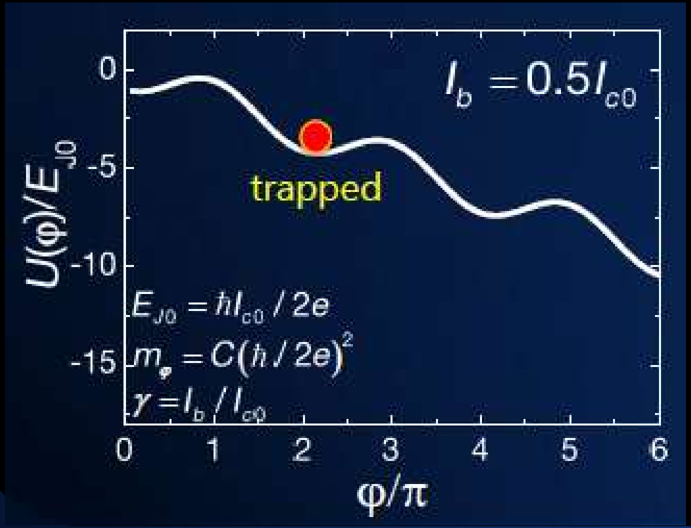

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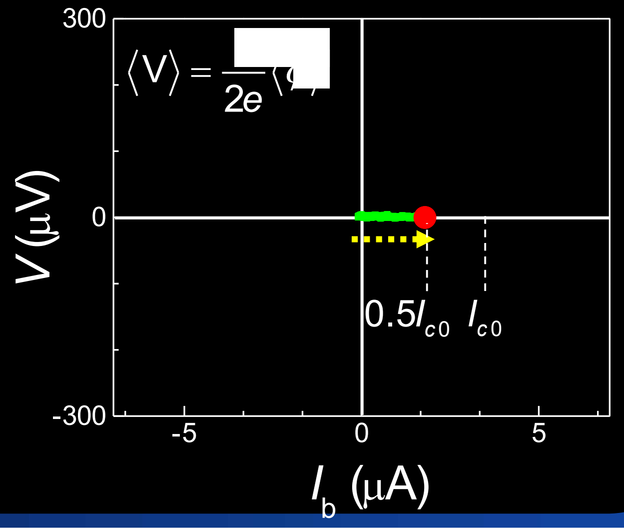

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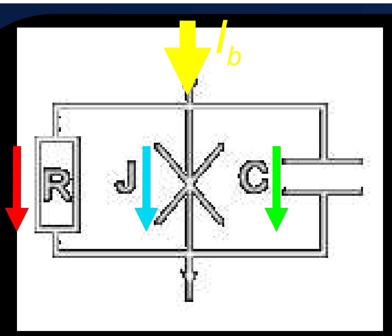

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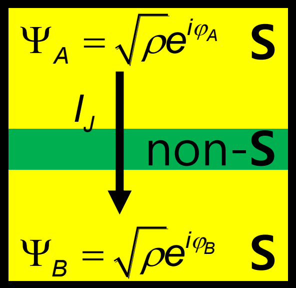

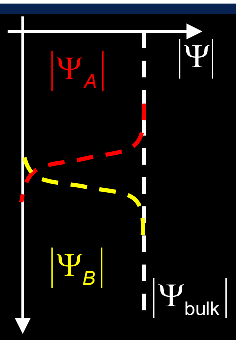

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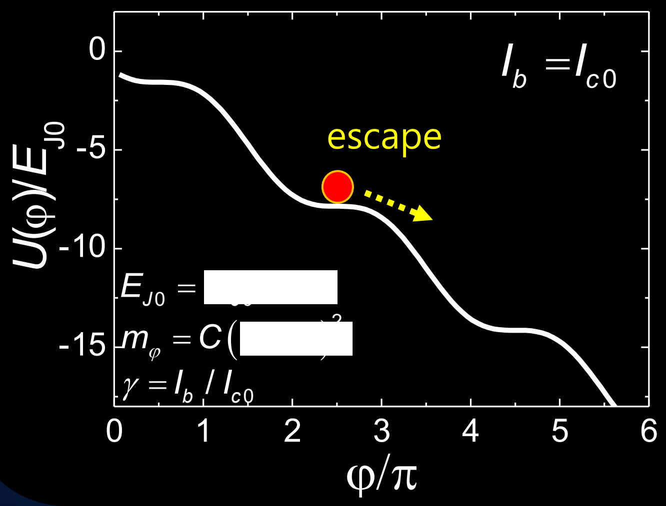

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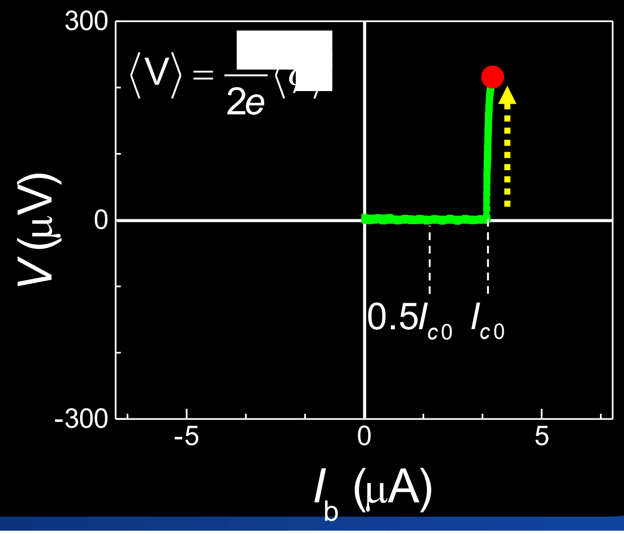

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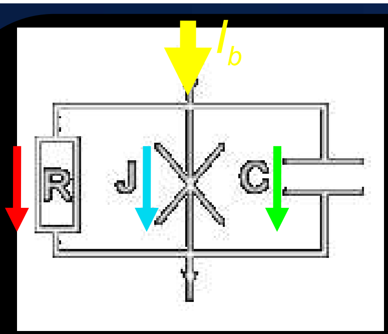

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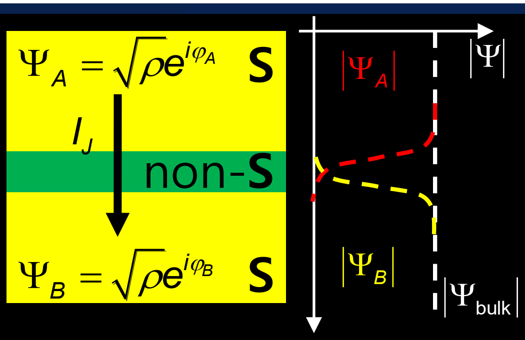

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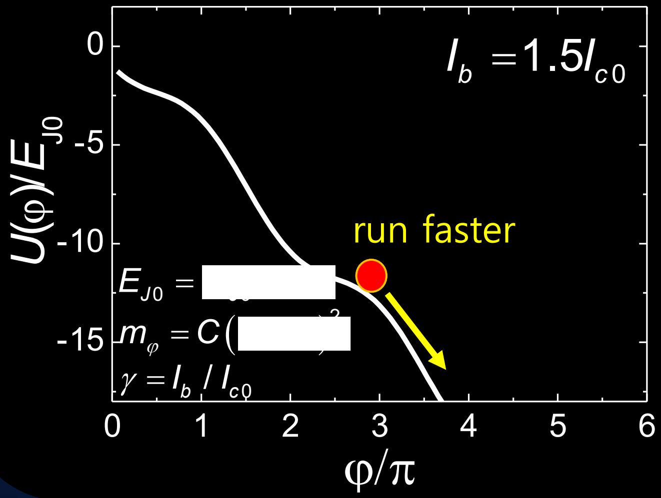

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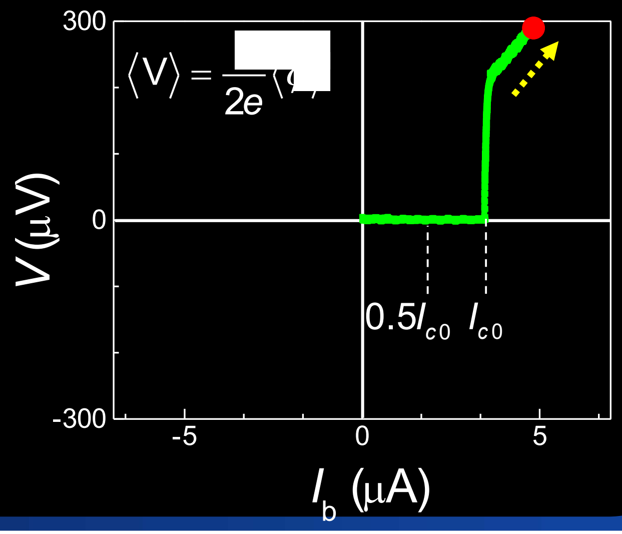

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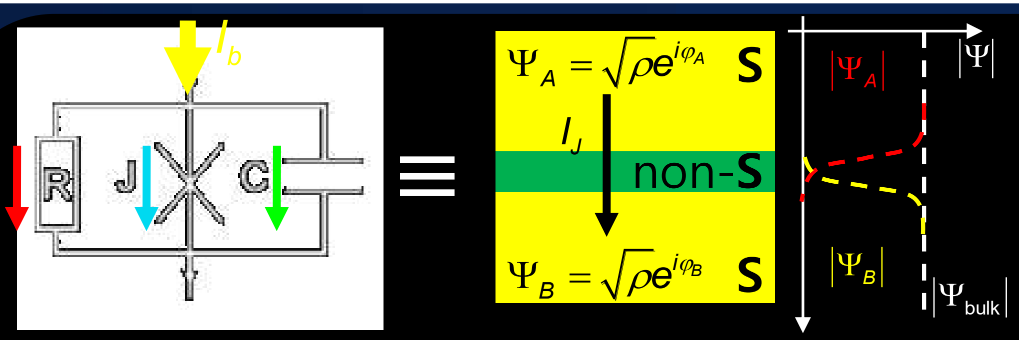

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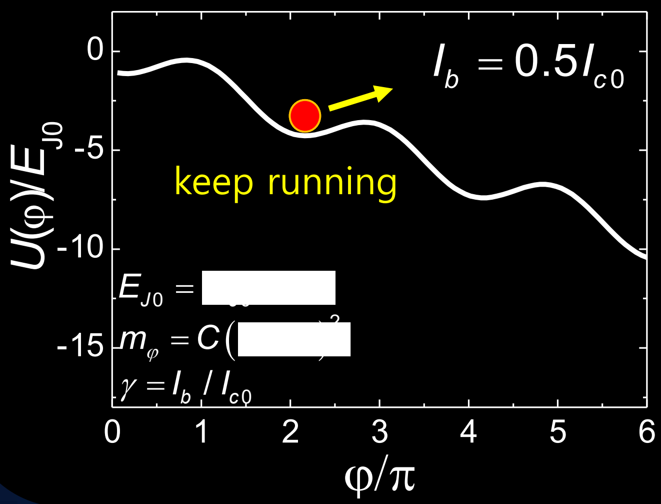

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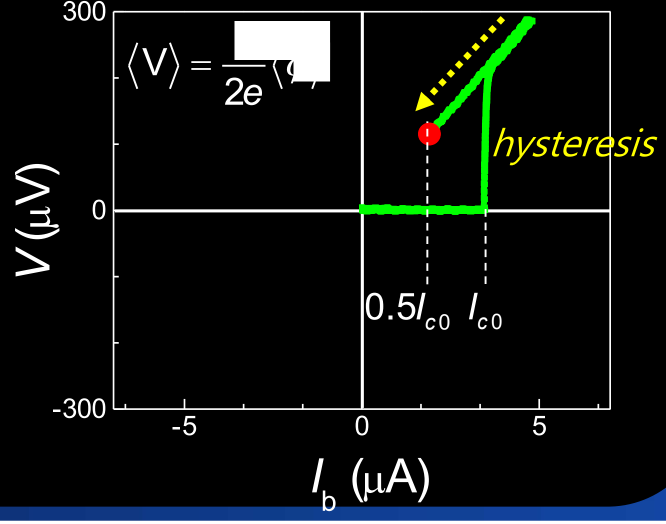

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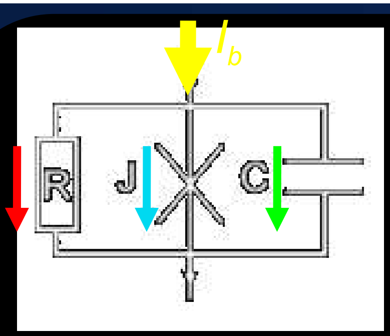

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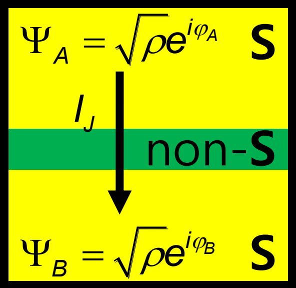
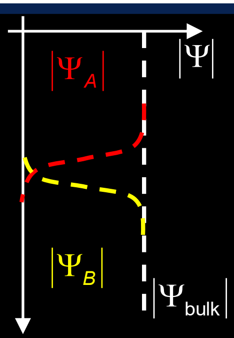




Josephson relations

$I_J = I_{c0}\sin\varphi$, V = … / 2e

Phase difference

$\varphi = \varphi_B - \varphi_A$

Brian Josephson
Nobel prize in 1973

$$I_b = \frac{V}{R} + I_{c0}\sin\varphi + C\frac{dV}{dt}$$

m_φ … $\frac{1}{RC}$ … $\frac{\partial U(\varphi)}{\partial \varphi} = 0$

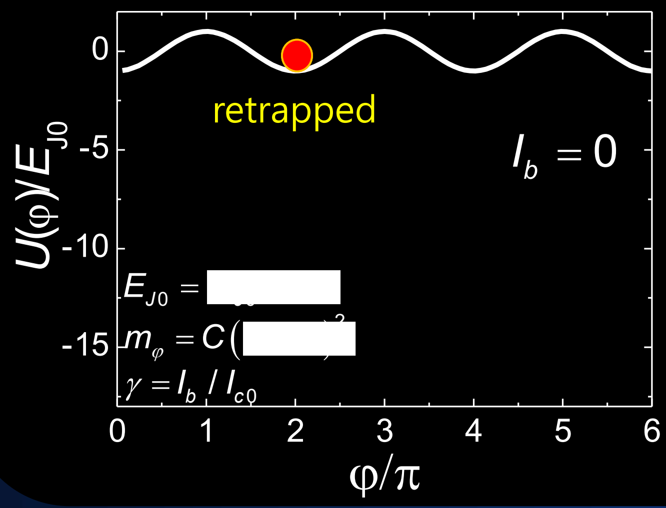
kinetic — damping — potential

Equation of motion of the phase particle in

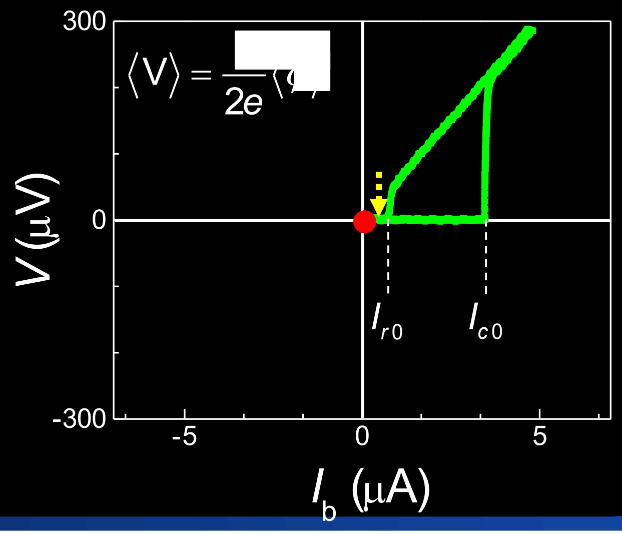
$$U(\varphi) = E_{J0}(-\cos\varphi - \gamma\varphi)$$

- φ: position of phase particle
- m_φ : mass of phase particle

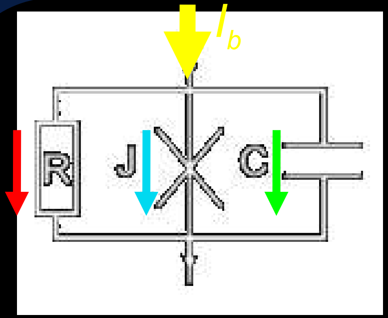
washboard potential



current-voltage characteristic



Josephson Junction



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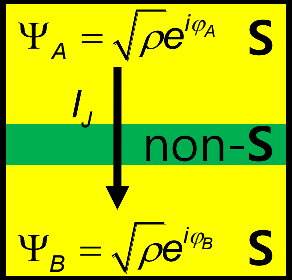
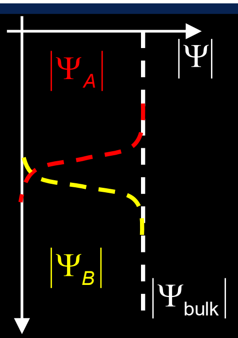




Josephson relations

$I_J = I_{c0} \sin\varphi$, V = [redacted]

Phase difference

$\varphi = \varphi_B - \varphi_A$

Brian Josephson
Nobel prize in 1973

$$I_b = \frac{V}{R} + I_{c0} \sin\varphi + C \frac{dV}{dt}$$

m_φ [redacted] $= 0$

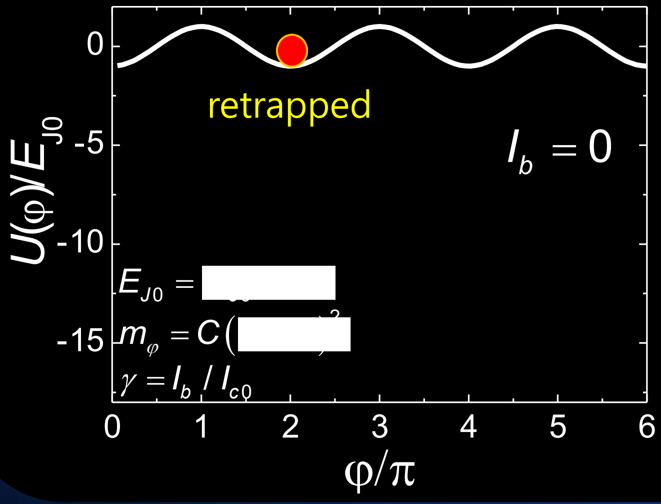
kinetic — damping — potential

Equation of motion of the phase particle in

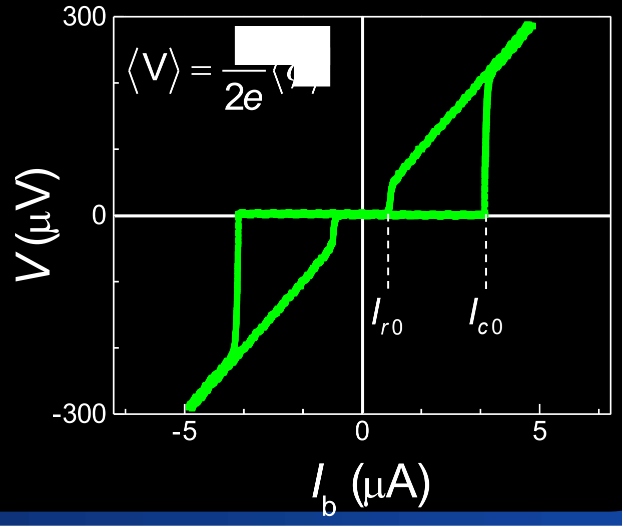
$U(\varphi) = E_{J0}(-\cos\varphi - \gamma\varphi)$

- φ: position of phase particle
- m_φ : mass of phase particle

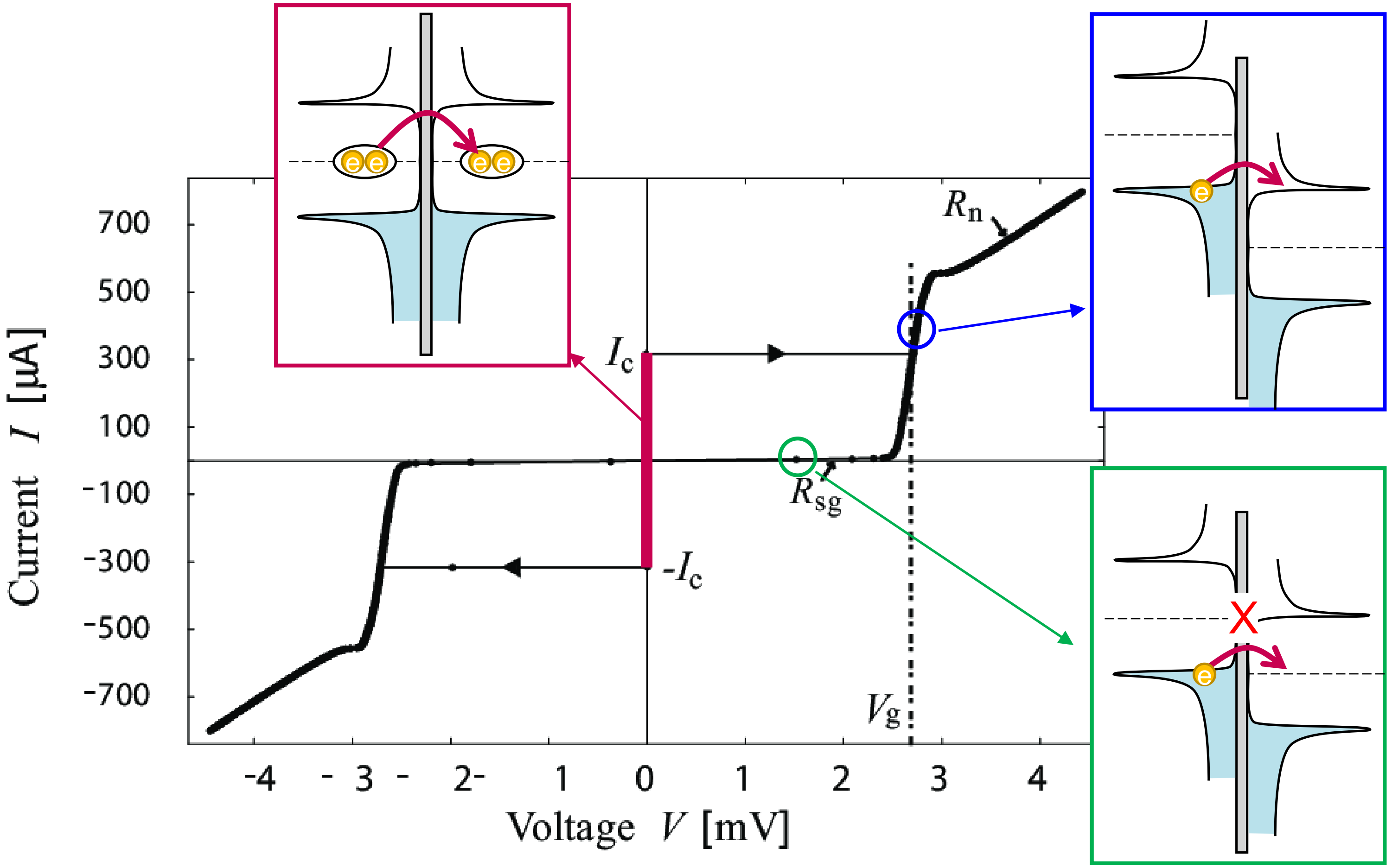
washboard potential



current-voltage characteristic

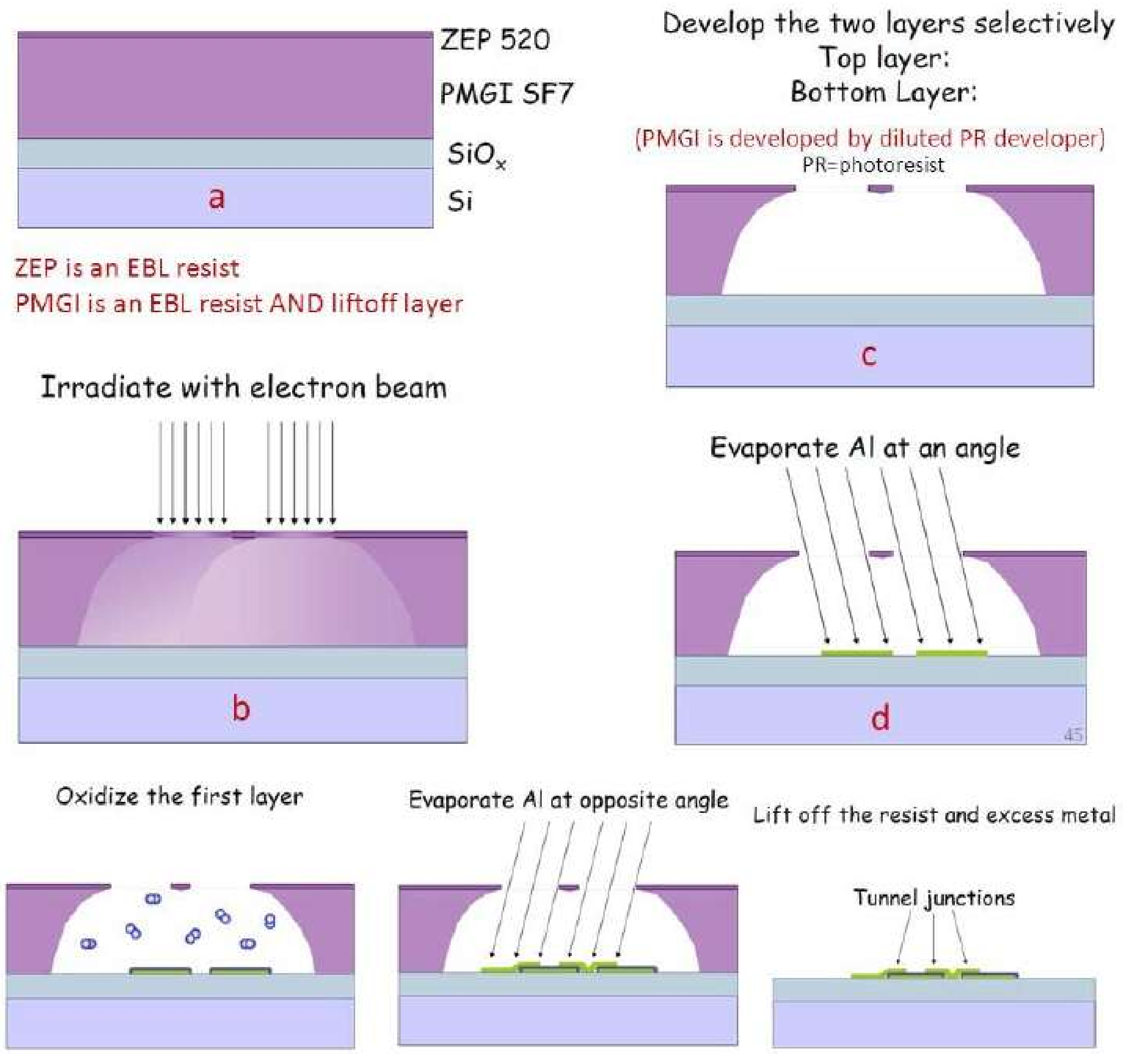


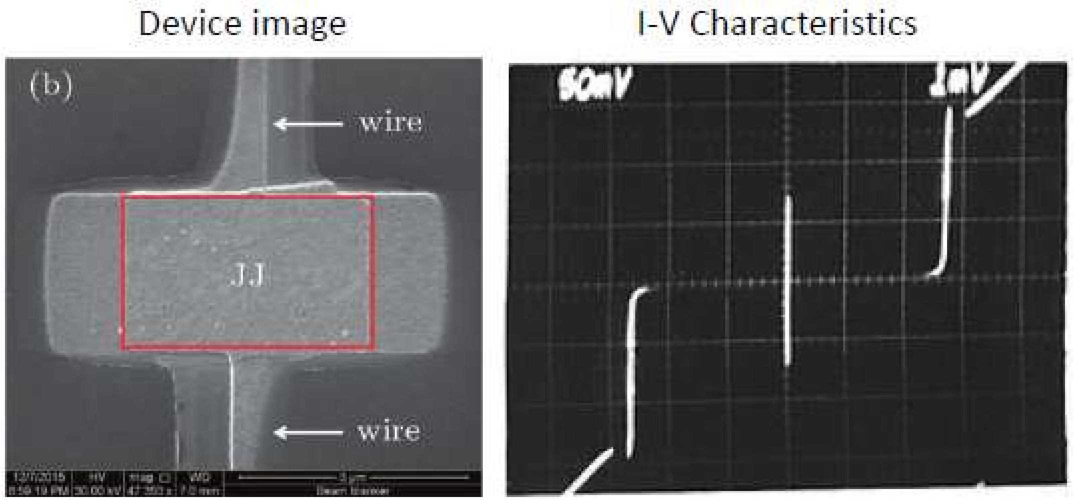
Typical Current-Voltage Characteristics of JJ



POSTECH

Angle Evaporation for Tunneling JJ



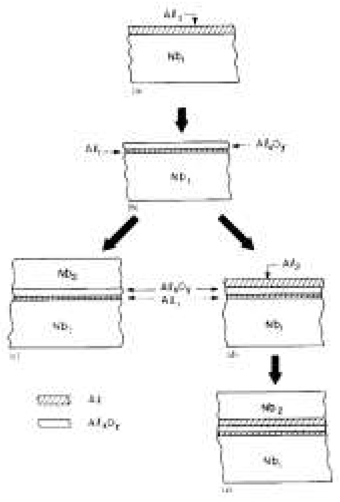


POSTECH

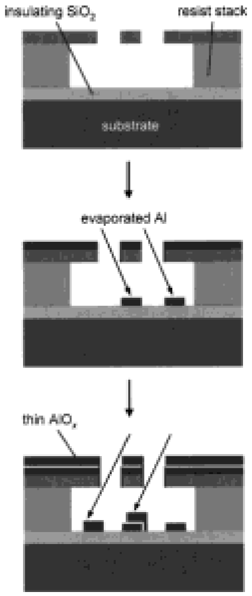
Various Types of Josephson Junction

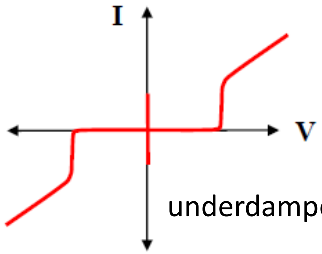
투과형 조셉슨 접합 (tunneling Josephson junction)

Tri-layer process

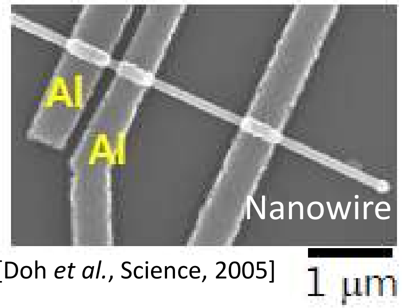


Double-angle evaporation

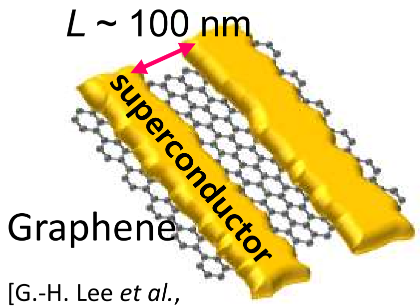




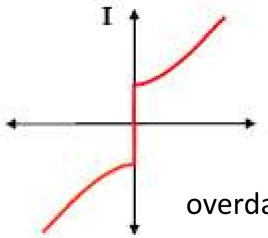
근접형 조셉슨 접합 (proximity Josephson junction)



[Doh *et al.*, Science, 2005]

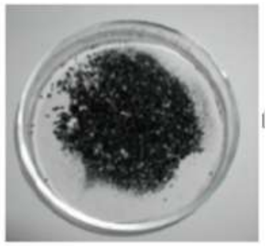


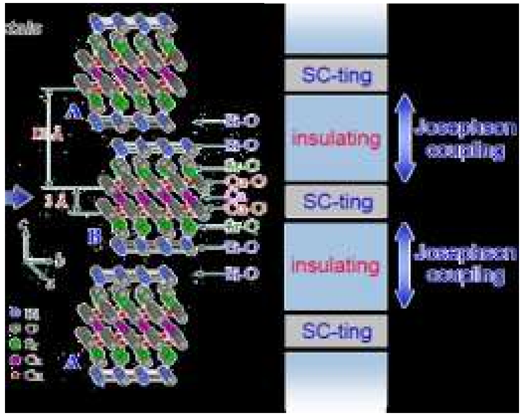
[G.-H. Lee *et al.*, *Nature Comm.*, 2015]

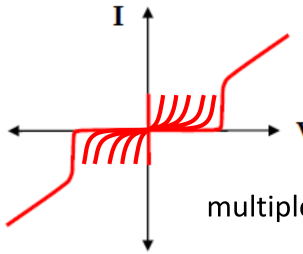


선천성 조셉슨 접합 (intrinsic Josephson junction)

$Bi_2Sr_2CaCu_2O_{8+x}$





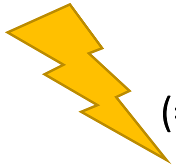


POSTECH

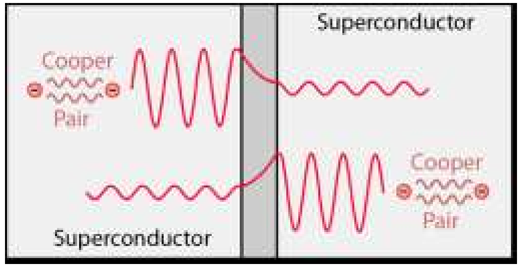
Josephson Junction with light

Shapiro step (1963)

$V(t) = V_{ac} \cos \omega t$



Microwave
(= AC biasing)



$$\frac{d\varphi(t)}{dt} = \frac{2eV(t)}{\hbar}$$

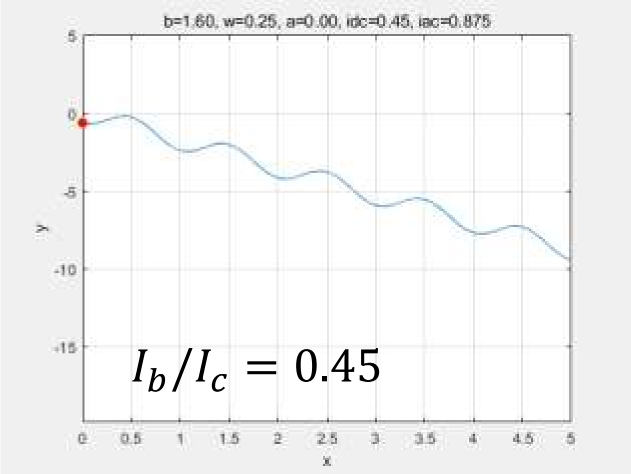
$$\varphi(t) \propto \sin \omega t$$

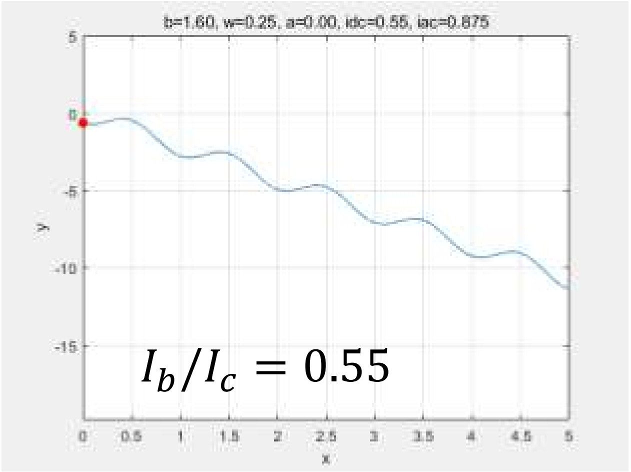
$$I_J = I_c \sin(\varphi(t)) \propto \sin(\sin \omega t)$$

Resonance of Josephson junction with microwave

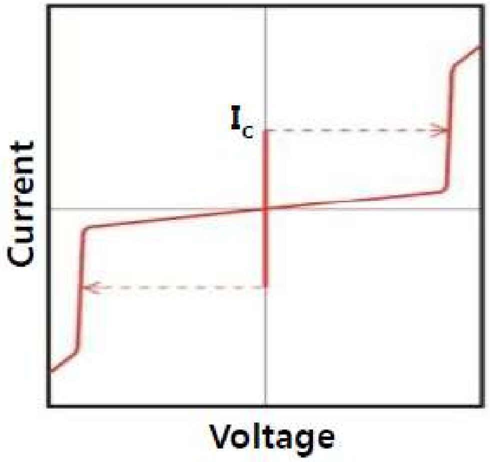
$$V_n = n\frac{\hbar\omega}{2e} \quad (n\text{=integer})$$

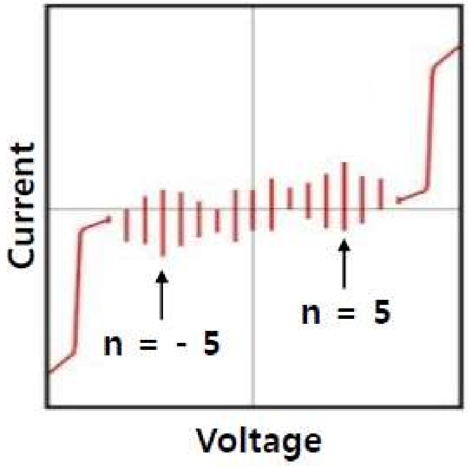
with same microwave frequency

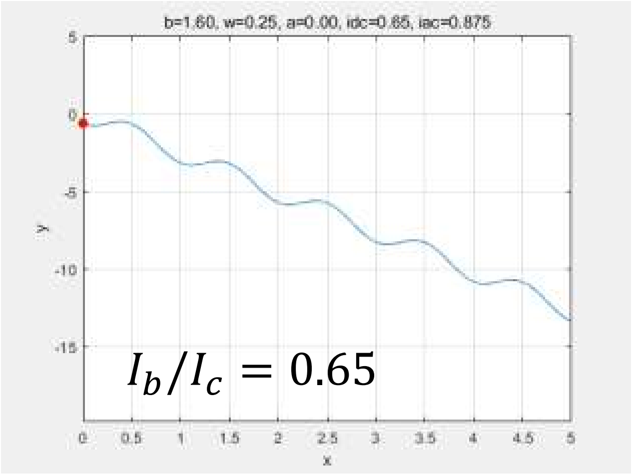




"AC→DC converter"







[from 박진호,최용빈]

POSTECH

Voltage Standard (전압 표준), “AC→DC”

$$V_n = n\frac{\hbar\omega}{2e}$$

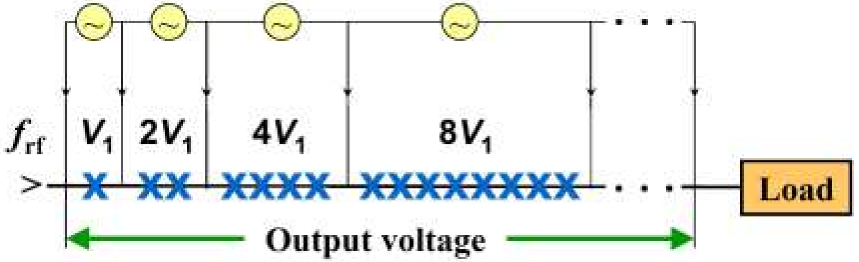
기본상수(Fundamental constant)

(n=integer)

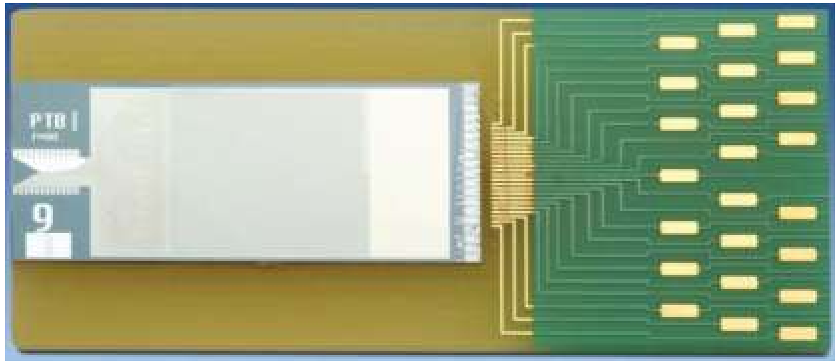
~2.07 μV/GHz

- Standard voltage generation using Shapiro steps

Schematics of Voltage Standard

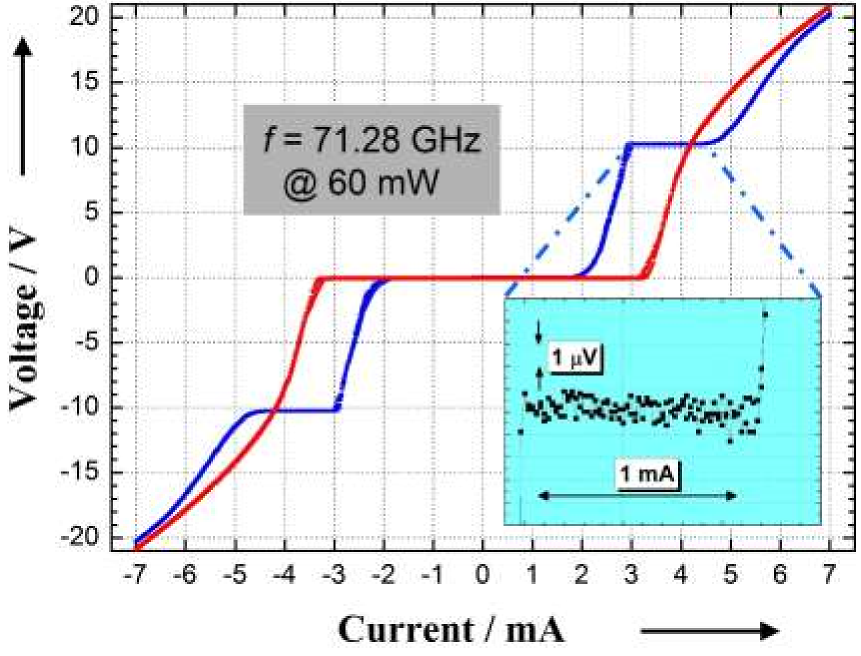


~70,000 Josephson Junctions in Series



[F. Mueller et al. (2009)]

10V voltage standard I-V curve



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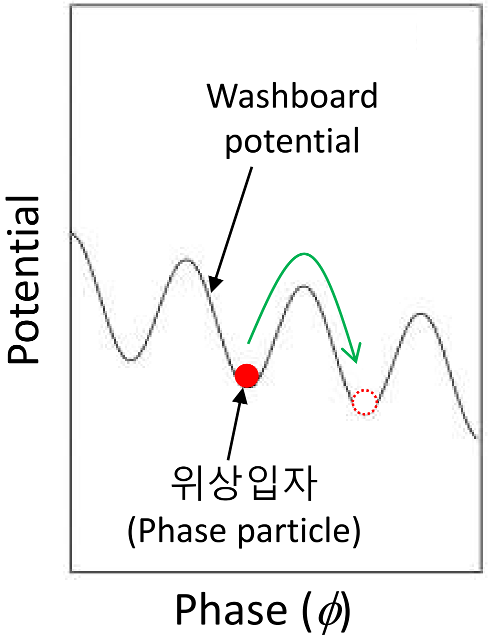
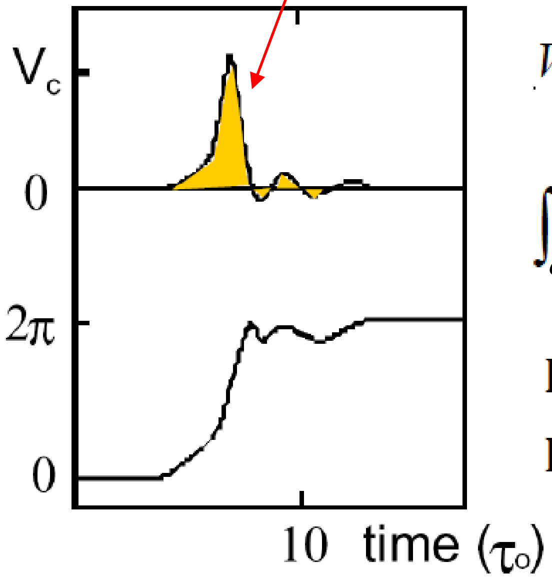
Voltage Pulse Generation

~1 ps-short voltage pulse generation



A voltage pulse generated when the phase particle crosses a potential barrier.



$$V(t) = \frac{\hbar}{2e}\frac{d\phi}{dt}$$

Flux quantum

$$\int_{\phi=0}^{\phi=2\pi} V(t)\,dt = \frac{h}{2e} = \Phi_o = 2.07\ \mathrm{mV\cdot ps}$$

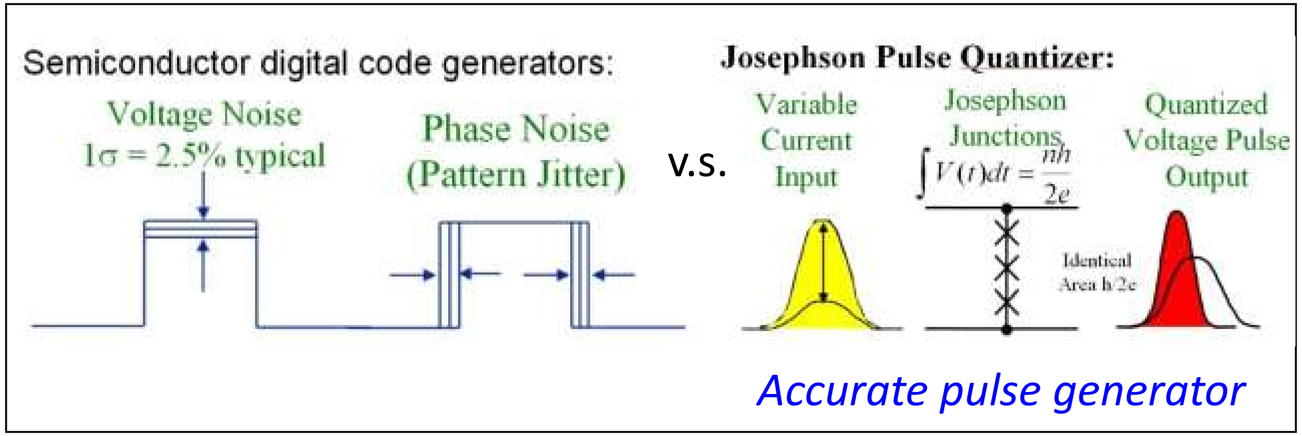
pulse height ~ $V_c = I_c R_n \sim \Delta_{sc}/e \sim k_B T_c$

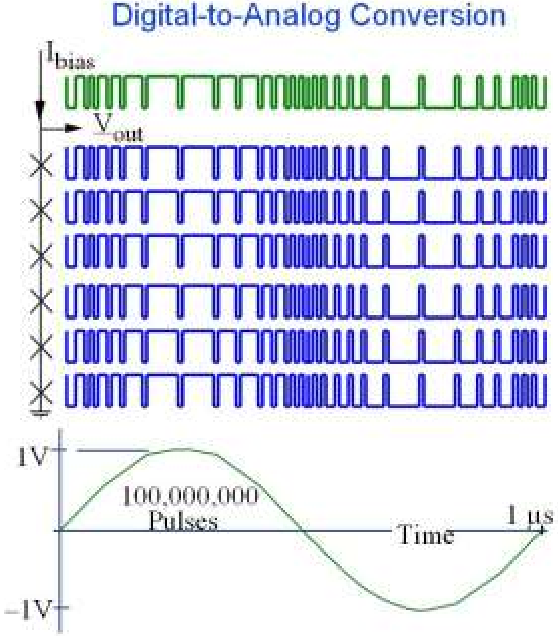
pulse width ~ Φ_o / V_c

POSTECH

JAWS & RSFQ

Josephson Arbitrary Waveform Synthesizer (JAWS)





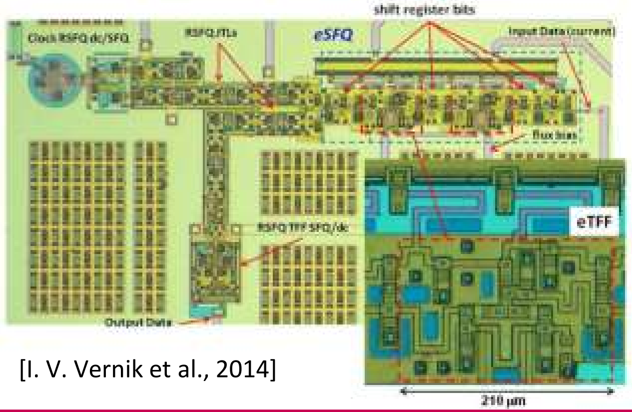
Rapid single flux quantum (RSFQ)

- A digital logic device using Josephson pulses instead of 0/5V voltages
- Data encoding, processing, and transmission are performed with ultra-short pulses (~1 ps)

  → enabling high-speed operations (100 GHz clock speed).
- Voltage pulses travel through superconducting wires

  → preventing heat generation and eliminating overheating issues.

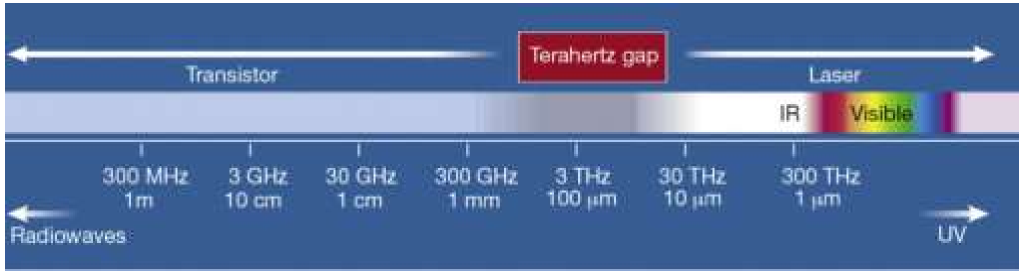
Example of RFSQ device



[I. V. Vernik et al., 2014]

Terahertz Generation

- The development of THz-band generation devices is still under progress.



$$\frac{d\varphi(t)}{dt} = \frac{2e}{\hbar} V_{\mathrm{DC}}$$

$$\varphi(t) = \frac{2e}{\hbar} V_{\mathrm{DC}} t$$

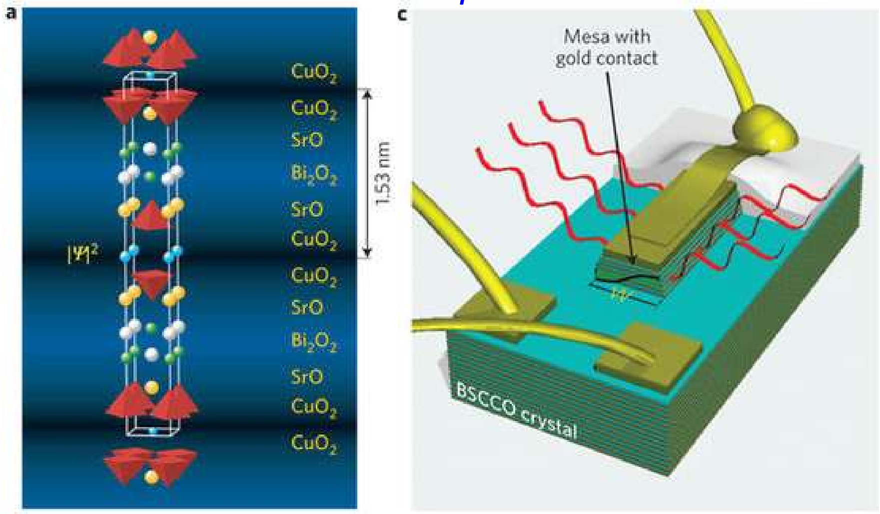
$$I_J = I_c \sin\left(\frac{2e}{\hbar} V_{\mathrm{DC}} t\right)$$

DC voltage
↓
AC current

Application of THz for security

Safer to human body than X-ray

Intrinsic Josephson Junction

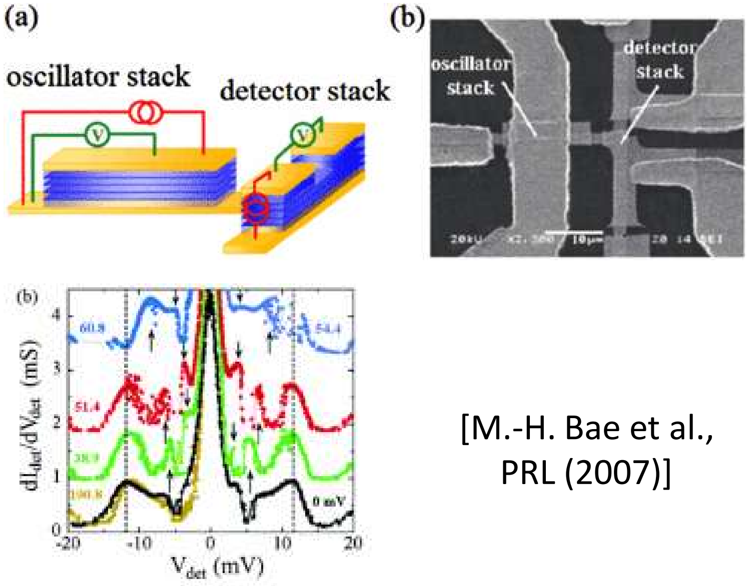


[U. Welp et al., 2013]

$$\frac{d\varphi}{dt} = \frac{2eV}{\hbar}$$, $2e/h$ ~ **0.4836 THz/mV**

10 mV DC voltage generates ~4.8THz wave

Josephson-vortex-flow THz emssion

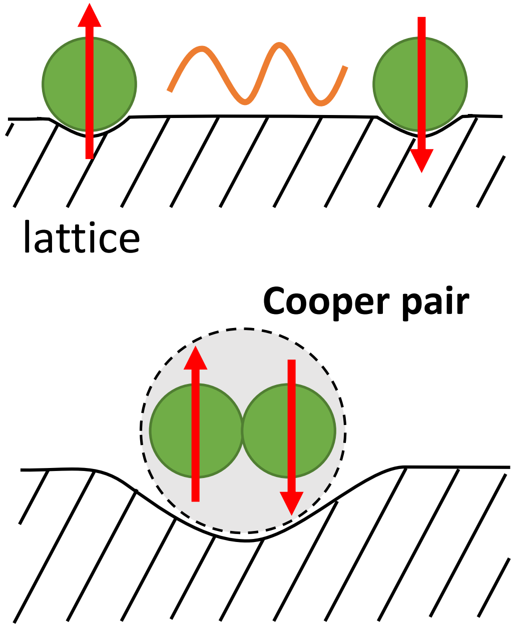


[M.-H. Bae et al., PRL (2007)]

POSTECH

SC-light interaction:
optical regime

SC - Many-body interacting QM system



in BCS superconductor

Spin-singlet (anti-sym.)

$$|S\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Orbital : *s*-wave (sym.)

10^{23} Cooper pairs are ***coherent***.

Particle number – quantum phase uncertainty:

$$\Delta N \Delta\varphi \sim 1$$

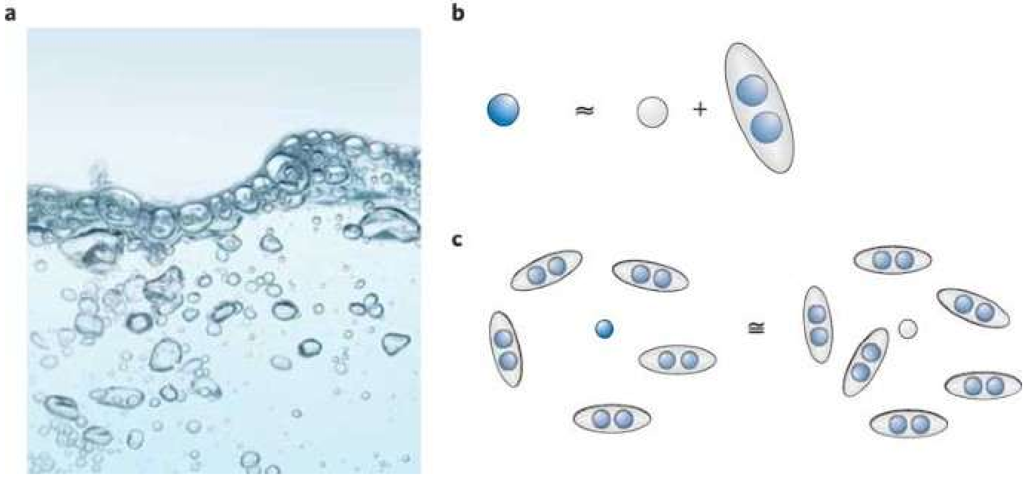
Similar to "Coherent state" in quantum optics (e.g., Laser)

$$|\alpha\rangle \sim e^{\alpha a^\dagger}|0\rangle$$

⇕

$$|\Psi_{\mathrm{BCS}}\rangle \sim e^{\sum_k \alpha_k P_k^\dagger}|0\rangle$$

Cooper pair creation operator: $P_k^\dagger = c_{k,\uparrow}^\dagger c_{-k,\downarrow}^\dagger$



Cooper pairs form a condensate at zero energy.

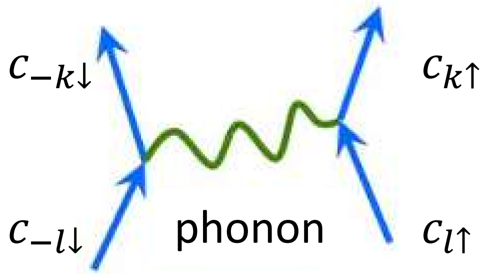
↓

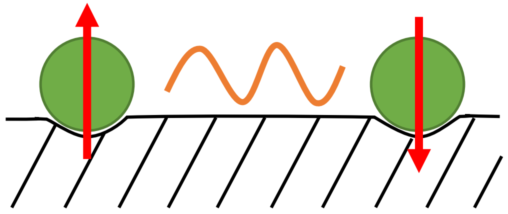
"Electron" and *"hole"* are similar in SC.

Mean-field BCS Hamiltonian*

* Optional slide

Electron-electron interaction via phonon exchange





BCS took *mean-field* approach, where *k* state depends only on the average of *k'≠k* states.

number operator : $n_{\mathbf{k}\sigma} = c^*_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}$

BCS Hamiltonian : $\mathscr{H} = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} n_{\mathbf{k}\sigma} + \sum_{\mathbf{kl}} V_{\mathbf{kl}} c^*_{\mathbf{k}\uparrow} c^*_{-\mathbf{k}\downarrow} c_{-\mathbf{l}\downarrow} c_{\mathbf{l}\uparrow}$

average — fluctuation

$$c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} = b_{\mathbf{k}} + (c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} - b_{\mathbf{k}})$$

$$b_{\mathbf{k}} = \langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle_{\text{av}}$$

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{l}} V_{\mathbf{kl}} b_{\mathbf{l}} = -\sum_{\mathbf{l}} V_{\mathbf{kl}} \langle c_{-\mathbf{l}\downarrow} c_{\mathbf{l}\uparrow} \rangle$$

Mean-field approximation

Mean-field BCS Hamiltonian :

$$\mathscr{H}_M = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} c^*_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} (\Delta_{\mathbf{k}} c^*_{\mathbf{k}\uparrow} c^*_{-\mathbf{k}\downarrow} + \Delta^*_{\mathbf{k}} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} - \Delta_{\mathbf{k}} b^*_{\mathbf{k}})$$

- 전체 상태는 쌍들로 구성된 위상-정합된(coherent) many-body 파동함수:

$$|\text{BCS}\rangle = \prod_{\mathbf{k}} (u_k + v_k c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow})|0\rangle$$

- 이 파동함수에는 모든 **k**에 대해 동일한 위상으로 쌍들이 포함됨
- 즉, 상태 자체가 **전자쌍을 만드는 연산자에 대해 coherent expectation value**를 가짐

$$\langle c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle = b_k \neq 0$$

→ 이건 실제로 **superconducting order parameter**의 **정의**에 해당:

$$\Delta_k \propto \downarrow {}_{\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow} \rangle$$

Now, particle number is *not* conserved.
(Grand canonical ensemble)

POSTECH

Diagonalizing H_M *

Mean-field BCS Hamiltonian :

$$\mathscr{H}_M = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}} c^*_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - \sum_{\mathbf{k}} \left(\boxed{\Delta_{\mathbf{k}} c^*_{\mathbf{k}\uparrow} c^*_{-\mathbf{k}\downarrow}} + \boxed{\Delta^*_{\mathbf{k}} c_{-\mathbf{k}\downarrow} c_{\mathbf{k}\uparrow}} - \Delta_{\mathbf{k}} b^*_{\mathbf{k}} \right)$$

$\Delta_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}$
↓ creating "electron" of $k,\uparrow$
↳ removing "hole" of $-k,\downarrow$

$\Delta^*_k c_{-k\downarrow} c_{k\uparrow}$
↓ creating "hole" of $-k\downarrow$
↳ removing "electron" of $k,\uparrow$

electron of $k,\uparrow$ ⇄ hole of $-k\downarrow$

Superconducting pairing is mixing electron and hole.

Choose a smart unitary transformation to have diagonal form of H_M

$$c_{\mathbf{k}\uparrow} = u^*_{\mathbf{k}} \gamma_{\mathbf{k}0} + v_{\mathbf{k}} \gamma^*_{\mathbf{k}1}$$
$$c^*_{-\mathbf{k}\downarrow} = -v^*_{\mathbf{k}} \gamma_{\mathbf{k}0} + u_{\mathbf{k}} \gamma^*_{\mathbf{k}1}$$

, where $|v_{\mathbf{k}}|^2 = 1 - |u_{\mathbf{k}}|^2 = \frac{1}{2}\left(1 - \frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\right)$, $E_{\mathbf{k}} = (\Delta^2_{\mathbf{k}} + \xi^2_{\mathbf{k}})^{1/2}$

Bogoliubov(-Valatin) transformation (1958).

$$\mathscr{H}_M = \sum_{\mathbf{k}} (\xi_{\mathbf{k}} - E_{\mathbf{k}} + \Delta_{\mathbf{k}} b^*_{\mathbf{k}}) + \sum_{\mathbf{k}} E_{\mathbf{k}} (\gamma^*_{\mathbf{k}0}\gamma_{\mathbf{k}0} + \gamma^*_{\mathbf{k}1}\gamma_{\mathbf{k}1})$$

First term → Constant (condensation energy)

Second term → Excitation of new-quasi-particles, called Bogoliubons

Bogoliubons

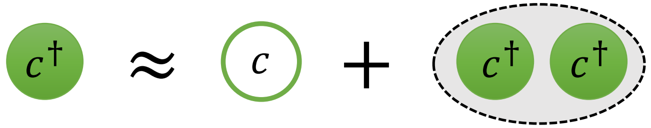
$$\gamma^*_{\mathbf{k}0} = u^*_{\mathbf{k}} c^*_{\mathbf{k}\uparrow} - v^*_{\mathbf{k}} c_{-\mathbf{k}\downarrow}$$
$$\gamma^*_{\mathbf{k}1} = u^*_{\mathbf{k}} c^*_{-\mathbf{k}\downarrow} + v^*_{\mathbf{k}} c_{\mathbf{k}\uparrow}$$

Vacuum : BCS ground state

Bogoliubon is a mixture of electron and hole.

Introduction to BCS

- Excitation in superconductor (Bogoliubov quasiparticles or 'Bogoliubon') = mixture of electron and hole in SC



- Bogoliubon for usual s-wave SC

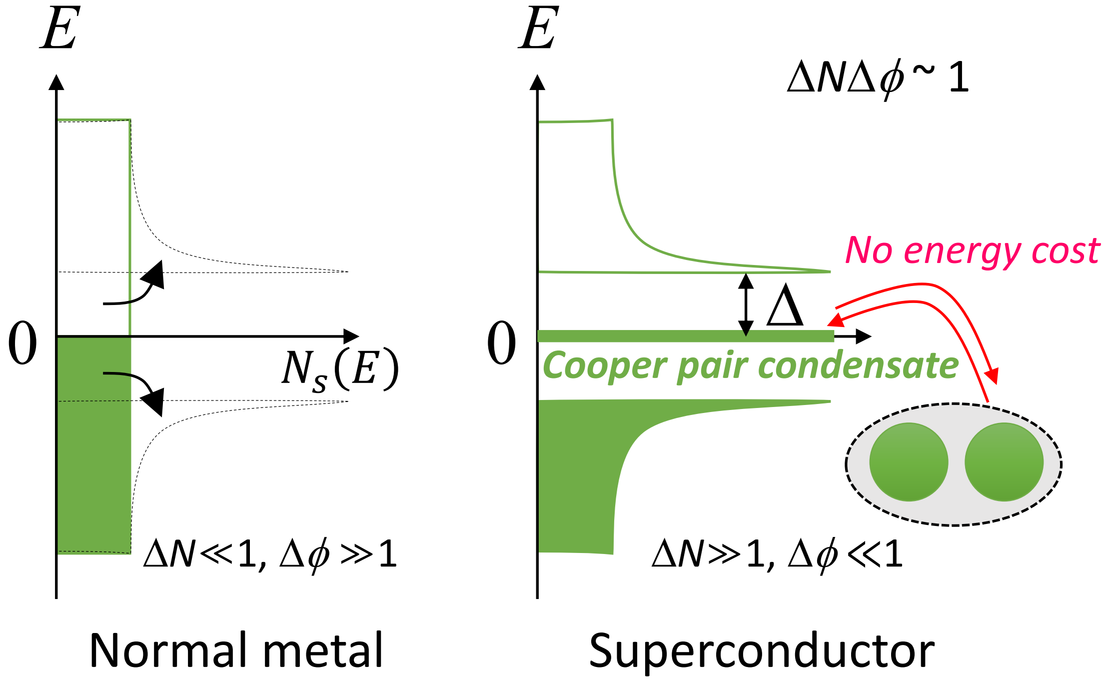
$$\begin{cases} \gamma_{k+}^\dagger = u_k\, c_{k\uparrow}^\dagger - v_k^* c_{-k\downarrow} & \text{: create } k \text{ and spin} \uparrow \\ \gamma_{k-}^\dagger = v_k\, c_{k\uparrow} + u_k\, c_{-k\downarrow}^\dagger & \text{: create } -k \text{ and spin} \downarrow \end{cases}$$

,with excitation energy $E = \sqrt{\xi^2 + \Delta^2}$

- ξ: the normal single-particle energy above E_F
- Δ: superconducting gap

- 1-to-1 btw. $N_s(E)$ and $N_n(\xi)$: $N_s(E)dE = N_n(\xi)d\xi$

If $N_n(E) = N_n(0)$, $N_s(E) = N_n(0)\frac{1}{dE/d\xi} = N_n(0)\frac{|E|}{\sqrt{E^2-\Delta^2}}$



* For a spinless case, in a special condition,

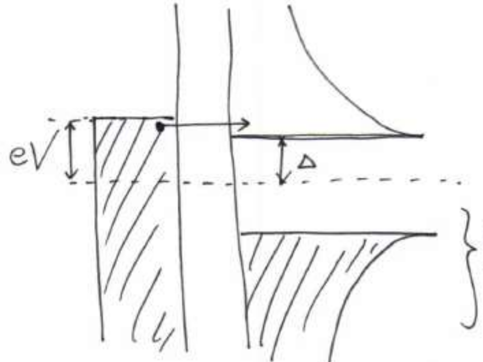
$\gamma_1 = c^\dagger + c = \gamma_1^\dagger = c + c^\dagger$ Majorana!

POSTECH

Tunneling Spectroscopy

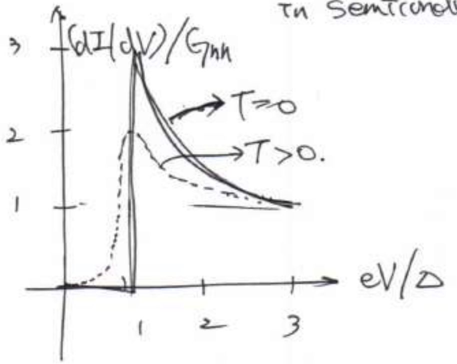
Normal metal – insulator – superconductor

* departure from the vacuum,



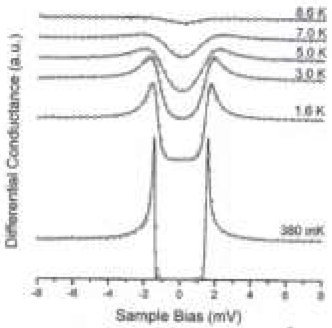
negative energy state is needed to reduce to normal metal case as Δ → 0.

quasi-particle density of state in superconductor in semiconducting model.

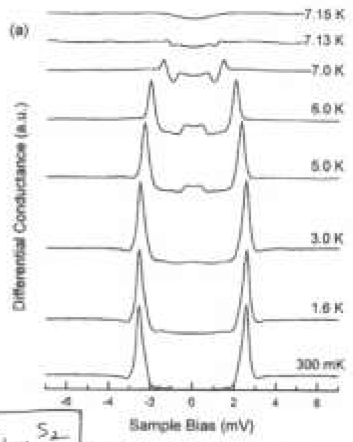


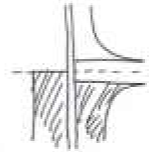
Vacuum tunneling of superconducting quasiparticles from atomically sharp scanning tunneling microscope tips

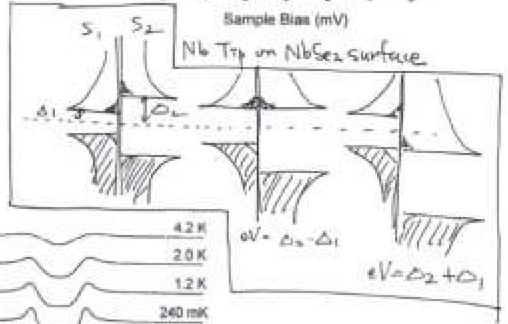
S. H. Pan, E. W. Hudson, and J. C. Davis[a]
Department of Physics, University of California, Berkeley, Berkeley, California 94720

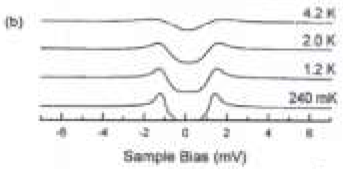


Nb Tip on Au surface









PtIr tip on NbSe2 surface
; broader peak than in Nb-NbSe2 case.

Note that superconducting tip gives sharper feature thanks to the sharp DOS of superconductor.

POSTECH

Superconducting Gap

(BCS theory in 1957)
predicts $\Delta = 1.763 k_B T_c$

- energy gap in Superconductor : Δ

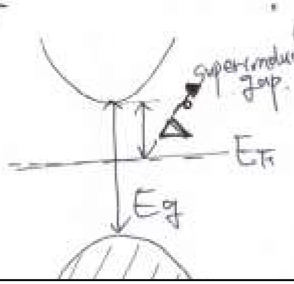
1) electronic heat capacity.

usually $C_{electron} \propto T$, $C_{phonon} \propto T^3$.

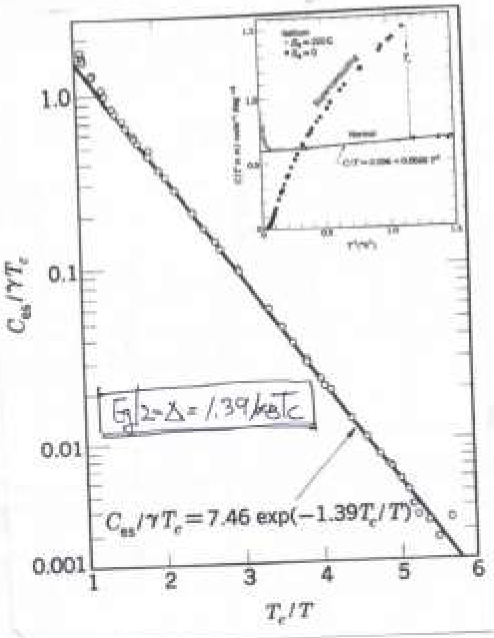
- <u>Exponential decay of C below T_c</u>

Show the <u>existence of energy gap</u> for quasi-particles who participate heat transfer.

(conductance, or heat capacity, ...)



$\propto \exp[-E_g/2k_BT]$

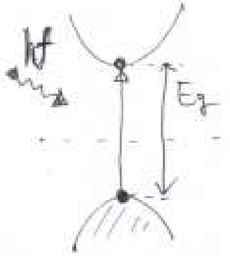


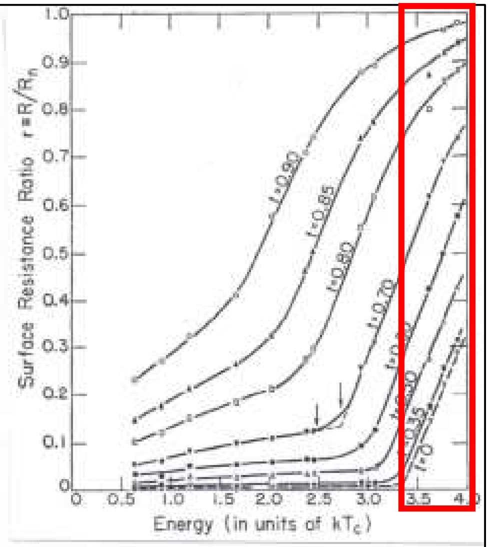
[N. E. Phillips, Phys. Rev. 134, A385 (1964)]

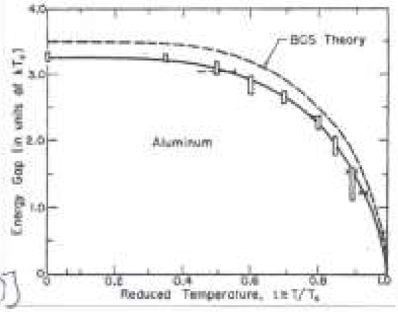
2) Photon absorption. (mm-waves)

There exist "threshold" energy for photons (electromagnetic wave) to be absorbed into superconductor.

$E_g \sim 3.3 k_B T_c$, → $\Delta \sim 1.7 k_B T_c$

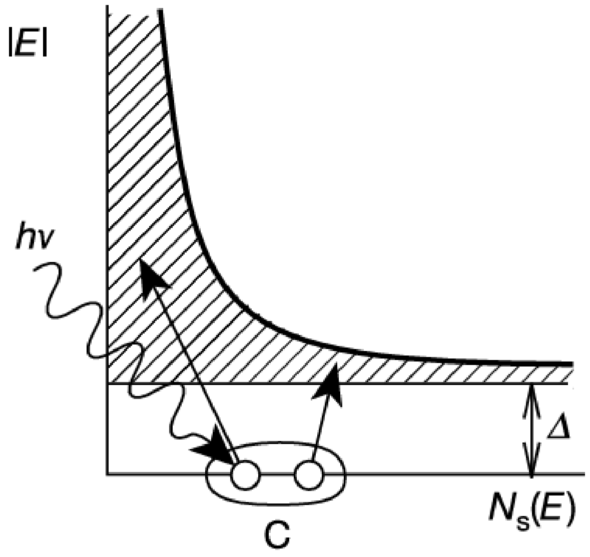


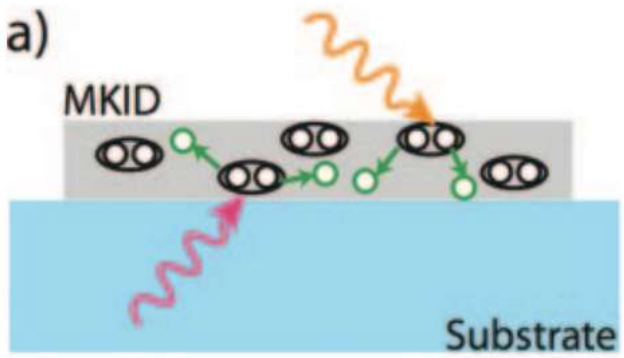




[Biondi & Garfunkel. Phys. Rev. (1959)]

Photons Break Cooper Pairs





Single photon hits superconductor.

↓

Two quasiparticles(qps)
with high energy are created.

↓

Phonons are emitted
while qp energy decreases.

→ some phonons escapes to substrate

↓

Lower η below 1

↓

More qps with lower energy are created.

↓

⋮

↓

Total # of qps: $N_{\mathrm{qp}} = \eta \cdot h\nu_{\mathrm{p}}/\Delta$

η: qp generation efficiency

Kinetic Inductance in Superconductor

[ChatGPT]

전류를 시간에 따라 바꾸려면 전자를
가속운동 시켜야 하는데 에너지 필요
→ "인덕턴스"에 해당

1. Drude 모델 운동 방정식

$m\frac{d\mathbf{v}}{dt} = e\mathbf{E}(t) - m\frac{\mathbf{v}}{\tau}$ (2.5)

- m: 전자의 질량
- e: 전자의 전하
- $\mathbf{E}(t)$: 시간에 따라 변하는 외부 전기장
- τ: 평균 자유 시간 (scattering or relaxation time)
- $\mathbf{v}(t)$: 전자의 평균 속도

2. Fourier 변환 (복소 주파수 영역으로 변환)

운동 방정식을 주파수 영역으로 Fourier 변환하면:

$$m(i\omega\mathbf{v}(\omega)) = e\mathbf{E}(\omega) - m\frac{\mathbf{v}(\omega)}{\tau}$$

정리하면:

$$\left(m(i\omega + \frac{1}{\tau})\right)\mathbf{v}(\omega) = e\mathbf{E}(\omega) \Rightarrow \mathbf{v}(\omega) = \frac{e}{m(i\omega + \frac{1}{\tau})}\mathbf{E}(\omega)$$

3. 전류 밀도와 속도의 관계

$$\mathbf{J}(\omega) = ne\mathbf{v}(\omega) \Rightarrow \mathbf{J}(\omega) = \frac{ne^2}{m(1/\tau + i\omega)}\mathbf{E}(\omega)$$

분자와 분모에 τ를 곱하면:

$$\mathbf{J}(\omega) = \frac{ne^2\tau}{m(1 + i\omega\tau)}\mathbf{E}(\omega) = \sigma(\omega)\mathbf{E}(\omega) \quad (2.6)$$

4. 복소 전도도 정의

위에서 정의된 복소 전도도는:

$$\sigma(\omega) = \frac{ne^2\tau}{m(1 + i\omega\tau)} = \sigma_1(\omega) - i\sigma_2(\omega)$$

분자와 분모를 $1 - i\omega\tau$로 곱해 유리화하면:

- **실수부:**

$$\sigma_1(\omega) = \frac{\sigma_0}{1 + \omega^2\tau^2}$$

- **허수부:**

$$\sigma_2(\omega) = \frac{\sigma_0\omega\tau}{1 + \omega^2\tau^2}$$

- 여기서 $\sigma_0 = \frac{ne^2\tau}{m}$: 정적 (dc) 전도도

$$\sigma_2(\omega) = \frac{ne^2\tau}{m}\frac{\omega\tau}{1 + \omega^2\tau^2} = \frac{ne^2}{m}\frac{\omega}{1/\tau^2 + \omega^2}$$

$\tau \rightarrow \infty$
$n \rightarrow n_s$: SC electrons

$$\sigma_2(\omega) = \frac{ne^2}{m\omega}$$

$$J(\omega) = \sigma(\omega)E(\omega) = \frac{1}{i\omega L_k}E(\omega)$$

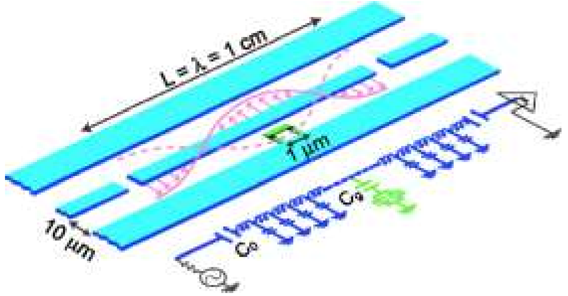
$$i\sigma_2 = \frac{1}{i\omega L_k}$$

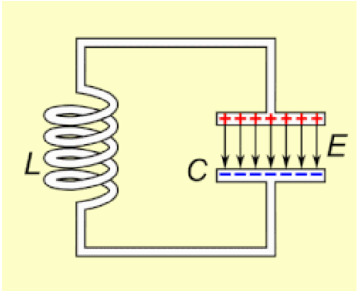
SC Kinetic inductance:

$$L_k = \frac{m}{n_s e^2}$$

Microwave Kinetic Inductance Detector (MKID)

coplanar waveguide (CPW)

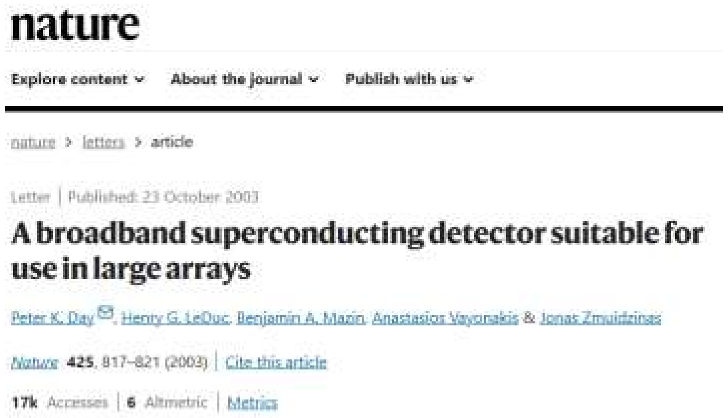




Resonance frequency:

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$n_s\downarrow, L\uparrow, \omega_r\downarrow$$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Letter | Published: 23 October 2003

A broadband superconducting detector suitable for use in large arrays

Peter K. Day, Henry G. LeDuc, Benjamin A. Mazin, Anastasios Vayonakis & Jonas Zmuidzinas

Nature 425, 817–821 (2003) | Cite this article

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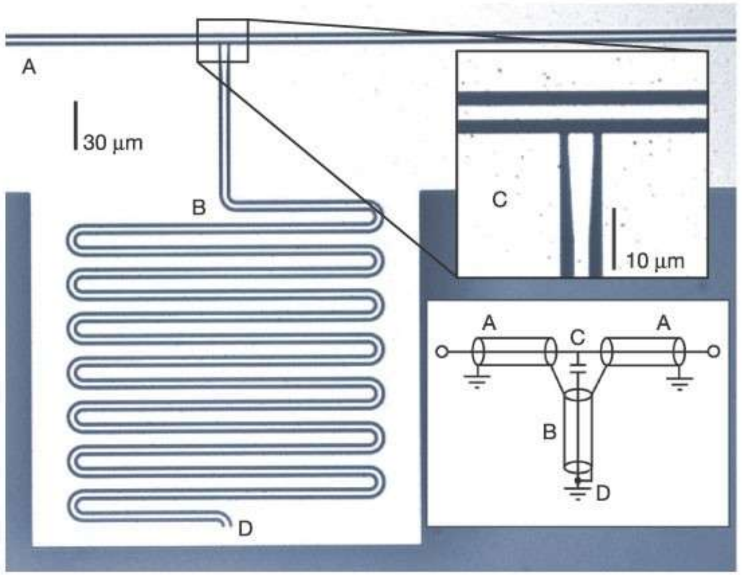
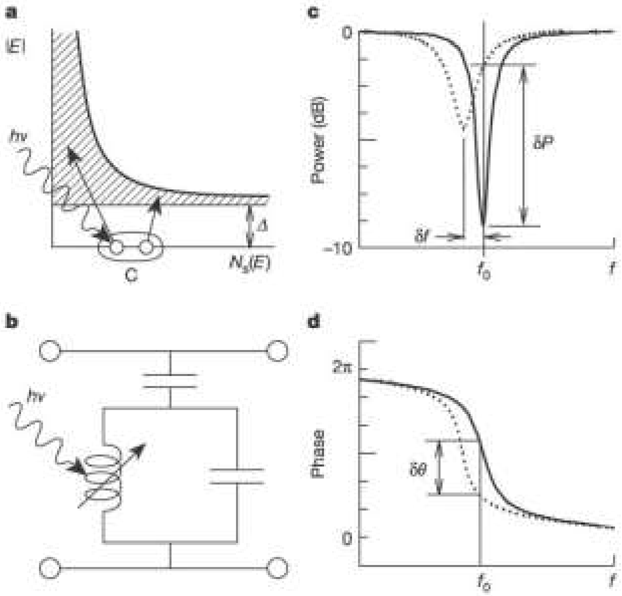
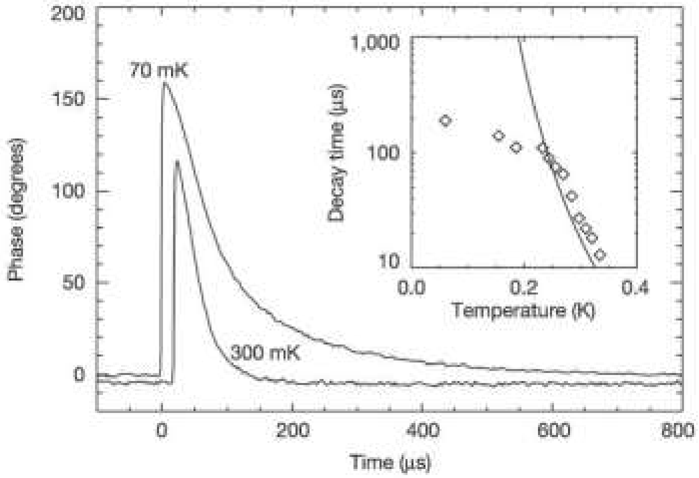


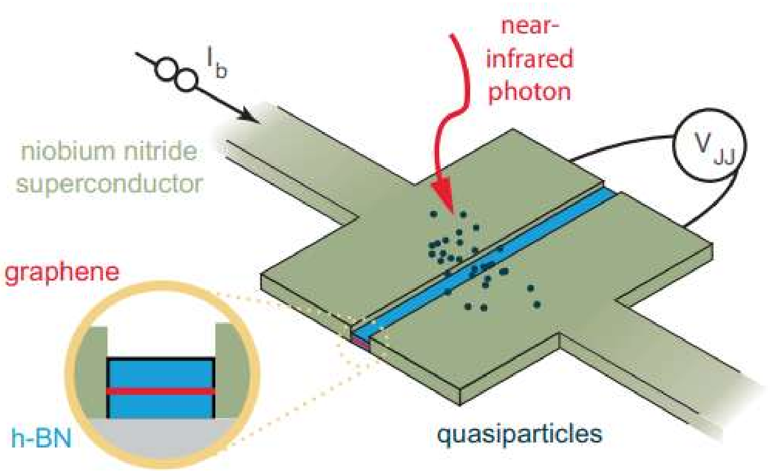


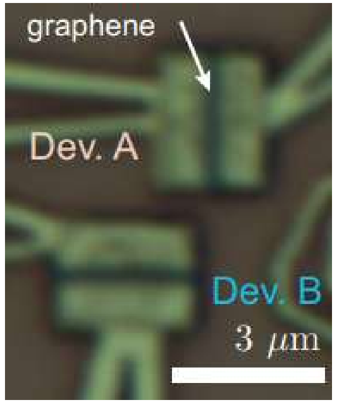


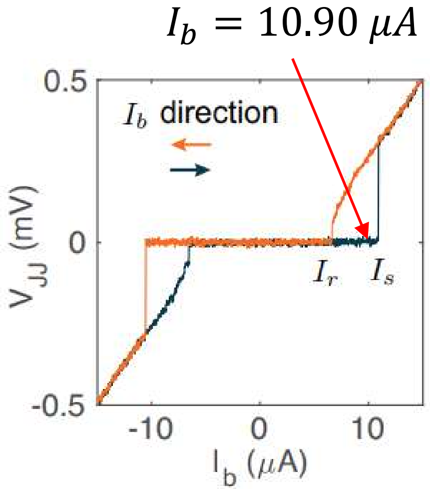
^{55}Fe source, which emits 5.9-keV X-rays

POSTECH

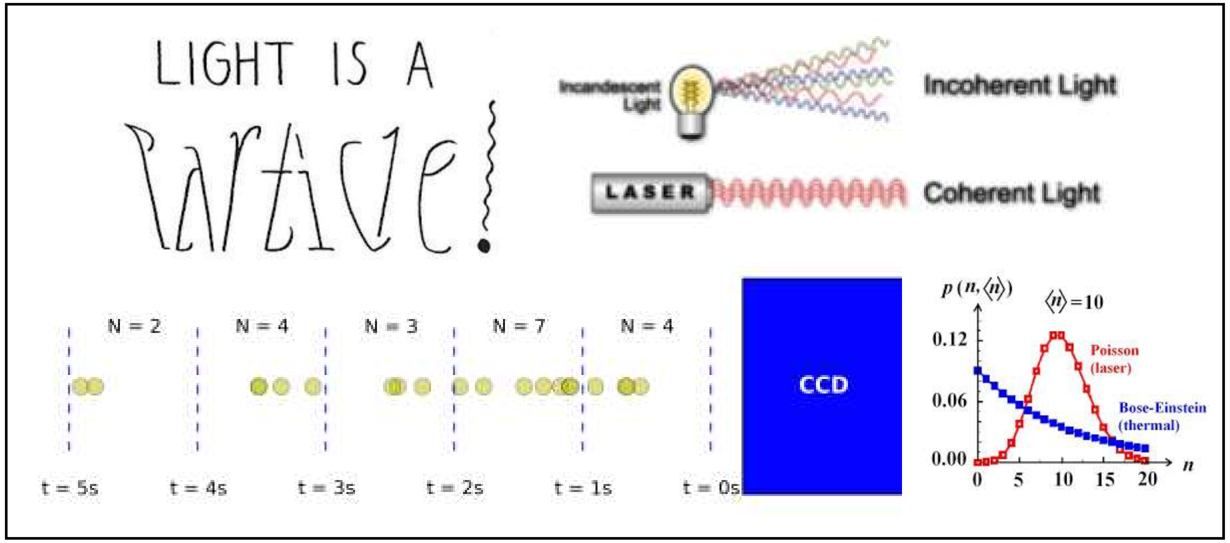
1,550-nm NIR Single Photon Detection

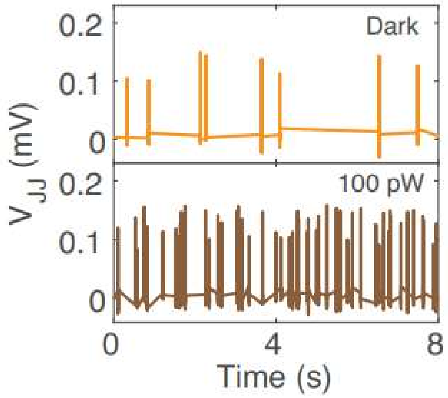


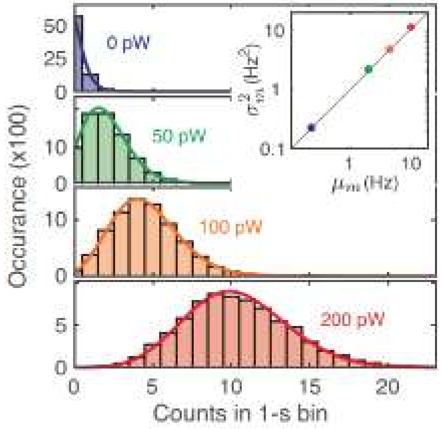




[Science 372, 409-412 (2021)]





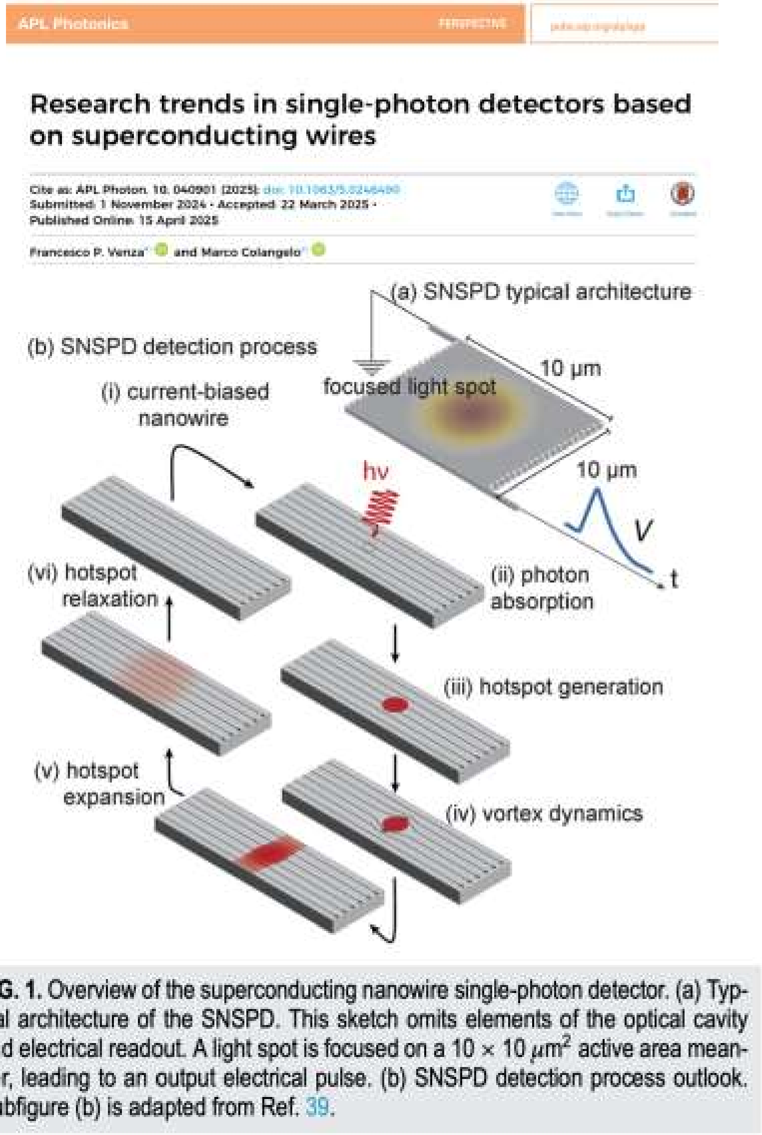


1. Poisson distribution
2. Mean count ∝ Laser power

Single photon detection

POSTECH

Superconducting Nanowire Single Photon Detector (SNSPD)

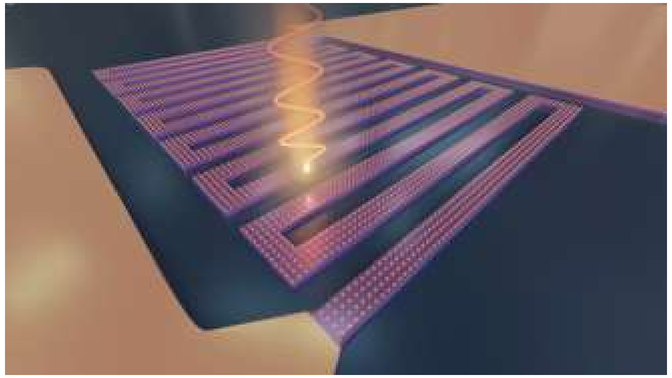

APL Photonics PERSPECTIVE

Research trends in single-photon detectors based on superconducting wires

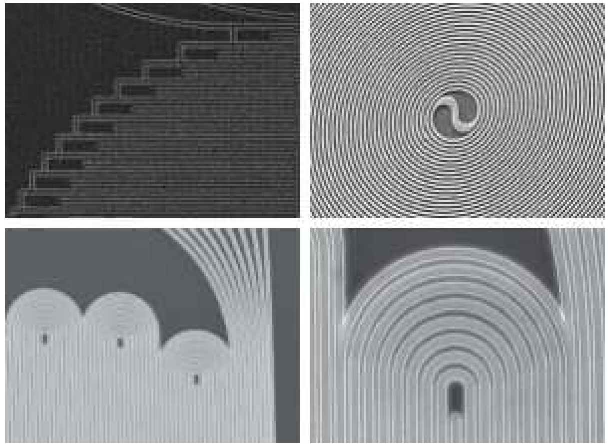
Cite as: APL Photon. 10, 040901 (2025); doi: 10.1063/5.0246490
Submitted: 1 November 2024 · Accepted: 22 March 2025 · Published Online: 15 April 2025

Francesco P. Venza and Marco Colangelo

FIG. 1. Overview of the superconducting nanowire single-photon detector. (a) Typical architecture of the SNSPD. This sketch omits elements of the optical cavity and electrical readout. A light spot is focused on a 10 × 10 μm^2 active area meander, leading to an output electrical pulse. (b) SNSPD detection process outlook. Subfigure (b) is adapted from Ref. 39.



[Single Quantum]

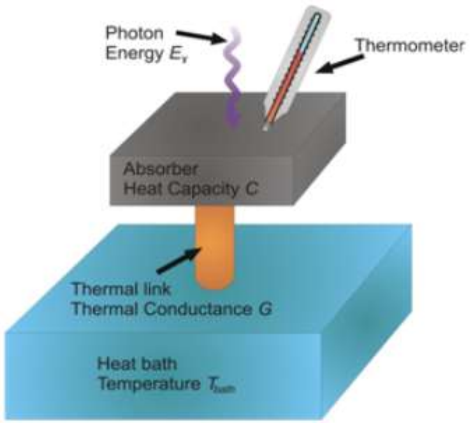


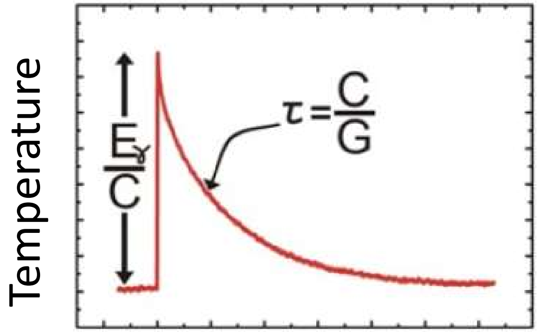
[IDQ]

POSTECH

Thermal Detector

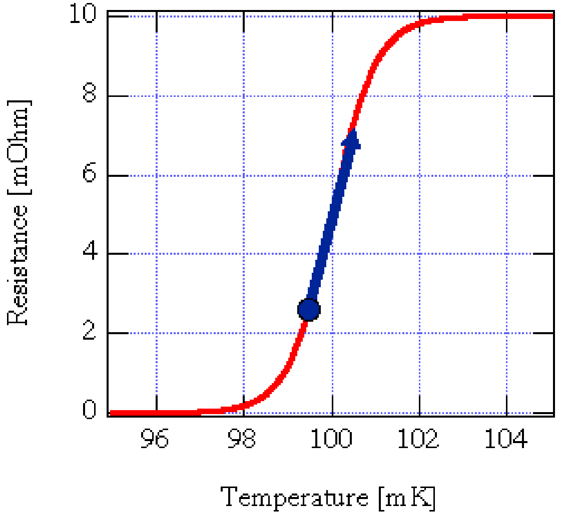
- No need to collect electron
- No threshold
- Robust against impurities
- More freedom to choose material



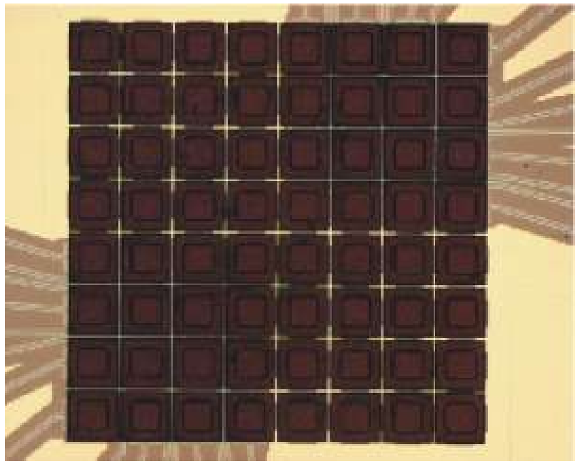


- E_γ: Photon energy
- C : Heat capacity
- G : Thermal conductance

Transition Edge Sensor (TES)



Array of TES

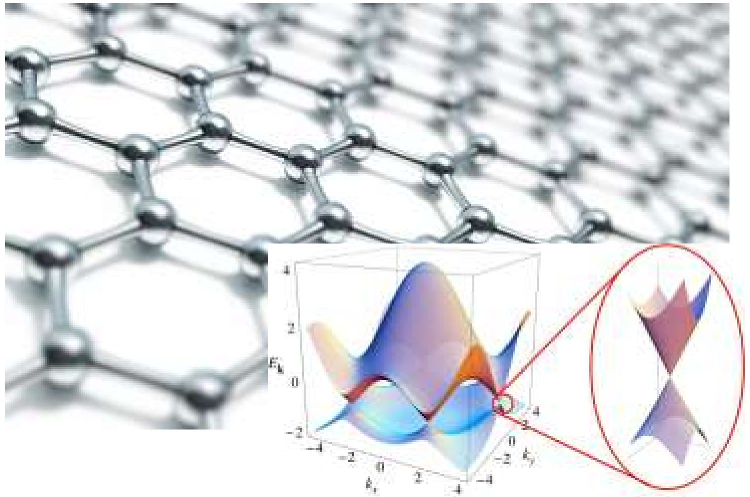


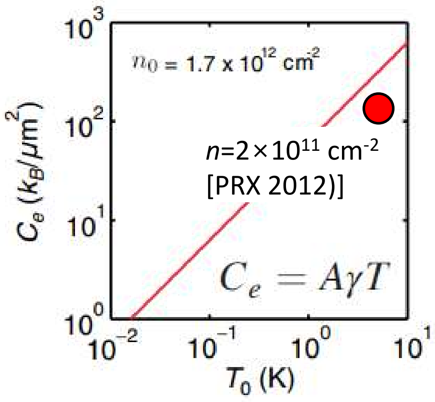
Smaller C
↓
Higher temperature rise
↓
Faster cooldown
(shorter deadtime)

POSTECH

Graphene for Absorber

Small Electronic Heat Capacity

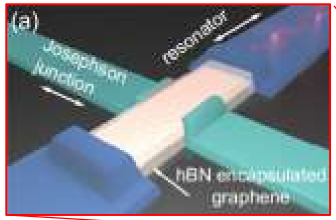


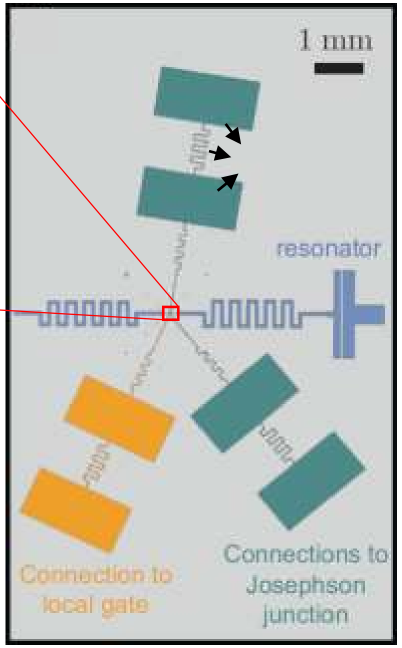


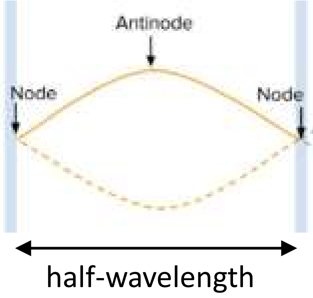
Specific heat capacity of monolayer graphene:

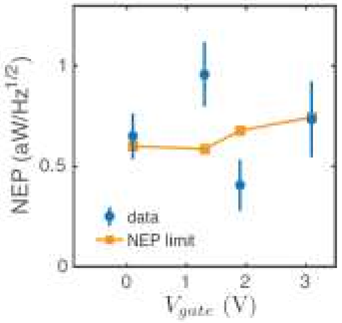
C_e ~ $10 k_B/\mu m^2$ (@ T=0.1 K, n=1.7×10^{12} cm^{-2})

Graphene JJ MW bolometer





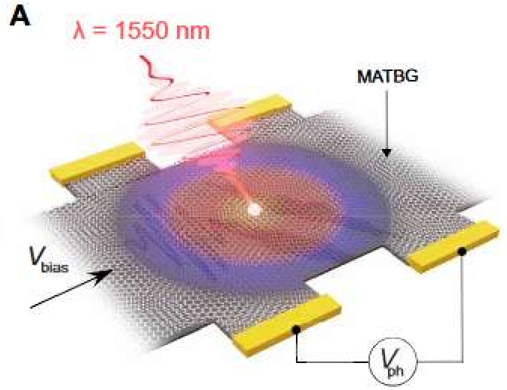


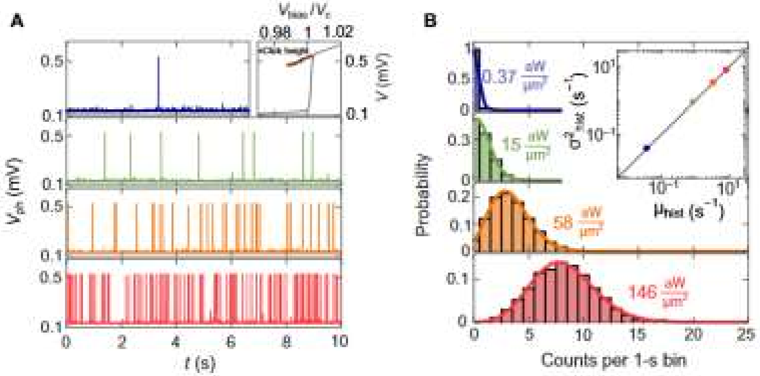


Fundamental limit of NEP is reached.

[Nature 586, 42–46 (2020)]

TBG bolometer





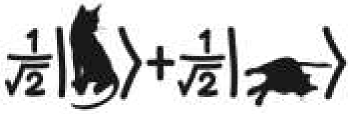
[Sci. Adv. 10, eadp3725 (2024)]

POSTECH

SC-light interaction: microwave regime

Quantum Computer (양자컴퓨터)

A computation device based on quantum mechanics.

Using **superposition** of quantum mechanics 

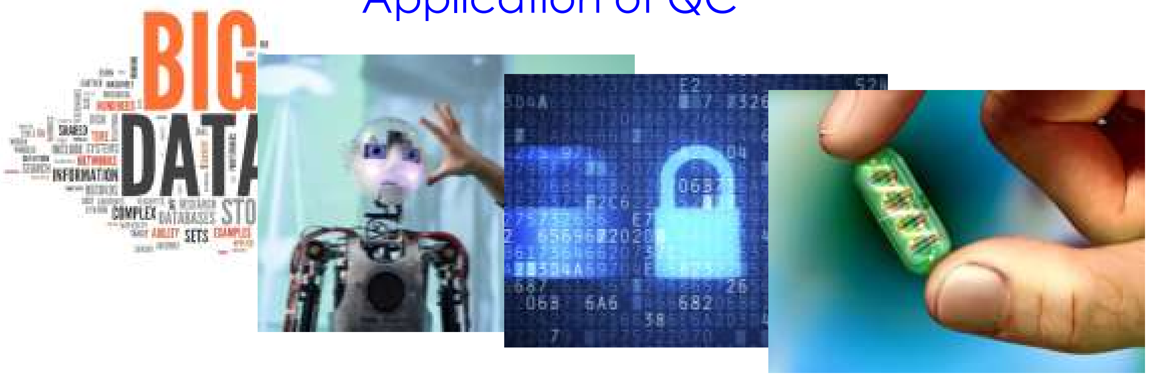
Quantum parallelism (양자 평행성)

- Classical computer: $f: 00 \rightarrow f(00)$
  $f: 01 \rightarrow f(01)$
  $f: 10 \rightarrow f(10)$
  $f: 11 \rightarrow f(11)$
- Quantum computer:

$$U_f : \frac{1}{\sqrt{4}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

$$\rightarrow \frac{1}{\sqrt{4}}(U_f|00\rangle + U_f|01\rangle + U_f|10\rangle + U_f|11\rangle)$$

Application of QC



Many groups working on QC

Google – superconducting QC

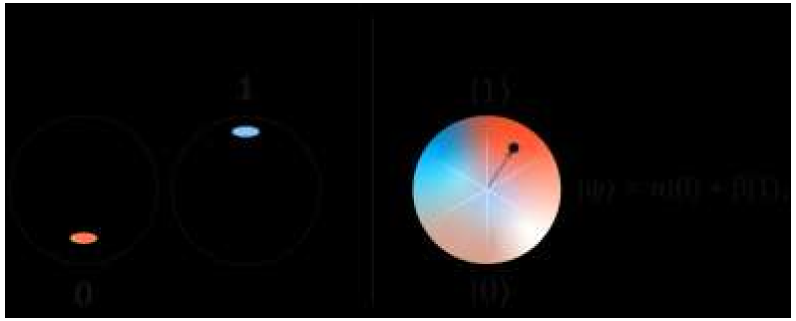
IBM – superconducting QC

Intel – Silicon-based QC

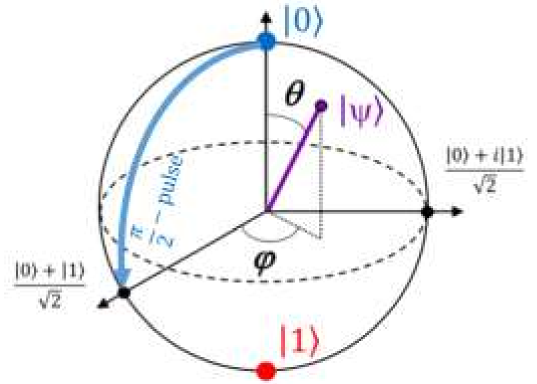
Microsoft – Topological QC

Quantum Bit, Qubit (큐빗)

Bit v.s. Qubit



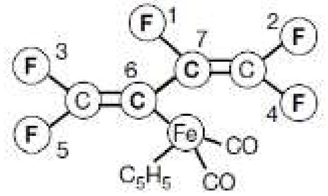
Bloch Sphere (1 qubit)



$|\psi\rangle = cos\frac{\theta}{2}|0\rangle + e^{i\varphi}sin\frac{\theta}{2}|1\rangle$

"traditional" quantum systems

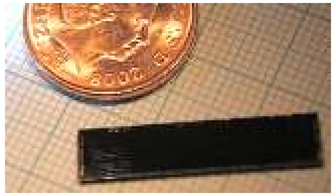
nuclear spins



Vandersypen et al., Nature (2001)

$|u\rangle$ $|d\rangle$

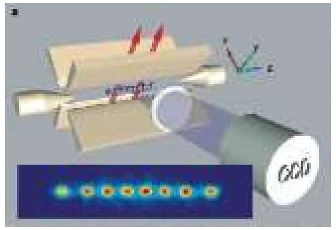
photons



Politi et al, Science (2009)

$|V\rangle$ $|H\rangle$

Trapped ions



Blatt et al, Nature (2008)

$|e\rangle$

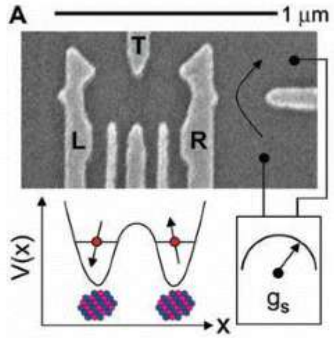
$|g\rangle$

Long quantum coherence time

Poor scalability, hard to control

"artificial" quantum systems

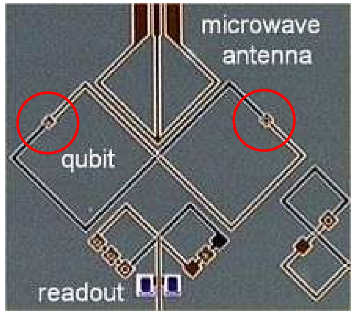
Semiconductor-based qubit



Electron spins in quantum dots of GaAs/AlGaAs heterostructure

[J. R. Petta et al., Science (2005)]

Superconductor-based qubit



courtesy of A. V. Ustinov

Macroscopic quantum coherence of superconductor

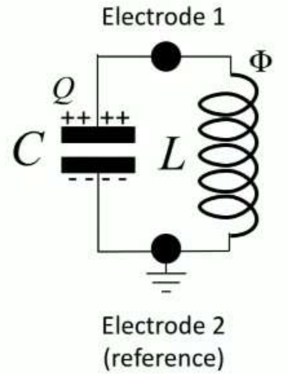
: Josephson junction

High scalability and flexibility in design

Difficult to remove decoherence

Quantum LC Resonator

The quantized *LC* oscillator



Hamiltonian:

$$\hat{H}_{\mathrm{LC}} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

Capacitive term ↗ ↖ Inductive term

Canonically conjugate variables:

$\hat{\Phi}$ = Flux through the inductor.

$\hat{Q}$ = Charge on capacitor plate.

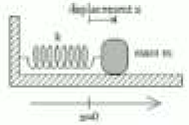
$$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

M. Devoret, Les Houches Session LXIII (1995)

Correspondence with simple harmonic oscillator

$$\hat{H}_{\mathrm{LC}} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C} \qquad \left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

$$\hat{H}_{\mathrm{SHO}} = \frac{k\hat{X}^2}{2} + \frac{\hat{P}^2}{2m} \qquad \left[\hat{X}, \hat{P}\right] = i\hbar$$



Correspondence: $\hat{\Phi} \leftrightarrow \hat{X}$, $\hat{Q} \leftrightarrow \hat{P}$, $L \leftrightarrow \frac{1}{k}$, $C \leftrightarrow m$, $\omega = \frac{1}{\sqrt{LC}} \leftrightarrow \sqrt{\frac{k}{m}}$

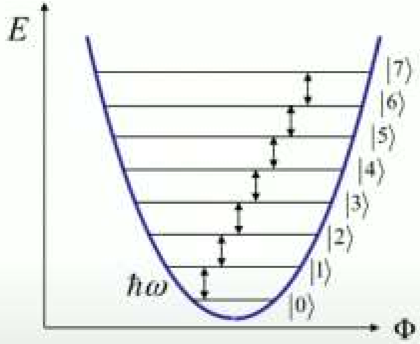
Solve using ladder operators:

$$\hat{a} = \left(\frac{\hat{Q}}{Q_{zpf}} - i\frac{\hat{\Phi}}{\Phi_{zpf}}\right) \qquad \Phi_{zpf} = \sqrt{2\hbar Z}$$

$$\hat{a}^\dagger = \left(\frac{\hat{Q}}{Q_{zpf}} + i\frac{\hat{\Phi}}{\Phi_{zpf}}\right) \qquad Q_{zpf} = \sqrt{2\hbar / Z}$$

$$Z = \omega L = \frac{1}{\omega C} = \sqrt{\frac{L}{C}}$$

$$\hat{H}_{\mathrm{LC}} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) \qquad \left[\hat{a}_r, \hat{a}_r^\dagger\right] = 1$$



M. Devoret, Les Houches Session LXIII (1995)

Josephson Inductance

Inductance describes voltage drop, V, induced by the change of current, dI/dt,

$$V = L \times (dI/dt).$$

For Josephson junction,

I changes in time

→ φ changes in time (DC Josephson relationship)

→ V appears (AC Josephson relationship)

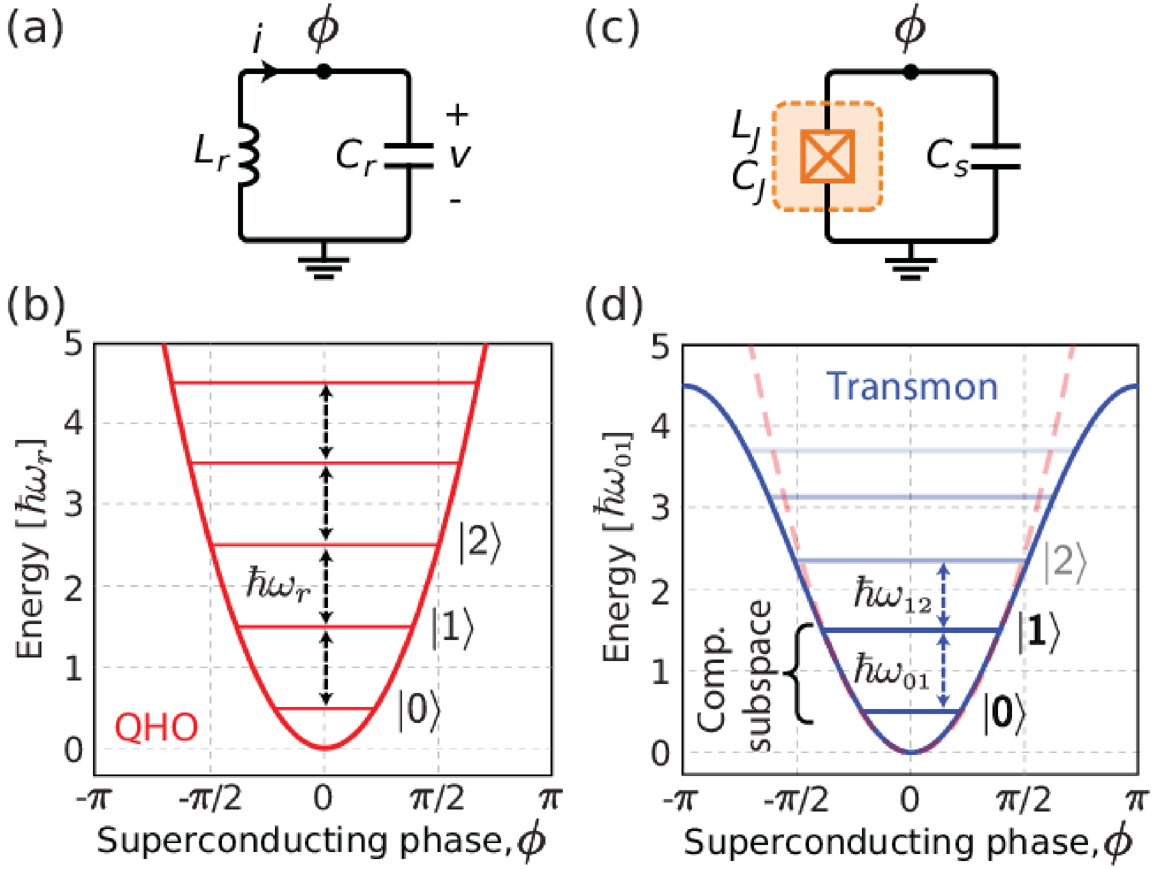
$$\frac{\partial I}{\partial \varphi} = I_c \cos\varphi, \quad \frac{\partial \varphi}{\partial t} = \frac{2\pi}{\Phi_0} V. \longrightarrow \frac{\partial I}{\partial t} = \frac{\partial I}{\partial \varphi}\frac{\partial \varphi}{\partial t} = I_c \cos\varphi \cdot \frac{2\pi}{\Phi_0} V, \longrightarrow V = \frac{\Phi_0}{2\pi I_c \cos\varphi}\frac{\partial I}{\partial t} = \boxed{L(\varphi)}\frac{\partial I}{\partial t}.$$

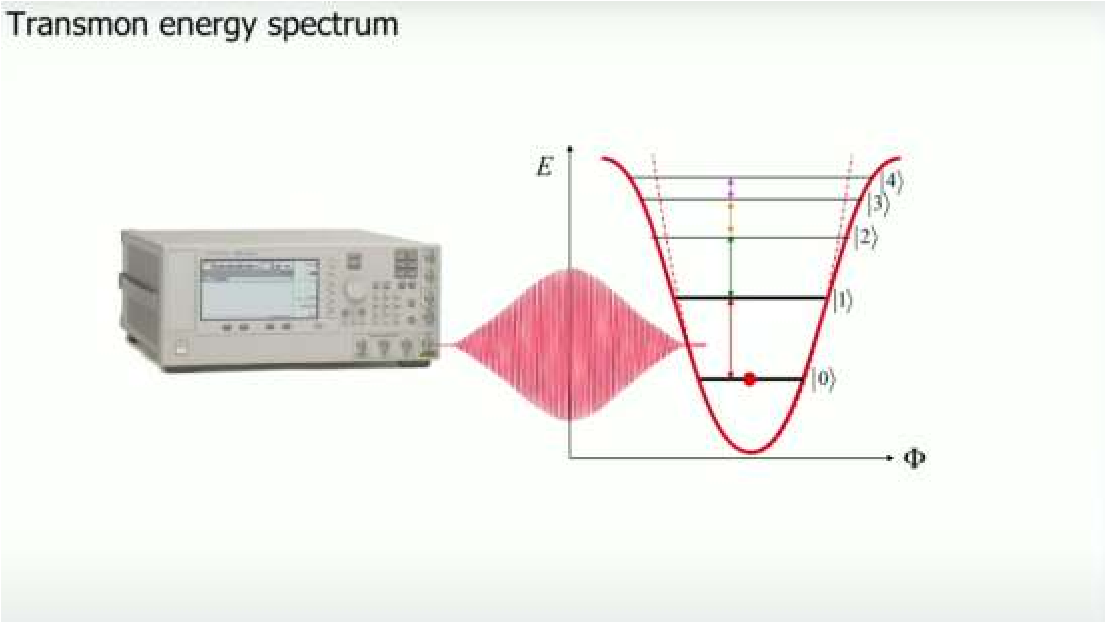
Josephson inductance

$$L(\varphi) = \frac{\Phi_0}{2\pi I_c \cos\varphi} = \frac{L_J}{\cos\varphi}. \qquad L_J = L(0) = \frac{\Phi_0}{2\pi I_c}$$

Josephson junction is a ‘quantum’ nonlinear inductor.

Anharmonic LC resonator





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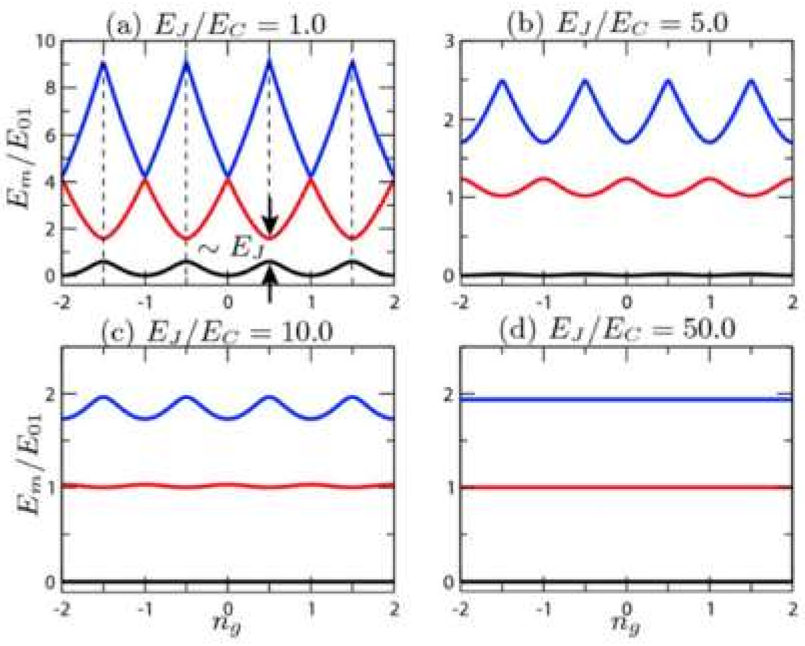
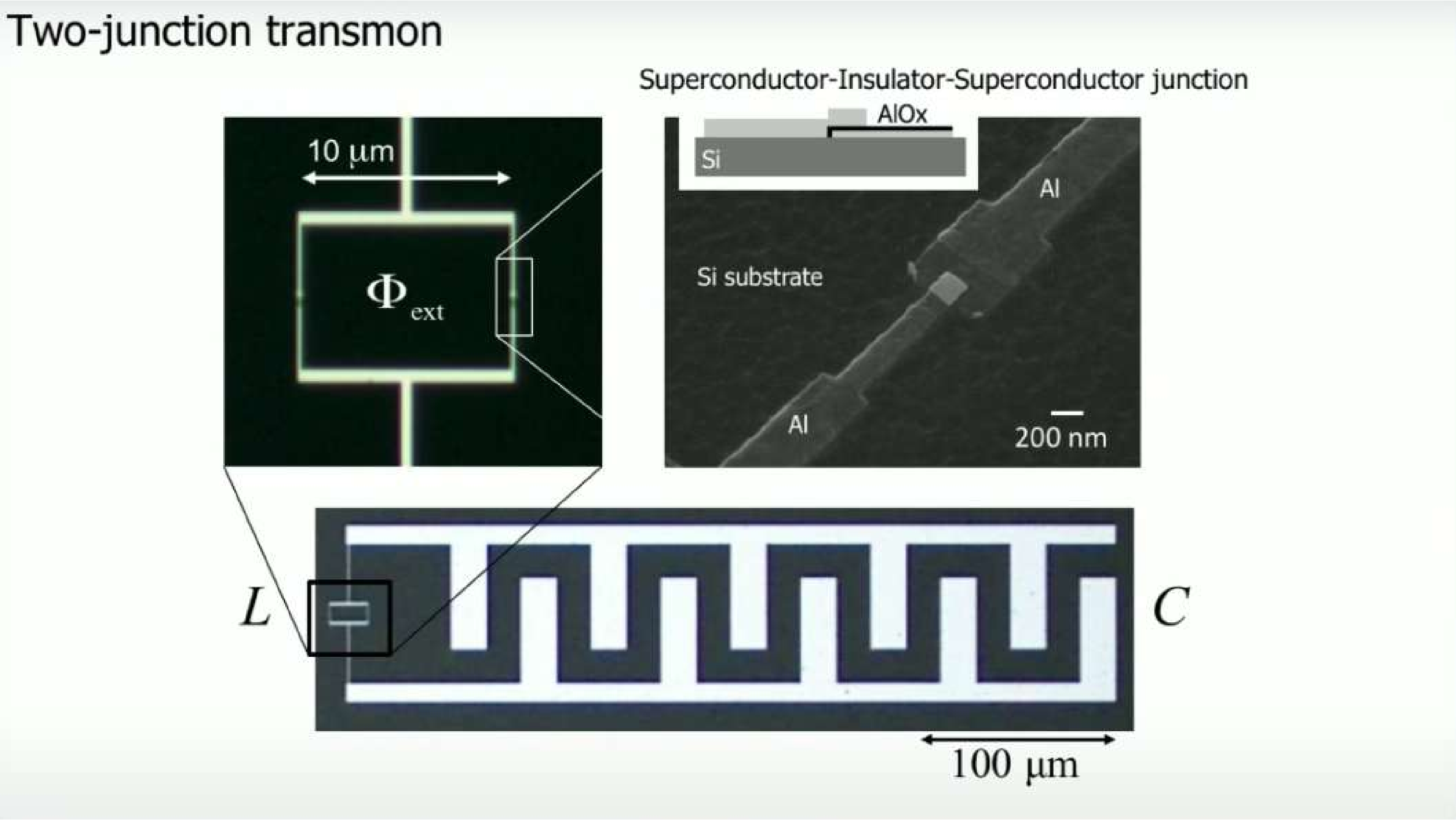
Shunting Capacitor

Energy stored in C: $E = \frac{(ne)^2}{2C} = E_C n^2$

To minimize effect charge noise, E_C was decreased by adding big capacitor.

Energy diagram as a function of charging





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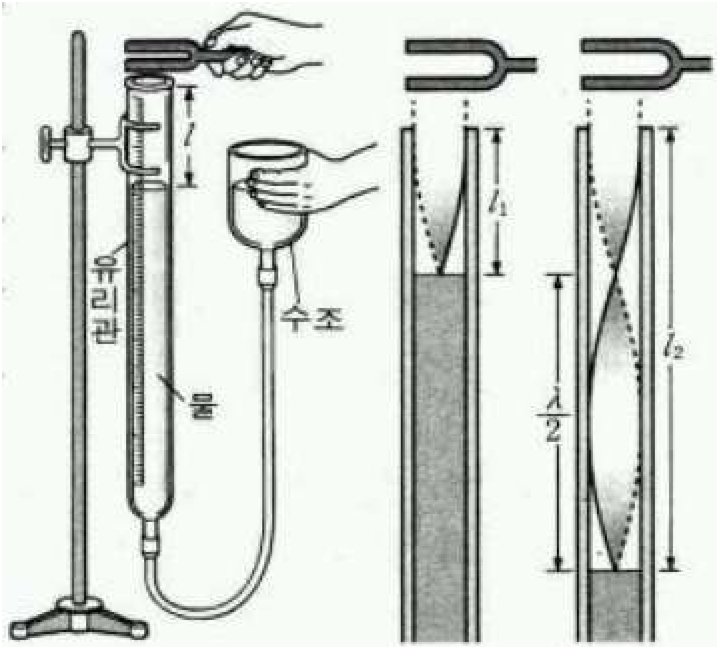
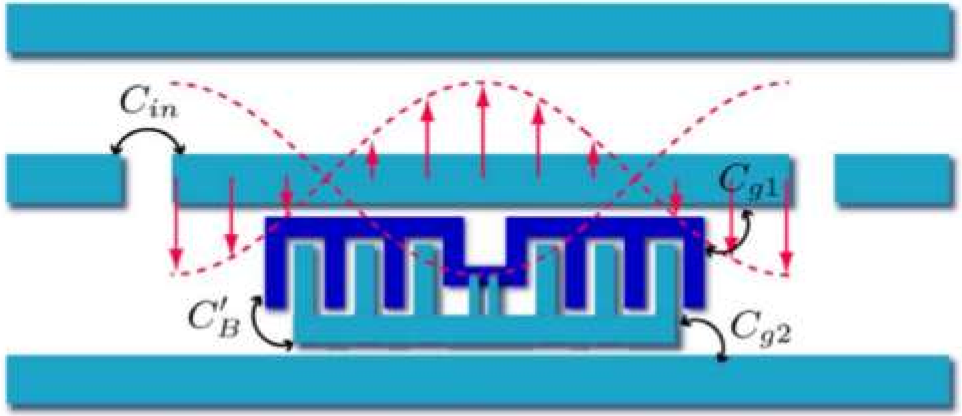
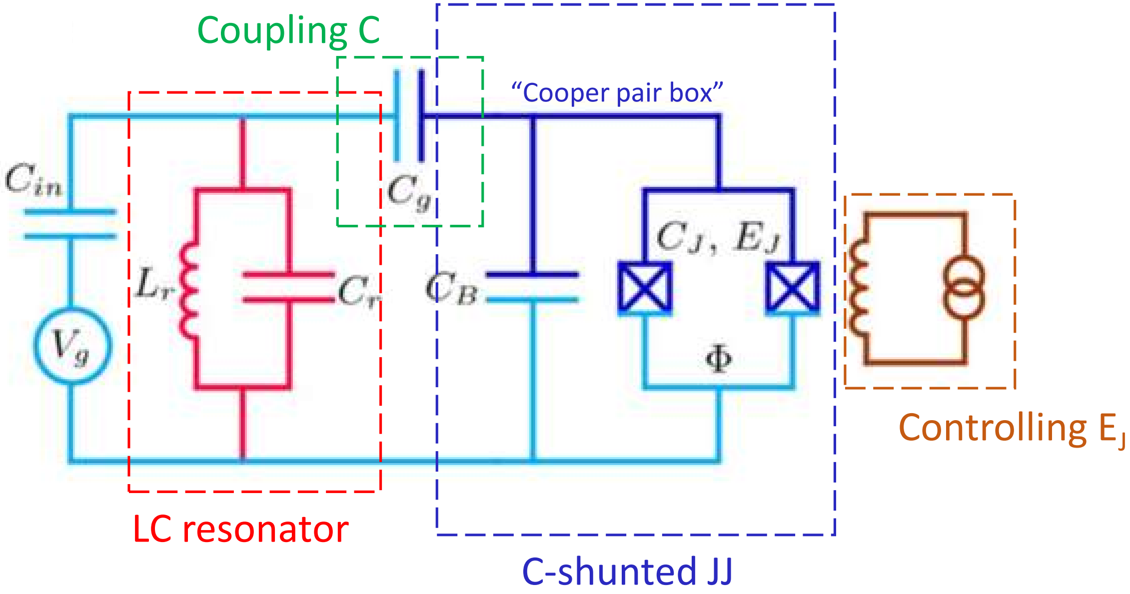
Coplanar Waveguide (CPW) coupled to Transmon



Air Column Resonance
(기주공명)





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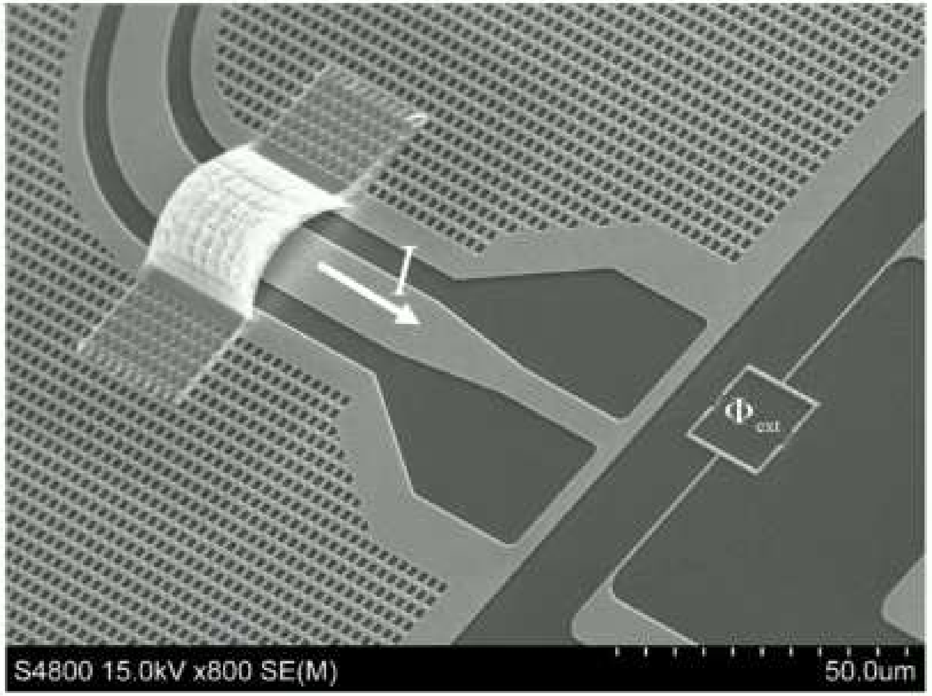
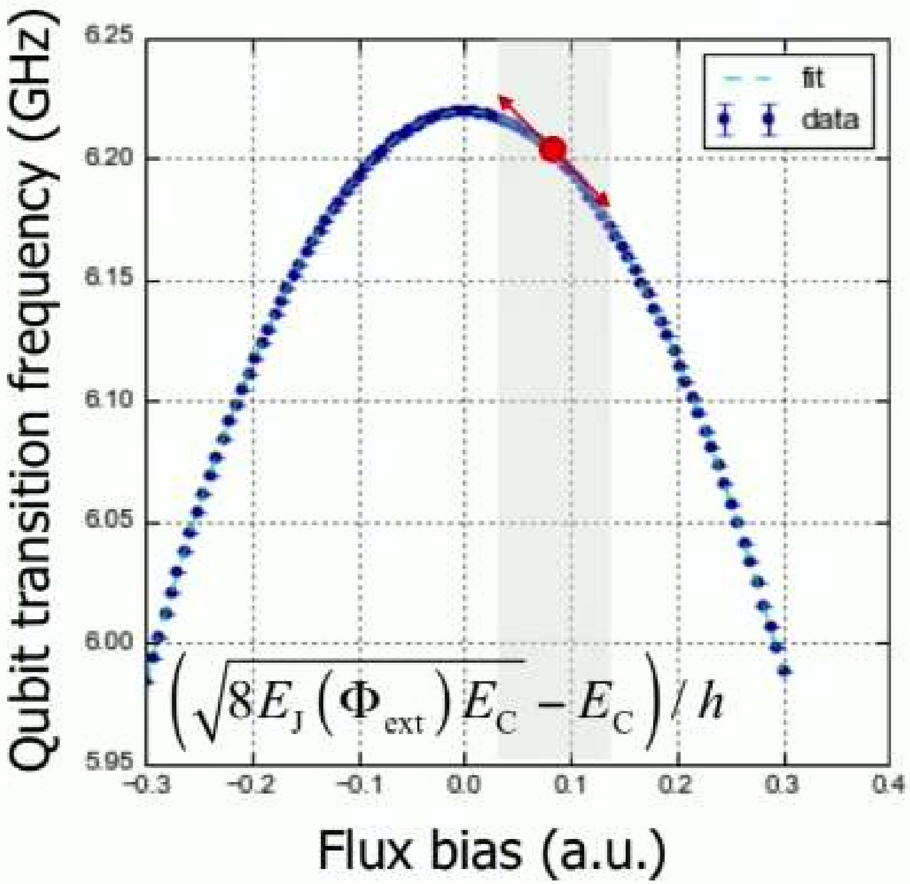
Flux control of Transmon Frequency

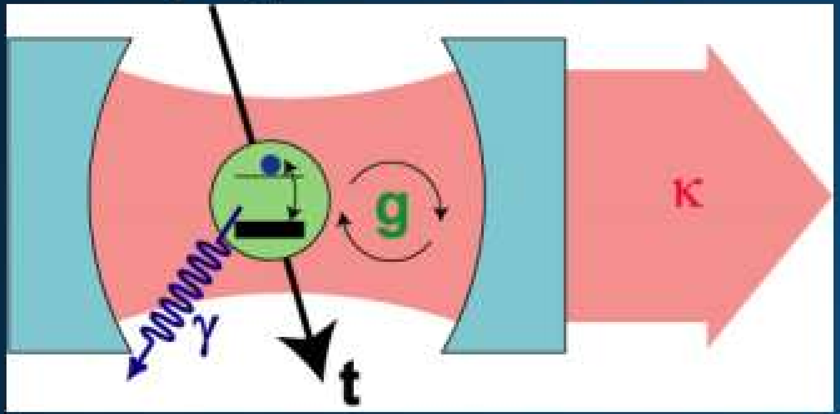
Short-circuited transmission line

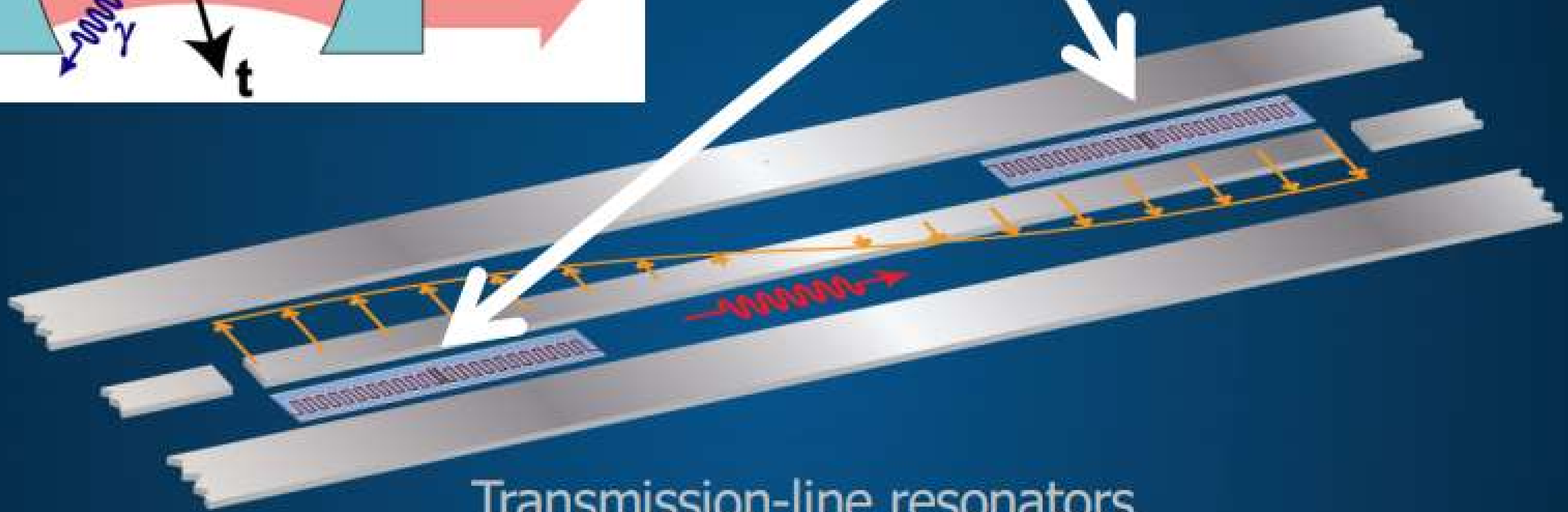


$$L_J = L(0) = \frac{\Phi_0}{2\pi I_c} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$



Circuit QED: Cavity QED with wires





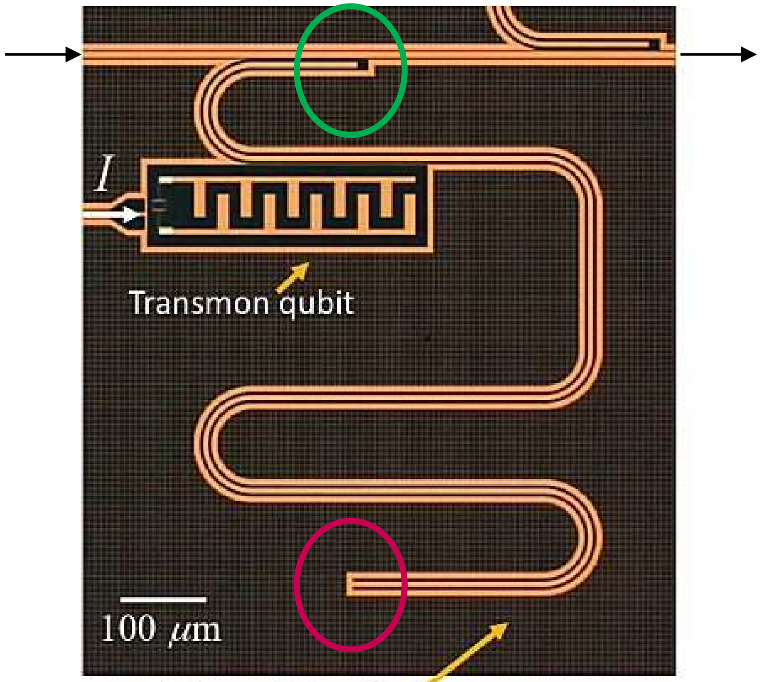
Transmission-line resonators

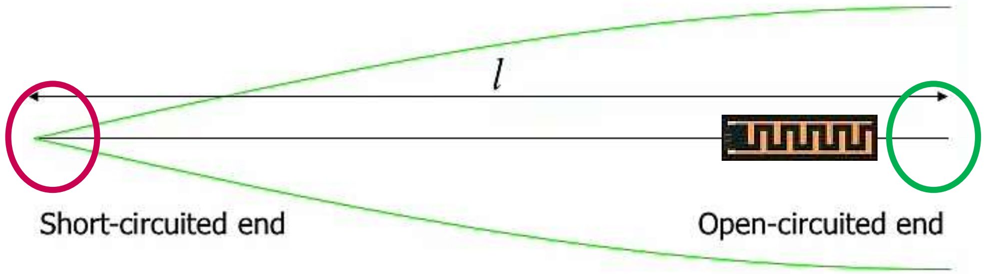
- mediate interaction between qubits
- allow qubit readout

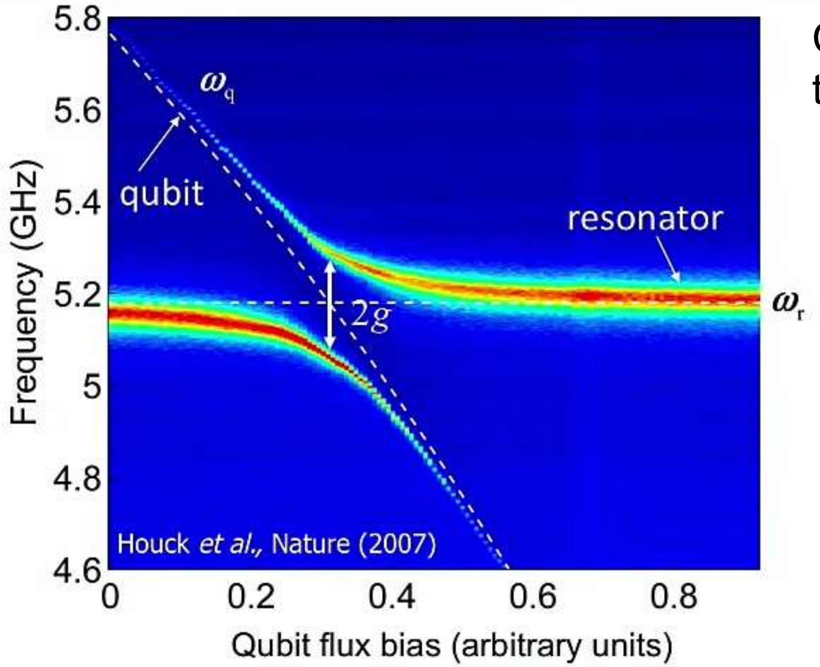
Blais *et al.*, Phys. Rev. A (2004)

Qubit-Resonator Interaction

Qubit and resonator are capacitively coupled.







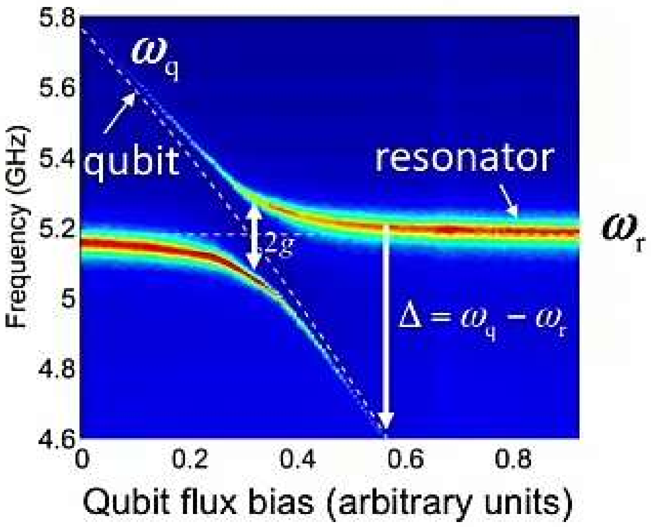
Color: absorption through readout line

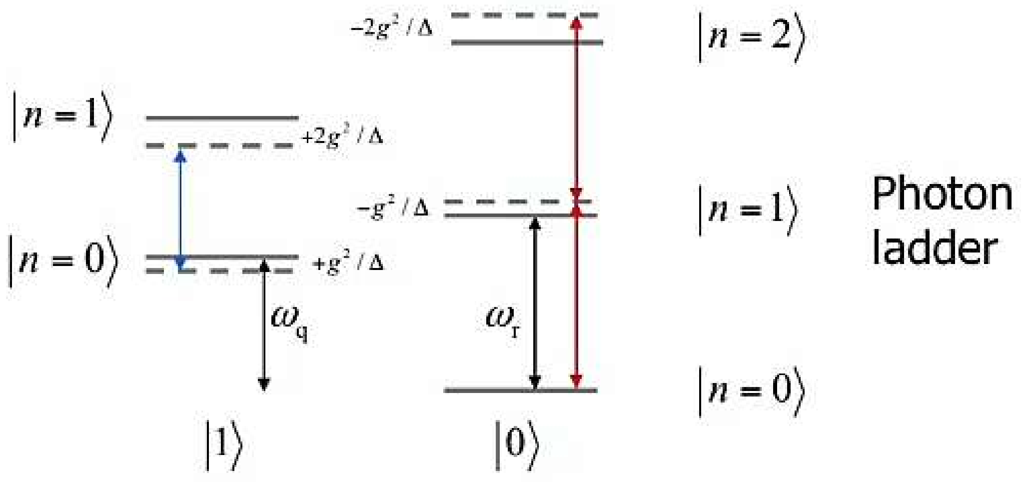
Decrease qubit state

$$\hat{H}_{\text{int}} = \hbar g \left(a_r \hat{\sigma}_+ + a_r^\dagger \hat{\sigma}_- \right)$$ Jaynes-Cummings interaction

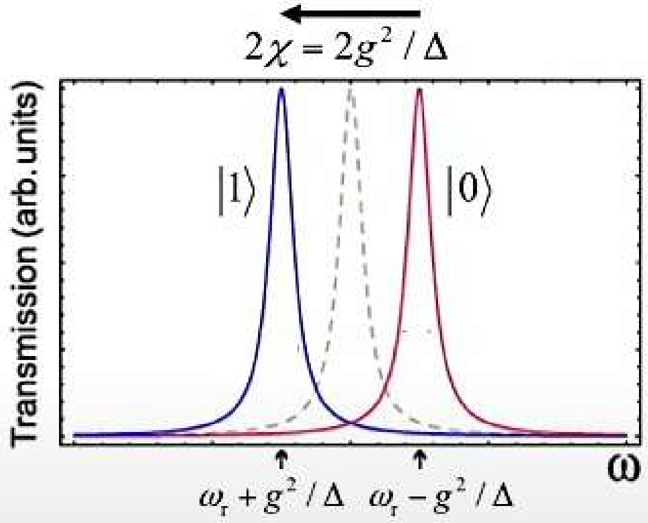
Increase resonator state

Qubit Measurement (Dispersive regime)





Dispersive regime: $|\Delta| = |\omega_q - \omega_r| >> g$



$$\hat{H}_{int} = -\hbar\chi\hat{\sigma}_Z a_r^{\dagger} a_r$$

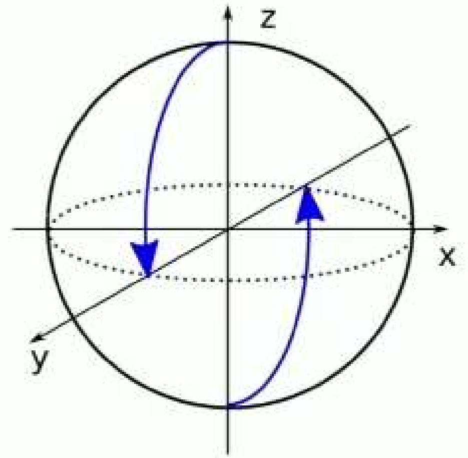
Qubit state

Resonator state

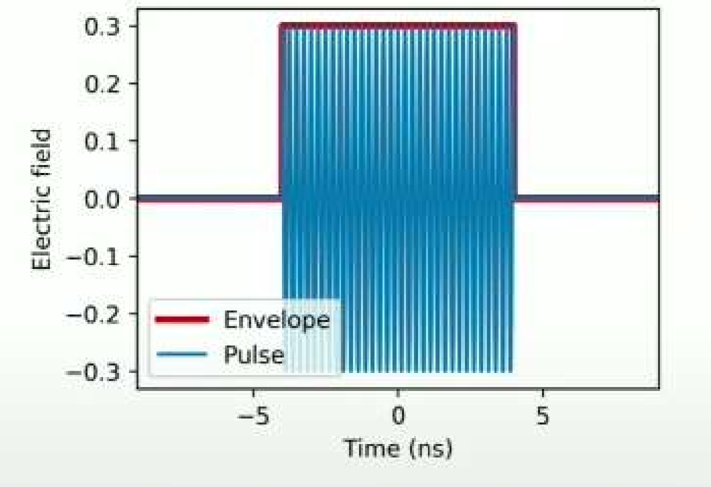
Single Qubit Operation

[Rabi oscillation]

– rotation around axis in x-y plane



Rotation angle
= speed × duration
= area of pulse envelope



[Flux-bias control]

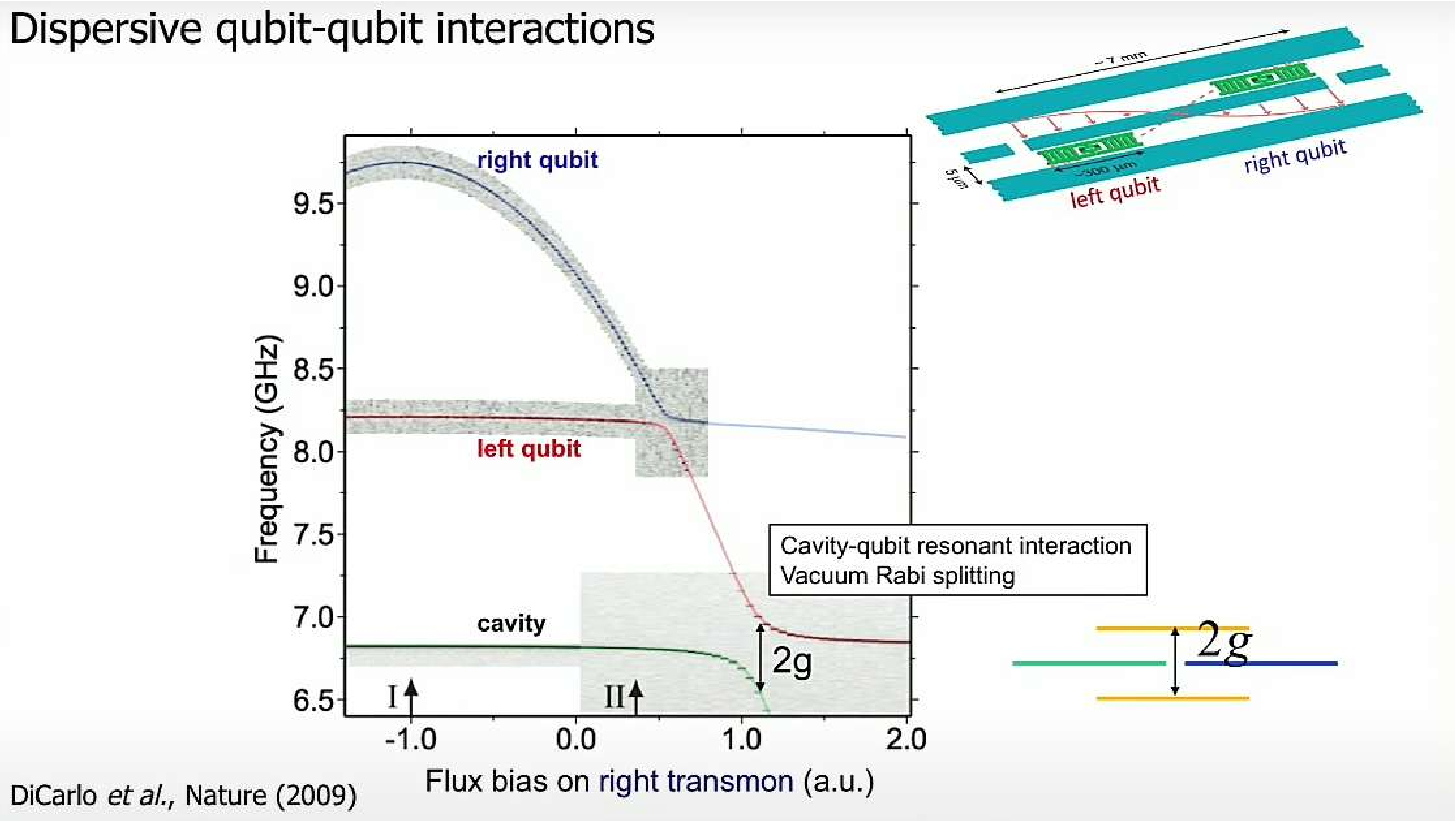
- Rotation around z-axis

- Time evolution give 'dynamic' phase accumulation as, $|\psi\rangle \rightarrow e^{iEt/\hbar}|\psi\rangle$.

- Then, energy difference gives phase difference between $|0\rangle$ and $|1\rangle$ as, $|0\rangle + |1\rangle \rightarrow e^{iE_0 t/\hbar}|0\rangle + e^{iE_1 t/\hbar}|1\rangle = e^{iE_0 t/\hbar}\left(|0\rangle + e^{i(E_1 - E_0)t/\hbar}|1\rangle\right)$.

- Flux-bias control can change the qubit frequency: $\hbar\omega_q = E_1 - E_0$

Qubit-Qubit Coupling



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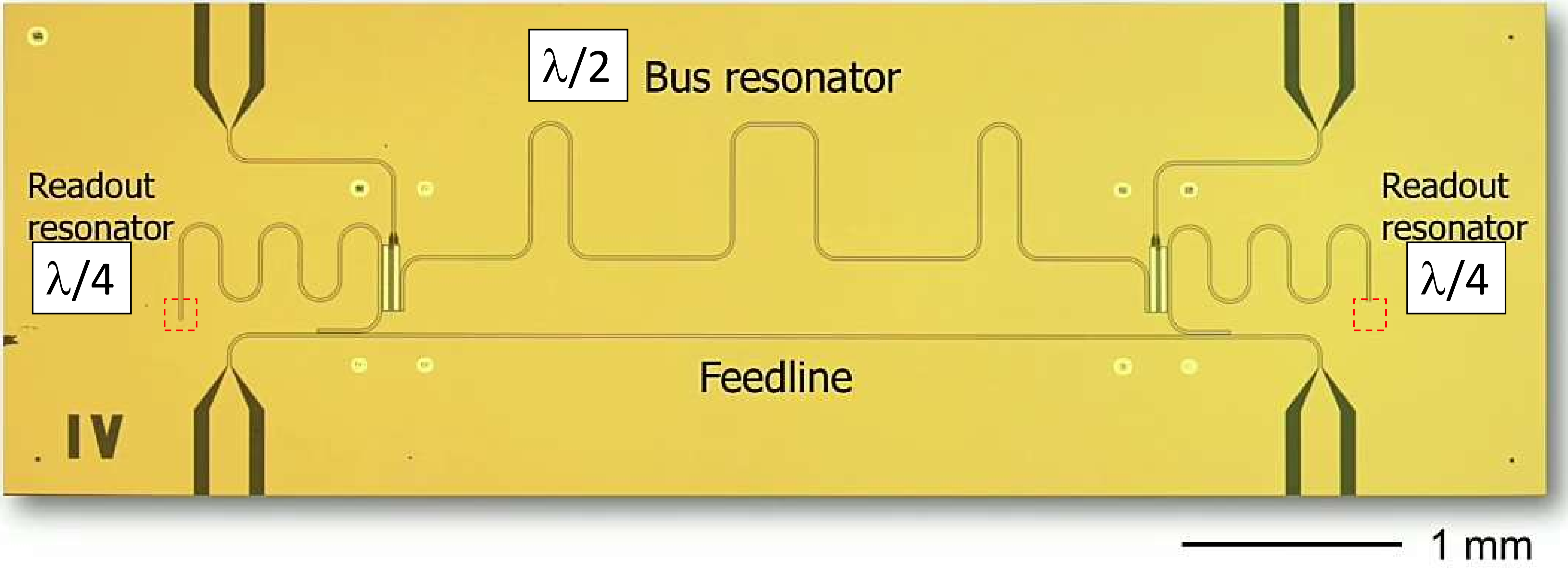
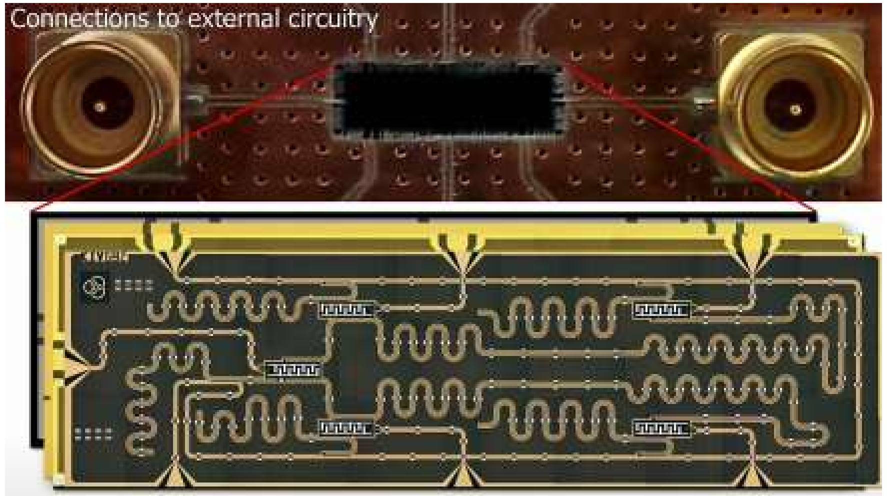
Two-Qubit Processor

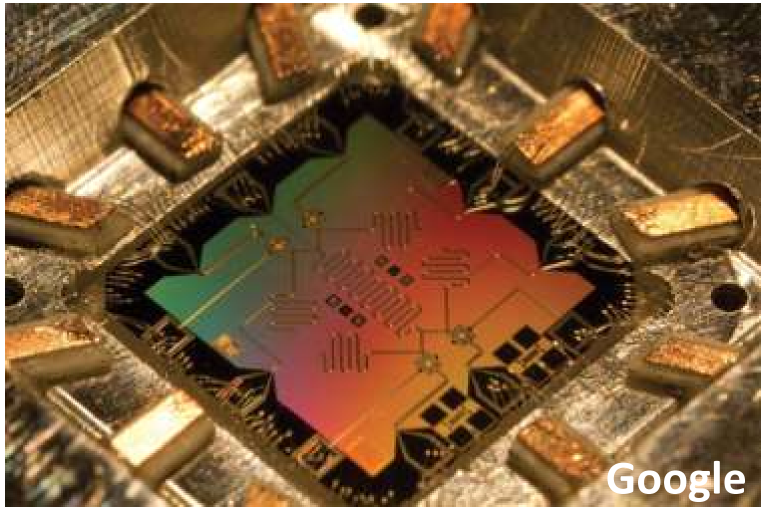


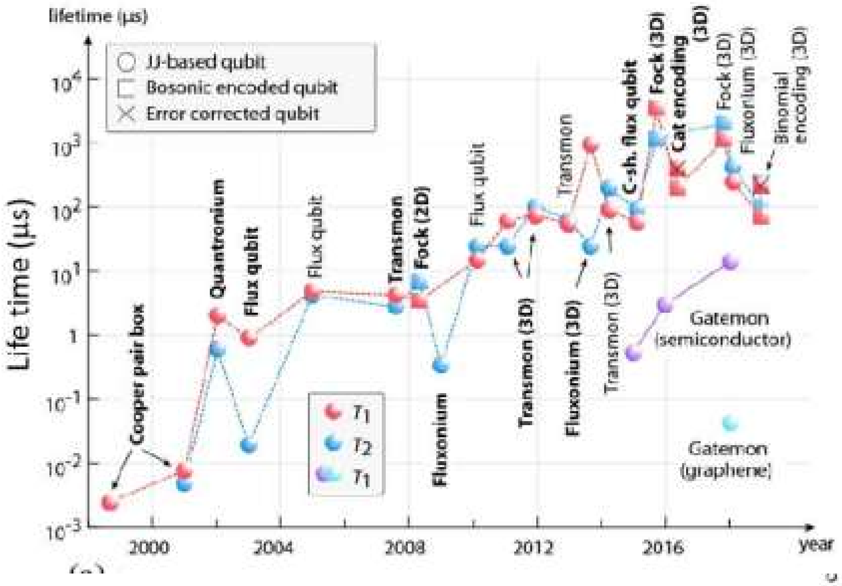
Different resonators for qubit-qubit coupling and for readout of each qubit

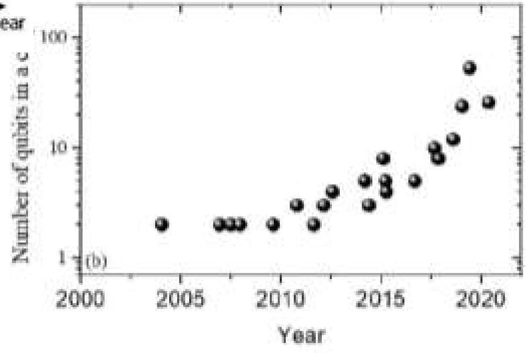
Groen *et al.*, Phys. Rev. Lett. (2013)

Packaging / Development Trend









The performance of qubits is tripling every year.

c.f.) Moore's Law: semiconductor integration density doubles every two years.

Hybrid system using coplanar waveguide