Josephson junction and diode effect

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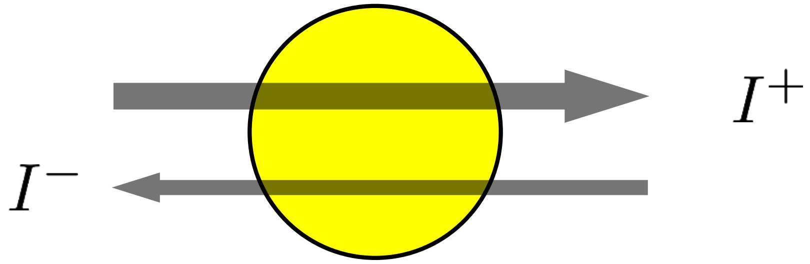
Nonreciprocal flow

Nonreciprocal element

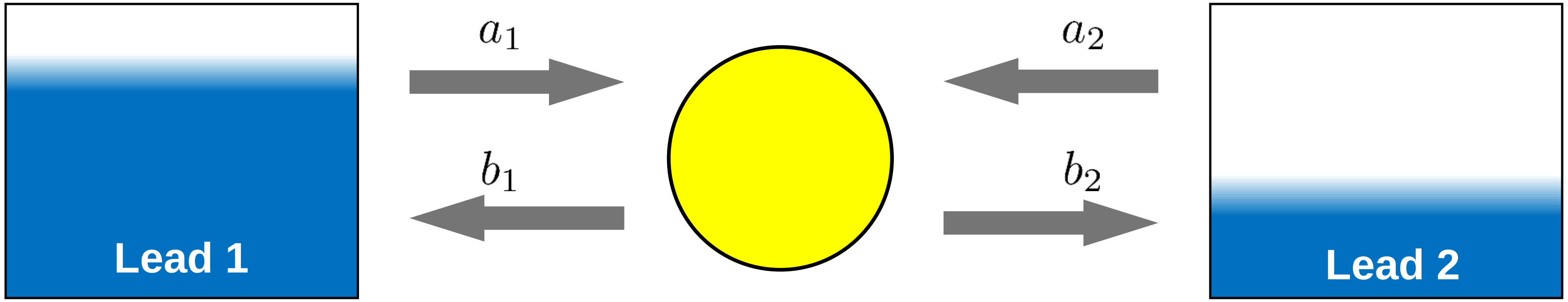


Nonreciprocal – forward and backward flows differ

$$I^{+} \neq I^{-}$$

Similarly, a check valve is a one-way mechanical device

Nonreciprocal flow



Input-output relation via the scattering matrix *S*

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = S \begin{pmatrix} a_1 \\ a_2 \end{pmatrix},$$

$$S = \begin{pmatrix} r_{11} & t_{12} \\ t_{21} & r_{22} \end{pmatrix}.$$

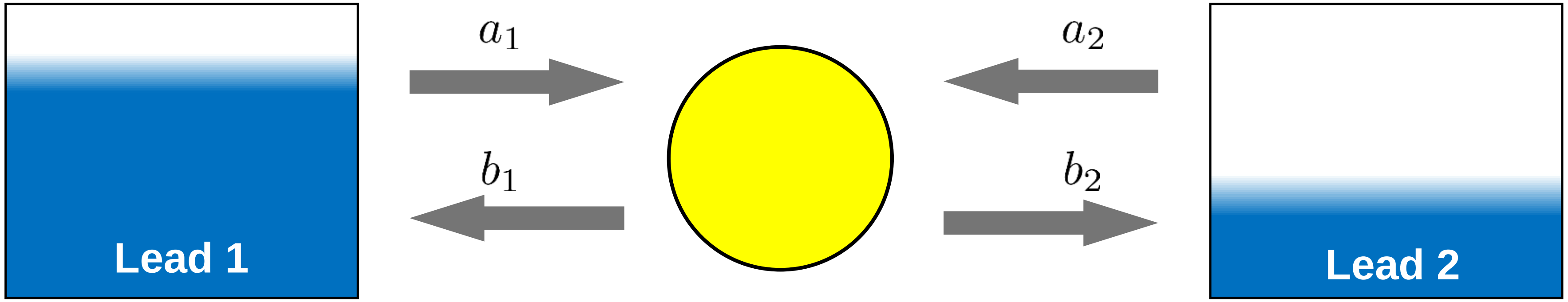
Time-reversal symmetry

$$S = S^T \rightarrow t_{21} = t_{12}$$

Inversion symmetry

$$S = \sigma_x S \sigma_x \rightarrow r_{11} = r_{22} \,\&\, t_{21} = t_{12}$$

Nonreciprocal flow



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Inversion symmetry

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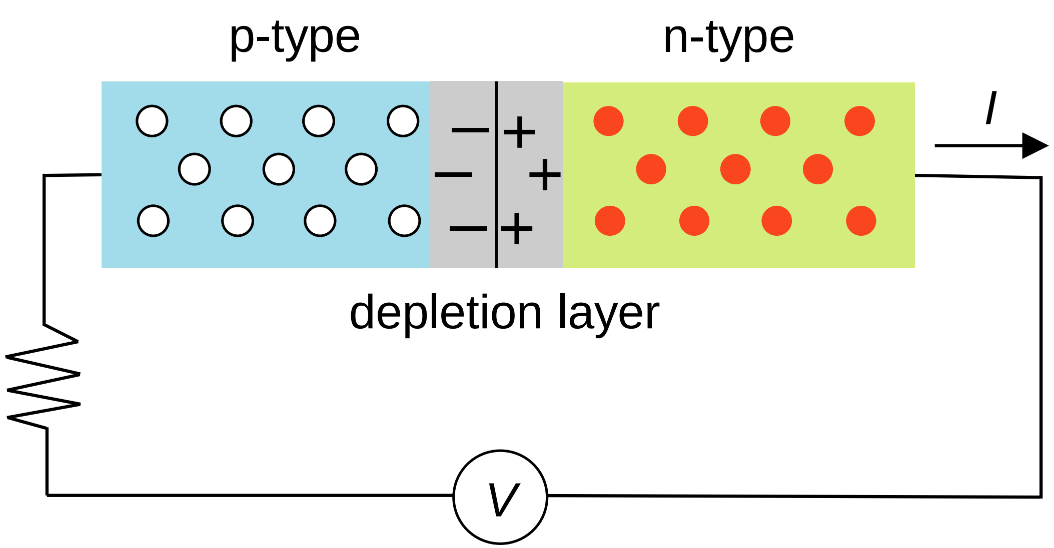
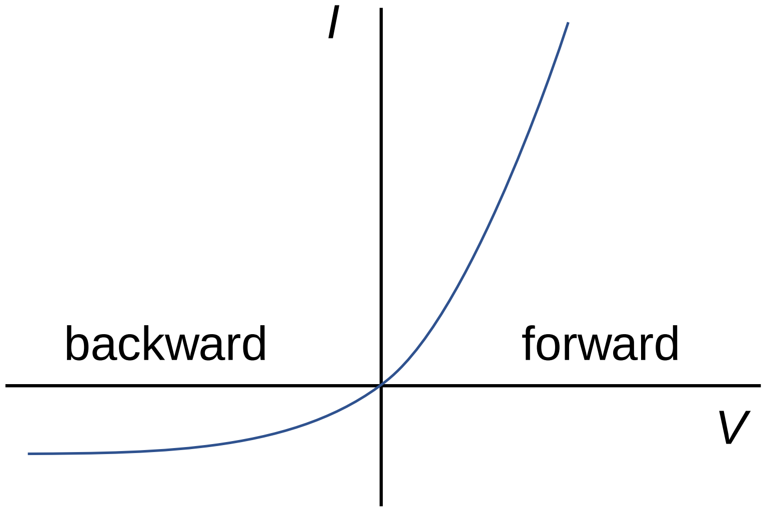
Nonreciprocal flow requires the breaking of at least one symmetry: time-reversal or inversion

P-N junction diode





$$I(V) \neq I(-V)$$

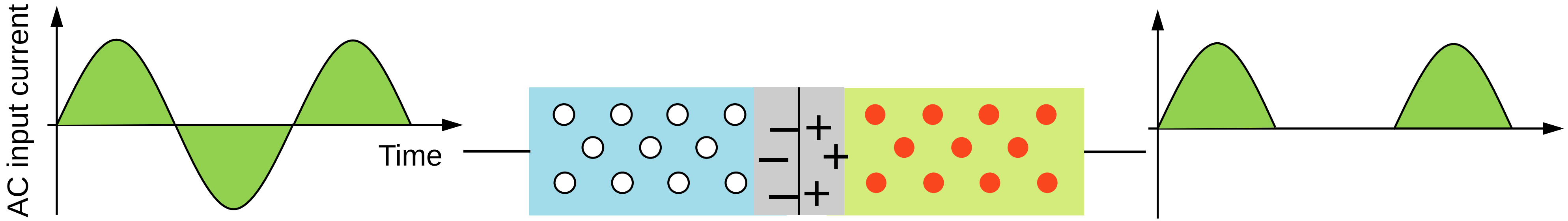
Key requirement for nonreciprocal charge transport: inversion symmetry breaking

$\Rightarrow$ current direction changes depletion layer thickness

$\Rightarrow$ asymmetric resistivity

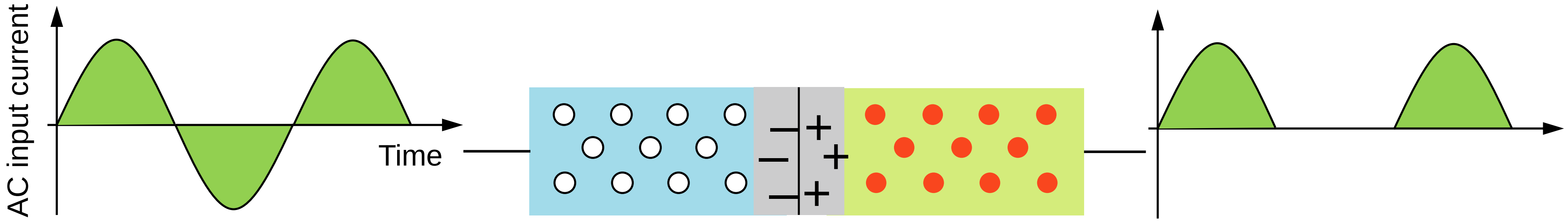
P-N junction diode

Rectifiers converting alternating signal (AC) into direct signal (DC)



P-N junction diode

Rectifiers converting alternating signal (AC) into direct signal (DC)



Nonreciprocal Cooper-pair flow

(superconducting dissipationless version of semiconducting diode)

⇒ **Josephson diode effect**

Outline

Part I. Josephson junction

Part II. Josephson diode mechanisms in superconducting hybrid systems

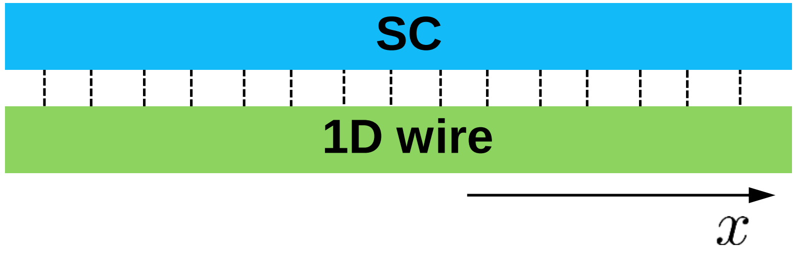
Part III. Overview of recent studies

Part I. Josephson junction

- Andreev states

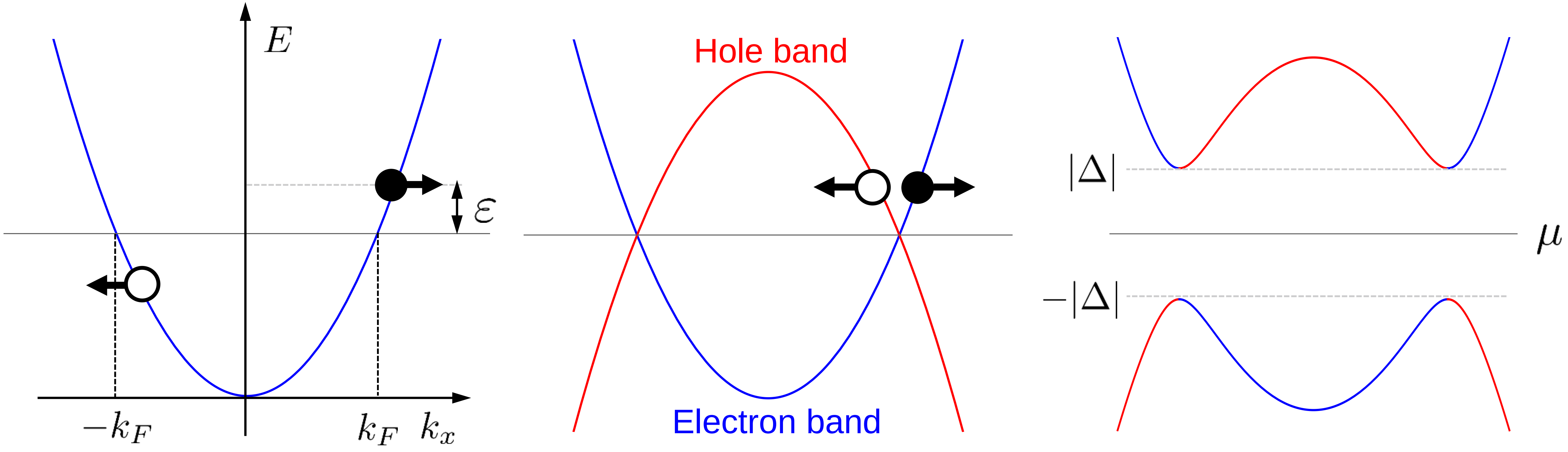
- supercurrent and current-phase relation

Bogoliubov-de Gennes picture

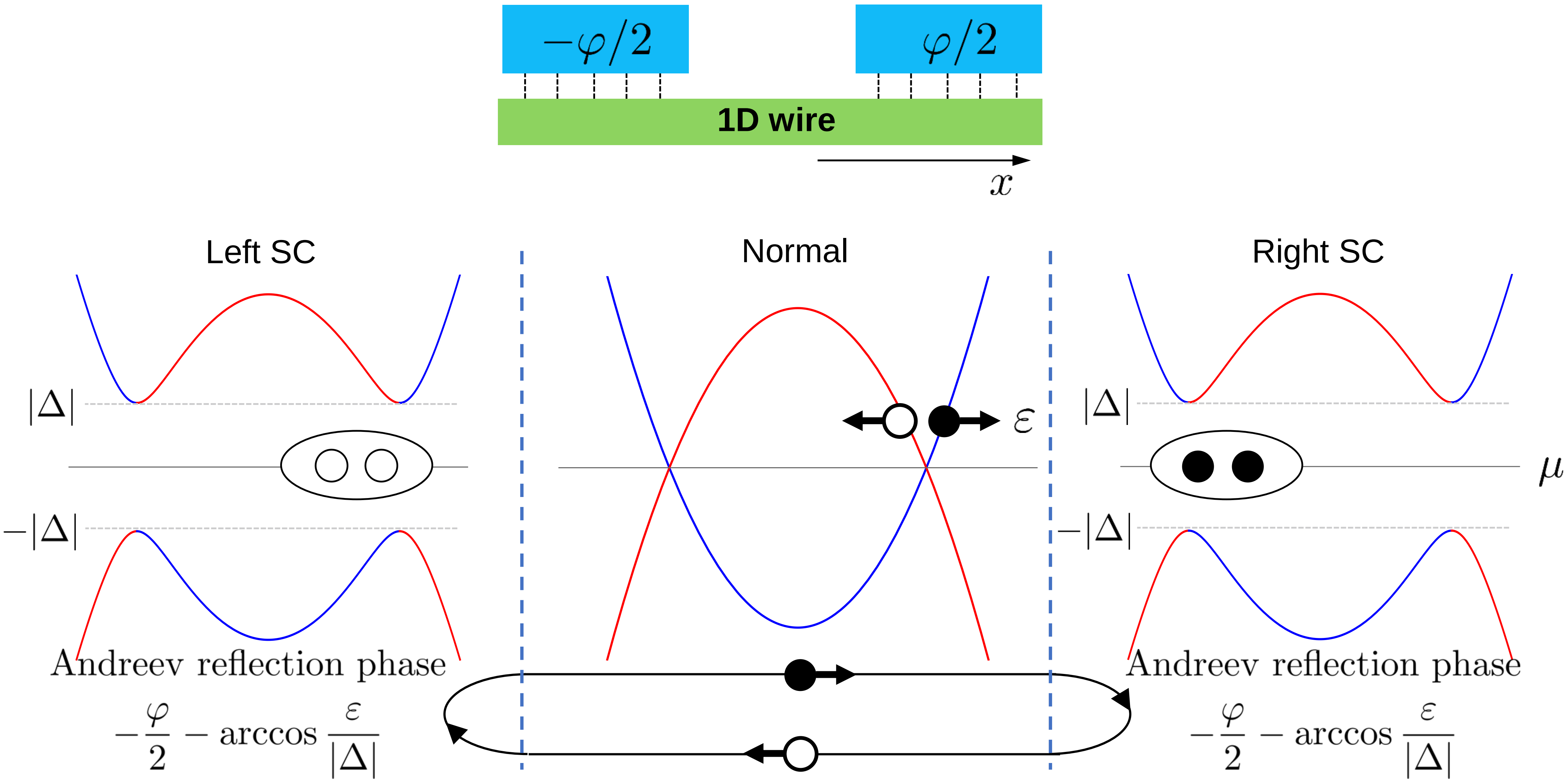


Proximity superconductivity

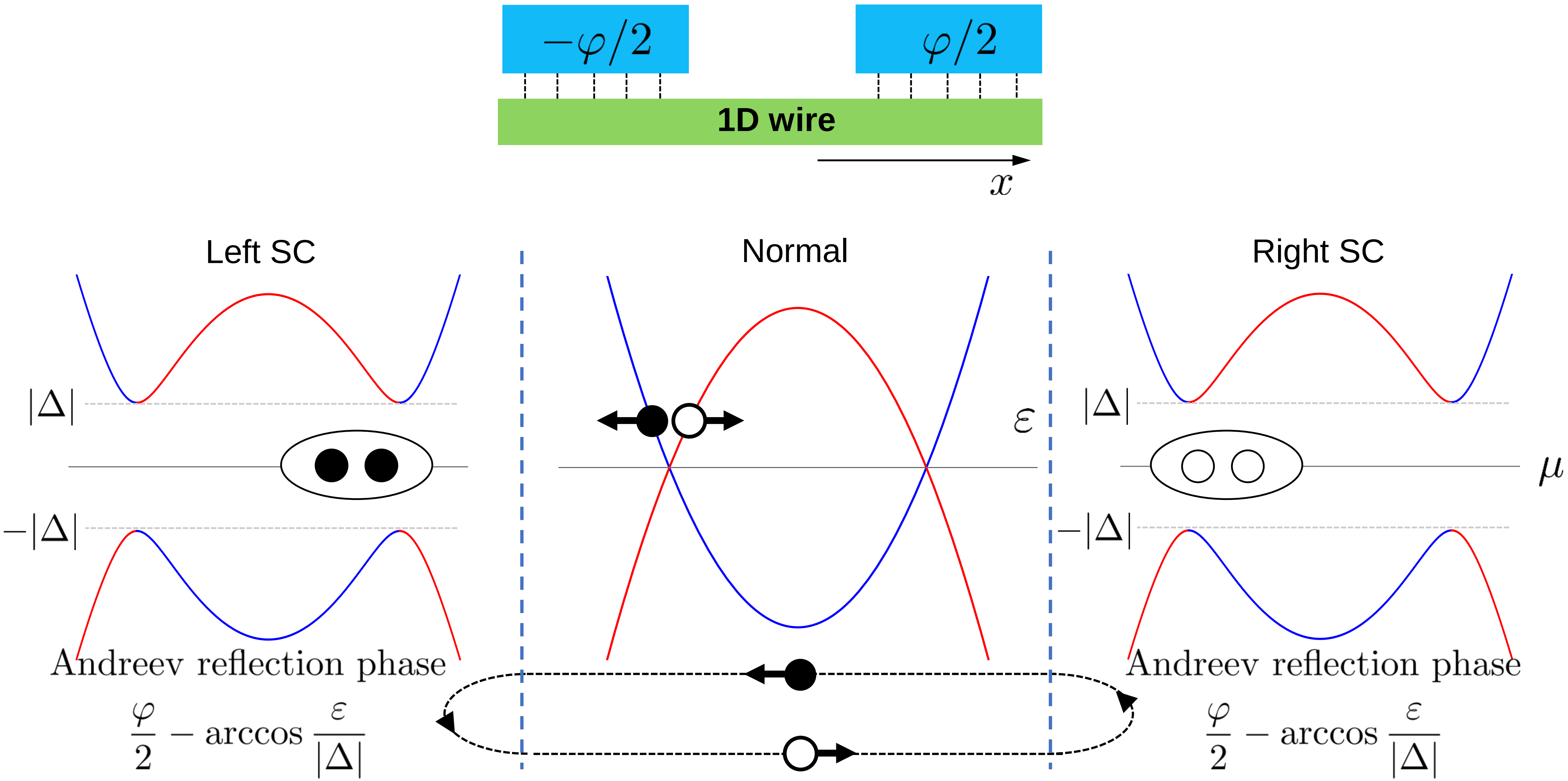
$$\begin{pmatrix} \frac{\hbar^2 k_x^2}{2m^*} - \mu & \Delta \\ \Delta^* & -\frac{\hbar^2 k_x^2}{2m^*} + \mu \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_h \end{pmatrix} = \varepsilon \begin{pmatrix} \psi_e \\ \psi_h \end{pmatrix}$$



Josephson junction - Andreev reflection and Andreev states

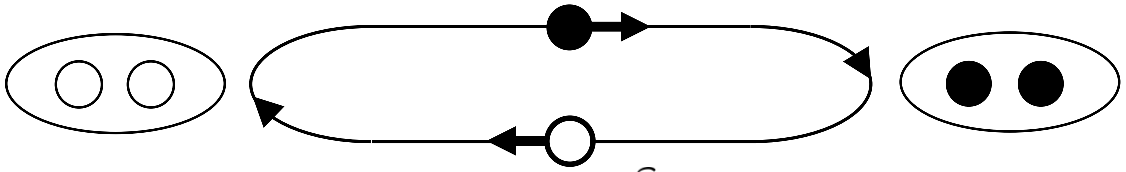


Josephson junction - Andreev reflection and Andreev states

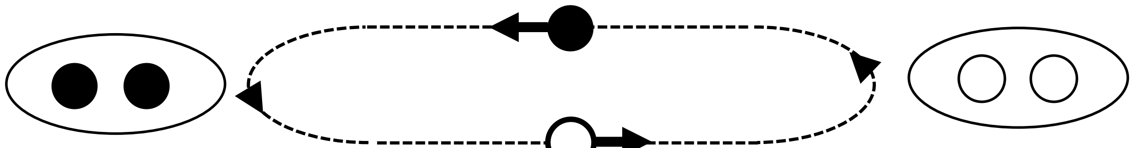


Short Josephson junction - Andreev spectrum

Phase accumulation and quantization

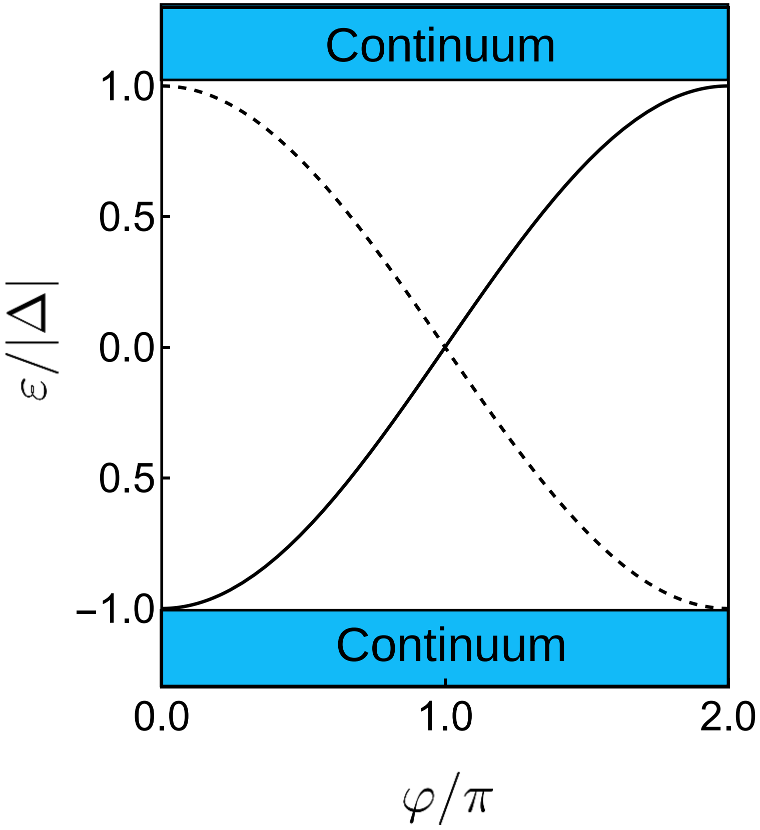


$$-\varphi - 2\arccos\frac{\varepsilon}{|\Delta|} = 2\pi n$$



$$\varphi - 2\arccos\frac{\varepsilon}{|\Delta|} = 2\pi n$$

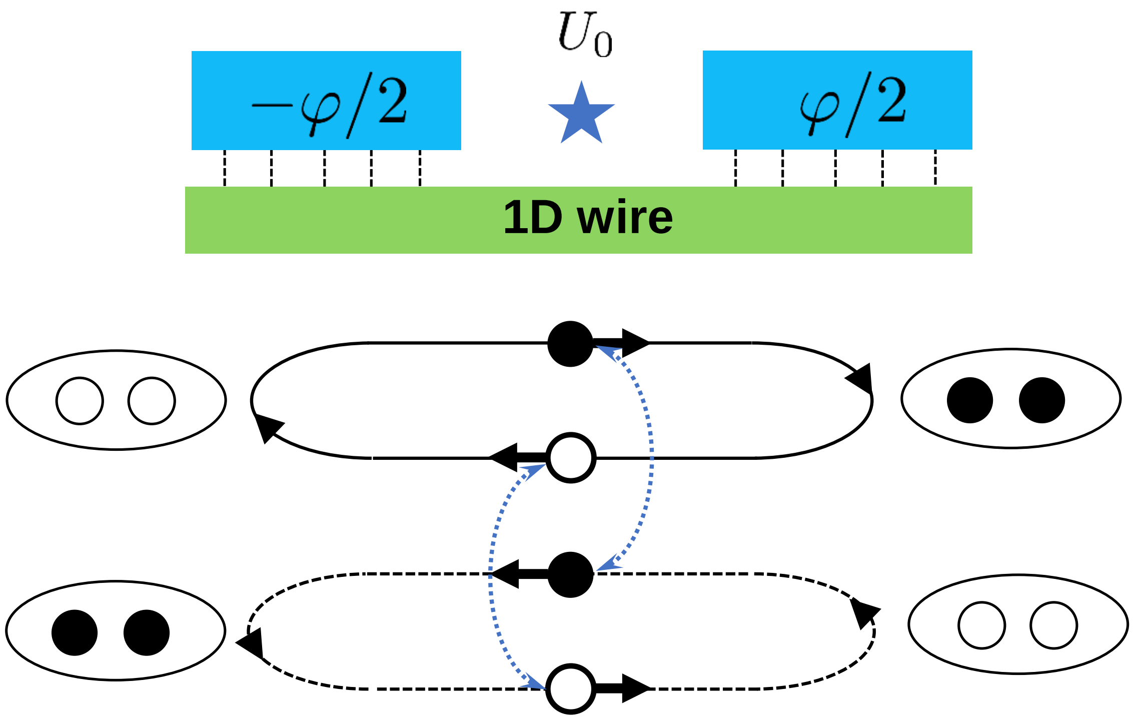
$$\varepsilon = \pm|\Delta|\cos\frac{\varphi}{2}$$



Bagwell, PRB 46, 12573 (1992)
Beenakker and Houten, PRL 66, 3056 (1991)
Kulik, JETP 30, 944 (1970)

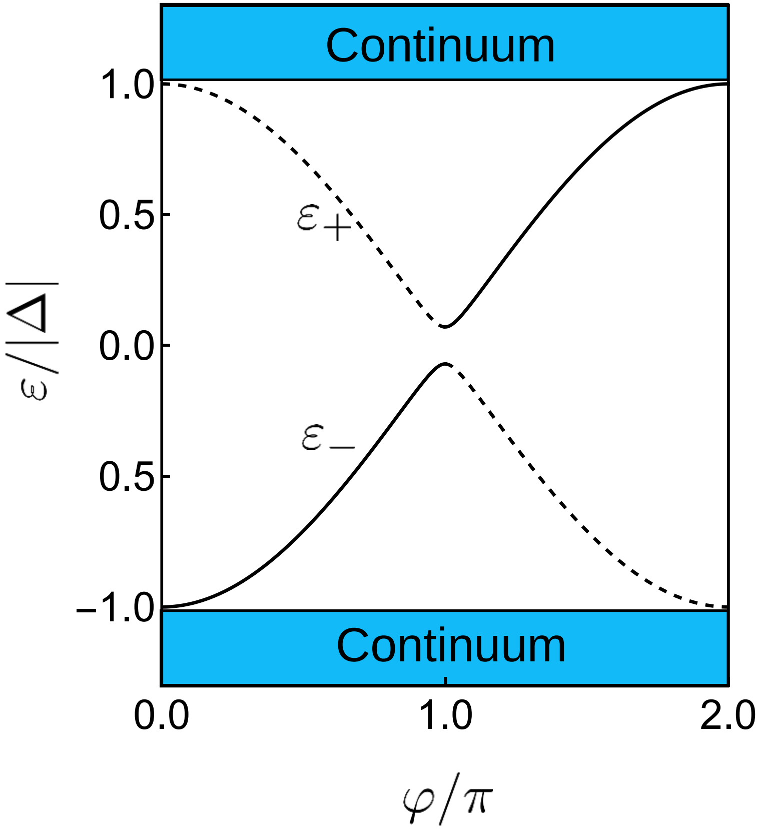
Short Josephson junction - Andreev spectrum

electron scattering



Transmission $T = \dfrac{1}{1 + \frac{U_0^2}{\hbar^2 v_F^2}}$

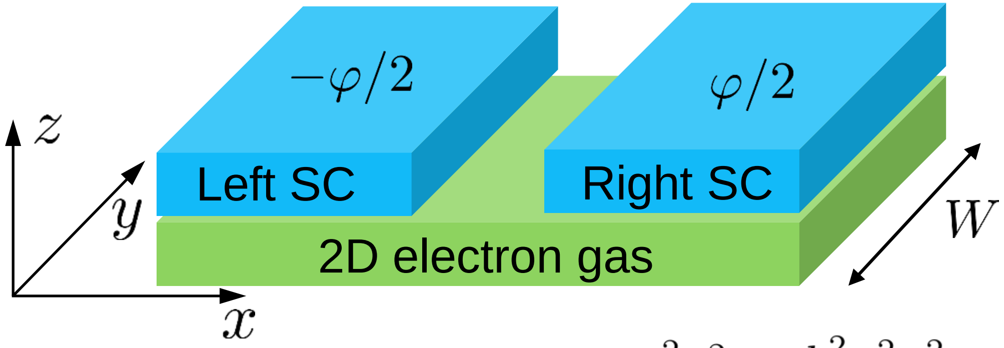
$$\varepsilon_\pm = \pm|\Delta|\sqrt{1 - T\sin^2\frac{\varphi}{2}}$$



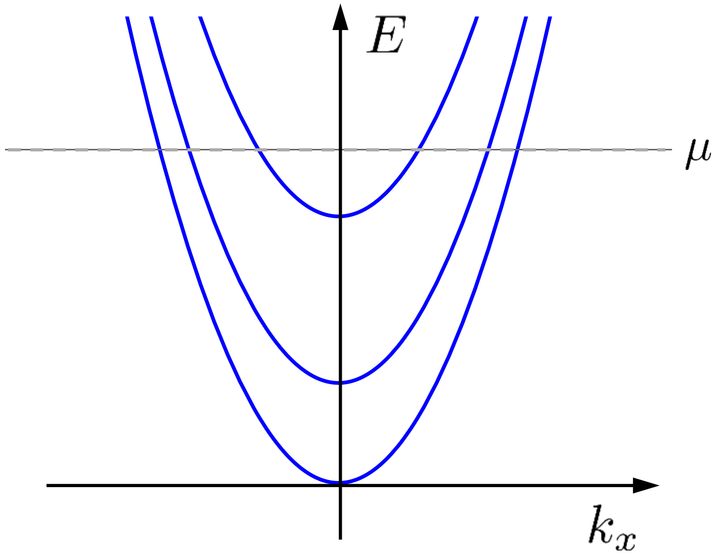
Bagwell, PRB 46, 12573 (1992)
Beenakker and Houten, PRL 66, 3056 (1991)
Kulik, JETP 30, 944 (1970)

Planar Josephson junction

$$\frac{\hbar^2}{2m^* W^2} \ll \mu$$

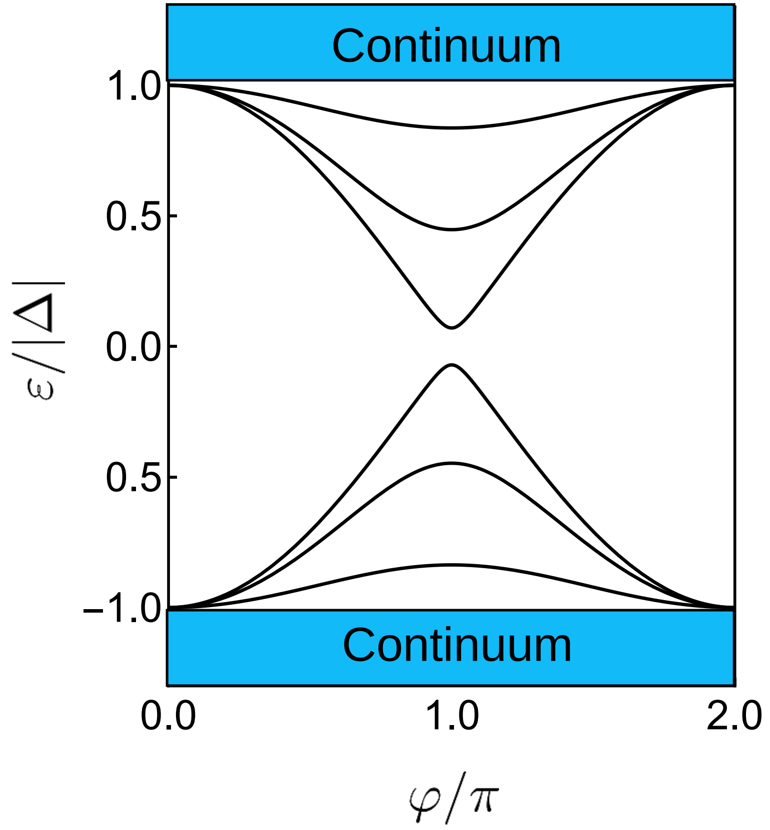


$$k_y \to \frac{\pi n_y}{W}, \;\; E_(k_x, n_y) = \frac{\hbar^2 k_x^2}{2m^*} + \frac{\hbar^2 \pi^2 n_y^2}{2m^* W^2}$$



Andreev spectrum

$$\varepsilon_{i\pm} = \pm|\Delta|\sqrt{1 - T_i \sin^2\frac{\varphi}{2}}, \;\; T_i = \frac{1}{1 + \frac{U_0^2}{\hbar^2 v_i^2}}$$



Planar Josephson junction – Josephson currents

$$\hat{H} = \frac{1}{2}\sum_n \varepsilon_n(\varphi)|n\rangle\langle n|$$

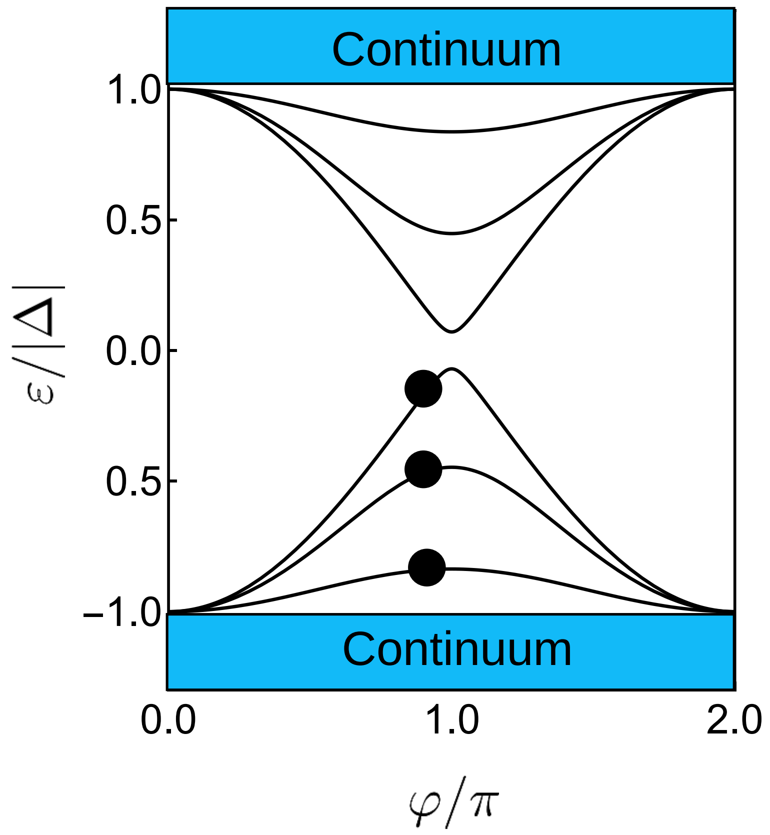
$$\hat{I} = -2e\frac{d\hat{n}}{dt} = -i\frac{2e}{\hbar}[\hat{H},\hat{n}] = \frac{2e}{\hbar}\frac{\partial \hat{H}}{\partial \varphi}$$

$$\text{where } \hat{n} = \frac{1}{i}\frac{\partial}{\partial \varphi}$$

Josephson current

$$I(\varphi) = \frac{2e}{\hbar}\frac{\partial E_g(\varphi)}{\partial \varphi}, \quad \text{where } E_g(\varphi) = \frac{1}{2}\sum_{n<0}\varepsilon_n(\varphi)$$

$$I_c^+ = \max_\varphi[I(\varphi)], \;\; I_c^- = \min_\varphi[I(\varphi)]$$



Critical current

$$I_c^+ = \max_\varphi[I(\varphi)],\ I_c^- = \min_\varphi[I(\varphi)]$$

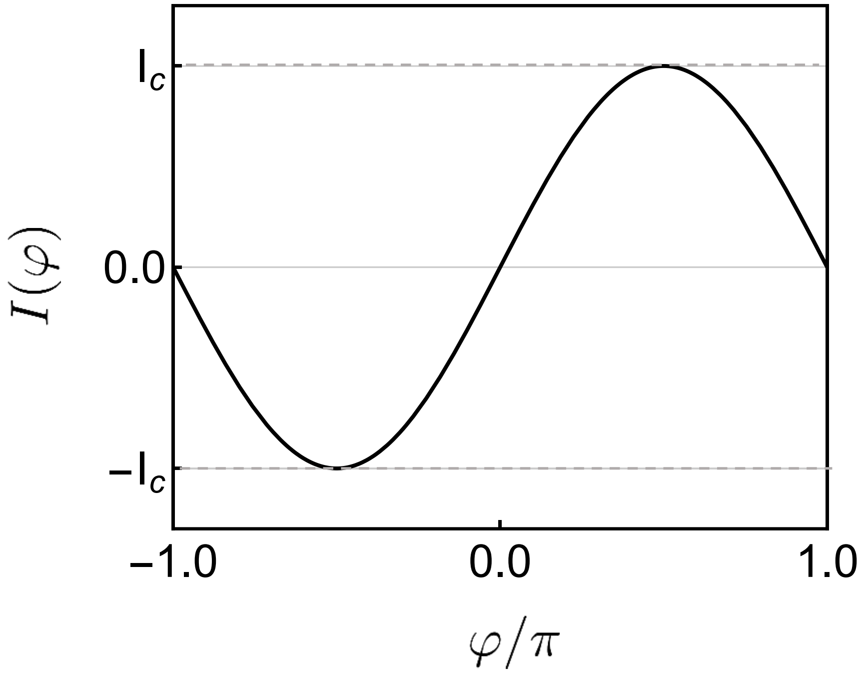
In the limit where $T \ll 1$,

$$E_g(\varphi) \sim -\frac{T|\Delta|}{4}\cos\varphi,$$

$$I(\varphi) \sim I_c \sin\varphi,$$

where $I_c = \dfrac{T|\Delta|}{4}$

$$I_c^+ = I_c,\ I_c^- = -I_c$$



Time-reversal symmetry guarantees $I(\varphi) = -I(-\varphi)$

Yokoyama, Eto, Nazarov, PRB 89, 195407 (2014)

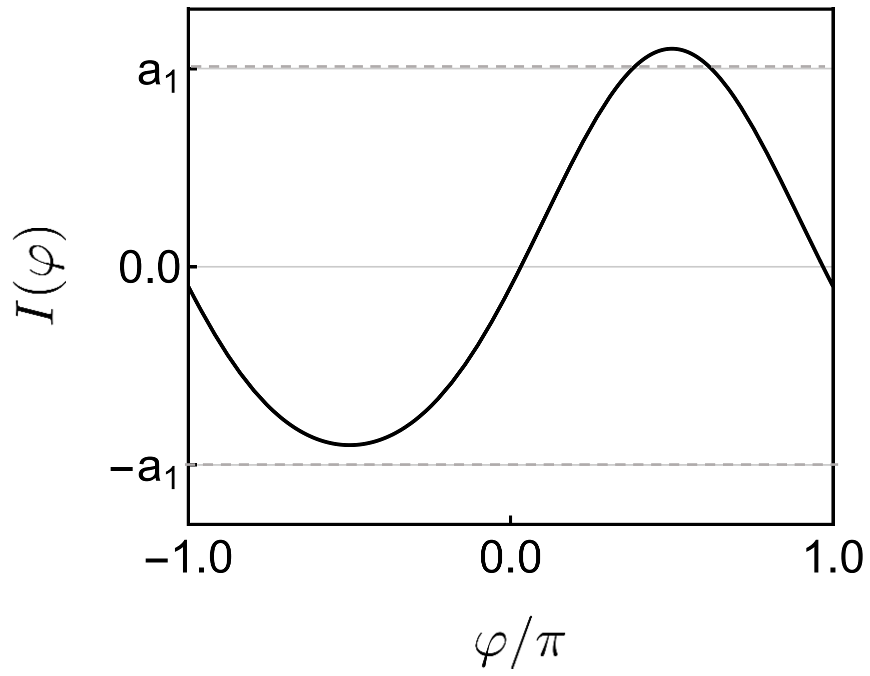
Minimal model for nonreciprocal critical current

$$I(\varphi) = a_1 \sin(\varphi) + a_2 \sin(2\varphi + \delta), \text{with } \delta \neq 0, \pi$$

In the limit where $|a_2| \ll |a_1|$,

$I_c^+ \approx a_1 - a_2 \sin\delta,$

$I_c^- \approx -a_1 - a_2 \sin\delta.$



$$\frac{a_2}{a_1} = 0.1,\ \delta = -0.5\pi$$

Pal et al., Nature Phys. 18, 1228 (2022)

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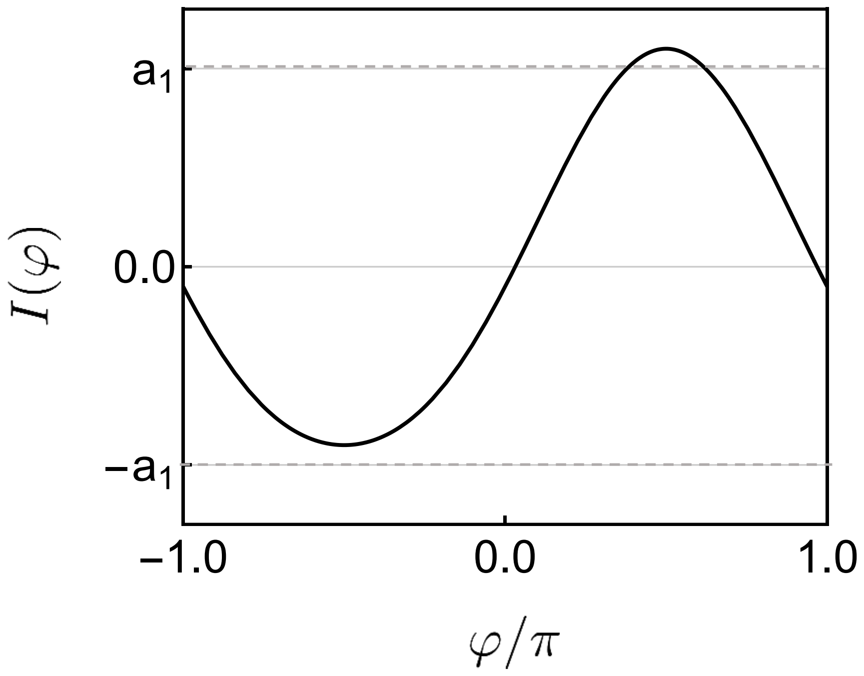


$$\frac{a_2}{a_1} = 0.1, \ \delta = -0.5\pi$$

How to realize such a nontrivial CPR in a real system?

Pal et al., Nature Phys. 18, 1228 (2022)

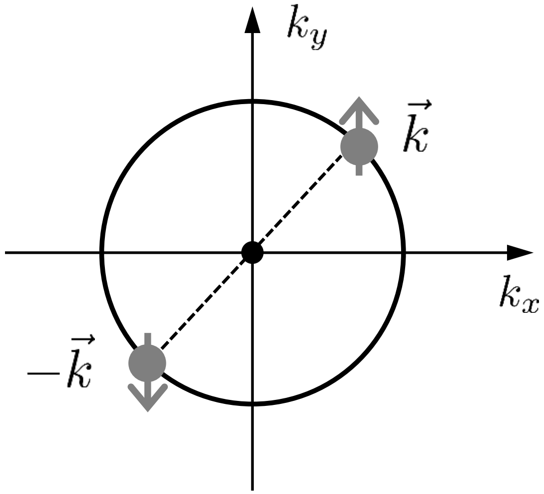
Part II. Josephson diode mechanisms in superconducting hybrid systems

- finite Cooper-pair momentum

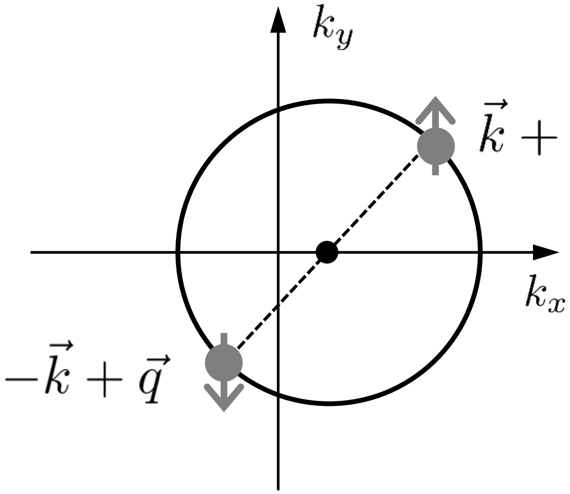
- spin-orbit coupling and Zeeman field

Finite Cooper-pair momentum

Zero momentum pairing

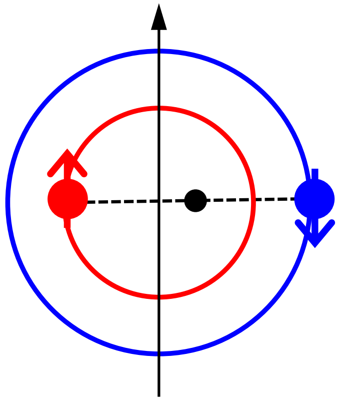


Finite momentum pairing $\Delta \to \Delta e^{i\vec{q}\cdot\vec{r}}$



Ginzburg-Landau current $\vec{J} \propto \nabla\varphi + \frac{2e}{\hbar}\vec{A}$

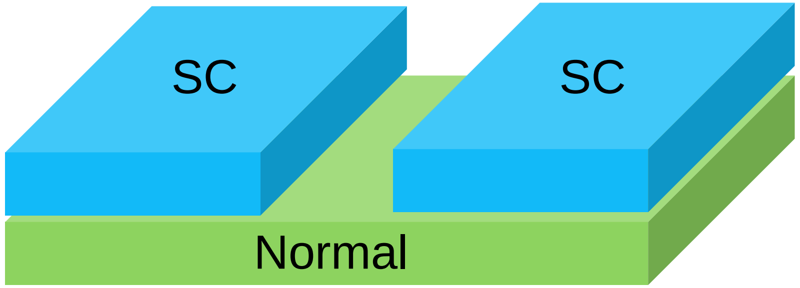
FFLO states



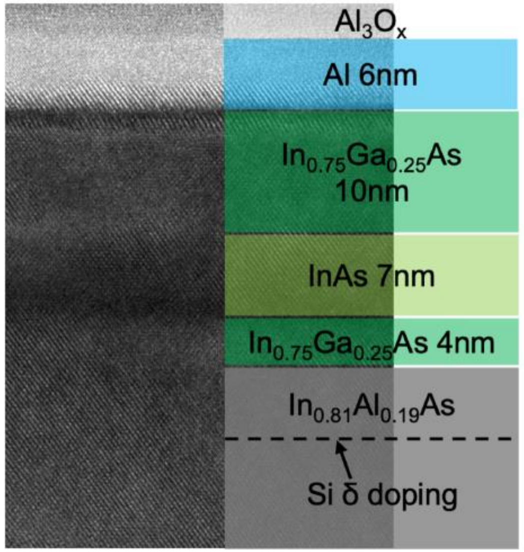
Fulde and Ferrell, Phys. Rev. 135, A550 (1964)

Superconducting Al-InAs hybrid system

Schematic of a planar Josephson junction



Cross section of the device

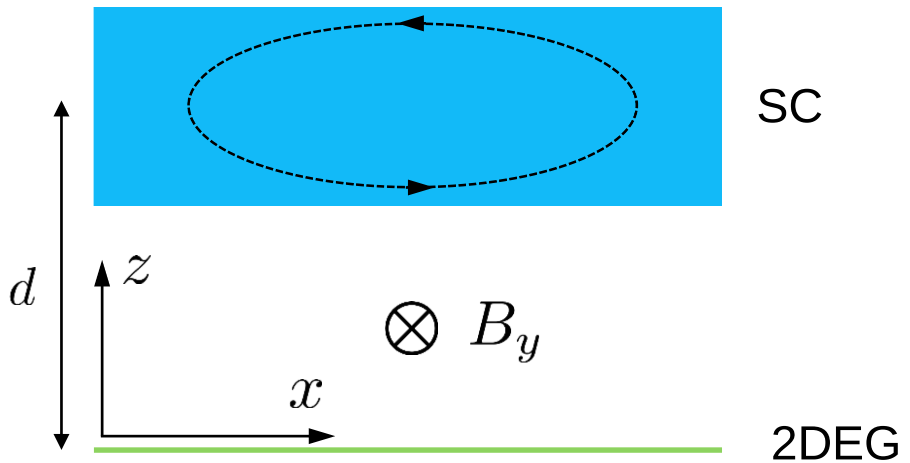


From conventional to topological and nonreciprocal Josephson currents:

Nature 569, 89 (2019)
Nat. Commnun. 11, 212 (2020)
Nat. Nanotechnol. 17, 39 (2022)
Nat. Nanotechnol. 18, 1266 (2023)
Phys. Rev. Lett. 131, 196301 (2023)
…

Orbital-induced finite Cooper-pair momentum

Meissner screening current $\vec{j}$



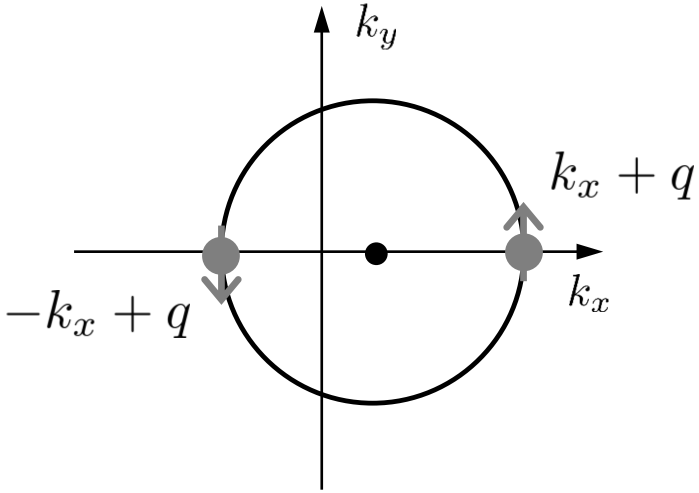
Using Ginzburg-Landau equation,

$$\vec{j} = -\frac{1}{\mu_0 \lambda^2}\left(\vec{A} + \frac{\Phi_0}{2\pi}\nabla\varphi\right)$$

$$\vec{A} = B_y z \hat{x}$$

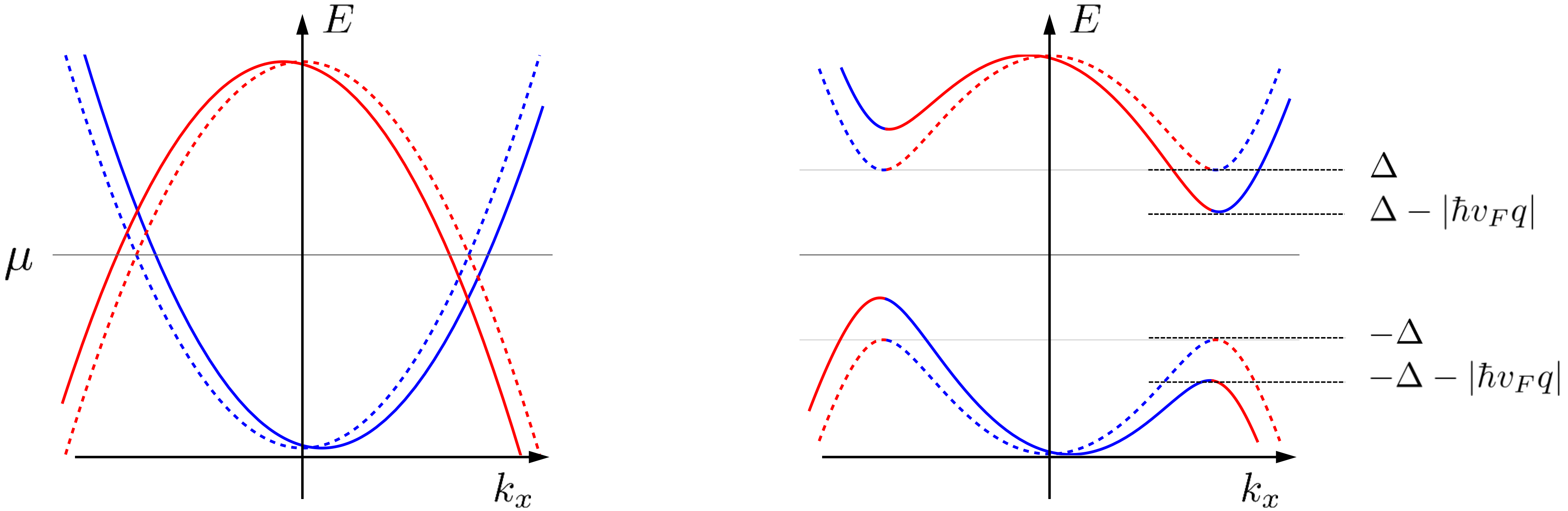
$$\Delta \to \Delta e^{i2qx}, \text{ where } q = -\frac{\pi B_y d}{\Phi_0}$$

Shift of the Fermi surface



Davydova, Prembabu, and Fu, Sci. Adv. 8, eabo0309 (2022)

Orbital-induced finite Cooper-pair momentum



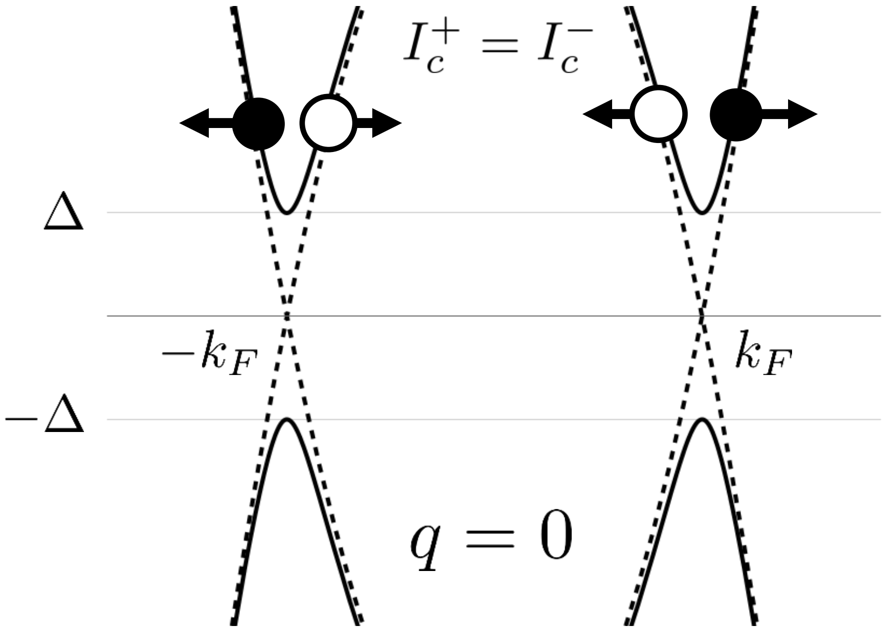
$$U = \begin{pmatrix} e^{-iqx} & 0 \\ 0 & e^{iqx} \end{pmatrix}$$

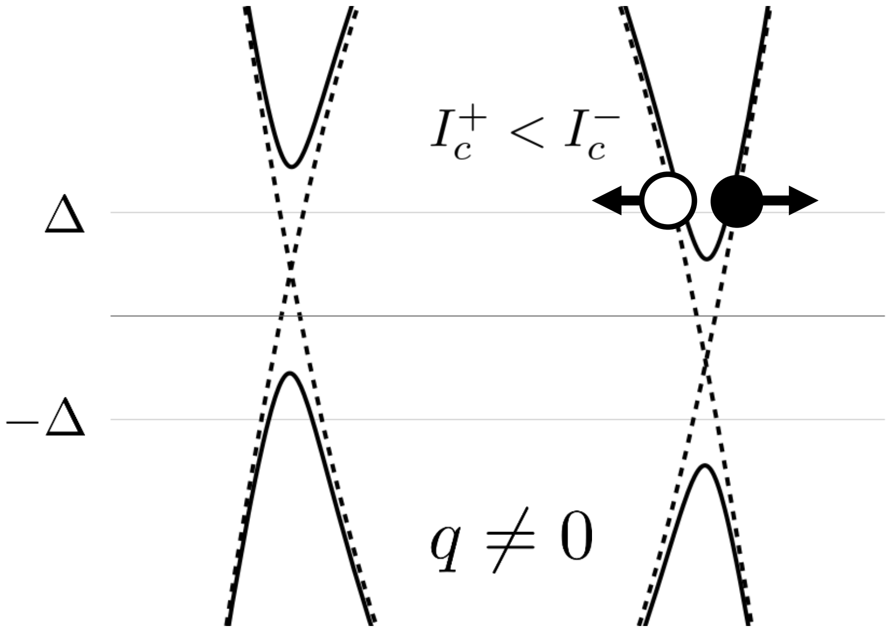
$$\begin{pmatrix} \frac{\hbar^2 k_x^2}{2m^*} - \mu & \Delta e^{i2qx} \\ \Delta e^{-i2qx} & -\frac{\hbar^2 k_x^2}{2m^*} + \mu \end{pmatrix} \Longrightarrow \begin{pmatrix} \frac{\hbar^2 (k_x + q)^2}{2m^*} - \mu & \Delta \\ \Delta & -\frac{\hbar^2 (k_x - q)^2}{2m^*} + \mu \end{pmatrix}$$

Davydova, Prembabu, and Fu, Sci. Adv. 8, eabo0309 (2022)

Diode effect by the finite Cooper pair momentum

Doppler shift: $\Delta \to \Delta \pm \hbar v_F q$





$$I(\varphi) = -\frac{e}{\hbar}\int_0^\infty dE\, E\, \frac{\partial}{\partial\varphi}\rho(E,\varphi)$$

$T = 1$ case:

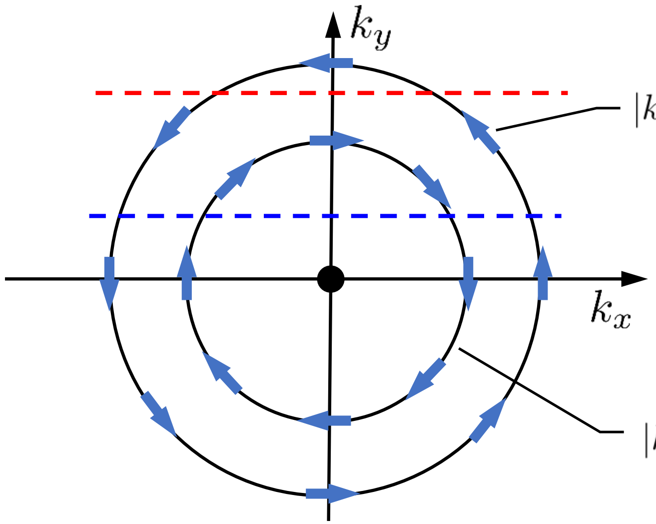
$$\Delta I_c = \mathrm{sgn}(q)\left(\frac{4e\,|\,q\,|\,v_F}{\pi\hbar} - \frac{e\Delta}{\hbar}\left[1 - \sqrt{1 - \left(\frac{qv_F}{\Delta}\right)^2}\,\right]\right)$$

Davydova, Prembabu, and Fu, Sci. Adv. 8, eabo0309 (2022)

Rashba spin-orbit coupling and Zeeman field

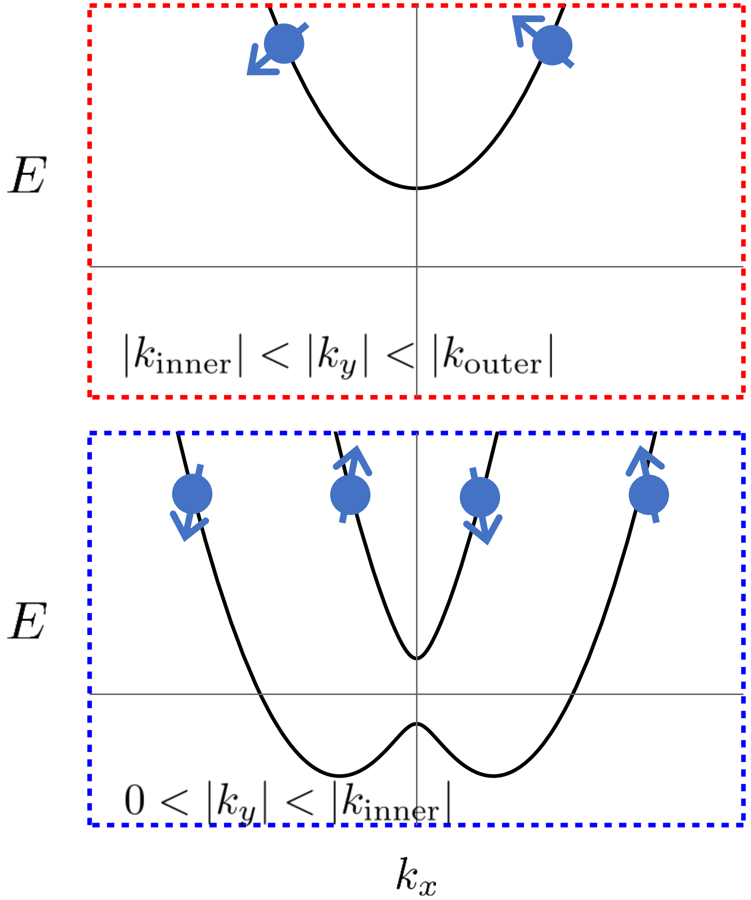
Rashba induced spin-split Fermi surface

$$H = \frac{\hbar^2 (k_x^2 + k_y^2)}{2m^*} + \alpha_R \left(-k_x \sigma_y + k_y \sigma_x\right)$$



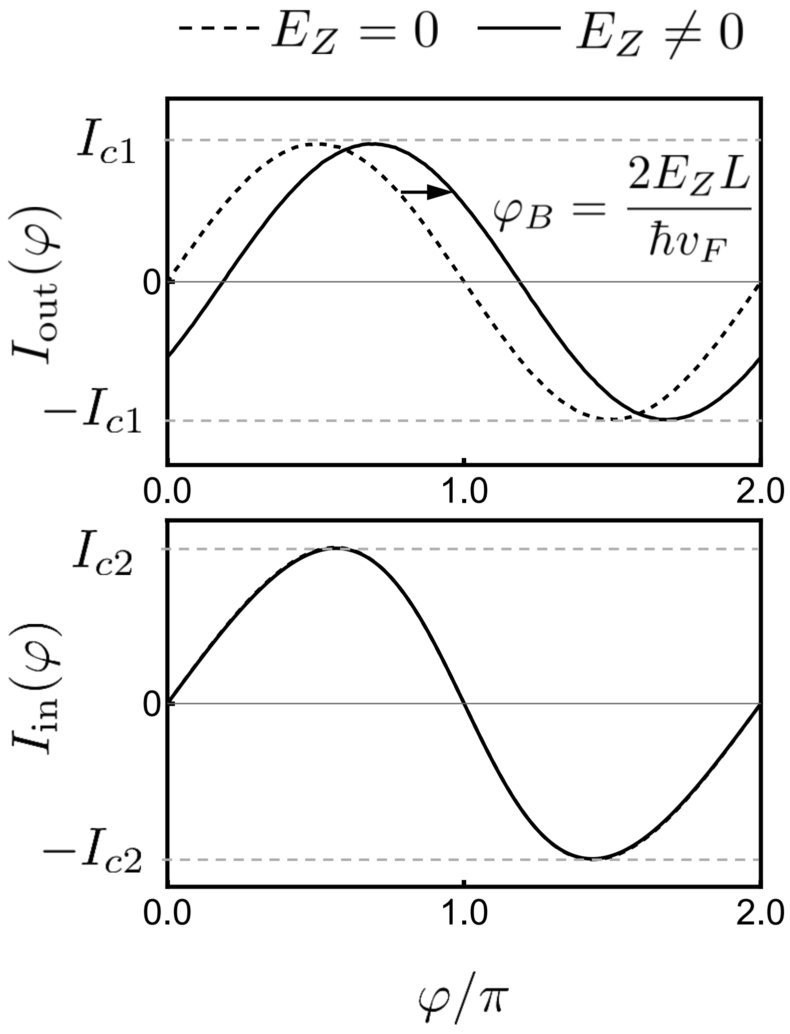
$$|k_{\mathrm{outer}}| = \sqrt{\frac{2m^*\mu}{\hbar^2} + \frac{m^{*2}\alpha_R^2}{\hbar^2}} + \frac{m^*\alpha_R}{\hbar}$$

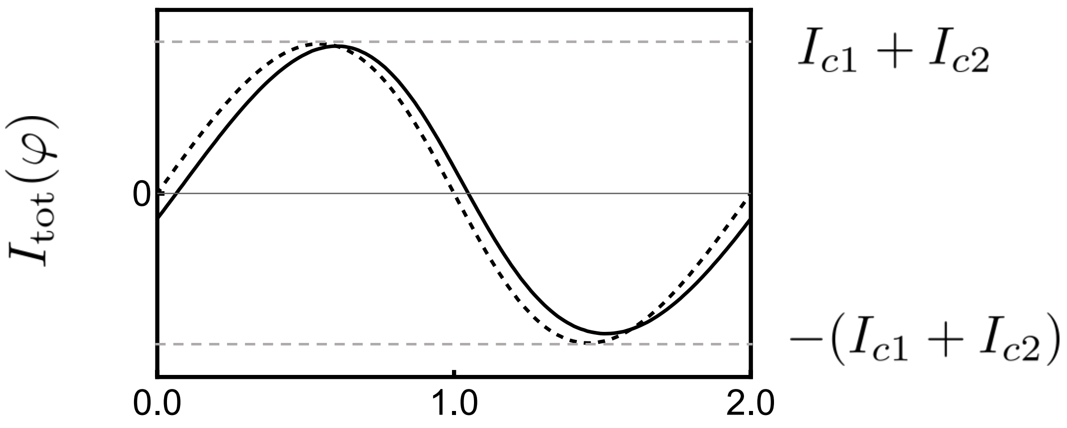
$$|k_{\mathrm{inner}}| = \sqrt{\frac{2m^*\mu}{\hbar^2} + \frac{m^{*2}\alpha_R^2}{\hbar^2}} - \frac{m^*\alpha_R}{\hbar}$$



Rashba, Sov. Phys. Solid State 2, 1109 (1960)
Dercioux and Lucignano, Rep. Prog. Phys. 78, 106001 (2015)

Rashba spin-orbit coupling and Zeeman field





$$I_{\text{tot}}(\varphi) = I_{\text{out}}(\varphi) + I_{\text{in}}(\varphi)$$

In the tunneling limit $T \ll 1$,

$$I_{\text{out}}(\varphi) \approx I_{c1} \sin(\varphi - \varphi_B),$$

$$I_{\text{in}}(\varphi) \approx I_{c2} \sin(\varphi).$$

Yokkoyama, Eto, and Nazarov, PRB 89, 195407 (2014)

Part III. Overview of recent studies on sign reversal of the Josephson diode effect

- C. Strunk group at Regensburg

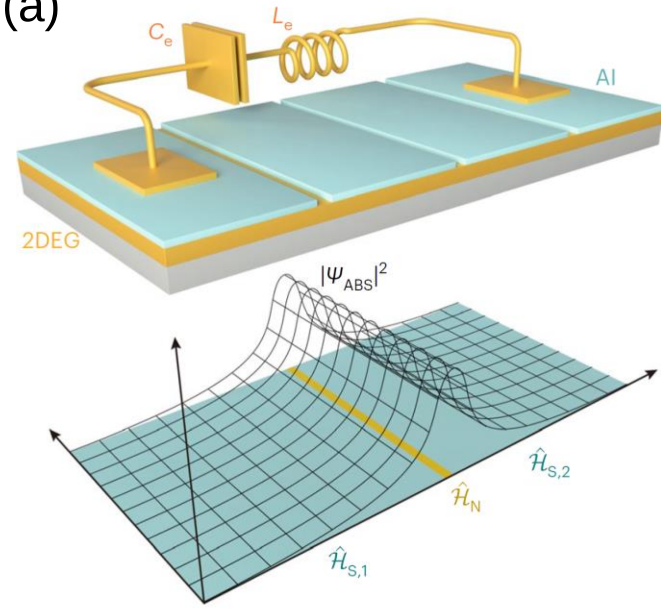
- C. Marcus group at Copenhagen

- J. Suh group at Postech

Experiments by the C. Strunk group

Schematic of the device

(a)

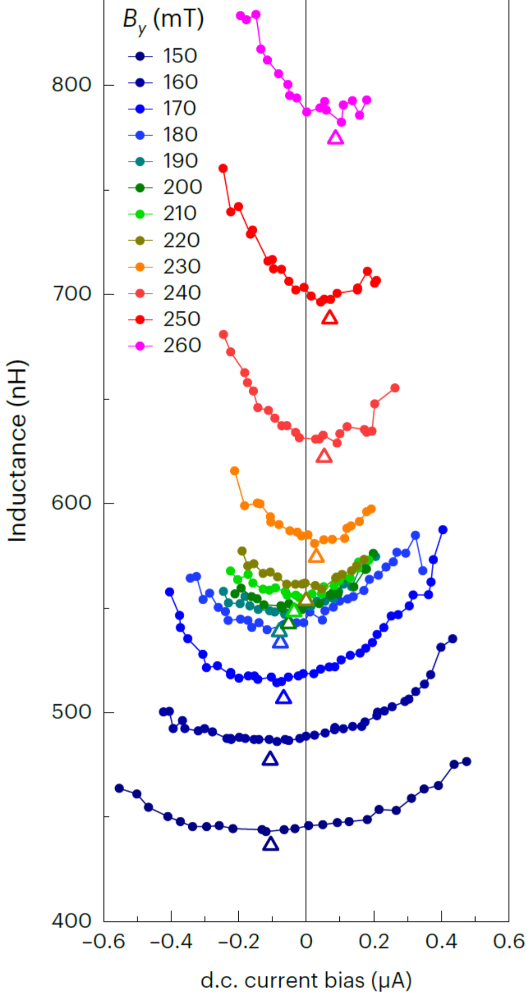


$$L = \frac{\hbar}{2e}\frac{d\varphi}{dI} \approx L_0 + L_0' I$$

Nat. Nanotechnol. 17, 39 (2022)
Nat. Nanotechnol. 18, 1266 (2023)

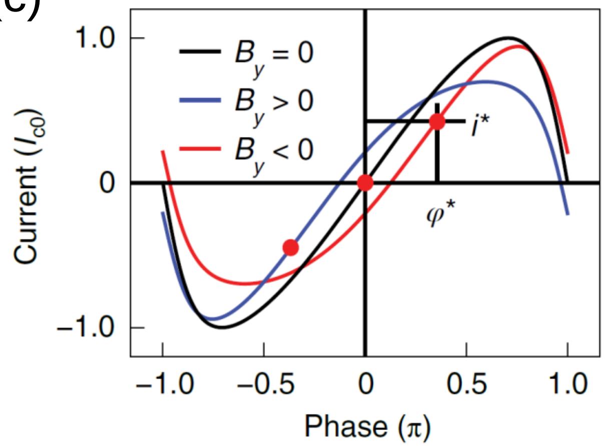
Josephson inductance

(b)

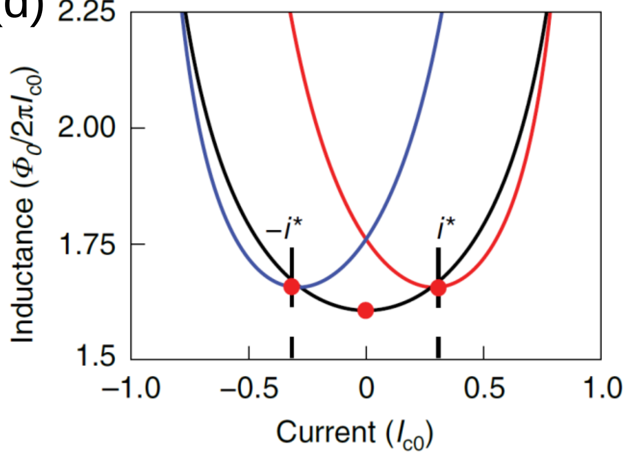


Inflection point of the CPR

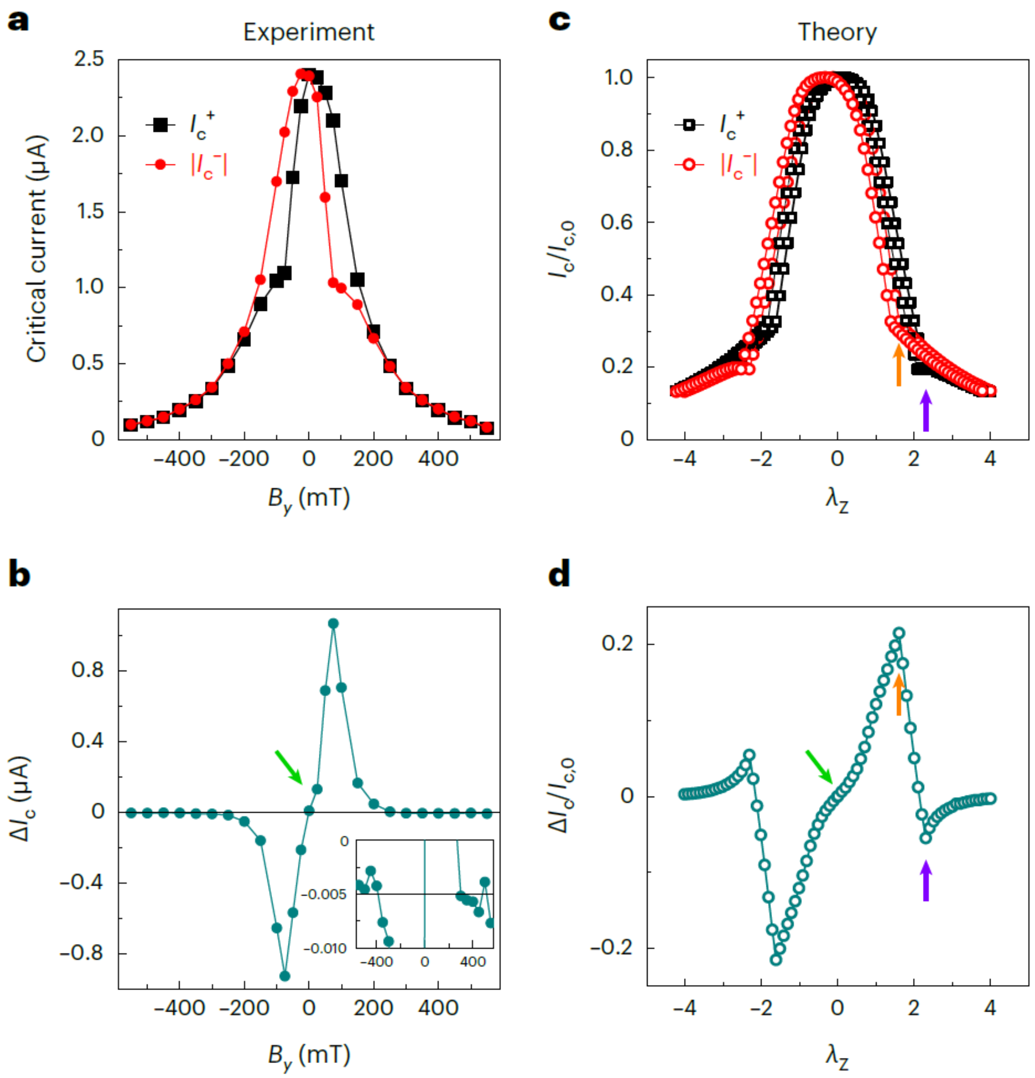
(c)

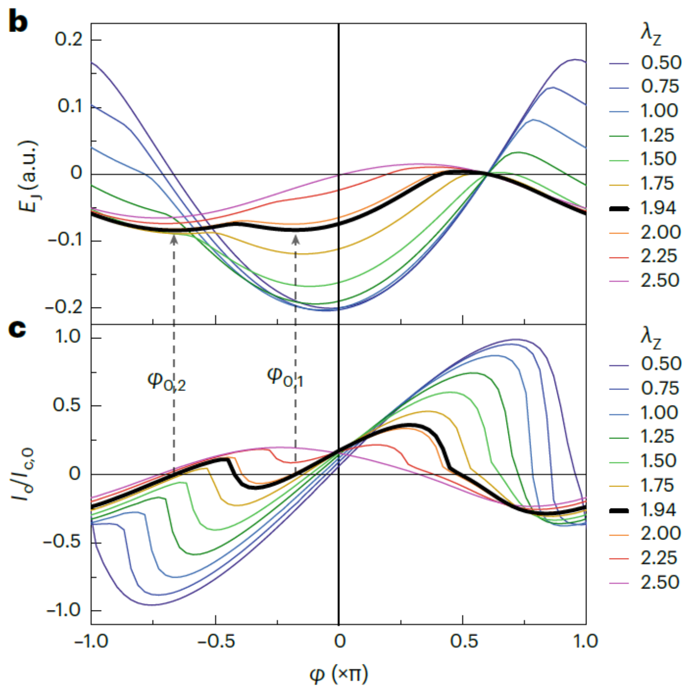


(d)



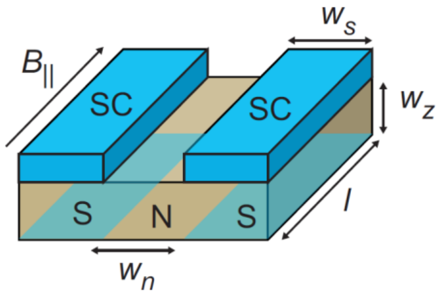
Experiments by the C. Strunk group

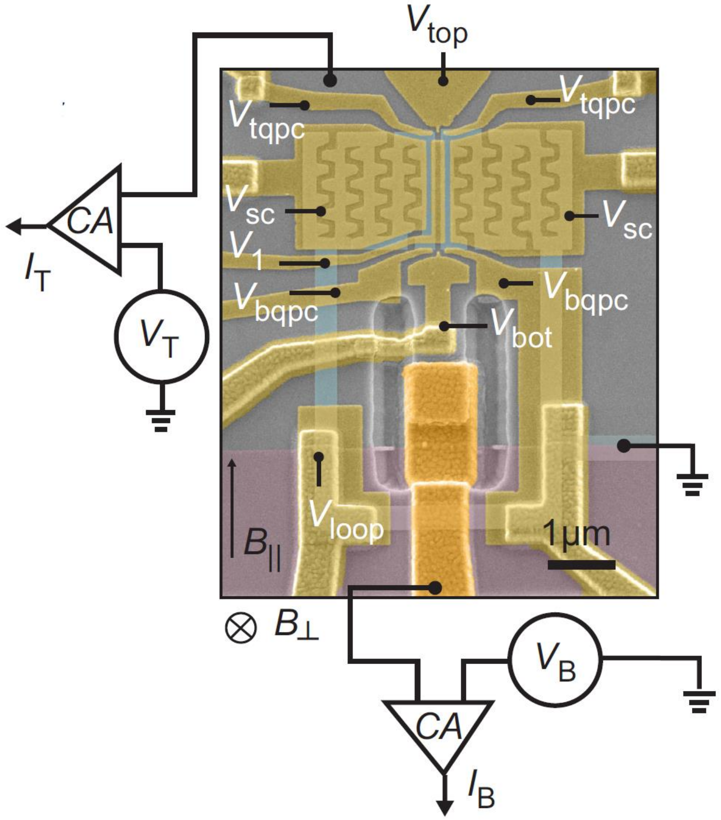




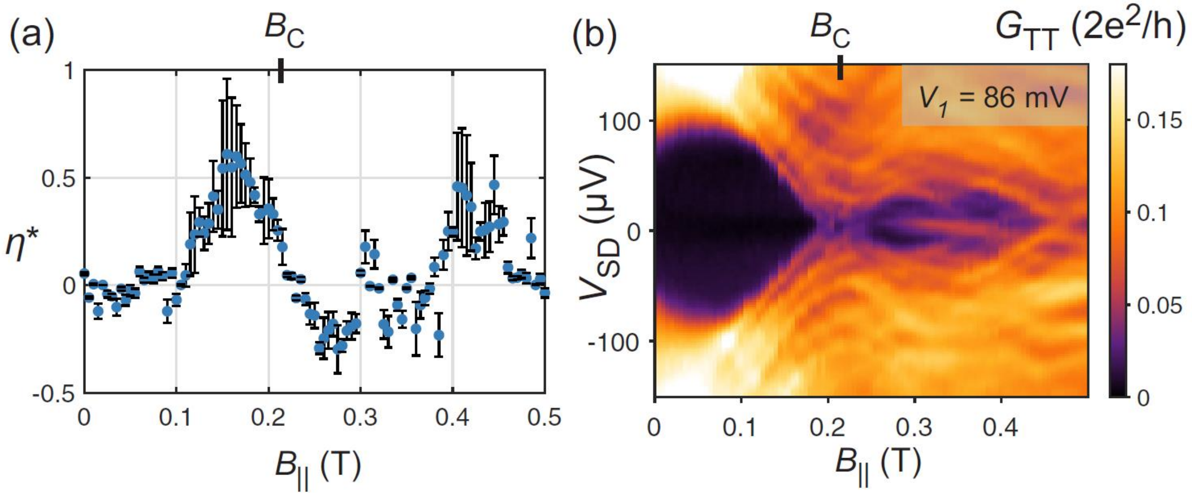
- Theory model with α_R and E_Z
- Sign reversal linked to the 0-π transition
- $\lambda_z = \dfrac{2m^* E_Z L}{\hbar^2 k_F}$ and $\lambda_{\rm SOI} = \dfrac{m^* \alpha}{\hbar^2 k_F} = 0.66$

Experiments by the C. Marcus group





Tunneling spectroscopy



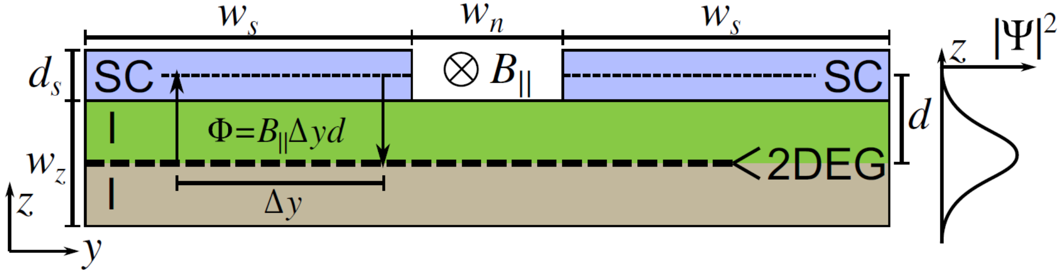
The sign reversal is correlated with a topological phase transition

Phys. Rev. Lett. 131, 196301 (2023)

Experiments by the C. Marcus group

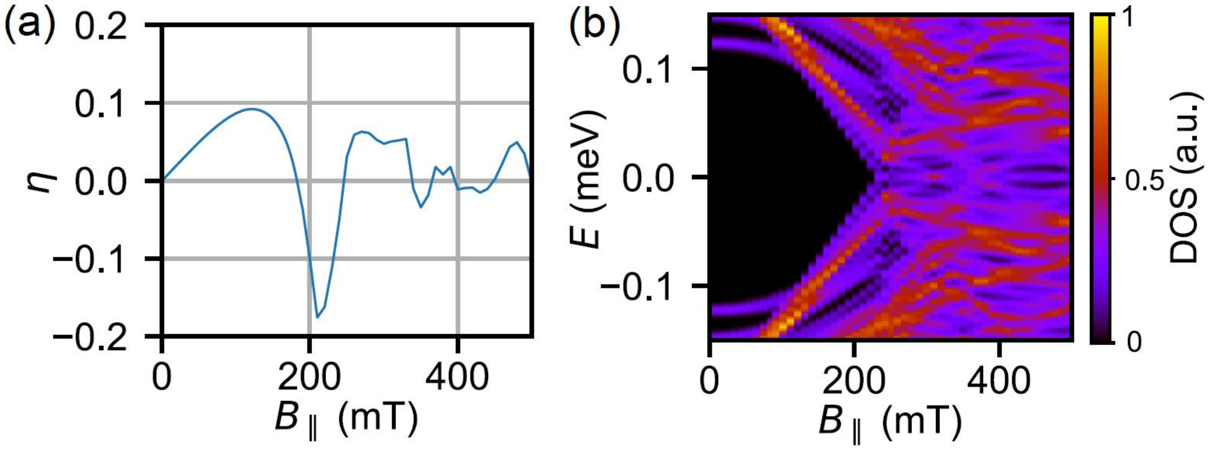
Tight-binding calculation incldues orbital-induced finite Cooper-pair momentum q and Rashba SOC α_R + Zeeman field E_Z

Cross section of the model



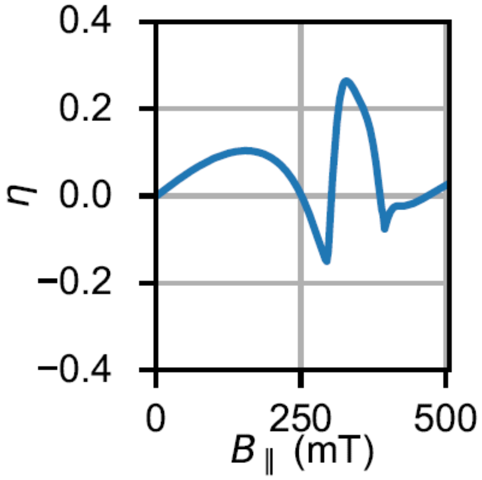
Best agreement

$\alpha = 15$ meVnm
$g = -10$
$d = 15$ nm
$w_s = 400$ nm



SC lead width effect

$\alpha = 0$ meVnm
$g = 0$
$d = 15$ nm
$w_s = 400$ nm

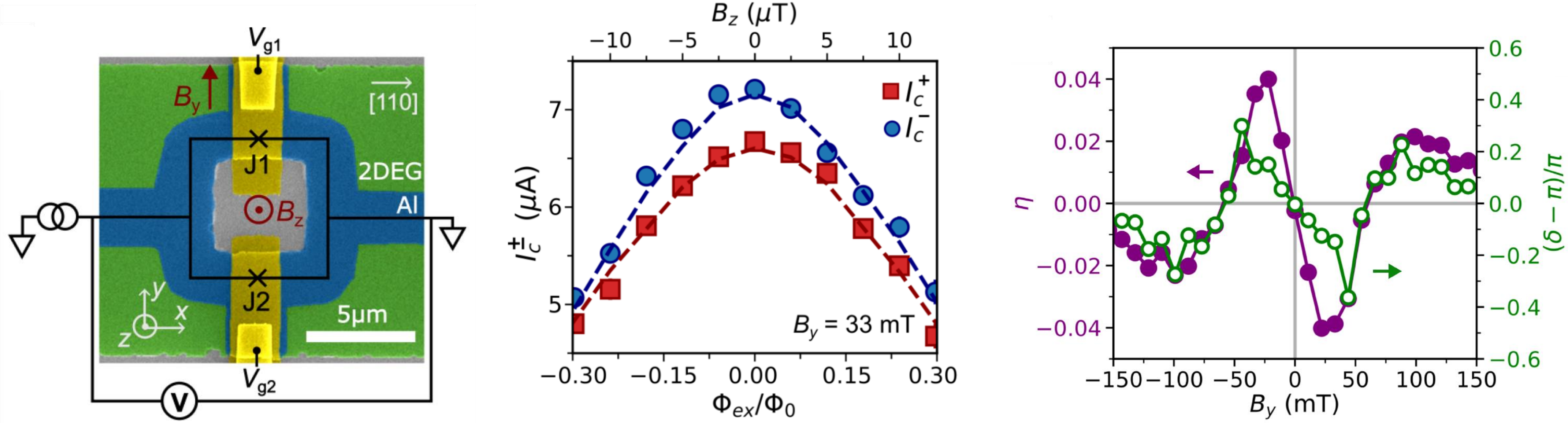


Phys. Rev. Lett. 131, 196301 (2023)

Experiments by the J. Suh group

POSTECH

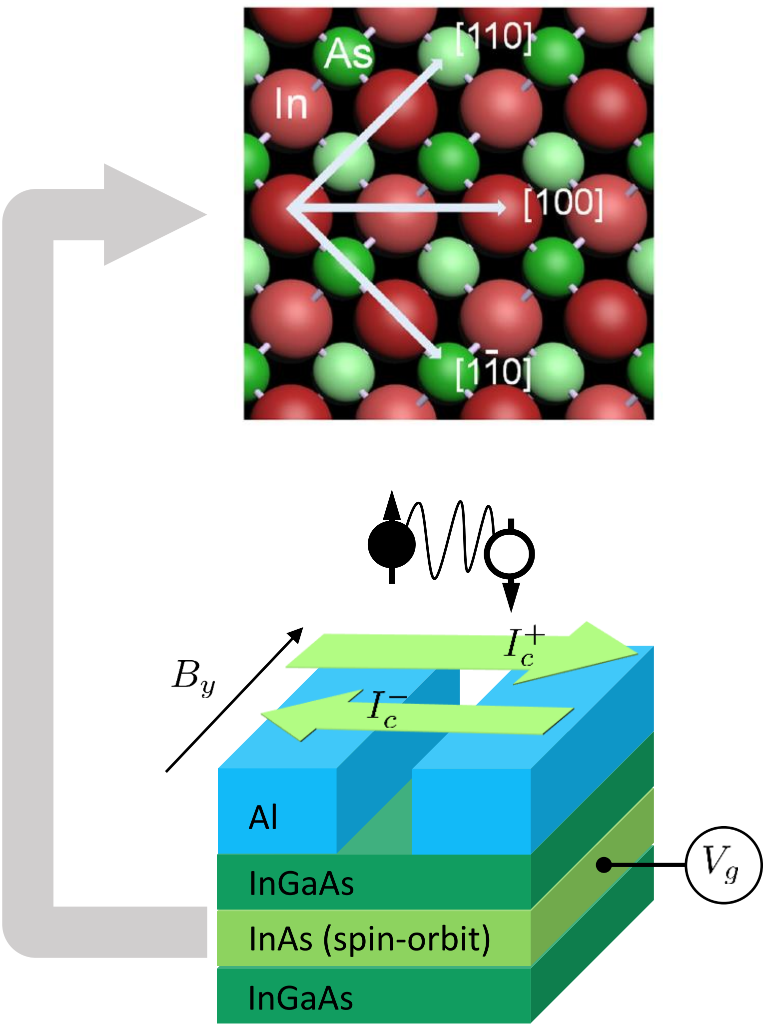
Dr. Junghyun Shin
Prof. Junho Suh

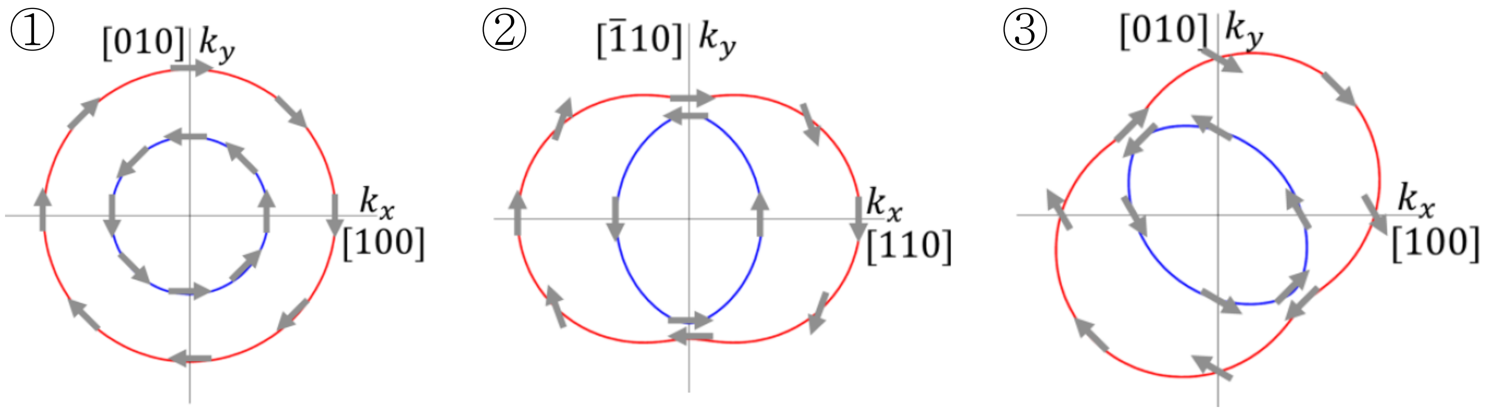


- Gate-tunable symmetric SQUID
- The diode efficiency η changes its sign at $B_y \approx 36\text{mT}$ when $V_g = 0$
- The anomalous phase difference δ is extracted by fitting the data with $a_1 \sin(\varphi) + a_2 \sin(2\varphi + \delta)$
- The polarity reversal of η coincides with $\delta = \pi$

arXiv:2409.17820 (2024)

Experiments by the J. Suh group

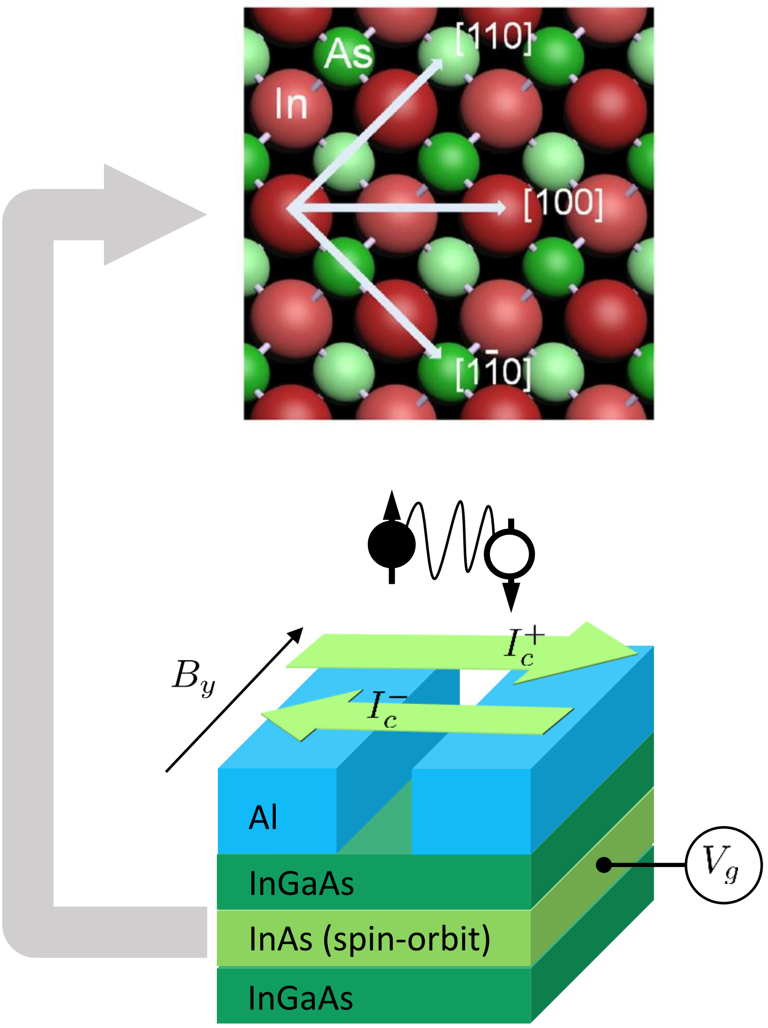


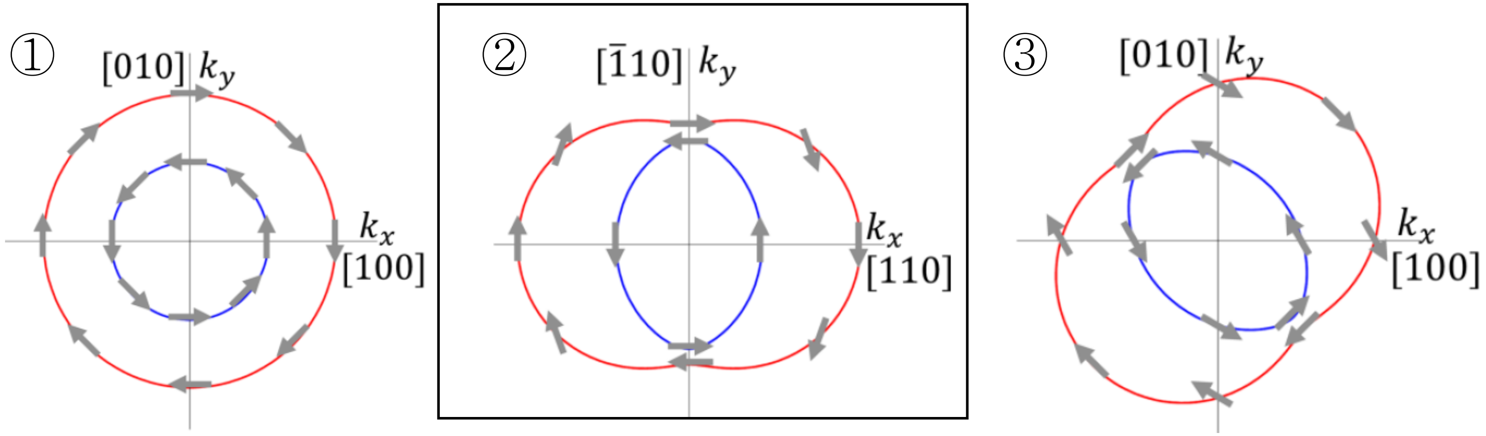


Rashba SOC

Rashba + Dresselhaus SOC

Experiments by the J. Suh group





Rashba SOC

Rashba + Dresselhaus SOC

Anisotropic SOC

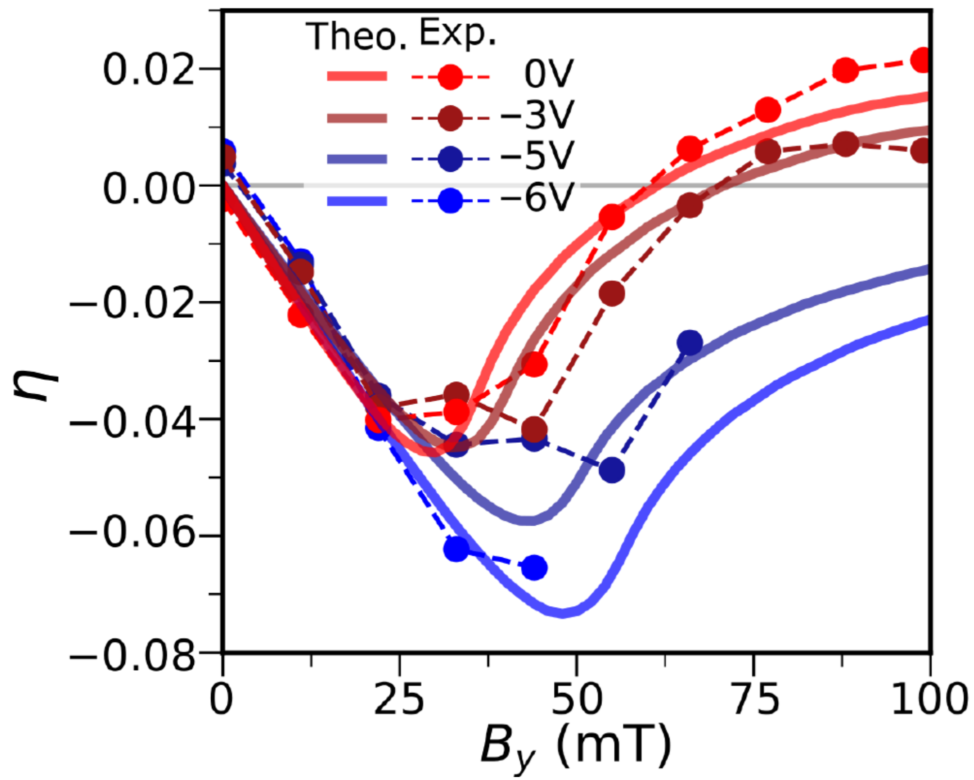
$$H_{\mathrm{SOC}} = -(\alpha_R + \alpha_D)k_x\sigma_y + (\alpha_R - \alpha_D)k_y\sigma_x$$

J. Phys: Condens. Matter 34, 154005 (2022)

Experiments by the J. Suh group

- Electrically controllable Josephson diode polarity
- Theory model with α_R, α_D and E_Z
- At low fields, finite Cooper-pair momentum mechanism dominates
- At high fields, the ratio between α_R and α_D determines the diode efficiency

$$\alpha_R = 5.08(-6V) - 7.53(0V) \text{ meV nm}$$
$$\alpha_D = 4.23 \text{ meV nm}$$
$$q = 1.42 \times 10^{-5} B_y \text{ mT}^{-1}\text{nm}^{-1}$$



arXiv:2409.17820 (2024)

Summary

- The Josephson diode effect can occur by breaking time-reversal and inversion symmetries in Al -InAs heterostructure.
- Mechanism 1. Orbital-induced finite Cooper-pair momentum
- Mechanism 2. Spin-orbit coupling (Rashba and Dresselhaus) and Zeeman field
- The diode polarity is controllable via magnetic or electric fields