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14th APCTP-BLTP JINR Joint Workshop
- Memorial Workshop in Honor of Prof. Yongseok Oh :
Modern problems in nuclear and elementary particle physics

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Cosmological constraints of Dark Matter in the Extended Gravity

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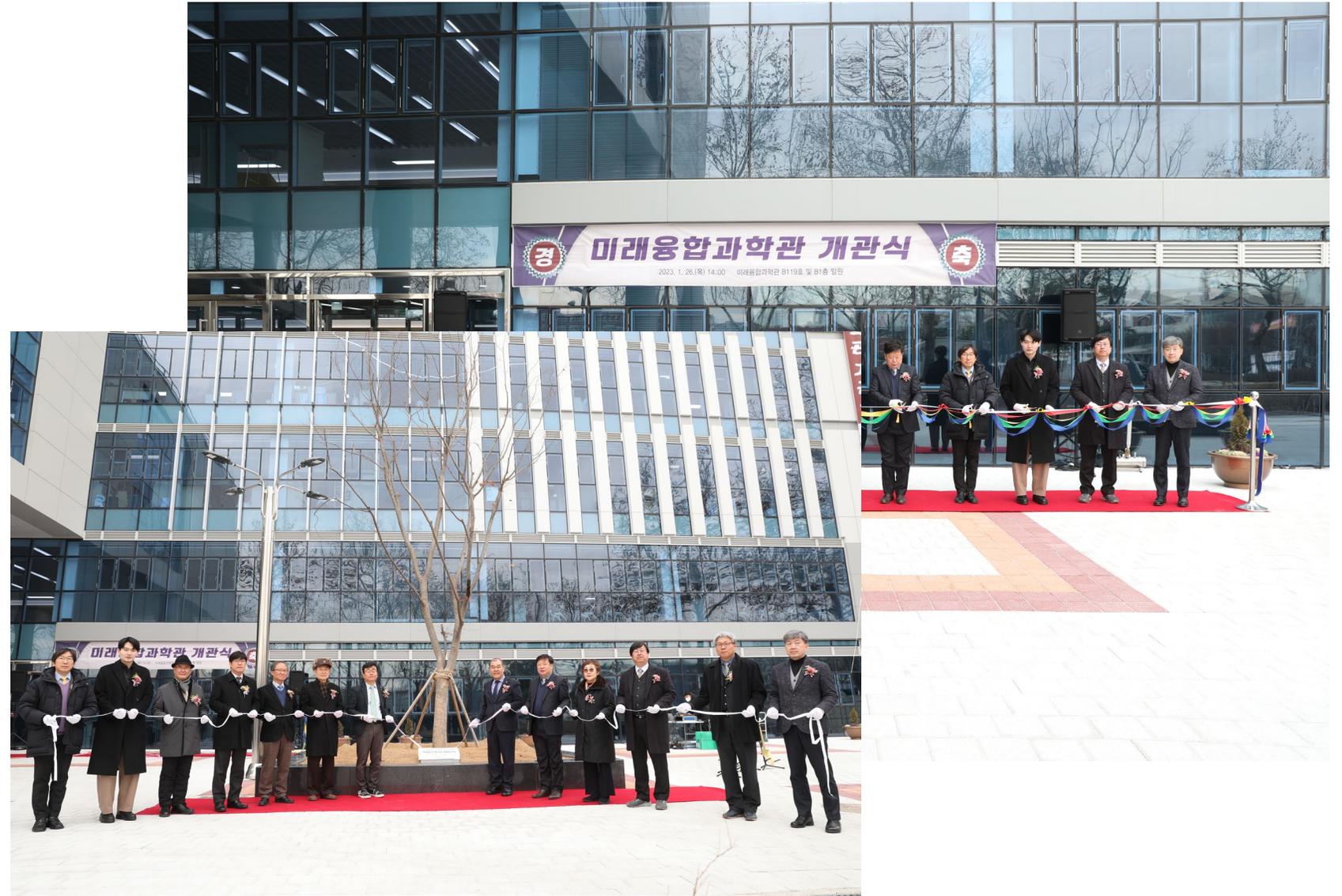
A. Biswas, A. Kar,
BHL, H. Lee, W. Lee,
S. Scopel, L. Yin
arXiv:2303.05813



I deeply mourn the late Professor Yongseok Oh.

경북대

각종 보직
초청-컬로퀴엄 등



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- BLTP 등과의 국제협력
 - 코오디네이터
- 정부 및 연구기관
KNO etc.



I express my deepest condolences
to the late Professor Yongseok Oh.



1. Extended Gravity - Motivation

Q : Is Extended Gravity beyond Einstein needed?

I. Theoretical Aspect

- GR is an **effective theory** valid below some ultraviolet cut-off, $M_{Pl} \sim 10^{19} GeV$
The String theory at low energy \rightarrow Einstein Gravity + higher curvature terms
- Standard Model of Cosmology (Λ CDM) : Is it satisfactory?
extremely fine-tuned ($\Lambda = 2,888 \times 10^{-122} \ell_P^{-2}$)
- Holography :
(asymptotic) AdS Black Hole in $d+1$ dim.
 \leftrightarrow Quantum System in d dim.

Holographic QCD

(asymptotic AdS Space)



QCD

II. Observational Aspect

1) testing gravity ex) gravitational waves

In the “long” distance or low energy scale, Einstein Grav is good. How about in other scales?

2) Cosmology requires new physics (beyond the Standard Model of particles & Λ CDM):

- What is the fundamental physics behind DM and DE? (Accelerating Expansion & LSS)

Note) Particle & Nuclear Phys vs cosmology & astrophysics

Ex) # of (light) ν -families

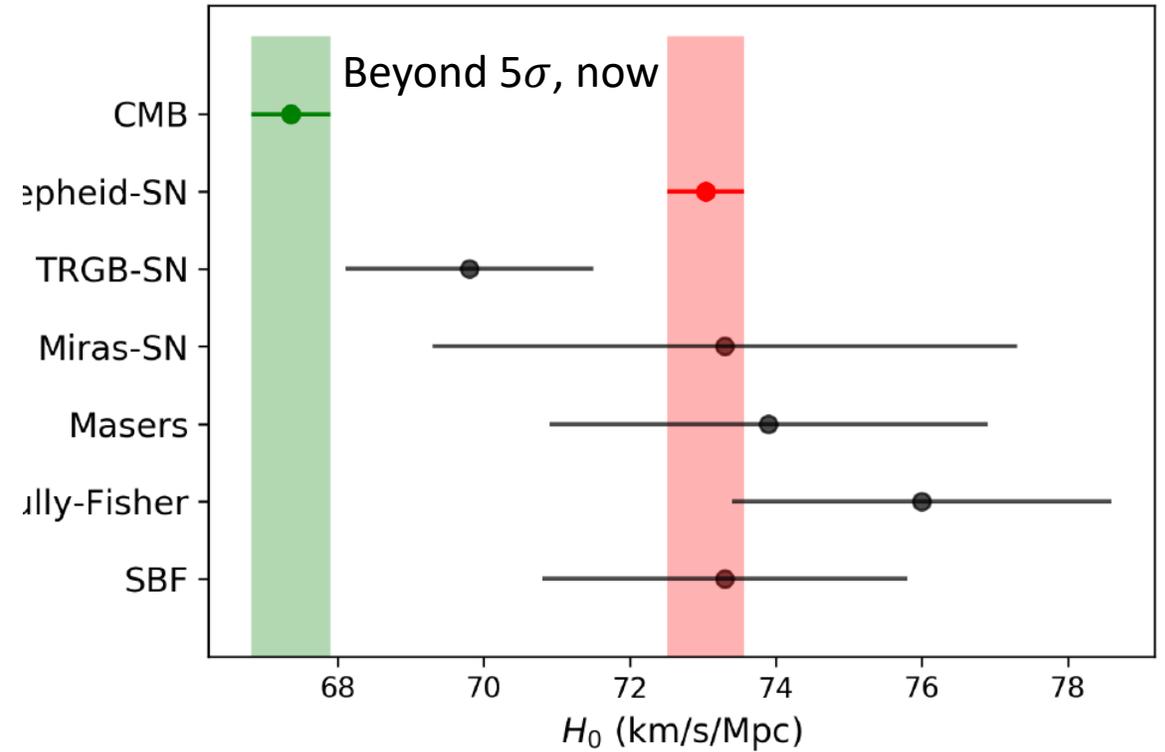
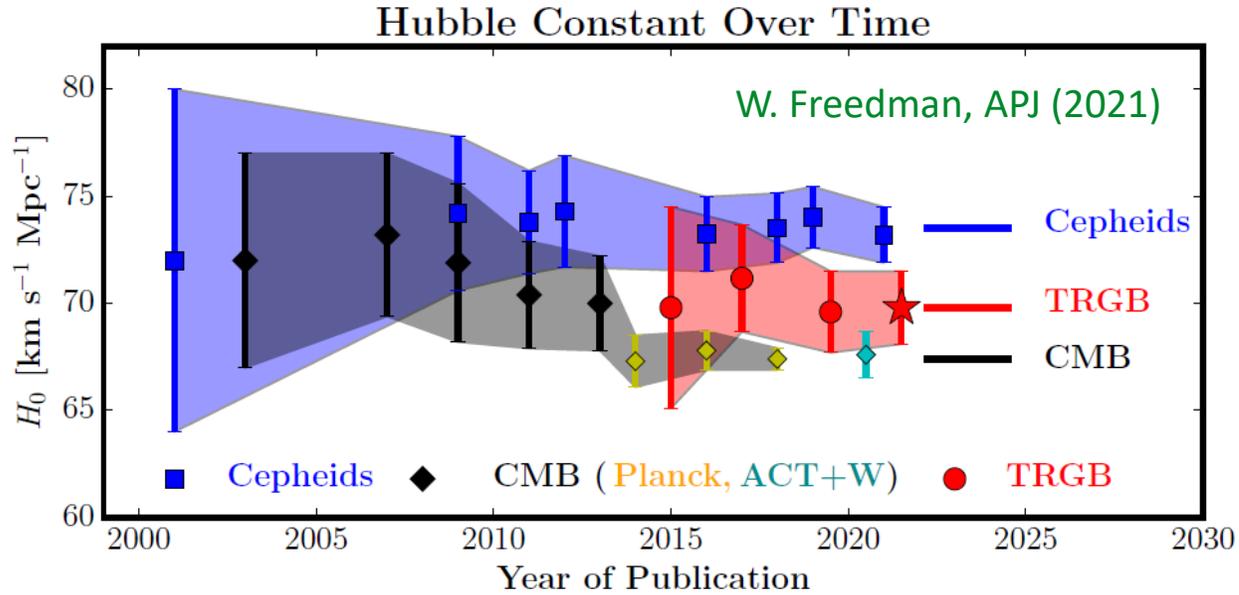
Nuclear Physics and BBN, Neutron Stars, etc.

Particle Physics (WIMP etc) in the early universe

(*) Observational Cosmic Tension

$$H_0 = 67.4 \pm 0.5 \text{ km/s/Mpc (CMB)}$$

$$= 73.5 \pm 1.4 \text{ km/s/Mpc (SN \& Cepheids)}$$



Q : Is Modified Gravity beyond Einstein better working? We have been investigating through

- 1) the Black Hole properties &**
- 2) the implication to the cosmology.**

1) Effects to the Black Holes - New Properties
 Hair, minimum mass, (in)stability, GW etc.

Soliton Star?

Black Holes



$M=0$

M_{\min}

$\rightarrow \infty$

2) Cosmological effects in the Early Universe

during the inflation, reheating period, Radiation Dominating period etc.

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5. Summary

2. DEGB Cosmology

2-1) Horndeski Theory

2-2) Standard Cosmology (Λ CDM Model)

2-3) DEGB cosmology

2-1) Horndeski Theory - the most general scalar-tensor theory
w/ 2nd-order field eqn in 4 dim. -

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\square\phi + G_4(\phi, X)R + G_{4X}[(\square\phi)^2 - \phi_{\mu\nu}\phi^{\nu\mu}] \\ + G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} - \frac{G_{5X}}{6}[(\square\phi)^3 - 3\square\phi\phi_{\mu\nu}\phi^{\nu\mu} + 2\phi_{\mu\nu}\phi^{\nu\lambda}\phi_{\lambda}{}^{\mu}]$$

Note : Horndeski theory is classified by 4 arbitrary functions
{ $G_i(\phi, X)$, $i = 2,3,4,5$ }.

Examples: ($G_5 = 0$)

(i) Einstein Gravity is obtained by taking $G_4 = \frac{M_P^2}{2}$ (other $G_i = 0$)

$$S = \int d^4x \sqrt{-g} \frac{M_P^2}{2} R \quad \text{Linear in curvature scalar}$$

(ii) Brans-Dicke/ $f(R)$ gravity by taking $G_2(\phi, X)$, $G_3 = 0$, $G_4 = f(\phi)$

$$S = \int d^4x \sqrt{-g} [G_2(\phi, X) + f(\phi)R]$$

(*) Nonminimally coupled Gravity by $G_4 = f(\phi)$ (other $G_i = 0$)

$$S = \int d^4x \sqrt{-g} f(\phi)R$$

(iii) k-inflation/k-essence by keeping $G_2(\phi, X)$, $G_3 = 0$, $G_4 = \frac{M_P^2}{2}$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R + G_2(\phi, X) \right] \quad \text{(including } f(R) \text{ gravity)}$$

(*) Quintessence by keeping $G_2(\phi, X) = X - V(\phi)$, $G_3 = 0$, $G_4 = \frac{M_P^2}{2}$

higher derivative theories
may have ghosts and
Ostrogradsky instability :

$$G_i(\phi, X) = \sum C_i^{\ell m} \phi^\ell X^m$$

∞ - number of parameters

Horndeski, *Int. J. Theor. Phys.*

10 363-84 (1974)

Charmousis, Copeland, Padilla &

Saffin *Phys. Rev. Lett.* **108**

051101 (2012)

(iv) kinetic gravity braiding(KGB)/G-inflation by taking $G_2(\phi, X), G_3(\phi, X), G_4 = \frac{M_P^2}{2}$

$$\mathcal{L} = \frac{M_P^2}{2} R + G_2(\phi, X) + G_3(\phi, X) \square \phi$$

(v) Nonstandard kinetic term $G^{\mu\nu} \phi_{,\mu\nu}$ obtained by taking $G_5 \propto \phi$

$$S = \lambda \int d^4x \sqrt{-g} G^{\mu\nu} \phi_{,\mu\nu}$$

(vi) Gauss-Bonnet Term

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} \xi(\phi) R_{GB}^2 \quad \text{where} \quad R_{GB}^2 = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

can be shown to be realized (at the level of the e.o.m) by

$$\xi^{(n)} = \partial^n \xi / \partial \phi^n$$

$$G_2 = 8\xi^{(4)}X^2(3 - \ln X) \quad G_3 = 4\xi^{(3)}X(7 - 3\ln X) \quad G_4 = 4\xi^{(2)}X(2 - \ln X) \quad G_5 = -4\xi^{(1)} \ln X$$

Model in this talk : the Dilaton-Einstein-Gauss-Bonnet (DEGB) Gravity

$$S_{DEGB} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R + \mathcal{L}_m^{rad} + f(\phi) R_{GB}^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

$$V(\phi) = 0$$

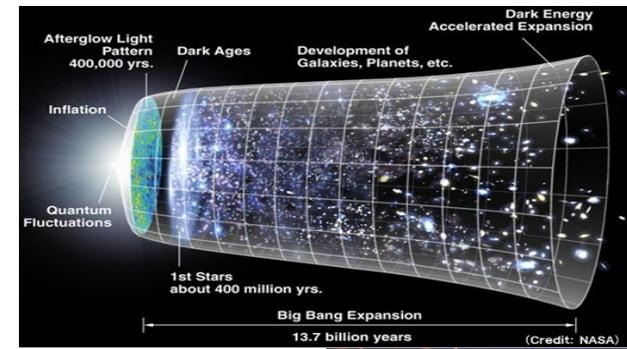
$f(\phi) = \alpha e^{\gamma\phi}$: Coupling of the Gauss-Bonnet term

2-2) Standard Cosmology (Λ CDM Model)

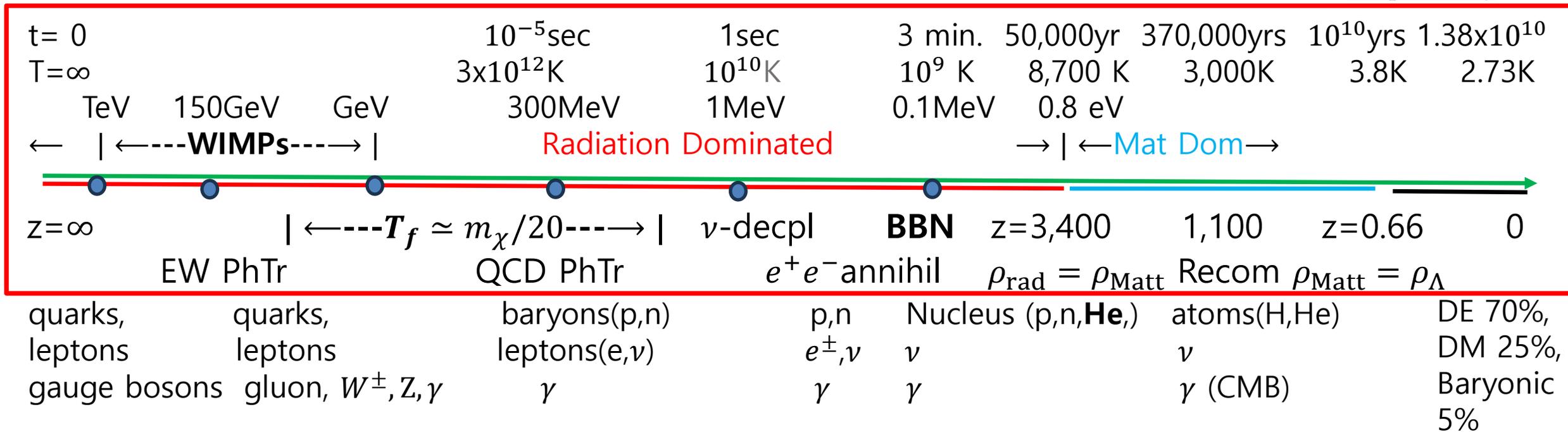
Gravity : Einstein Gravity
 Matter : Standard Model
 + (C)DM
 + DE

- An action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R + \sum_{m=StdMod} \mathcal{L}_m + \mathcal{L}_{CDM} - \frac{1}{\kappa} \Lambda \right]$$



<wikipedia.org>



Big Bang Nucleosynthesis (BBN)

BBN ($T_{BBN} \approx 1 \text{ MeV}$) strongly constrains any departure from Standard Cosmology.

(the earliest process in Cosmology providing a successful confirmation of both GR and the SM)

All events that take place at $T > T_{BBN}$ can be used to shed light on physics beyond GR and the SM.

Goal : Constrain the **Modified Gravity (dEGB)** based on the physics of **WIMPs decoupling**

2-3) DEGB cosmology

A. Biswas, A. Kar, **BHL**, H. Lee, W. Lee,
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- An action of the Dilaton-Einstein-Gauss-Bonnet(DEGB) cosmology :

$$S_{dEGB} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R + \mathcal{L}_m^{rad} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + f(\phi) R_{GB}^2 \right]$$

Note:

- 1) In the Standard Cosmol, WIMPs decouple during the rad dom era
- 2) If $f(\phi) = \text{const}$, no role of R_{GB}^2 (surface term) and
 The theory is reduced to a quintessence model (rad dom. era).

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + \mathcal{L}_m^{rad} \right]$$

$$\kappa \equiv 8\pi G = 1/M_{PL}^2$$

$$R_{GB}^2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2$$

Gauss-Bonnet term

$f(\phi)$: The coupling btw ϕ and GB

$$f(\phi) = \alpha e^{-\gamma\phi(r)} \quad \text{our choice}$$

- Equations of motion

- Gravity (metric)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa \left(T_{\mu\nu}^{rad} + T_{\mu\nu}^\phi + T_{\mu\nu}^{GB} \right) = \kappa T_{\mu\nu}^{tot}$$

$$T_{\mu\nu}^{rad} = -2 \frac{\delta \mathcal{L}_m^{rad}}{\delta g_{\mu\nu}} + \mathcal{L}_m^{rad} g_{\mu\nu}$$

$$T_{\mu\nu}^{GB} = 4 \left(R \partial_\mu \partial_\nu f(\phi) - g_{\mu\nu} R \square f(\phi) \right)$$

$$T_{\mu\nu}^\phi = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} g^{\rho\sigma} \partial_\rho \phi \partial_\sigma \phi + V \right)$$

- scalar field

$$-8 \left(R_\nu^\rho \partial_\rho \partial_\mu f(\phi) + R_\mu^\rho \partial_\rho \partial_\nu f(\phi) - R_{\mu\nu} \square f(\phi) - g_{\mu\nu} R^{\rho\sigma} \partial_\rho \partial_\sigma f(\phi) + R_{\mu\rho\nu\sigma} \partial^\rho \partial^\sigma f(\phi) \right)$$

$$\square \phi - V' + f' R_{GB}^2 = 0$$

- The energy density and the pressure

$$-\rho_I = T_I^0{}_0 \quad p_I \delta_j^i = T_I^i{}_j \quad \text{where } I = \{\phi, GB, rad\}$$

- FLRW Universe metric:

$$ds^2 = - dt^2 + a^2(t) \left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right)$$

$a(t)$: the scale factor

- The energy density and the pressure

for the radiation

$$\rho_{rad} = \frac{\pi^2}{30} g_* T^4$$

$$p_{rad} = \frac{1}{3} \rho_{rad}$$

$$g_* = \sum_B g_B \left(\frac{T_B}{T_\gamma} \right)^4 + \frac{7}{8} \sum_F g_F \left(\frac{T_F}{T_\gamma} \right)^4$$

of effective relativistic d.o.f
in thermal equilibrium

for the scalar

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi)$$

for the Gauss-Bonnet term:

$$\rho_{GB} = -24\dot{f}H^3 = -24f'\dot{\phi}H^3 = -24\alpha\gamma e^{\gamma\phi} \dot{\phi}H^3$$

$$p_{GB} = 8(f''\dot{\phi}^2 + f'\ddot{\phi})H^2 + 16f'\dot{\phi}H(\dot{H} + H^2)$$

$$= 8 \frac{d(fH^2)}{dt} + 16\dot{f}H^3 = 8 \frac{d(fH^2)}{dt} - \frac{2}{3} \rho_{GB}$$

$$\dot{f} = f'\dot{\phi}$$

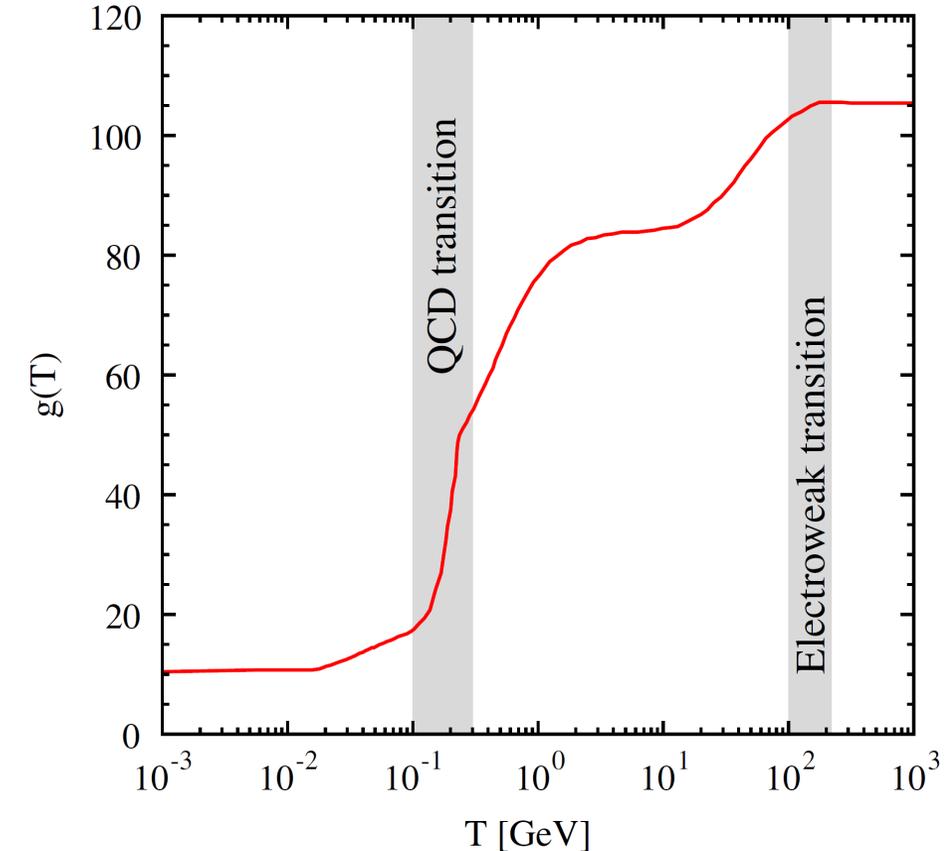
$$\ddot{f} = f''\dot{\phi}^2 + f'\ddot{\phi}$$

$$f(\phi) = \alpha e^{-\gamma\phi(r)}$$

$$H(t) = \frac{\dot{a}}{a}$$

Hubble parameter
(Universe expansion rate)

Note: The signature of $\rho_{\{\phi+GB\}}$ and $p_{\{\phi+GB\}}$ is not necessarily positive.



The Einstein and scalar Eqs. ($V(\phi) = 0$)

$$H^2 = \frac{\kappa}{3} (\rho_{\{\phi+GB\}} + \rho_{rad})$$

$$= \frac{\kappa}{3} \left(\frac{1}{2} \dot{\phi}^2 - 24fH^3 + \rho_{rad} \right) = \frac{\kappa}{3} \rho_{tot}$$

$$\dot{H} = -\frac{\kappa}{2} [(\rho_{\{\phi+GB\}} + p_{\{\phi+GB\}}) + (\rho_{rad} + p_{rad})]$$

$$= -\frac{\kappa}{2} \left[\dot{\phi}^2 + 8 \frac{d(fH^2)}{dt} - 8fH^3 + (\rho_{rad} + p_{rad}) \right]$$

$$\equiv -\frac{\kappa}{2} (\rho_{tot} + p_{tot}) = -\frac{\kappa}{2} \rho_{tot} (1 + w_{tot})$$

$$\ddot{\phi} + 3H\dot{\phi} + V'_{GB} = 0$$

where:

$$V'_{GB} \equiv -f' R_{GB}^2 = -24f'H^2(\dot{H} + H^2) = 24\alpha\gamma e^{\gamma\phi} q H^4$$

$$R_{GB}^2 = 24H^2(\dot{H} + H^2) \equiv -24H^4 q$$

The acceleration (deceleration) of expansion

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} \equiv -H^2 q$$

$$= -\frac{\kappa}{6} [(\rho_{\{\phi+GB\}} + 3p_{\{\phi+GB\}}) + (\rho_{rad} + 3p_{rad})]$$

$$= -\frac{\kappa}{6} \rho_{tot} (1 + 3w_{tot}) = -\frac{1}{2} H^2 \rho_{rad}$$

geometric units $\kappa = 8\pi G = 1$, $c = 1$

Then $[\alpha] = m^2$, $[\phi] = [\gamma] = \text{dimensionless}$.

or $H^2(t) = H_0^2 \left(\rho_{\{\phi+GB\}}(t)/\rho_0 + \rho_{rad}(t)/\rho_0 \right)$

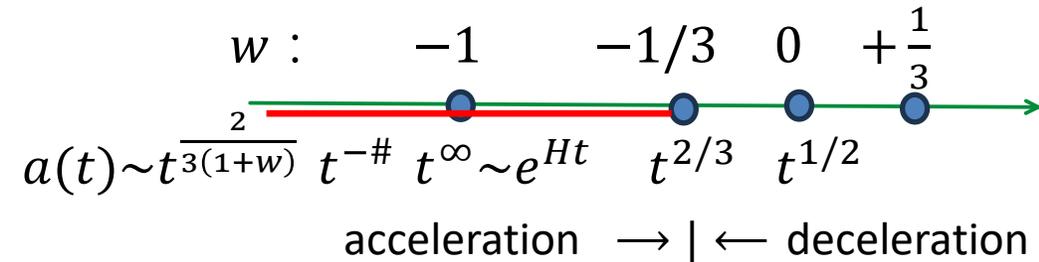
$$= H_0^2 \left(\Omega_{\{\phi+GB\}} a^{-3(1+w_{\{\phi+GB\}})} + \Omega_{rad} a^{-4} \right)$$

$$w_i = \frac{p_i}{\rho_i} \quad \Omega_i = \rho_{i0}/\rho_0$$

$$\frac{\dot{\rho}}{\rho} + 3(1+w) \frac{\dot{a}}{a} = 0 \quad \rho \sim a^{-3(1+w)}$$

$$H^2 = \frac{\kappa}{3} \rho_{tot} \quad \rightarrow a(t) \sim t^{\frac{2}{3(1+w)}}$$

$$H \sim t^{-1} \sim T^{3(1+w)/2}$$



$$\dot{f} = f' \dot{\phi}$$

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{2} (1 + 3w_{tot})$$

Deceleration parameter

the continuity equation

radiation:

$$\dot{\rho}_{rad} + 3H(\rho_{rad} + p_{rad}) = \dot{\rho}_{rad} + 3H(1 + w_{rad})\rho_{rad} = 0$$

the sum of the scalar & GB can be shown to satisfy:

$$\begin{aligned} & \dot{\rho}_{\{\phi+GB\}} + 3H(\rho_{\{\phi+GB\}} + p_{\{\phi+GB\}}) \\ &= \dot{\rho}_{\{\phi+GB\}} + 3H(1 + w_{\{\phi+GB\}})\rho_{\{\phi+GB\}} = 0 \end{aligned}$$

$$\begin{aligned} \rho_{GB} &= -24\dot{f}H^3 \\ p_{GB} &= 8\frac{d(\dot{f}H^2)}{dt} - \frac{2}{3}\rho_{GB} \end{aligned}$$

Note : 1) The scalar and Gauss-Bonnet contributions don't satisfy the continuity equation separately

2) The signature of $\rho_{\{\phi+GB\}}$ and $p_{\{\phi+GB\}}$ is not necessarily positive.

The only boundary condition is $\dot{\phi}_{BBN} \geq 0$ (Initial conditions at the BBN temperature $T = T_{BBN} = 1$ MeV)

Note : a shift of ϕ_{BBN} is equivalent to a redefinition of the α parameter

\rightsquigarrow can choose $\phi_{BBN}=0$ with $\alpha = \tilde{\alpha}$

Note : The Friedmann Eqns are invariant under a simultaneous change of signs of $\dot{\phi}_{BBN}$ & γ

\rightsquigarrow Can choose such that $\dot{\phi}_{BBN} \geq 0$

The contribution of $\rho_{\phi}(T_{BBN}) = \frac{1}{2}\dot{\phi}_{BBN}^2$ to ρ_{BBN} is constrained by $N_{eff} \leq 2.99 \pm 0.17$,

This can be converted into

$$\rho_{\phi}(T_{BBN}) \leq 3 \times 10^{-2} \rho_{BBN} \equiv \epsilon_{max} \rho_{BBN} \quad (\text{with } \rho_{BBN} = \rho_{rad, BBN})$$

3. WIMPs in cosmology

3-1) WIMPs in cosmology

3-2) Indirect detection bounds on WIMP annihilation

3-3) Solutions ($V(\phi) = 0$)

3-1) WIMPs in cosmology

WIMPs (Weakly Interacting Massive Particle)

- are the most popular candidates of the Cold Dark Matter (CDM).
- The SM provides no candidate for CDM.
- $\text{GeV} \lesssim m_\chi \lesssim \text{TeV}$ ($50\text{MeV} \lesssim T_f \approx m_\chi/20 \lesssim 50\text{GeV}$)
- the observed present **DM relic density**,

$$\Omega_\chi h^2 = 0.12 \quad (\text{assuming all DM are WIMPs})$$

- the **annihilation cross section** with SM particles for the observed CDM relic density

$$\langle \sigma v \rangle_f \approx 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1} \quad (\text{in Standard Cosmology}).$$

$$\text{at } 50\text{MeV} \lesssim T_f (\approx \frac{m_\chi}{20}) \lesssim 50\text{GeV} \quad (T_f : \text{the freeze-out temperature})$$

the **WIMP annihilation rate** to SM particles $\Gamma = n_\chi \langle \sigma v \rangle$

Expanding $\langle \sigma v \rangle$ in powers of $\frac{v^2}{c^2} \ll 1 \ll 1$

$$\langle \sigma v \rangle \approx a + 6b \frac{T}{m_\chi} \sim \begin{cases} 1 \text{ for s-wave } (a \neq 0) (\ell = 0) \\ T \text{ for p-wave } (a = 0) (\ell = 1) \end{cases}$$

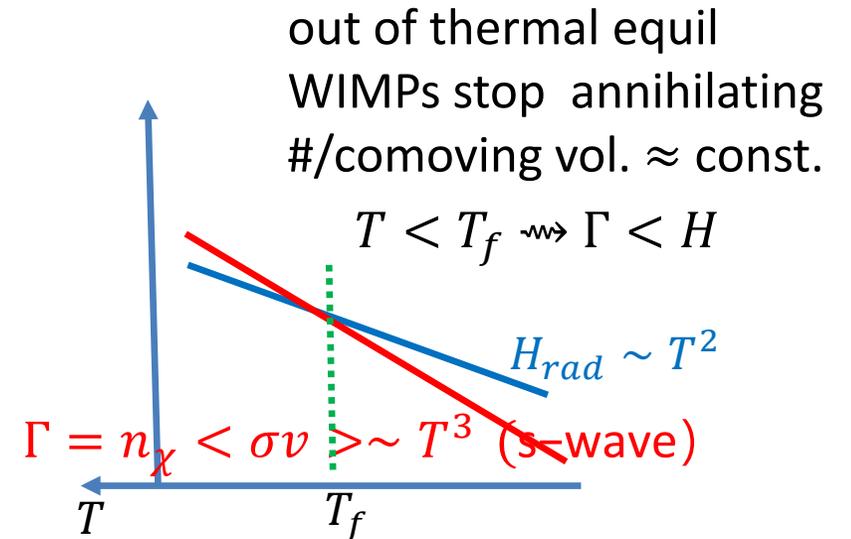
one gets

$$\frac{\Gamma}{H_{rad}} = n_\chi \langle \sigma v \rangle / H_{rad} \sim \begin{cases} T^1 \text{ for s-wave } (a \neq 0) \\ T^2 \text{ for p-wave } (a = 0) \end{cases}$$

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n_χ : the WIMP # density

$T_f \approx m_\chi/20$, decoupling temperature



Note : $n_\chi \sim a^{-3} \sim T^3$ and
 $H_{rad} \sim T^2$ (Standard Cosmol)

The **Boltzmann equation** : the evolution of n_χ

$$\frac{d(n_\chi a^3)}{a^3 dt} = \frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma v \rangle (n_\chi^2 - (n_\chi^{eq})^2)$$

Note : If in thermal equilibrium, then $n_\chi = n_\chi^{eq}$
 And its abundance would decrease exponentially
 the entropy density

$$s = \frac{2\pi^2}{45} g_{*s} T^3$$

Isoentropic expansion $S \equiv sa^3 = const$,

(the entropy in a comoving volume is conserved
 in the absence of phase transitions)

The number density n_χ^{eq}

$$n_\chi^{eq} = \begin{cases} \frac{3\zeta(3)}{4\pi^2} g_\chi T^3, \zeta(3) \approx 1.202 \text{ (extremely relativistic)} \\ g_\chi \left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} \text{ (nonrelativistic } T \lesssim m) \end{cases}$$

Rewrite the **Boltzmann equation** in terms of Y_χ

$$Y_\chi \equiv \frac{n_\chi}{s} \quad \text{the comoving density}$$

$$Y_\chi^{eq} \equiv \frac{n_\chi^{eq}}{s} \quad \text{the corresponding equilibrium value}$$

$$x = \frac{m_\chi}{T}$$

the change of variable from time to temperature

$$\frac{dT}{dt} = -\frac{HT}{\beta} \quad \beta = \left(1 + \frac{1}{3} \frac{d \ln g_{*s}}{d \ln T}\right)$$

The **Boltzmann equation**

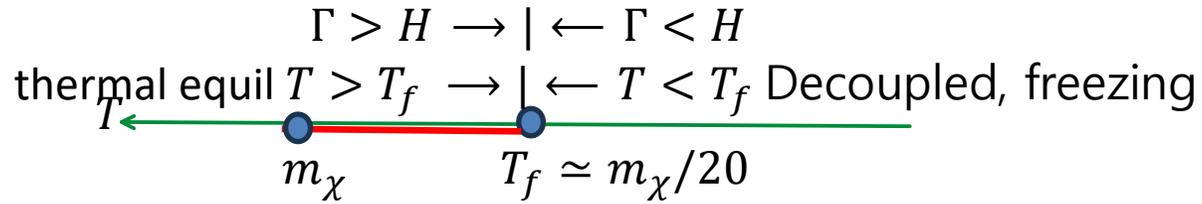
can be rewritten as

$$\frac{dY_\chi}{dx} = -\frac{\beta s}{Hx} \langle \sigma v \rangle (Y_\chi^2 - (Y_\chi^{eq})^2)$$

Goal : Solve the above equation to get Y_χ^0 ,
 the WIMP comoving density at present time

WIMP thermal relic density

- the thermal decoupling scenario

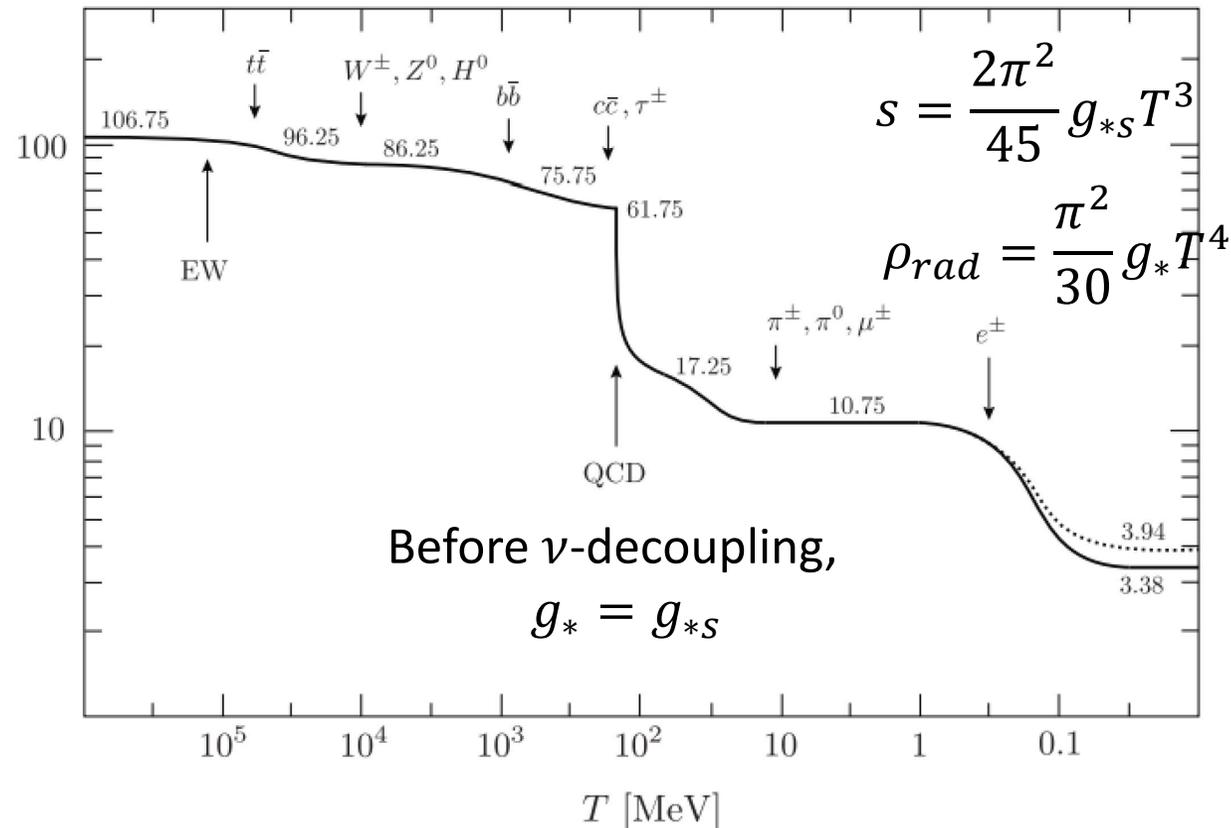


$n_\chi(T) = n_{eq}(T)$ $n_\chi(T)a^3(T) \approx n_\chi(T_f) a^3(T_f)$

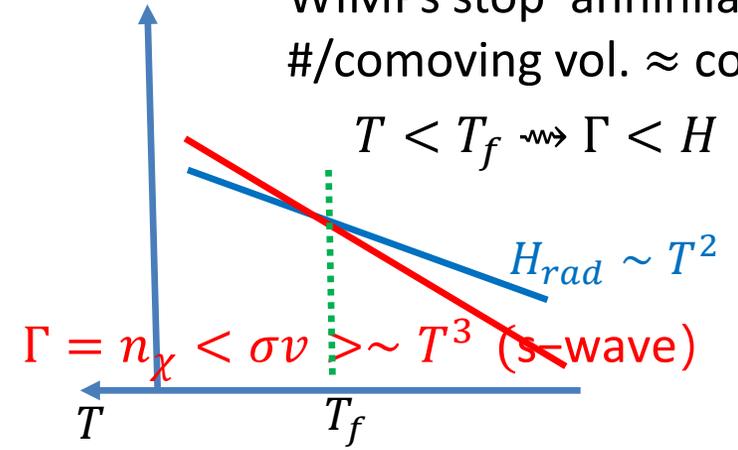
n_{eq} : the WIMP # density in thermal equil

$T_f \approx m_\chi/20$, decoupling temperature

$\Gamma = n_\chi \langle \sigma v \rangle$: the WIMP annihilation rate to SM particles

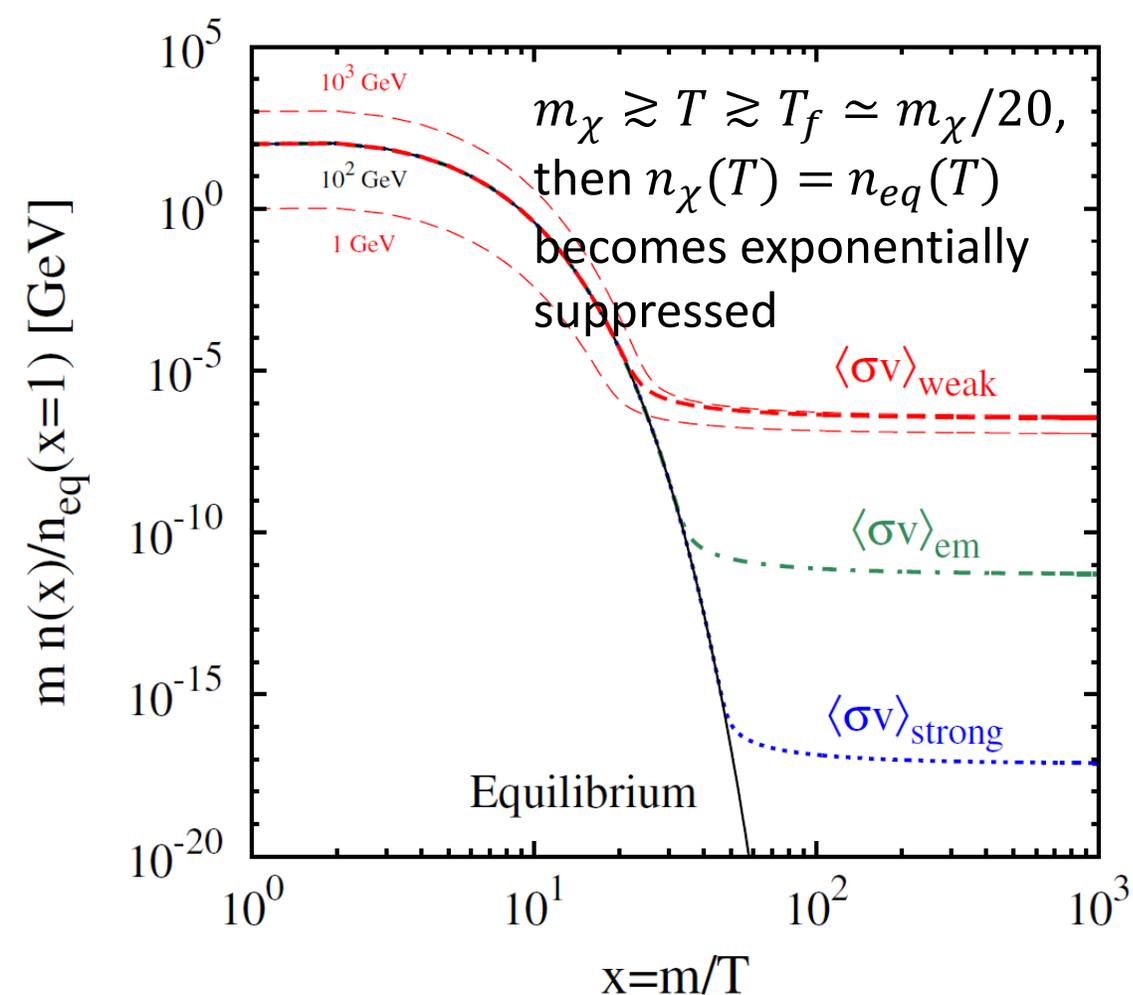


out of thermal equil
WIMPs stop annihilating
#/comoving vol. \approx const.



$$g_{*S} = \sum_B g_B \left(\frac{T_B}{T_\gamma}\right)^3 + \frac{7}{8} \sum_F g_F \left(\frac{T_F}{T_\gamma}\right)^3$$

$$g_* = \sum_B g_B \left(\frac{T_B}{T_\gamma}\right)^4 + \frac{7}{8} \sum_F g_F \left(\frac{T_F}{T_\gamma}\right)^4$$



Note) the WIMP relic density :

$$\Omega_\chi h^2 = \frac{\rho_\chi}{\rho_0} h^2 = 2.755 \times \left(\frac{m_\chi}{\text{GeV}} \right) Y_\chi^0$$

The standard scenario, the WIMP freezes out in a rad bg $\Omega h^2 \simeq 0.12$ (the observed CDM density) is obtained for $\langle \sigma v \rangle_{\text{relic}} \simeq 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$.

A modified cosmol scenario

changes the expansion rate $H(T)$, giving $A(T) = \frac{H(T)}{H_{\text{rad}}}$

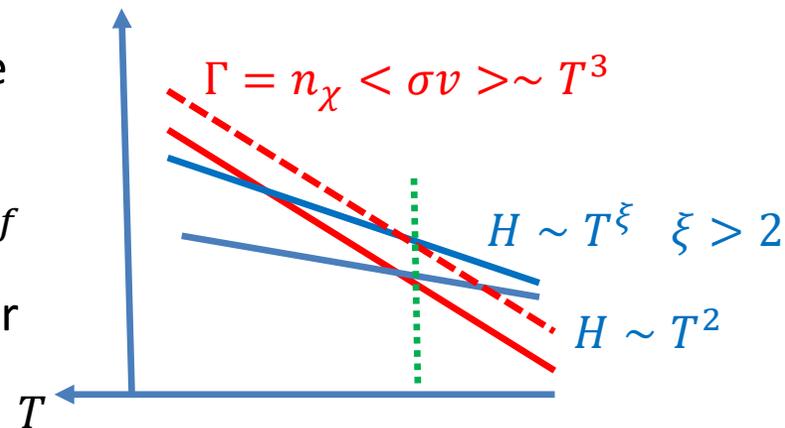
- If $A(T) > 1$, then the WIMP freeze-out $\frac{\Gamma}{H} = 1$ at a larger T ,
- so that at fixed $\langle \sigma v \rangle_f$, the relic density $n_\chi(\text{relic})$ is increased.

- Then the correct relic density is achieved for a **larger value of** $\langle \sigma v \rangle_f = \langle \sigma v \rangle_{\text{relic}}$ compared to the standard case.

If $\langle \sigma v \rangle_f$ larger, n_χ follows n_{eq} (exponentially suppressed) for a longer time \rightsquigarrow smaller $n_\chi(T_f)$ & smaller relic density.

The relic density is anti-correlated to the annihilation rate, $\Omega h^2 \sim 1/\langle \sigma v \rangle_f$

Note) Nonobservation of the WIMP annihilation rate (indirect searches) in our Galaxy today to $\gamma, e^- (e^+), p, \bar{p}$ and $\nu, \bar{\nu}$ fluxes constrain the dEGB scenario.



3-2) Indirect detection bounds on WIMP annihilation

If $A(T_f) > 1$, $\langle \sigma v \rangle_{relic}$ becomes larger

Nonobservation of the WIMP annihilation in our Galaxy today to γ , e^\pm , p , \bar{p} , & ν , $\bar{\nu}$ constrain the dEGB scenario.

WIMPs annihilations in the halo of our Galaxy

- the primary annihilation channels (depending on m_χ) can be

e^+e^- , $\mu^+\mu^-$, $\tau^+\tau^-$, $b\bar{b}$, $t\bar{t}$, $\gamma\gamma$, W^+W^- , ZZ , etc.

- secondary final states :

γ , e^\pm , p , \bar{p} , and ν , $\bar{\nu}$ etc.

The amount of e^+ or γ with energy E per unit t , V & E produced by WIMP annihilations $Q_{e^+/\gamma}(r, E)$

$$Q_{e^+/\gamma}(r, E) = \langle \sigma v \rangle_{gal} \frac{\rho_\chi^2(r)}{2m_\chi^2} \sum_F B_F \frac{dN_{e^+\gamma}^F}{dE}(E, m_\chi) \quad (\text{Assuming a self-conjugate WIMP})$$

where

$\langle \sigma v \rangle_{gal}$: the WIMP annihilation cross section in our Galaxy, $(\sum_F B_F = 1)$

$\frac{\rho_\chi(r)}{m_\chi}$: the number density of DM at the location r of the annihilation process,

B_F : the branching fraction to the primary annihilation channel F ,

$\frac{dN_{e^+\gamma}^F}{dE}$: the e^+ or γ energetic spectrum per F annihilation.

Experiments

- **AMS** : e^+ or \bar{p} with $E \gtrsim$ a few tens of GeV.
- **Fermi LAT** : γ in the Galactic center or in dwarf spheroidal galaxies (dSphs).
- **Planck data** : anisotropies of CMB due to ionizing particles

The non-observation of a significant excess over b.g.

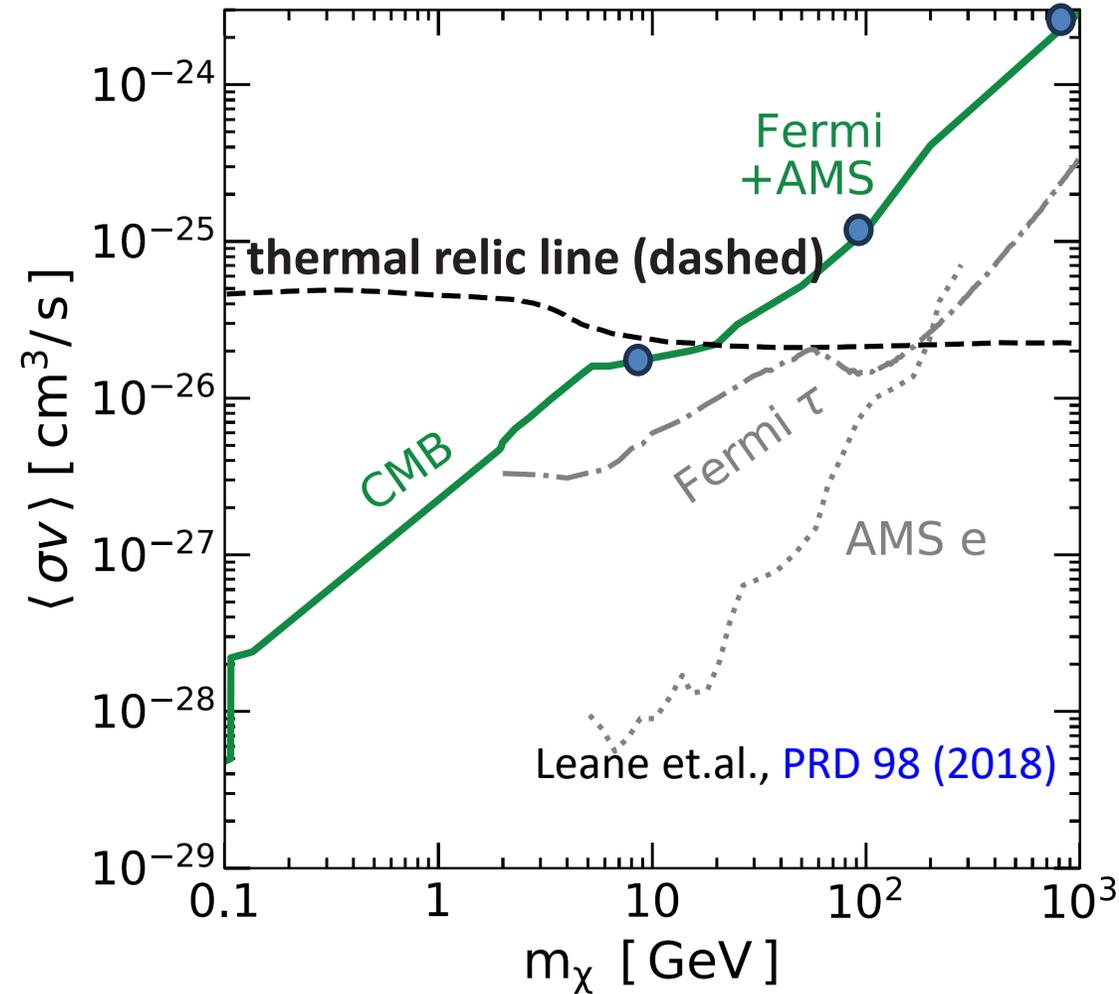
\rightsquigarrow an upper bound $\langle \sigma v \rangle_{ID}$ on $\langle \sigma v \rangle_{gal}$ as a fn of m_χ .

m_χ [GeV]	$\langle \sigma v \rangle_{ID}$ [$\text{cm}^3 \text{s}^{-1}$]
10	1.8×10^{-26}
100	10^{-25}
1000	3×10^{-24}

Indirect detection bounds on WIMP annihilation :

the numerical values corresponding to the three benchmark WIMP masses

$m_\chi = 10 \text{ GeV}, 100 \text{ GeV} \ \& \ 1 \text{ TeV}$



combined limit on $\langle \sigma v \rangle_{gal}$ for WIMP DM (solid)

comparison with the standard 100% cases for Fermi τ (dot-dashed) and AMS electrons (dotted).

3-3) Solutions ($V(\phi) = 0$) $f(\phi) = \alpha e^{-\gamma\phi(r)}$

Step 1) Solve the Eqns to get $H(T)$ & $\dot{\phi}(T)$, especially the enhancement factor $A(T) = \frac{H(T)}{H_{rad}}$

Changing variable from t to T using $\frac{dT}{dt} = -\frac{HT}{\beta}$ $\beta = \left(1 + \frac{1}{3} \frac{d \ln g_*}{d \ln T}\right)$

Boundary Condition $\dot{\phi}_{BBN} \geq 0$

BBN ($T_{BBN} \simeq 1 \text{ MeV}$) strongly constrains any departure from Standard Cosmology.

adopt three benchmarks for $\dot{\phi}_{BBN}$ corresponding to

$$\rho_\phi(T_{BBN}) = 0, \quad \rho_\phi(T_{BBN}) = 10^{-4} \rho_{BBN}, \quad \rho_\phi(T_{BBN}) = 3 \times 10^{-2} \rho_{BBN} \equiv \epsilon_{max} \rho_{BBN}$$

Numerical solutions for ϕ , $\dot{\phi}$, and H are obtained from $T_{BBN} = 1 \text{ MeV}$ to $T = 100 \text{ TeV}$.

Step 2) Use the physics of WIMPs decoupling to probe dEGB Cosmologies.

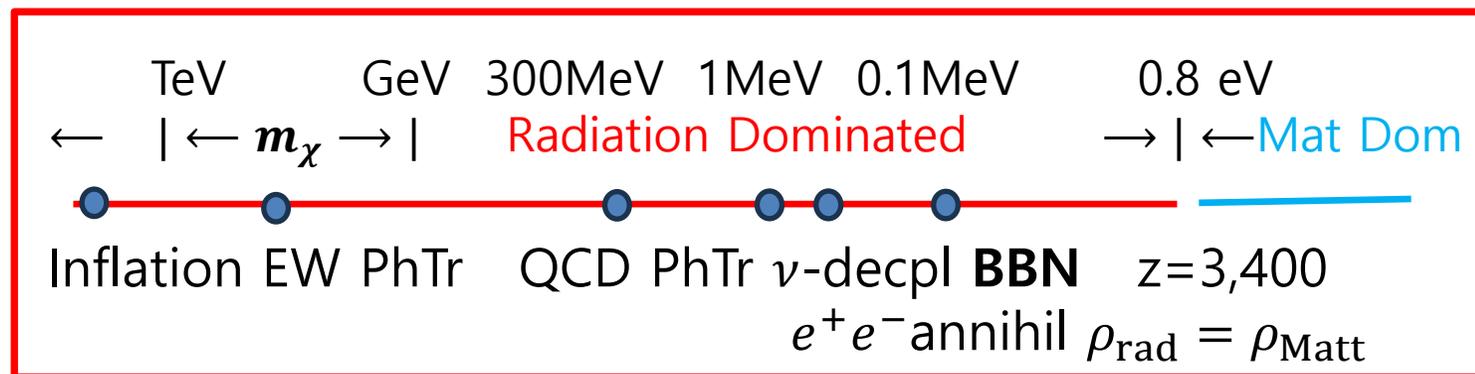
If $A(T) > 1$,

then the $\langle \sigma v \rangle_f = \langle \sigma v \rangle_{relic}$

is driven to higher values

compared to the standard case.

the dEGB parameter space will be constrained by bounds from WIMP indirect searches



Consider the solution for $\dot{f} = 0$ ($\tilde{\alpha}$ and/or $\gamma = 0$)

(i.e., for a theory of the radiation and the scalar kinetic term with vanishing dEGB term).

$$H^2 = \frac{\kappa}{3} \left(\frac{1}{2} \dot{\phi}^2 + \rho_{rad} \right)$$

$$\dot{H} = -\frac{\kappa}{2} \left[\dot{\phi}^2 + (\rho_{rad} + p_{rad}) \right]$$

$$\ddot{\phi} + 3H\dot{\phi} = 0$$

Boundary condition $\rho_\phi(T_{BBN}) = \frac{1}{2} \dot{\phi}_{BBN}^2 = \epsilon \rho_{BBN}$,

$$\rho_\phi \gtrsim \rho_{rad} \text{ for } T \gtrsim T_{cross} = \frac{T_{BBN}}{\sqrt{\epsilon}},$$

ρ_ϕ drives the Universe expansion, with the

enhancement factor $A(T) = \sqrt{1 + \epsilon \left(\frac{T}{T_{BBN}} \right)^2}$.

Continuity Eq. for the scalar $w_\phi = 1$

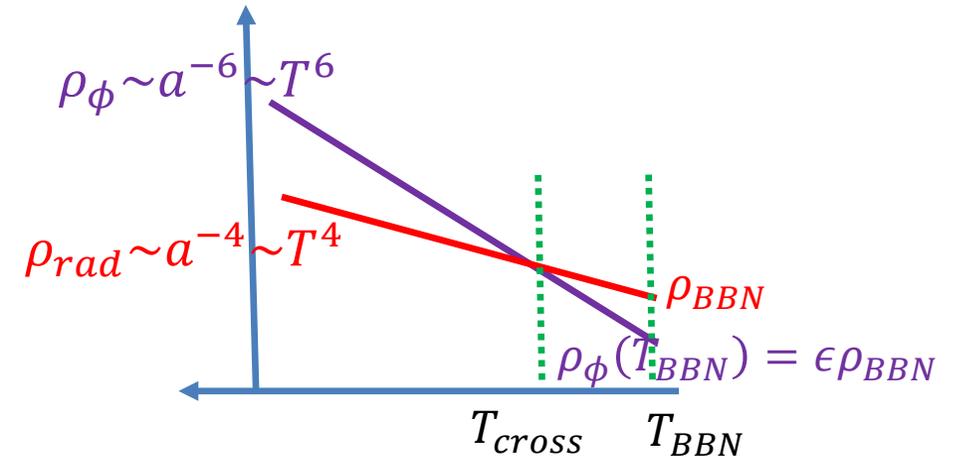
$$\dot{\rho}_\phi + 3H(1 + 1)\rho_\phi = 0$$

$$\rightsquigarrow \rho \sim a^{-3(1+w)} \sim a^{-6}$$

Energy density

$$\rho_{rad} \sim a^{-4} \sim T^4,$$

$$\rho_\phi \sim a^{-6} \sim T^6$$



Ex) For $\epsilon = 3 \times 10^{-2}$, $T_{cross} \simeq 5.8$ MeV

$80 \lesssim A(T) \lesssim 8000$ for $500 \text{ MeV} \leq T(= T_f) \leq 50 \text{ GeV}$,
 $A(T)$ require $\langle \sigma v \rangle_{gal} = \langle \sigma v \rangle_f$ exceeding the
bound $\langle \sigma v \rangle_{ID}$ unless $\epsilon \ll \epsilon_{max}$

Fig. 5 shows that both values

$\rho_\phi(T_{BBN}) = 10^{-4} \rho_{BBN}$ (the black dashed line)
and

$\rho_\phi(T_{BBN}) = 3 \times 10^{-2} \rho_{BBN}$ (the black solid).
are excluded for the theory w/o GB term,

while many combinations of $\tilde{\alpha} \neq 0, \gamma \neq 0$ are allowed
The dEGB mitigates the kination dynamics,
slowing down the scalar field evolution and reducing
the enhancement factor $A(T)$.

Notice also that if $\rho_\phi(T_{BBN}) = 0$,
 ρ_ϕ vanishes at all T for $\tilde{\alpha}$ and/or $\gamma = 0$ (kination only),
while this is no longer true in presence of the dEGB term.

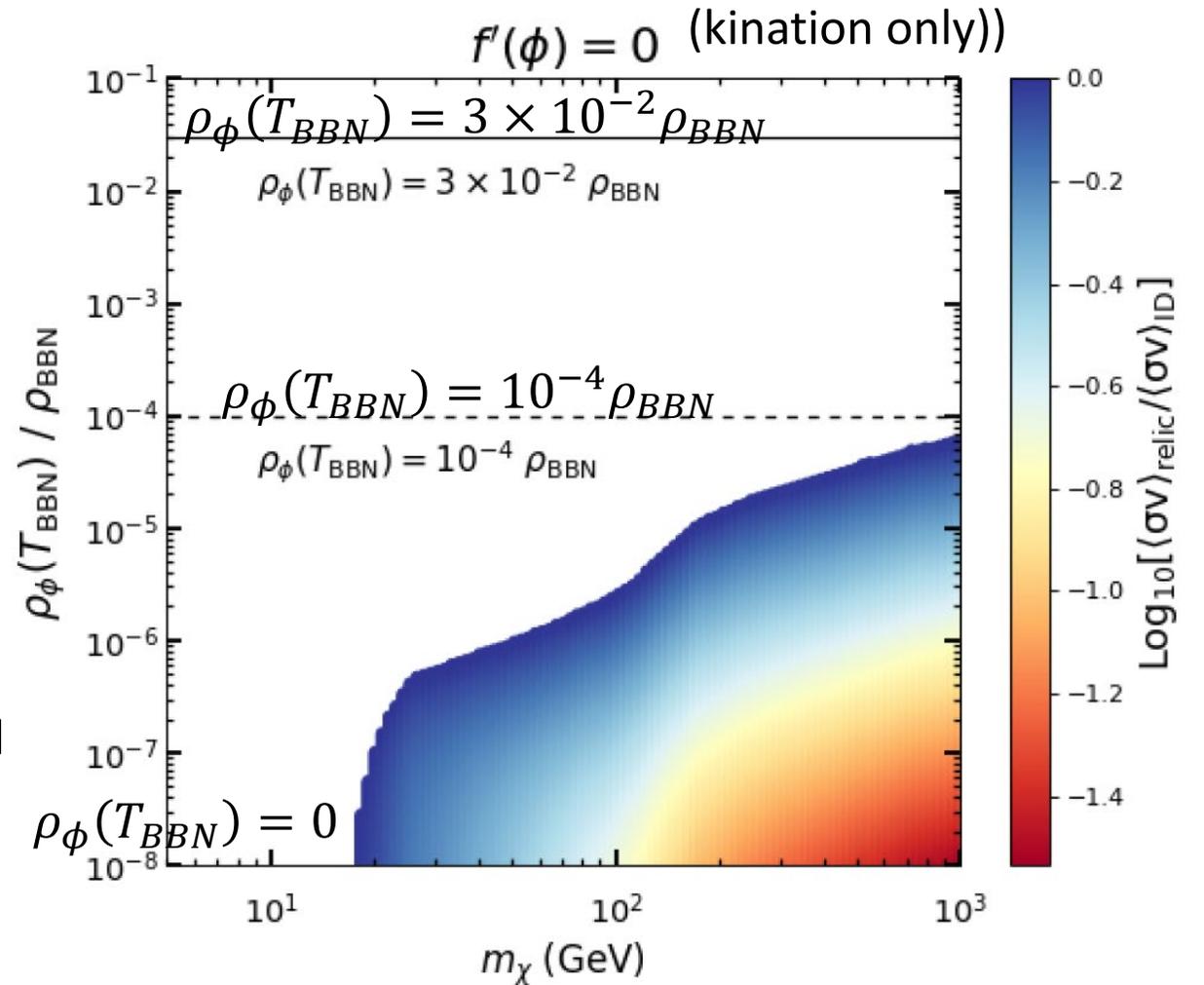
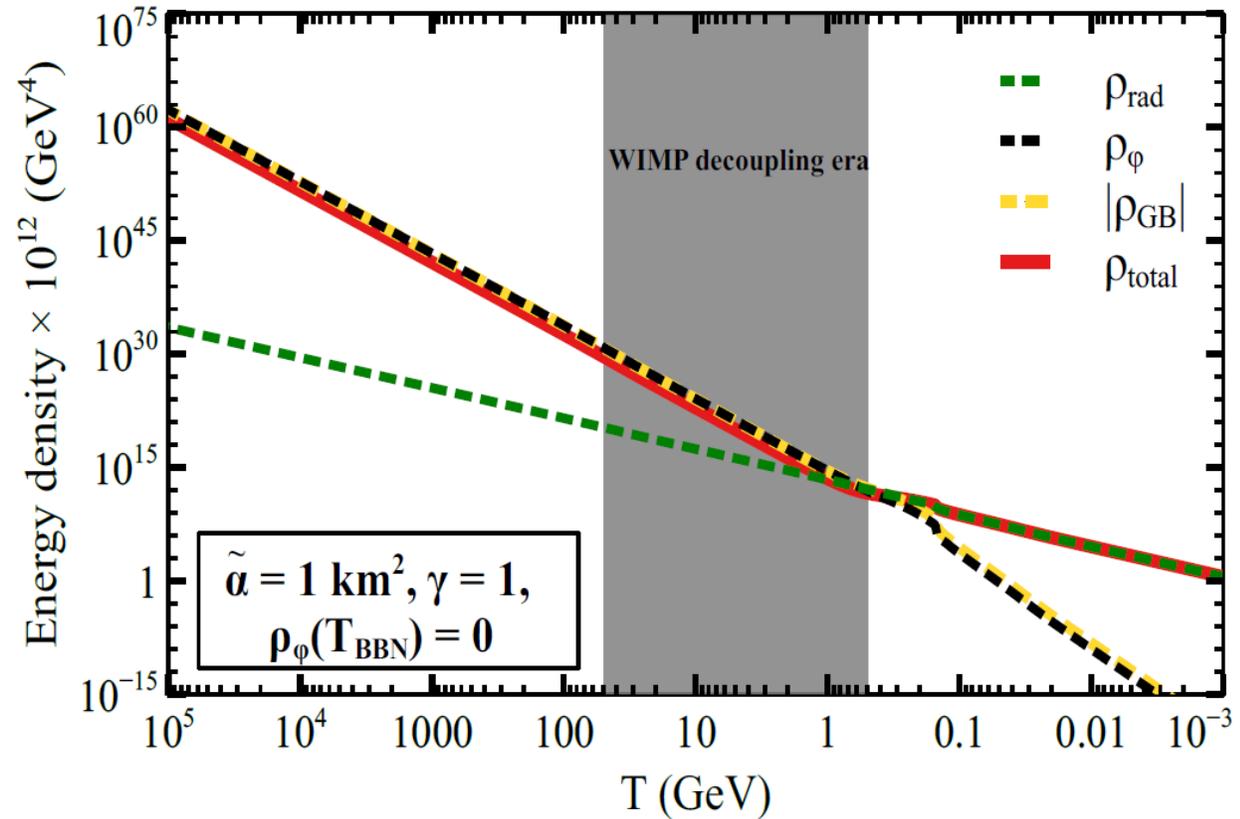
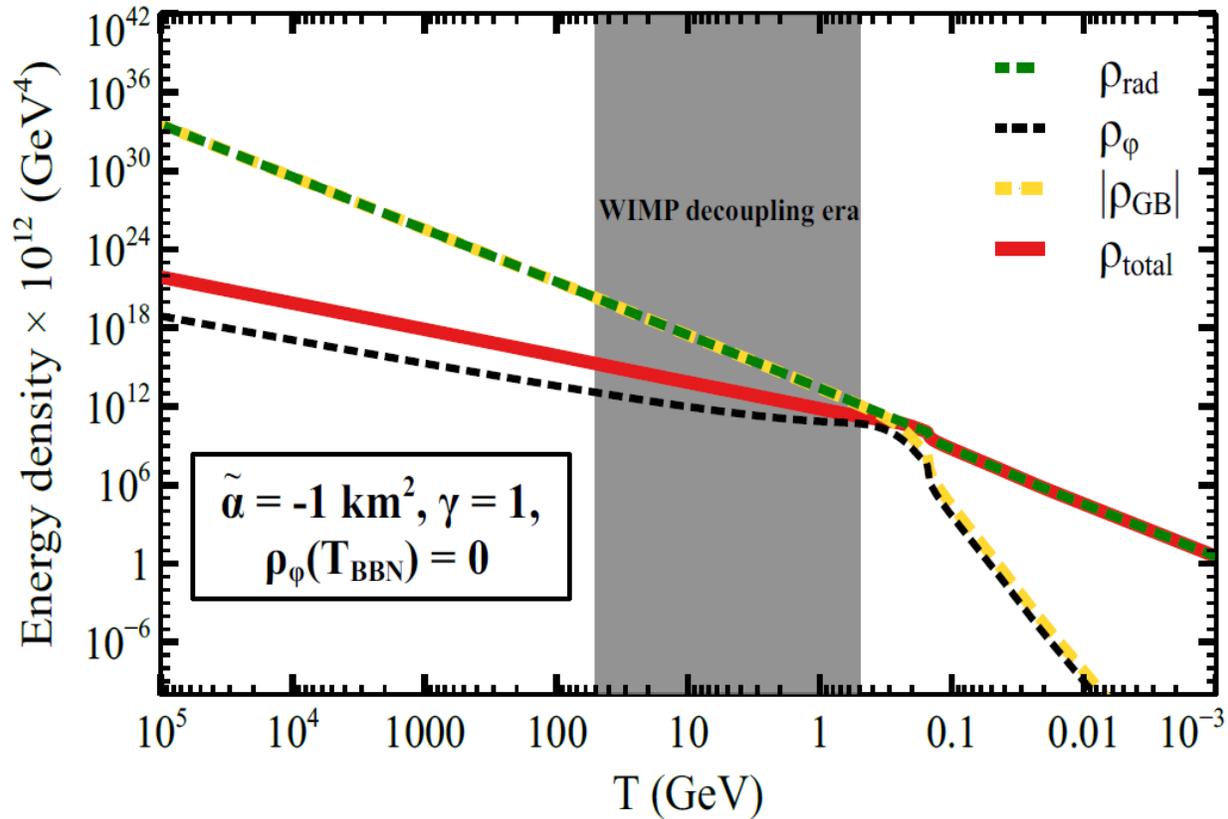


Fig. 5. the upper bound on $\rho_\phi(T_{BBN})$ or values of
 $\rho_\phi(T_{BBN})$ favoured by WIMP indirect detection (ID)

Assume s-wave annihilation ($a \neq 0$) and
 $\langle \sigma v \rangle_{gal} = \langle \sigma v \rangle_f$

Fig.1 Evolution of $\rho_{rad}, \rho_\phi, \rho_{GB}$ and ρ_{total} for $\dot{\phi}(T_{BBN}) = 0$.
 Recall that only ρ_{rad} and the $\rho_{GB} + \rho_\phi$ are physical quantities.



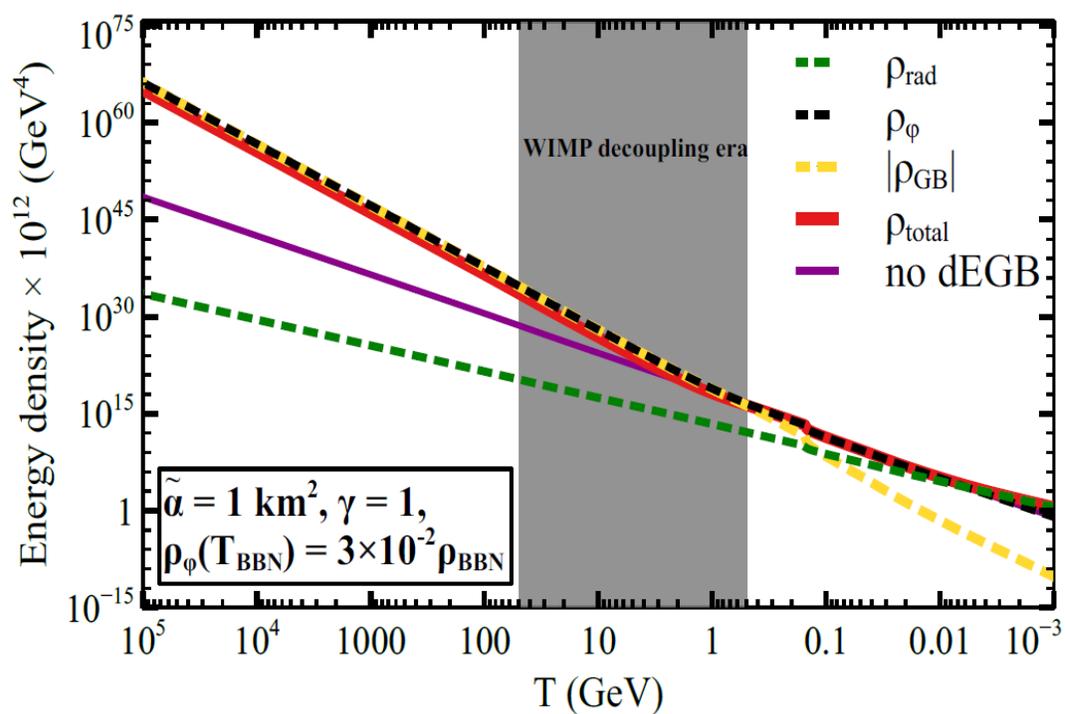
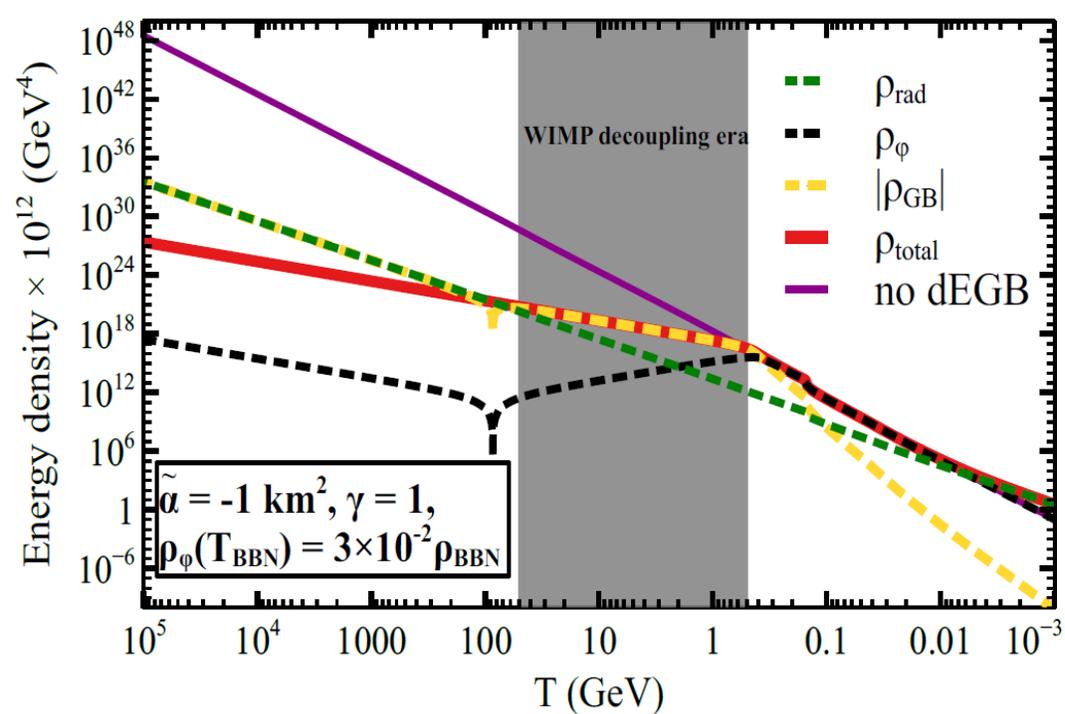
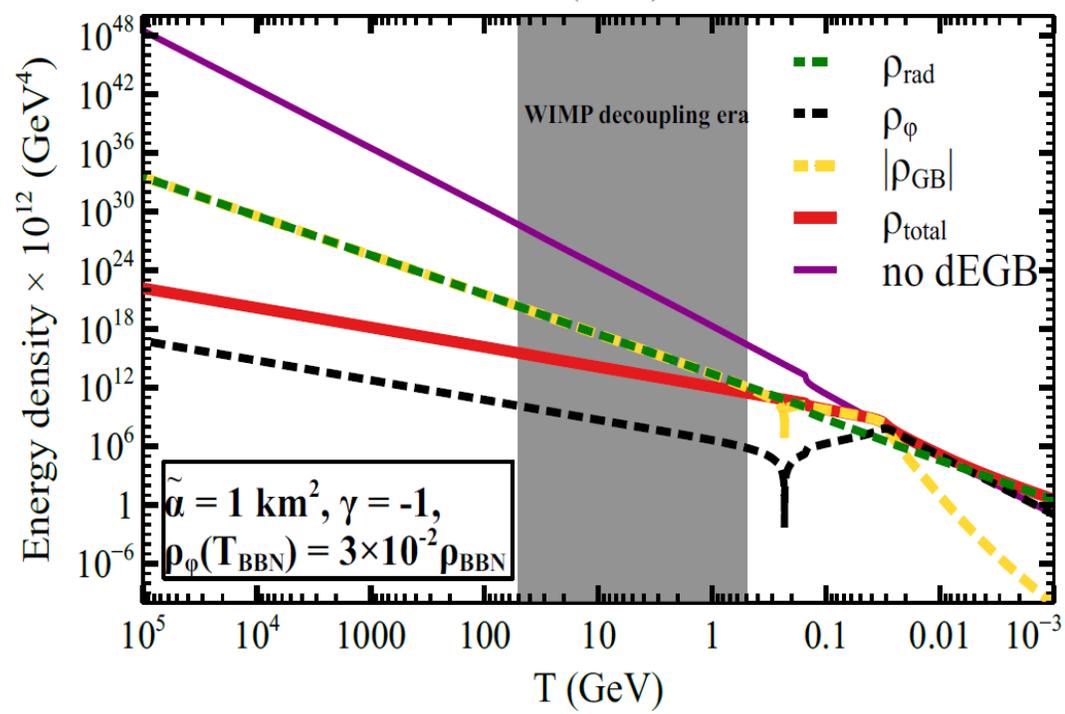
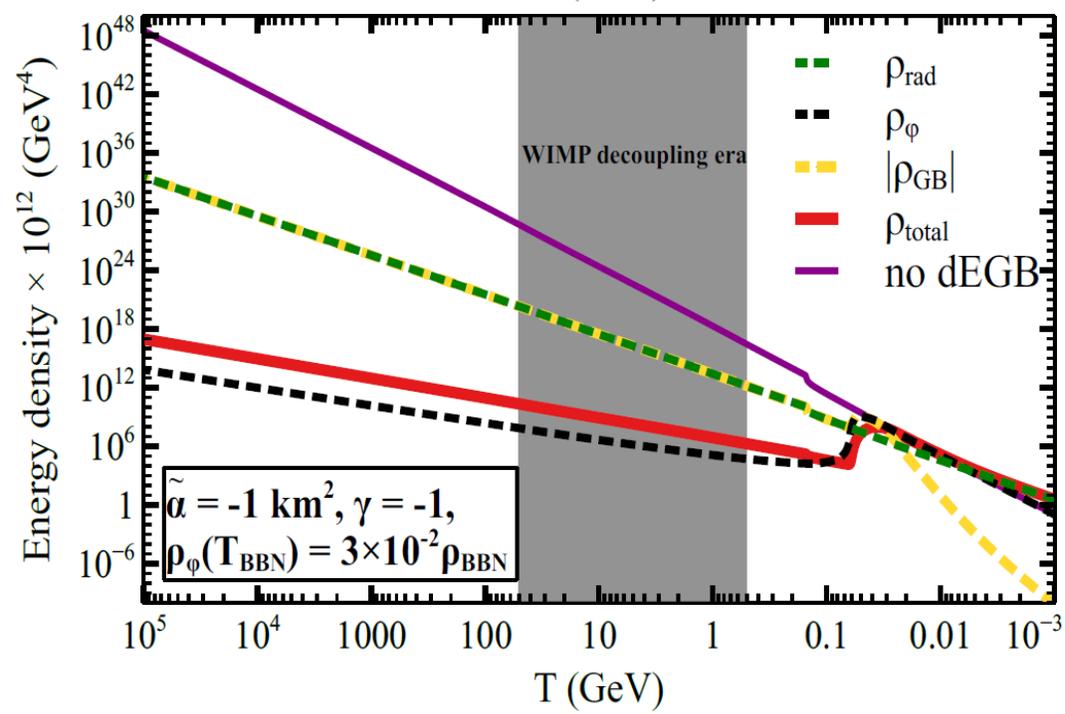


Fig.2 Evolution of $\rho_{rad}, \rho_{\phi}, \rho_{GB}$ & ρ_{total}

$$\rho_{\phi}(T_{BBN}) = 3 \times 10^{-2} \rho_{BBN}$$

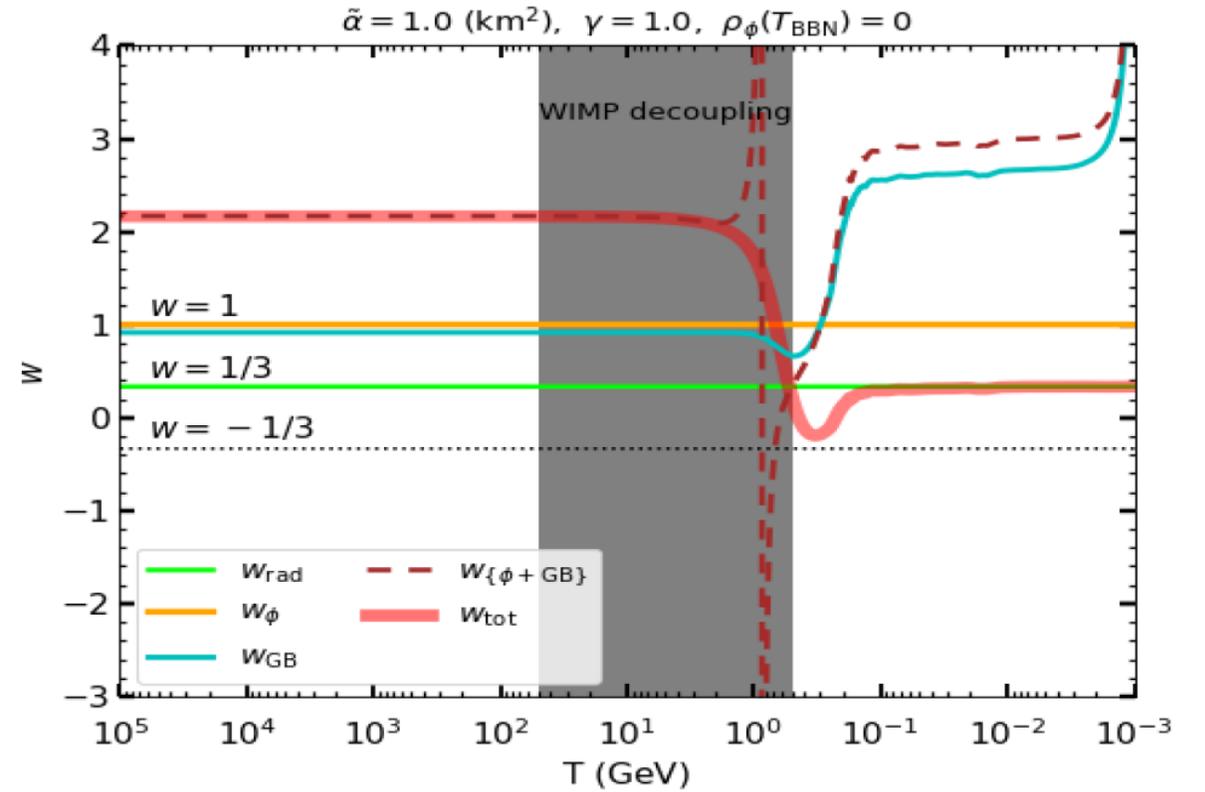
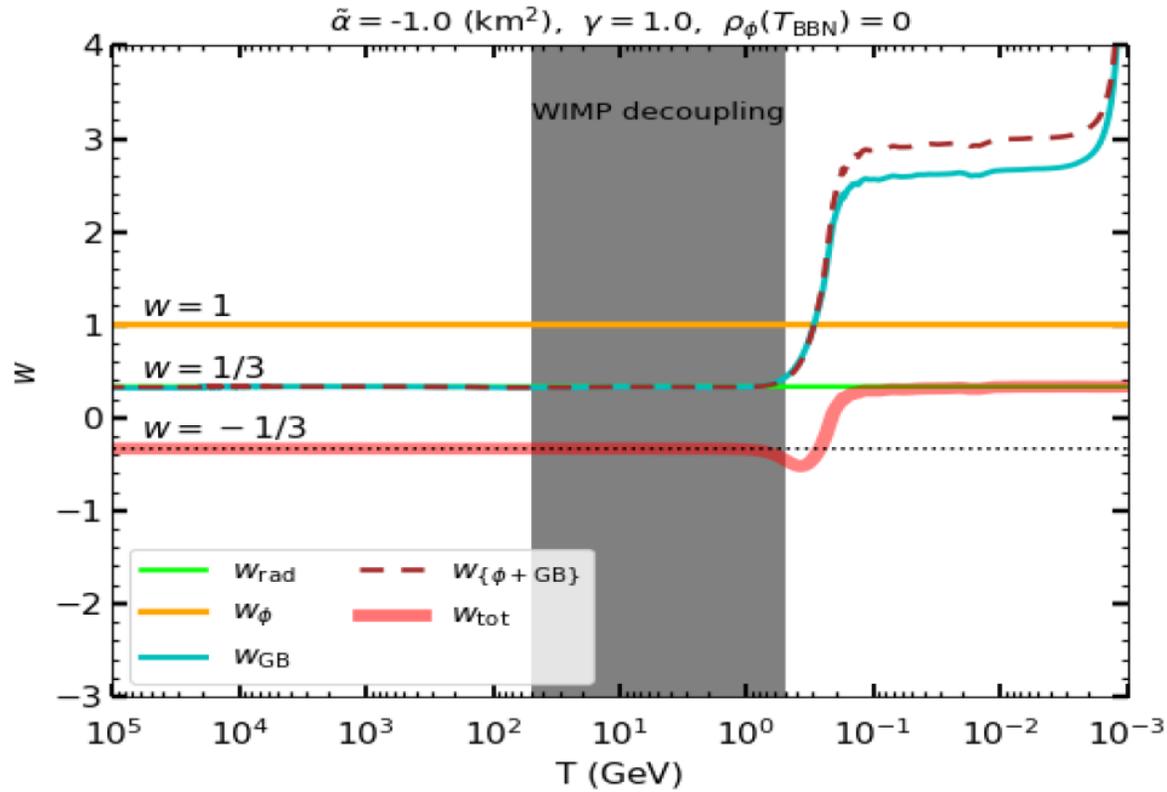


Recall that only ρ_{rad} and $\rho_{GB} + \rho_{\phi}$ are physical quantities.

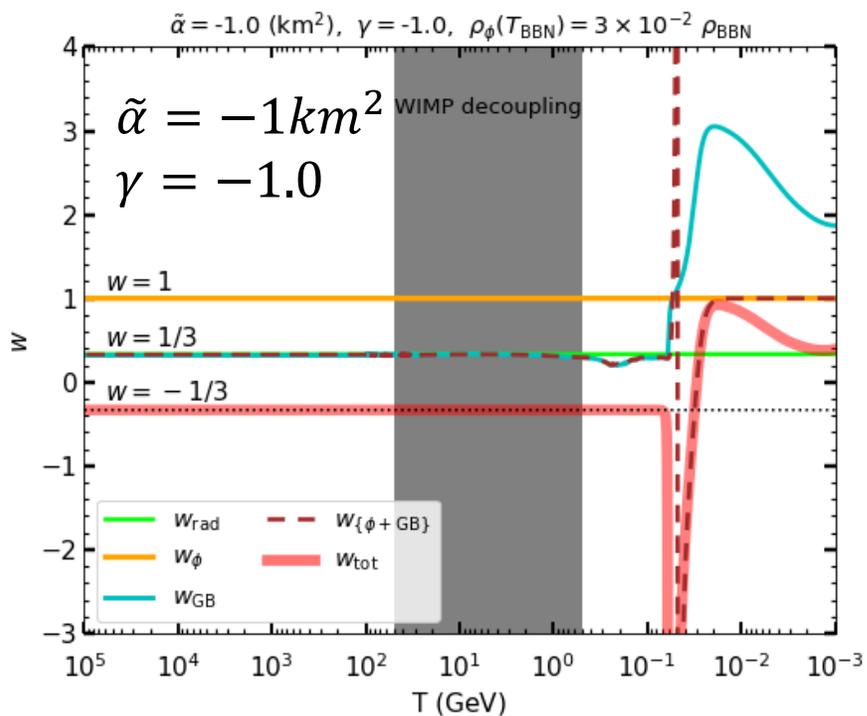
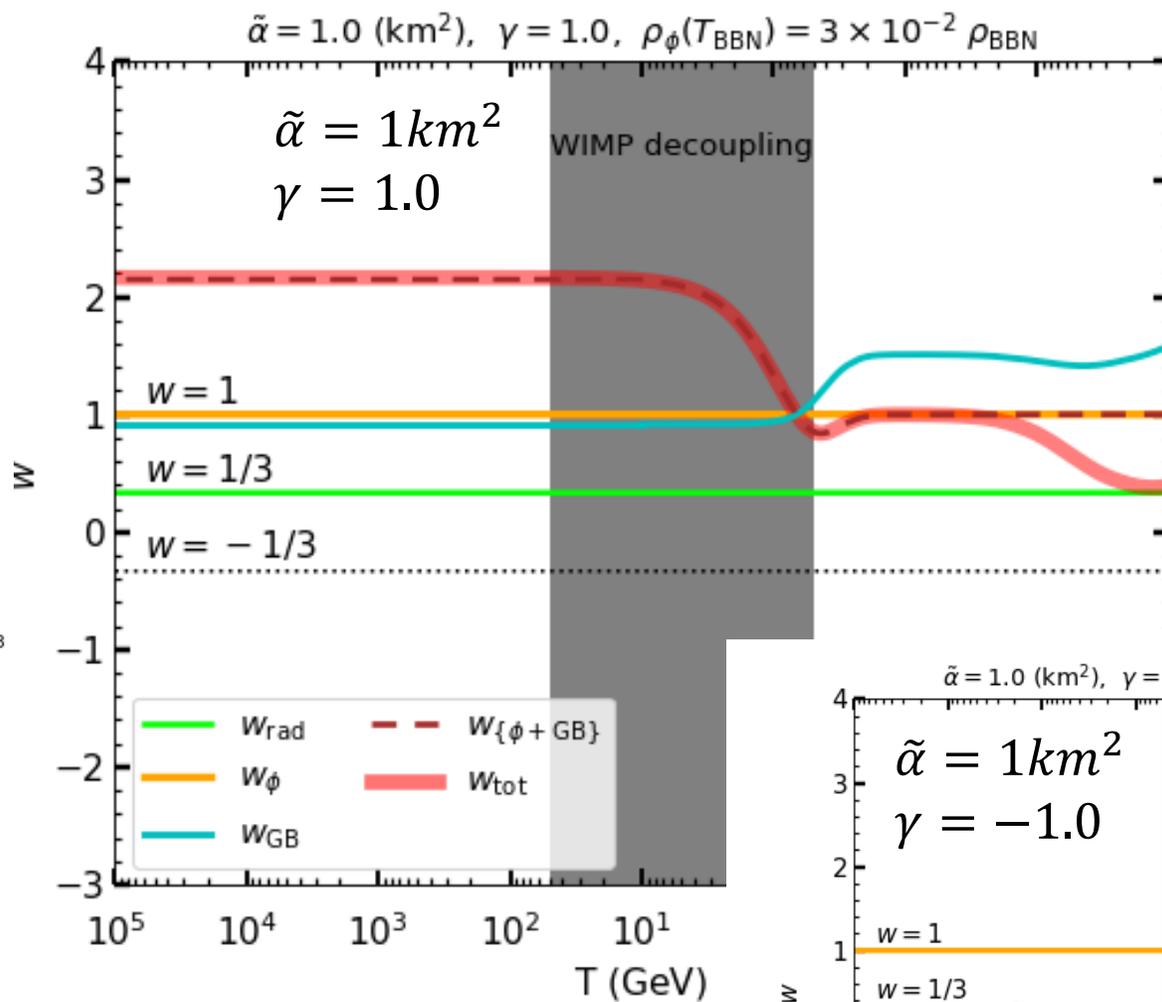
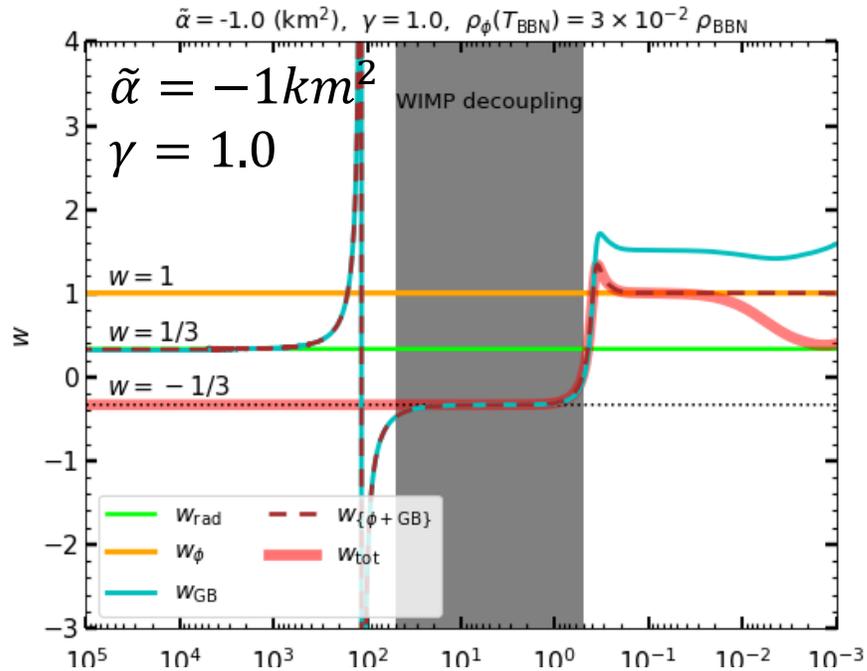
- For slow $\dot{\phi}$ solutions, the corresponding boundary conditions at high temperature correspond asymptotically to an equation of state $w = -1/3$ and a vanishing deceleration parameter q .
- This implies that in this class of solutions the effect of dEGB at high T is to add an accelerating term that exactly cancels the deceleration predicted by GR.
- In this regime the density of the Universe is driven by $\rho_{tot} \simeq \rho_{rad} + \rho_{GB}$, with a large cancellation between ρ_{rad} and $\rho_{GB} < 0$.

- For the class of solutions that comply with WIMP indirect detection bounds, the dEGB term plays a mitigating role on the scalar field (kination) dynamics, slowing down the speed of its evolution and reducing A .
- The bounds that we found from WIMP indirect detection are nicely complementary to late-time constraints from compact binary mergers. This suggests that it could be interesting to use other Early Cosmology processes to probe the dEGB scenario.
- We note that dEGB plays an important role in the evolution of the Universe at high temperature. It would be interesting to study the implications of this on Inflation or the evolution of density perturbations.

Temperature evolution of $w_{tot} = p_{tot}/\rho_{tot}$ and $w_i = p_i/\rho_i$ ($\tilde{\alpha} = \pm 1 \text{ km}^2, \gamma = 1, \rho_\phi(T_{BBN}) = 0$)



In some cases $w_{\{\phi+GB\}}$ diverges, because $\rho_{\{\phi+GB\}}$ changes sign while maintaining a smooth T behaviour.
 The relation $\rho_i \sim T^{3(1+w_i)}$ only holds when w_i is constant and for components satisfying the continuity eqn.
 If, plots for $\gamma = 1$ and $\gamma = -1$ are identical.



$$\rho_\phi(T_{\text{BBN}}) = 3 \times 10^{-2} \rho_{\text{BBN}}$$

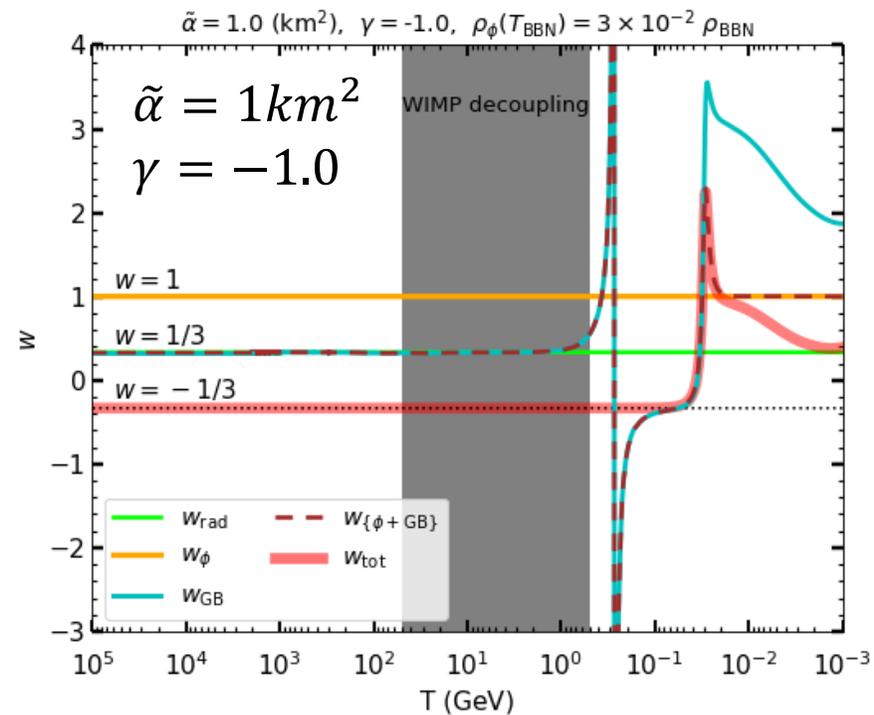
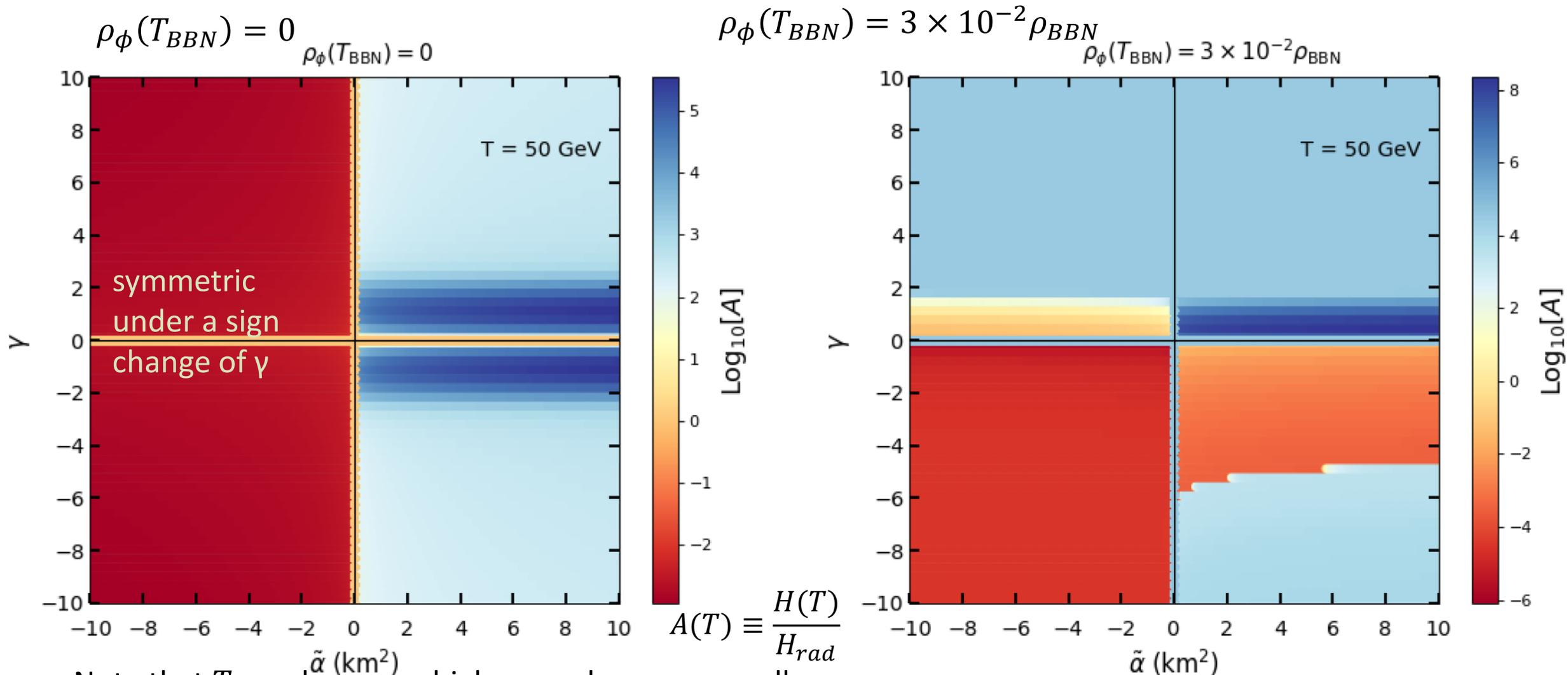


Figure 6. the enhancement factor $A \equiv H/H_{rad}$ at $T = 50\text{GeV}$



Note that T_{cross} becomes higher as ϵ becomes smaller.

Actually, when $\rho_\phi(T_{BBN}) = 0$, $\rho_\phi(T) = 0$ identically, reducing to Standard Cosmology where the evolution is simply by radiation. Hence, the enhancement factor $A(T) \equiv 1$.

This can be seen on the axes of the left hand plot of Fig. 6, where $\tilde{\alpha}\gamma = 0$.

4. Constraints on the dEGB scenario

4-1) the constraints from the GW signals
from BH-BH and BH-NS merger events

4-2) WIMP indirect detection

4. Constraints on the dEGB scenario

Bounds

- tests of gravity within the Solar System ([Sotiriou & Barausse, PRD \(2007\)](#))
- deviations of Kepler's formula for the motion of binary-pulsar systems
- dipole radiation emission from binary pulsars.

no competitive constraints on the dEGB scenario

4-1) the constraints from the GW signals from BH-BH and BH-NS merger events

- the waveforms of the different phases (inspiral, merger and ringdown) in the presence of dEGB gravity can be compared to the data and constraints can be obtained.
- Use the data from the LIGO-Virgo to put constraints on deviations from GR. a constraint on the GB term, of the order of $\alpha_{GB}^{1/2} \leq \mathcal{O}(2 \text{ km})$ or $\alpha_{GB}^{1/2} \leq 1.18 \text{ km}$ Lyu Jiang Yagi
PRD (2022)
- the scalar field ϕ eventually freezes at some asymptotic temperature $T_L \ll T_{BBN}$ to a constant background value $\phi(T_L)$, implying no departure from GR at the cosmological level for $T < T_L$.
- On the other hand, in the vicinity of a BH or a NS, ϕ is distorted compared to $\phi(T_L)$, leading to a local departure from GR that can modify the GW signal in a merger event.
- Near the BH or the NS the dEGB function $f(\phi)$ can be expanded up to the linear term in the small perturbation $\Delta\phi$ around the asymptotic value $\phi(T_L)$ of the scalar field at large distance:

$$f(\phi) = f(\phi(T_L)) + f'(\phi(T_L))\Delta\phi + \mathcal{O}((\Delta\phi)^2)$$

In this way the constraints from compact binary mergers is expressed in terms of f' ($\phi(T_L)$):

$$|f'(\phi(T_L))| \leq \sqrt{8\pi}\alpha_{GB}^{max}$$

with $\alpha_{GB}^{max} = (1.18)^2 \text{ km}^2$

At $T \lesssim T_{BBN}$ the scalar field equation is homogeneous,

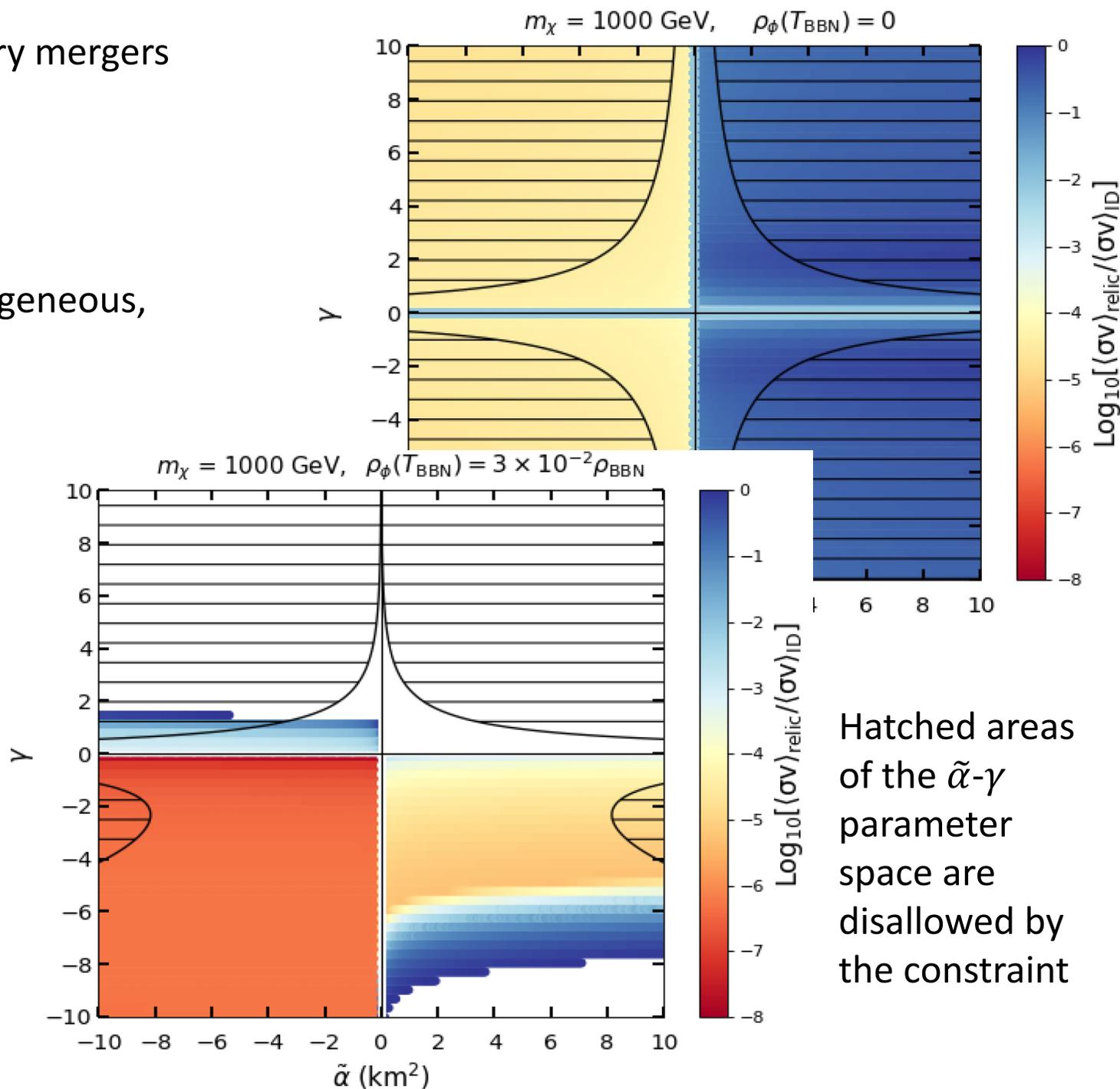
$$\ddot{\phi} + 3H\dot{\phi} = a^3 d/dt(a^3 \dot{\phi}) = 0$$

- If $\dot{\phi}(T_{BBN}) = 0$, then $\dot{\phi}(T) = 0$ for all $T < T_{BBN}$. Hence,

$$|\tilde{\alpha}\gamma| \leq \sqrt{8\pi}\alpha_{GB}^{max}$$

- If $\dot{\phi}(T_{BBN}) \neq 0$, then one needs to consider the residual evolution of ϕ below T_{BBN} to get

$$|\tilde{\alpha}\gamma e^{\gamma \frac{\phi_{BBN}}{H_{BBN}}}| \leq \sqrt{8\pi}\alpha_{GB}^{max}$$



4-2) WIMP indirect detection

- 1) For a given parameter, find $\langle \sigma v \rangle_{relic}$ of $\langle \sigma v \rangle_f$
which yields WIMP relic density $\Omega_\chi h^2 \simeq 0.12$, the observational CDM density.
- 2) Compare $\langle \sigma v \rangle_{gal} = \langle \sigma v \rangle_{relic}$
with $\langle \sigma v \rangle_{ID}$, the upper bound on the present annih cross sec in the halo of the Milky Way.
(consider an s-wave annihilation cross section, for which $\langle \sigma v \rangle_{gal} = \langle \sigma v \rangle_f$.)

The favoured regions of the Gauss-Bonnet parameter space by WIMP indirect detection is given by

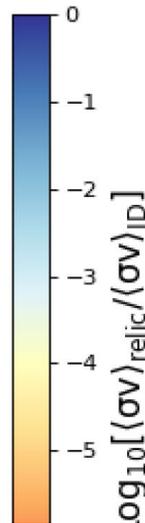
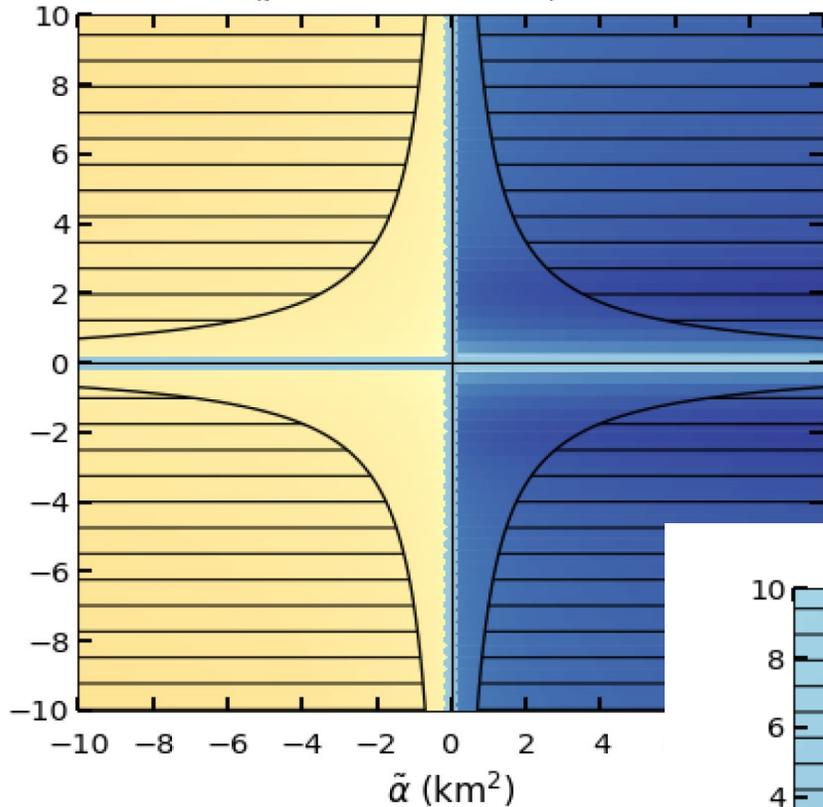
$$\langle \sigma v \rangle_{gal} / \langle \sigma v \rangle_{ID} \lesssim 1$$

The (white) regions of the parameter space $\frac{\langle \sigma v \rangle_{relic}}{\langle \sigma v \rangle_{ID}} > 1$ are disfavoured by indirect searches.

Note

- 1) the results for negative values of $\dot{\phi}_{BBN}$ can be obtained by flipping the sign of γ
- 2) the standard cosmological scenario is modified also for $\rho_\phi(T_{BBN}) = 0$, i.e. irrespective on the boundary conditions of the scalar field, as long as $\tilde{\alpha}\gamma \neq 0$.
- 3) for $\rho_\phi(T_{BBN}) = 0$, symmetry under the sign change of γ
- 4) the highest values of the enhancement factor A are found for $\tilde{\alpha} > 0$ and $\gamma > 0$ and this is confirmed by the plots, where the corresponding region is completely excluded for all three values of m_χ unless $\dot{\phi}_{BBN} = 0$.
- 5) For a vanishing dEGB term the value of A is very large and incompatible with WIMP bounds unless $\dot{\phi}_{BBN}$ is much lower than the present constraints.

$m_\chi = 1000 \text{ GeV}, \quad \rho_\phi(T_{\text{BBN}}) = 0$

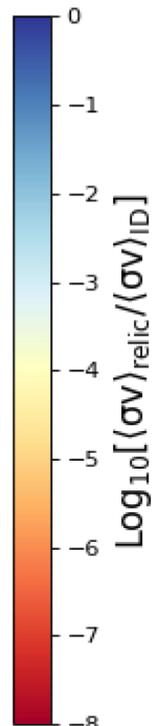
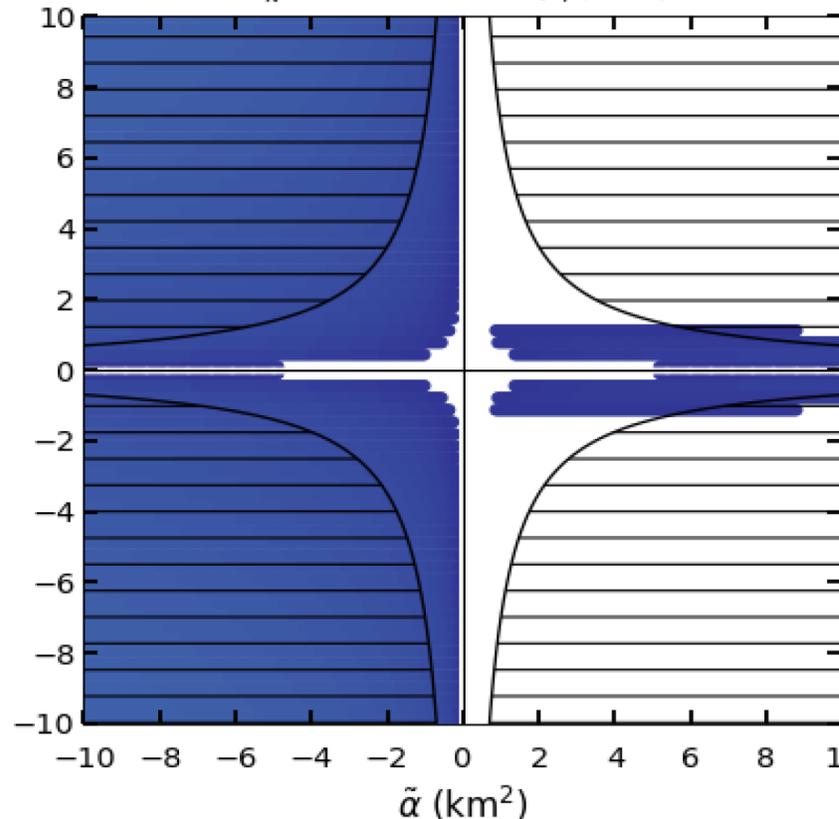


DEGB parameters
corresponding to
 $\Omega_\chi h^2 \simeq 0.12$.

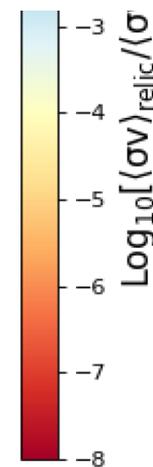
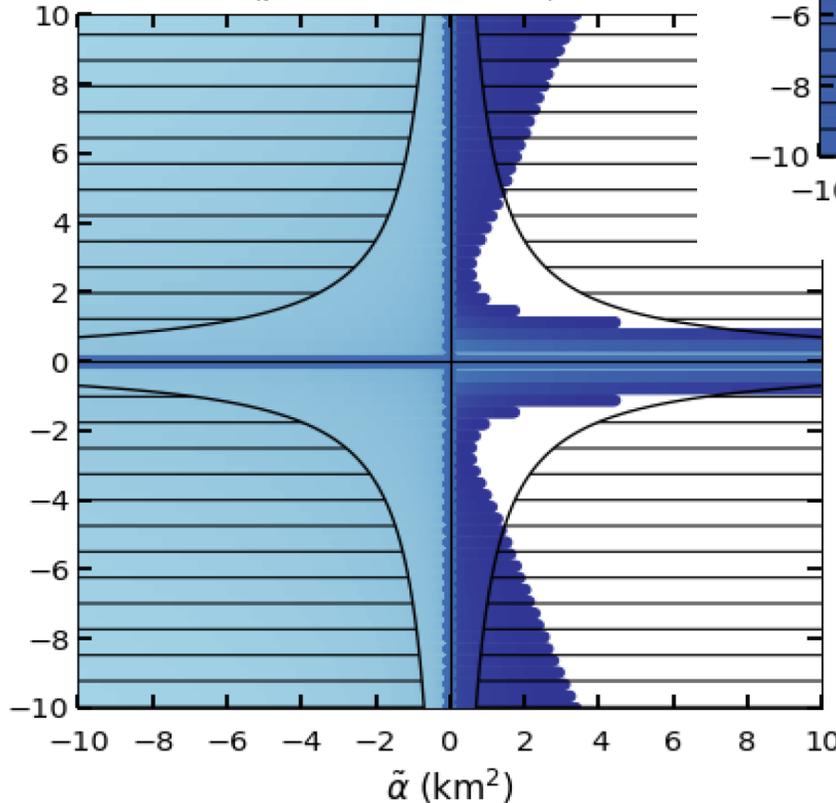
The color code
refers to the ratio

$$\text{Log}_{10} \left[\frac{\langle \sigma v \rangle_{\text{relic}}}{\langle \sigma v \rangle_{\text{ID}}} \right].$$

$m_\chi = 10 \text{ GeV}, \quad \rho_\phi(T_{\text{BBN}}) = 0$



$m_\chi = 100 \text{ GeV}, \quad \rho_\phi(T_{\text{BBN}}) = 0$

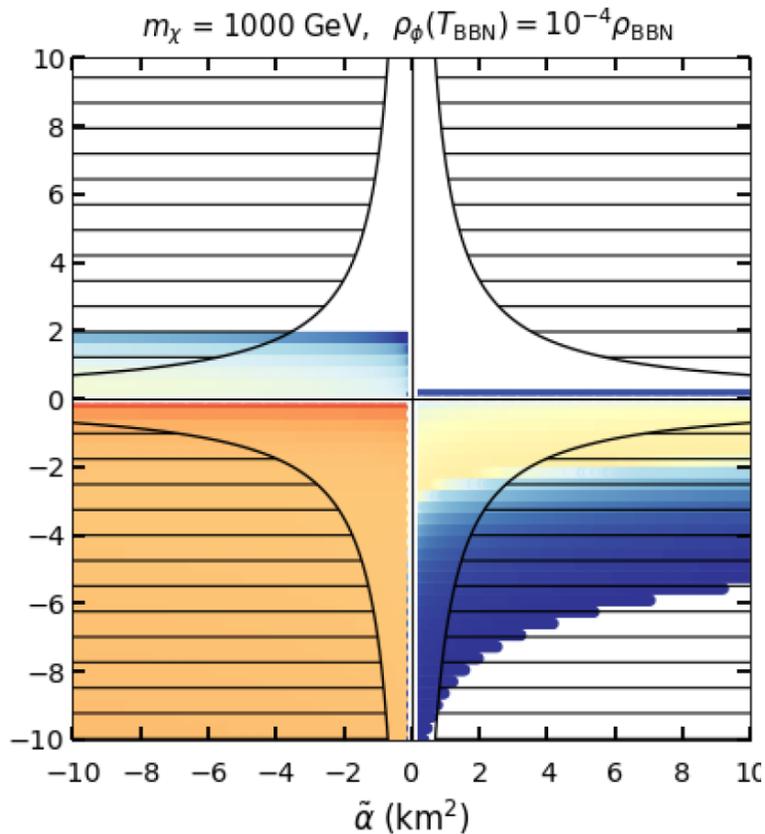


$$\rho_\phi(T_{\text{BBN}}) = 0$$

The **white regions** are
excluded by
WIMP indirect searches,
the **hatched** ones are ruled
out by the **GW detection from
compact binary mergers**.

Note

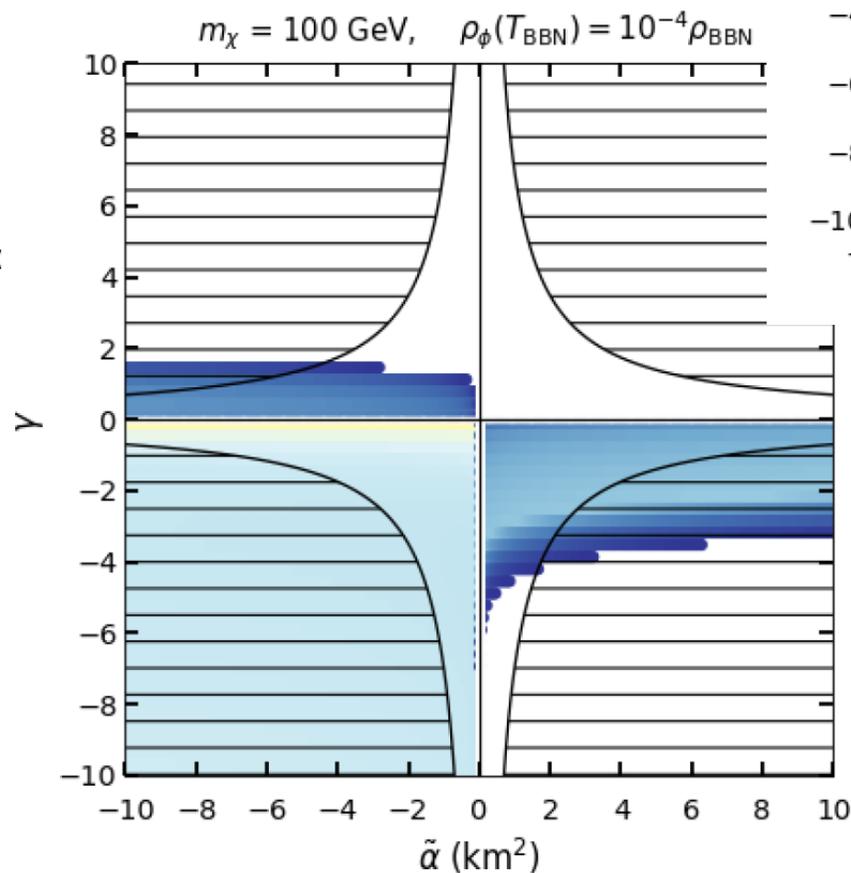
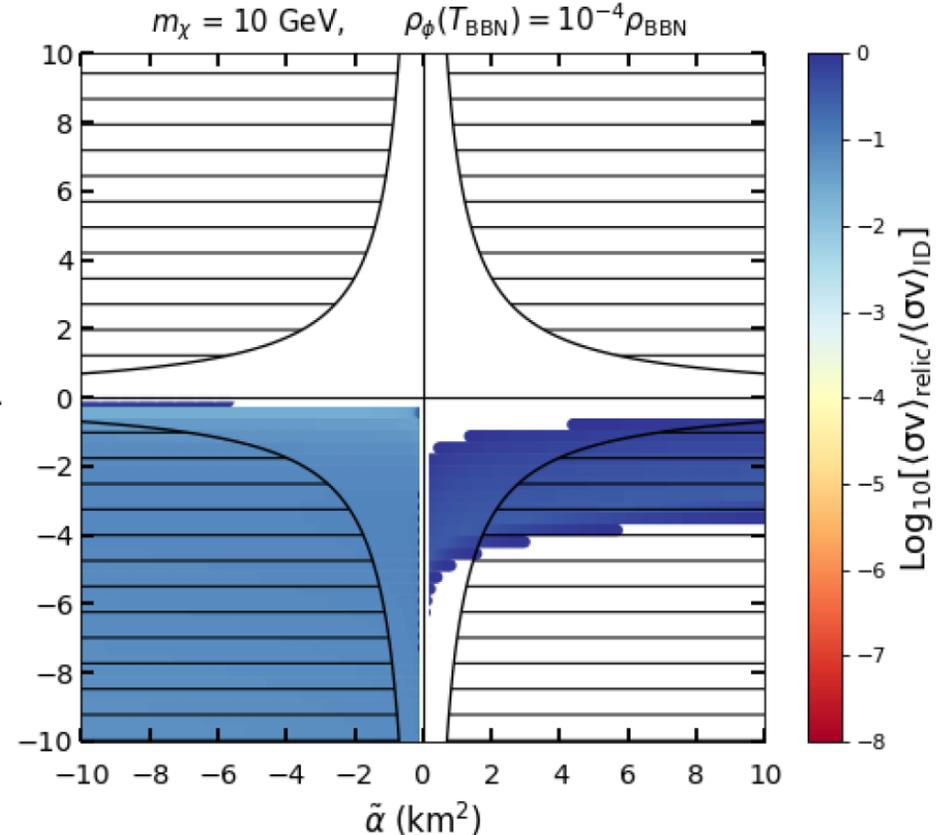
- 1) the standard scenario is modified as long as $\tilde{\alpha}\gamma \neq 0$.
- 2) symmetry under $\gamma \rightarrow -\gamma$
- 3) the highest values of the A for $\tilde{\alpha}\gamma > 0$



$$\Omega_\chi h^2 \simeq 0.12$$

The color code refers to the ratio

$$\text{Log}_{10} \left[\frac{\langle \sigma v \rangle_{\text{relic}}}{\langle \sigma v \rangle_{\text{ID}}} \right].$$

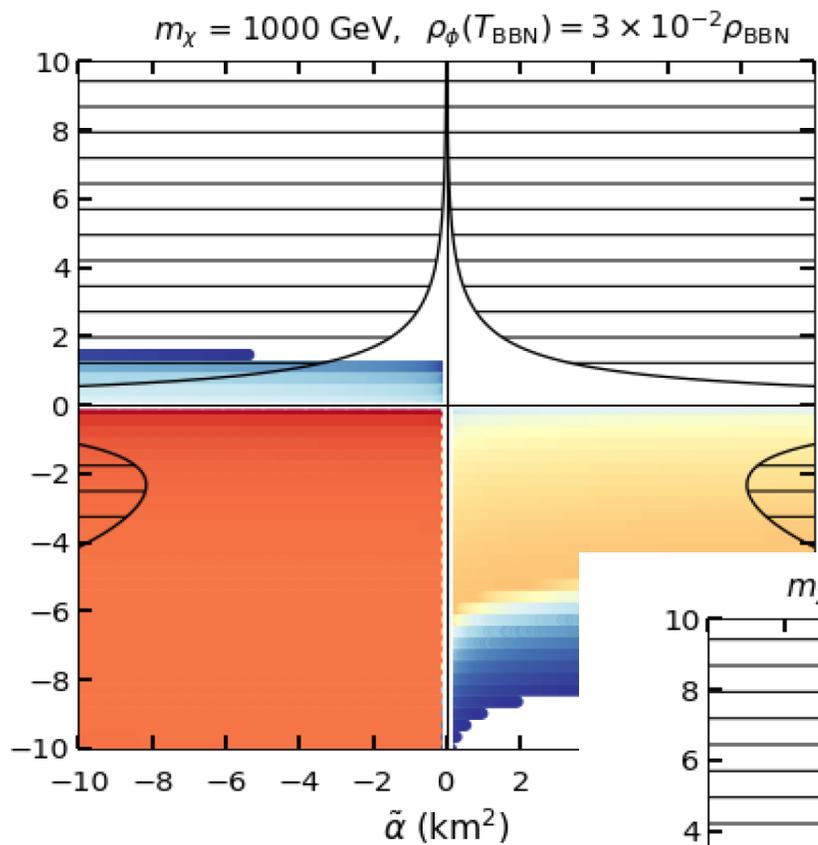


$$\rho_\phi(T_{\text{BBN}}) = 10^{-4} \rho_{\text{BBN}}$$

The **white regions** are excluded by **WIMP indirect searches**, the **hatched ones** are ruled out by the **GW detection from compact binary mergers**.

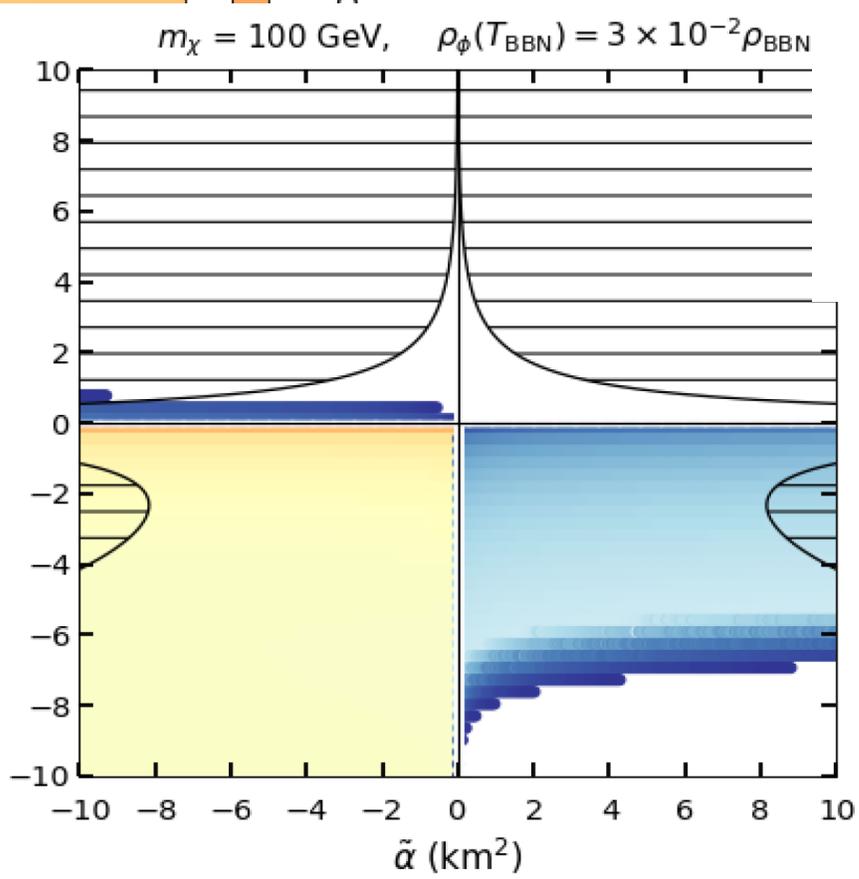
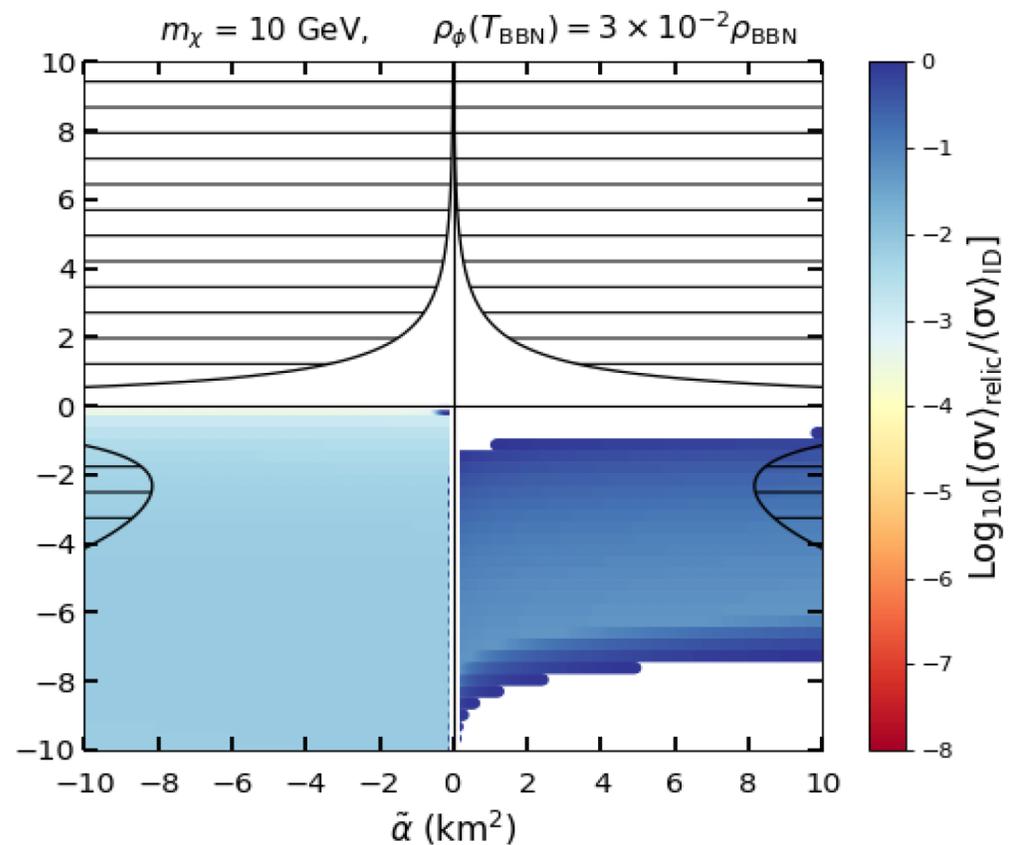
Note

- 1) the standard scenario is modified as long as $\tilde{\alpha}\gamma \neq 0$.
- 2) the highest values of the A for $\tilde{\alpha} > 0$ & $\gamma > 0$



The color code refers to the ratio $\text{Log}_{10} \left[\frac{\langle \sigma v \rangle_{\text{relic}}}{\langle \sigma v \rangle_{\text{ID}}} \right]$.

$\langle \sigma v \rangle_{\text{relic}}$ corresponds to the observed $\Omega_\chi h^2 \simeq 0.12$.



Note

- 1) the standard scenario is modified as long as $\tilde{\alpha}\gamma \neq 0$.
- 2) the highest values of the A for $\tilde{\alpha} > 0$ & $\gamma > 0$

$$\rho_\phi(T_{\text{BBN}}) = 3 \times 10^{-2} \rho_{\text{BBN}}$$

The **white regions** are excluded by **WIMP indirect searches**,

the **hatched ones** are ruled out by the detection of **GW from compact binary mergers**.

$\tilde{\alpha} > 0$ and $\gamma > 0$ both positive, values of the enhancement factor as high as $A \approx 10^5$ do not drive $\langle \sigma v \rangle_{gal}$ beyond $\langle \sigma v \rangle_{ID}$ its upper bound.

This shows that the value of $\langle \sigma v \rangle_{gal}$ does not scale directly with A and is a clear indication of the mitigating effect that the post-freeze-out WIMP annihilation process can have on the relic density and that is expected when the temperature evolution of H is faster than in the standard case.

the cases for $m_\chi = 10$ GeV and $\dot{\phi} \neq 0$ differ from those at higher values of m_χ , in that no allowed regions are found for $\tilde{\alpha} < 0$ and $\gamma > 0$.

In particular, at low WIMP masses the present bounds on $\langle \sigma v \rangle_{gal}$ have already reached the standard value $3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$, i.e. they already exclude the standard scenario.

In this case, at variance with what happens for higher WIMP masses, a modified cosmological scenario with $A > 1$ such as the dEGB model discussed in the present paper is actually required to reconcile the indirect detection bounds with the observed relic density.

the hatched regions excluded by the late-time constraints from compact binary mergers are nicely complementary to those from WIMP indirect detection, with cases allowed/excluded by both bounds or excluded by only one of the two.

5. Summary

Modified Gravity beyond Einstein needed?

Theoretical Aspect

Observational Aspect

H_0 tension, Cosmological Birefringence etc.

Holography (Holographic QCD, AdS/CMT etc.)

Modification of GR - needs to introduce additional d.o.f.

- higher derivatives is one way of introducing additional degree of freedom

Ostrogradsky instability :

The most general form of the Lagrangian for the scalar-tensor theory having second-order field equations has been known as the **Horndeski theory**

which is classified by 4 arbitrary functions $\{G_i(\phi, X), i = 2, 3, 4, 5\}$.

$$G_i(\phi, X) = \sum C_i^{\ell m} \phi^\ell X^m$$

the **Dilaton-Einstein-Gauss-Bonnet Gravity** belongs to Horndeski theory

$$S_{dEGB} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R + \mathcal{L}_m^{rad} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + f(\phi) R_{GB}^2 \right]$$

$$f(\phi) = \alpha e^{\gamma\phi} \quad V(\phi) = 0$$

5. Summary (continued)

- We have applied the physics of WIMP decoupling to probe dEGB Cosmology scenario ($V = 0$).
- In dEGB Cosmology scenario, standard cosmology is modified
- Mass 1 GeV to 1 TeV.
- BH-NS or BH-BH mergers Indirect

Bounds

- tests of gravity within the Solar System
 - deviations of Kepler's formula for the binary-pulsar
 - dipole radiation emission from binary pulsars.
- no competitive constraints on the dEGB scenario

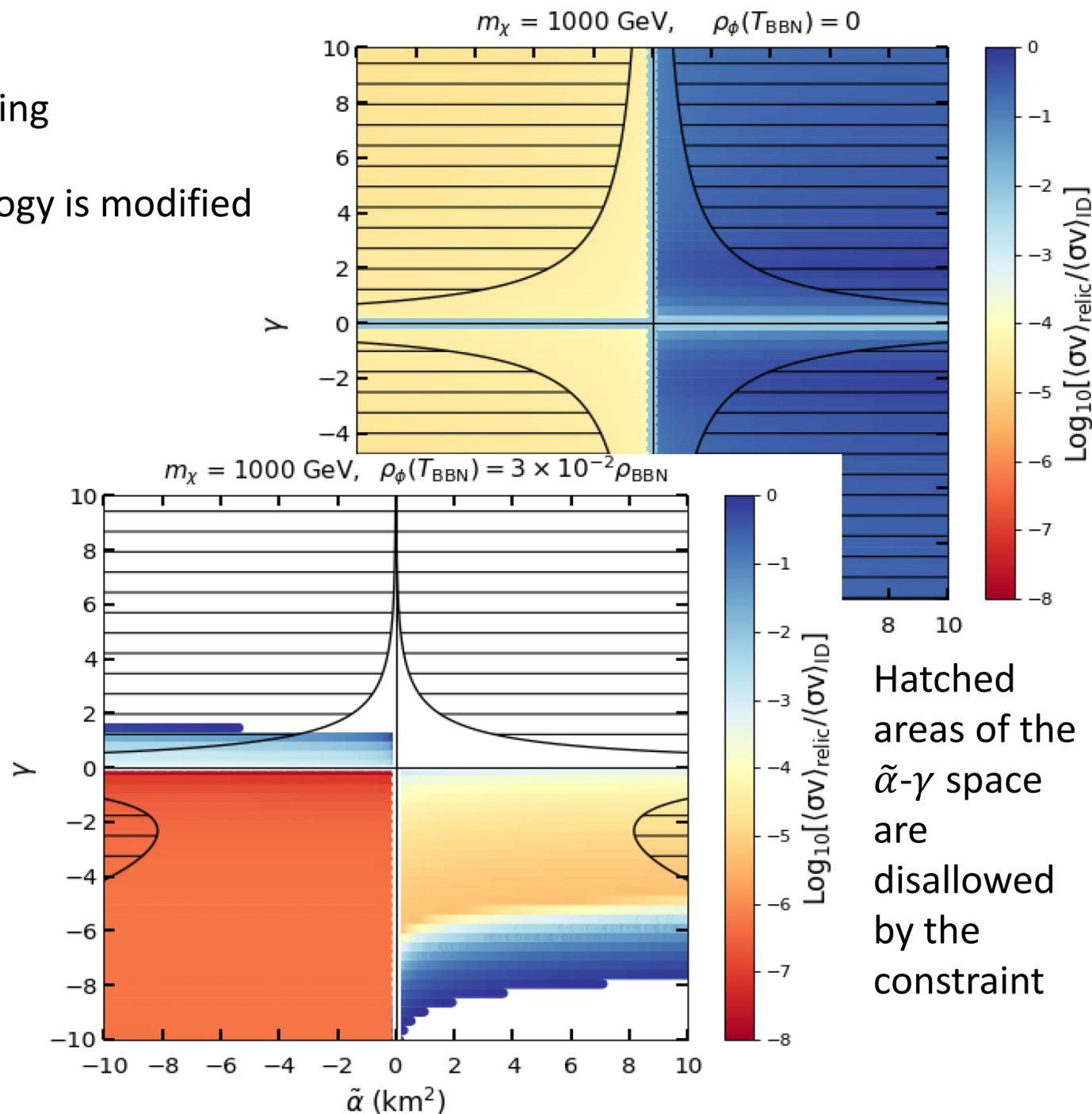
Bounds from GWs of BH-BH & BH-NS mergers

- Use the data from the LIGO-Virgo to put constraints on deviations from GR.

$$\alpha_{GB}^{1/2} \leq 1.18 \text{ km} = \alpha_{GB}^{max}$$

- If $\dot{\phi}(T_{BBN}) = 0$, $|\tilde{\alpha}\gamma| \leq \sqrt{8\pi}\alpha_{GB}^{max}$

- If $\dot{\phi}(T_{BBN}) \neq 0$, $|\tilde{\alpha}\gamma e^{\gamma \frac{\phi_{BBN}}{H_{BBN}}}| \leq \sqrt{8\pi}\alpha_{GB}^{max}$



WIMP indirect detection

- 1) For a given parameter, find $\langle \sigma v \rangle_{relic}$ of $\langle \sigma v \rangle_f$ which yields WIMP relic density $\Omega_\chi h^2 \simeq 0.12$.
- 2) Compare $\langle \sigma v \rangle_{gal} = \langle \sigma v \rangle_{relic}$ with $\langle \sigma v \rangle_{ID}$, the upper bound on the present annih cross sec in the halo of the Milky Way. (consider an s-wave annihilation cross section, for which $\langle \sigma v \rangle_{gal} = \langle \sigma v \rangle_f$.)

The favoured regions of the Gauss-Bonnet parameter space by WIMP indirect detection is given by

$$\langle \sigma v \rangle_{gal} / \langle \sigma v \rangle_{ID} \lesssim 1$$

- for the class of solutions that comply with WIMP indirect detection bounds we found that the dEGB term plays a mitigating role on the scalar field (kination) dynamics, slowing down the speed of its evolution and reducing A.
- The bounds that we found from WIMP indirect detection are nicely complementary to late-time constraints from compact binary mergers.
- It could be interesting to use other Early Cosmology processes to probe the dEGB scenario.

The standard scenario,

$$\Omega h^2 \simeq 0.12$$

(the observed CDM density) is obtained for

$$\langle \sigma v \rangle_f^{standard} \simeq 3 \times 10^{-26} cm^3 s^{-1}$$

Thank you!