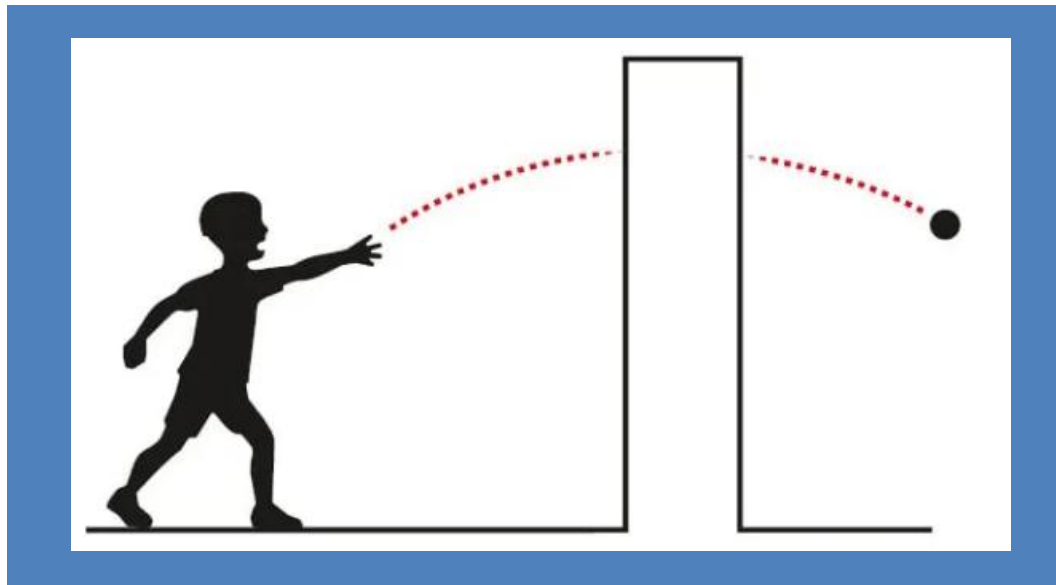


거시적 양자 투과 현상의 이해와 전망

도용주

(GIST 물리광학과)





The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics 2025 to

John Clarke

University of California, Berkeley, USA

Michel H. Devoret

Yale University, New Haven, CT and
University of California, Santa Barbara, USA

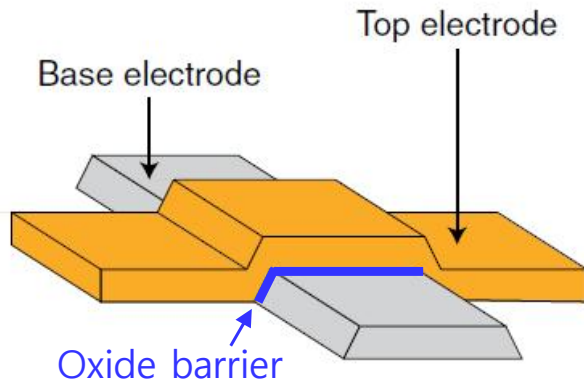
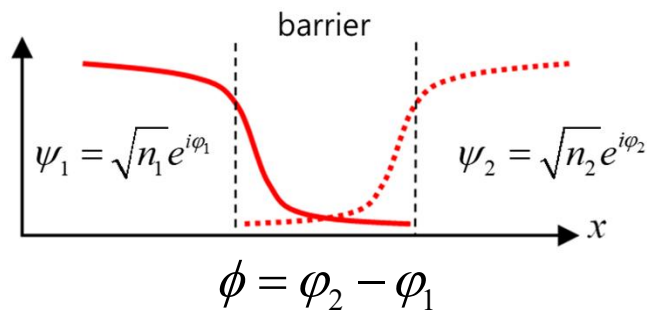
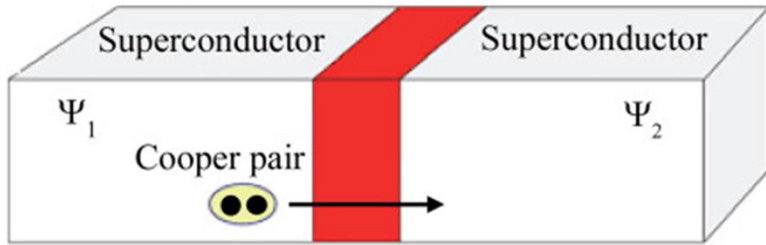
John M. Martinis

University of California, Santa Barbara
and Qolab, Los Angeles, CA, USA

“for the discovery of macroscopic quantum mechanical tunnelling and energy quantisation in an electric circuit”



SC-Insulator-SC

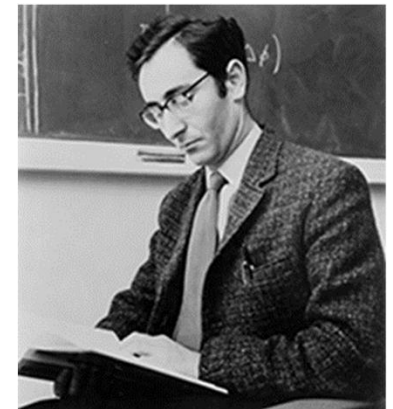


$$I_s = I_c \sin \phi \sim \text{supercurrent}^{[1]}$$

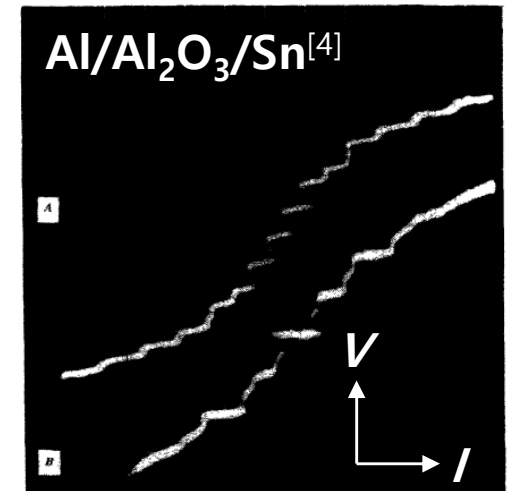
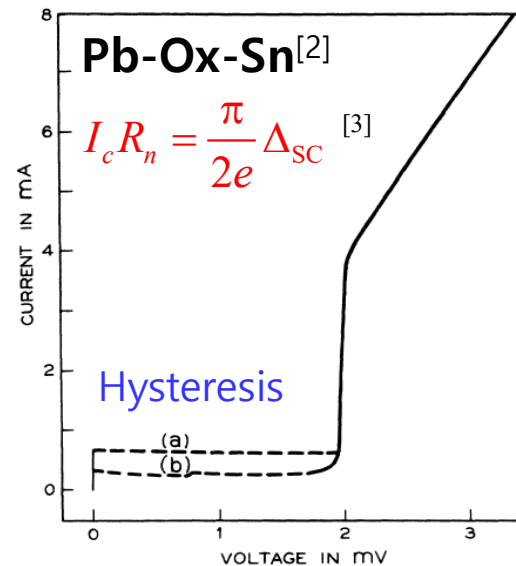
$$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$

$$I_s = I_c \sin \omega t, \quad \omega = \frac{2eV}{\hbar}$$

$$483.6 \text{ MHz} = 1 \mu\text{V}$$



B. D. Josephson
Nobel prize (1973)



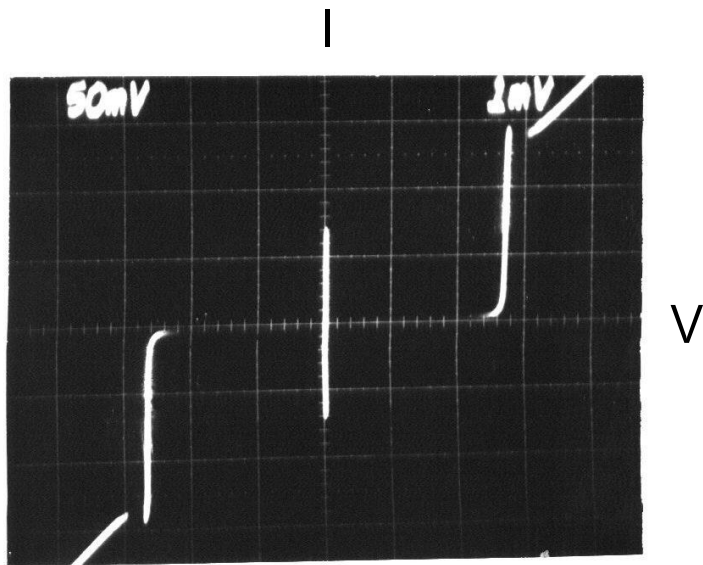
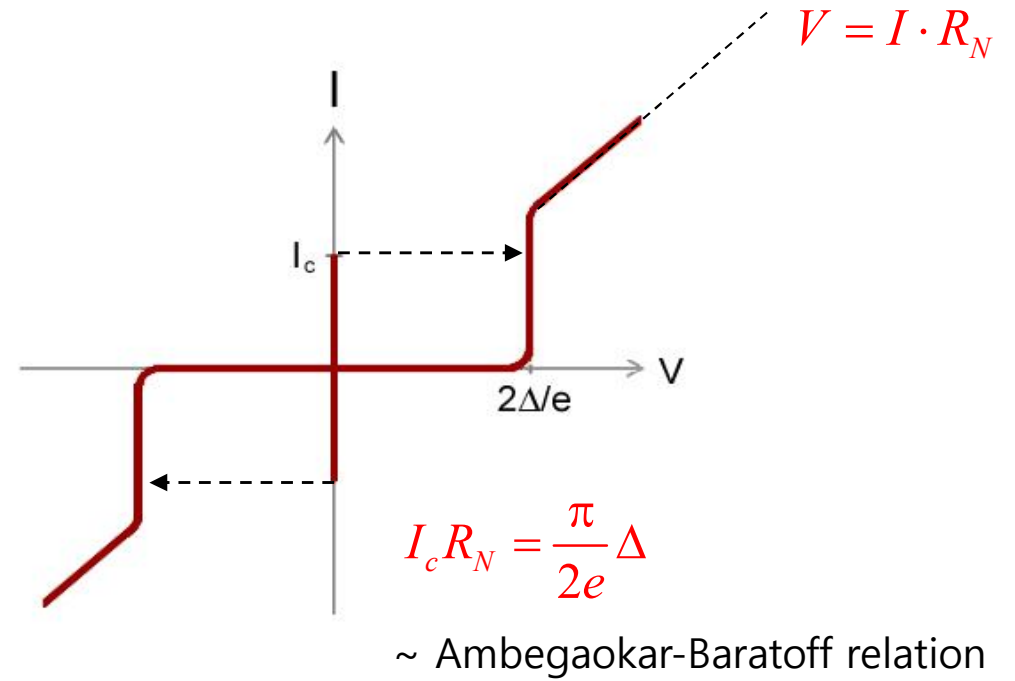
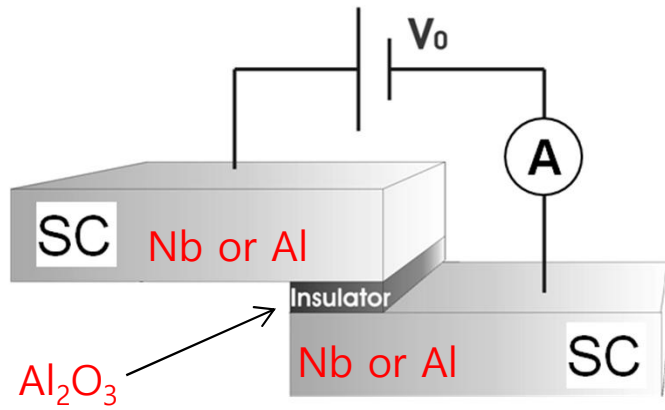
[1] B. D. Josephson, Physics Letters 1, 251 (1962). Possible new effects in superconductive tunneling.

[2] P. W. Anderson and J. M. Rowell, Phys. Rev. Lett. 10, 230 (1963). Probable observation of the Josephson SC tunneling ...

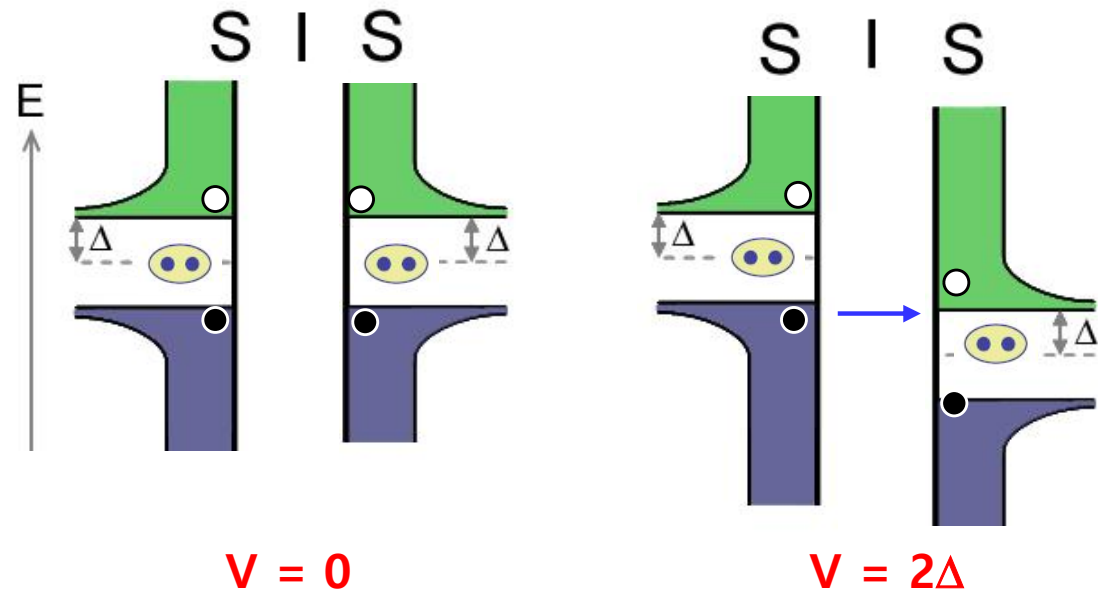
[3] V. Ambegaokar, A. Baratoff, Phys. Rev. Lett. 10, 486 (1963). Tunneling between superconductors

[4] S. Shapiro, Phys. Rev. Lett. 11, 80 (1963). Josephson currents in superconducting tunneling: the effect of microwaves ...

SIS Josephson junction



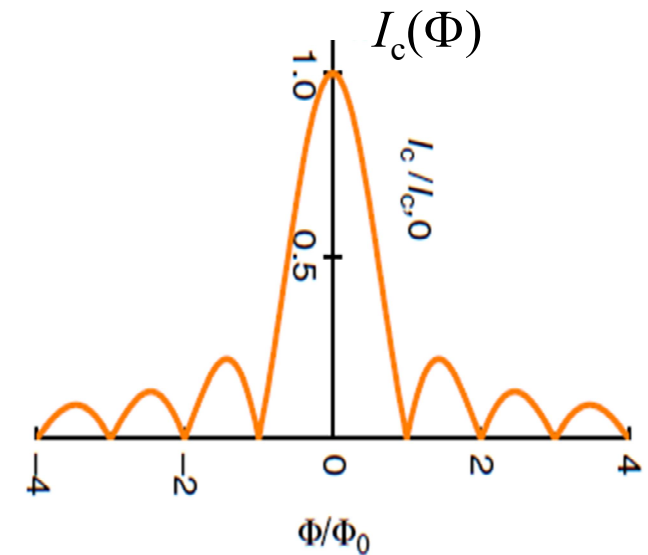
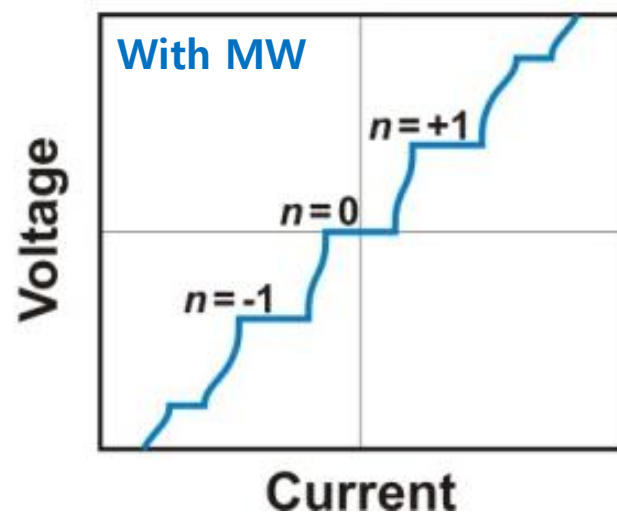
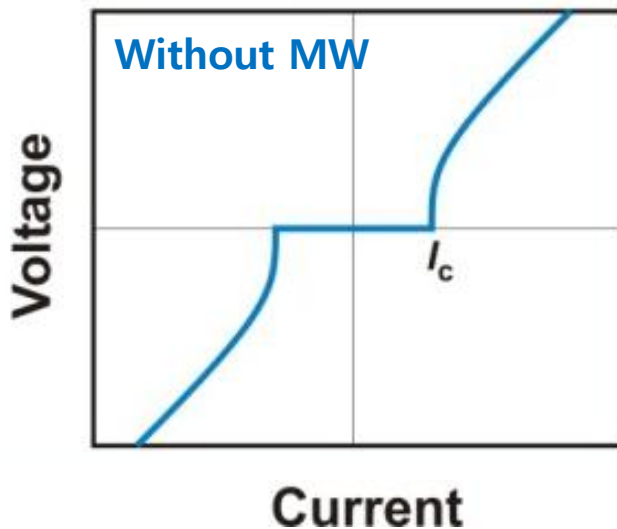
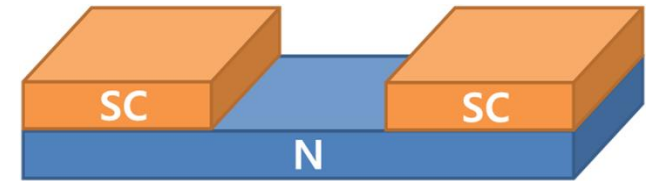
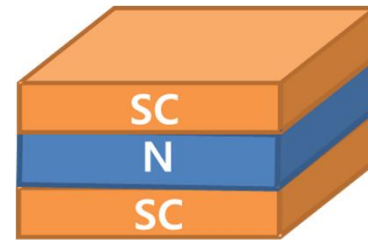
X-Y mode in oscilloscope



SNS Josephson junction ~ weak link or proximity-type junction

When two superconducting regions are separated by a thin normal region, effects similar to those considered here should occur and may be relevant to the theory of the intermediate state.

[1] B. D. Josephson, Physics Letters 1, 251 (1962).



$$I_c R_n \ll \frac{\pi}{2e} \Delta_{SC}$$

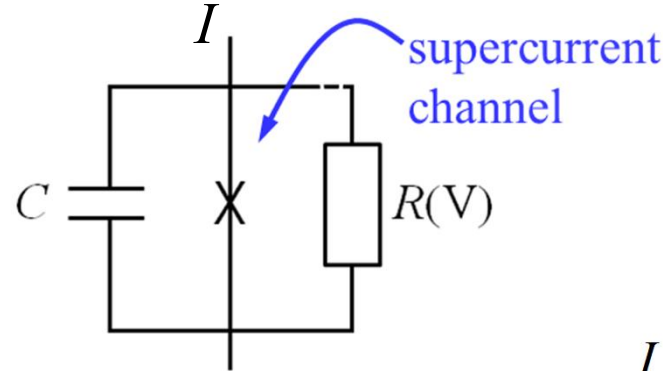
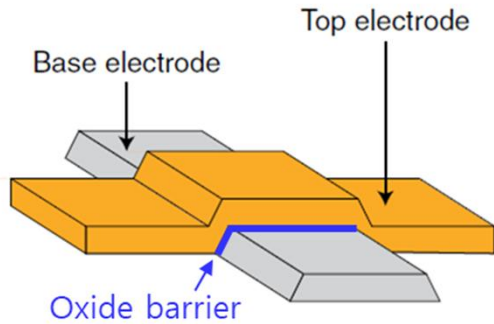
$$V_n = n \frac{hf_{MW}}{2e}, \quad n = \text{Integer}$$

No hysteresis

Fraunhofer pattern

$$\Phi_0 \equiv \frac{h}{2e} \sim \text{magnetic flux quantum}$$

Resistively and capacitively shunted junction (RCSJ) model



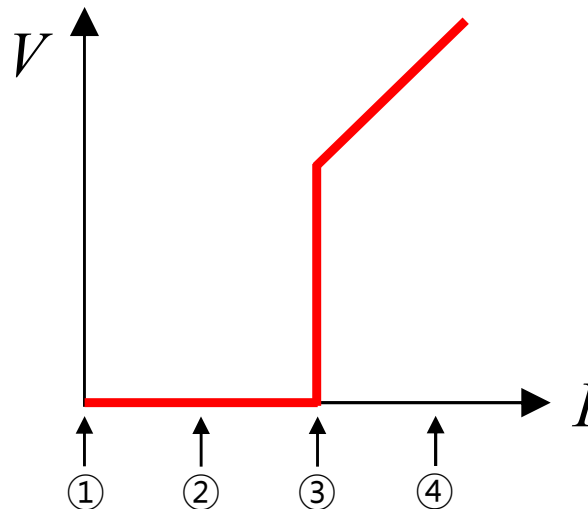
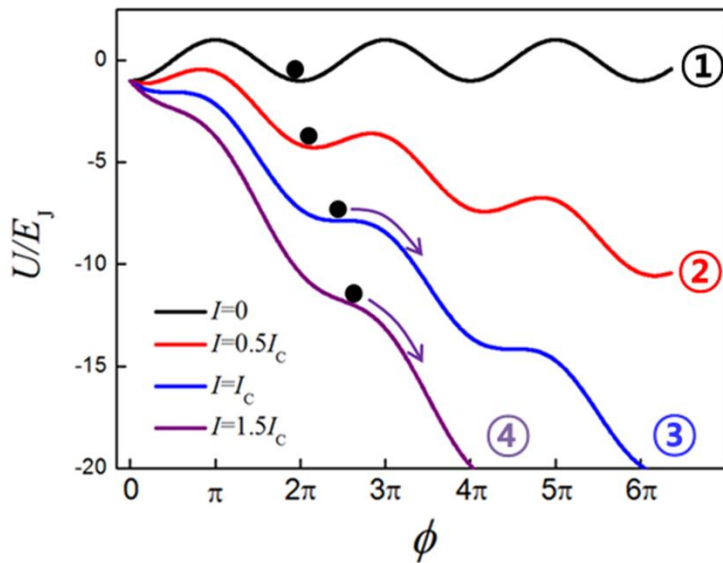
According to Kirchhoff's law,

$$I = I_c \sin \phi + \frac{V}{R} + C \frac{dV}{dt}$$

$$I_s = I_c \sin \phi, \quad \frac{d\phi}{dt} = \frac{2eV}{\hbar} \sim \text{Josephson Eq.}$$

$$\left(\frac{\hbar}{2e}\right)^2 C \frac{d^2 \phi}{dt^2} = -\frac{dU}{d\phi} - \left(\frac{\hbar}{2e}\right)^2 \frac{1}{R} \frac{d\phi}{dt}$$

$$\begin{cases} U(\phi) = -E_J \cos \phi - \frac{\hbar I}{2e} \phi \sim \text{washboard potential well} \\ E_J = \frac{\hbar I_c}{2e} \sim \text{Josephson coupling energy} \end{cases}$$



(a) Tilted washboard potential for a phase particle.

(b) Schematic current-voltage characteristics.

$$\left(\frac{\hbar}{2e}\right)^2 C \frac{d^2\phi}{dt^2} = -\frac{dU}{d\phi} - \left(\frac{\hbar}{2e}\right)^2 \frac{1}{R} \frac{d\phi}{dt}$$

In a dimensionless form,

$$i_{sc}(\phi) + \frac{d\phi}{d\tau} + \beta_c \frac{d^2\phi}{d\tau^2} = i_{dc} + i_{ac} \sin(\Omega\tau)$$

$$i_k = I_k / I_c \quad (k = sc, dc, ac)$$

$$\tau = 2eI_c R_n t / \hbar$$

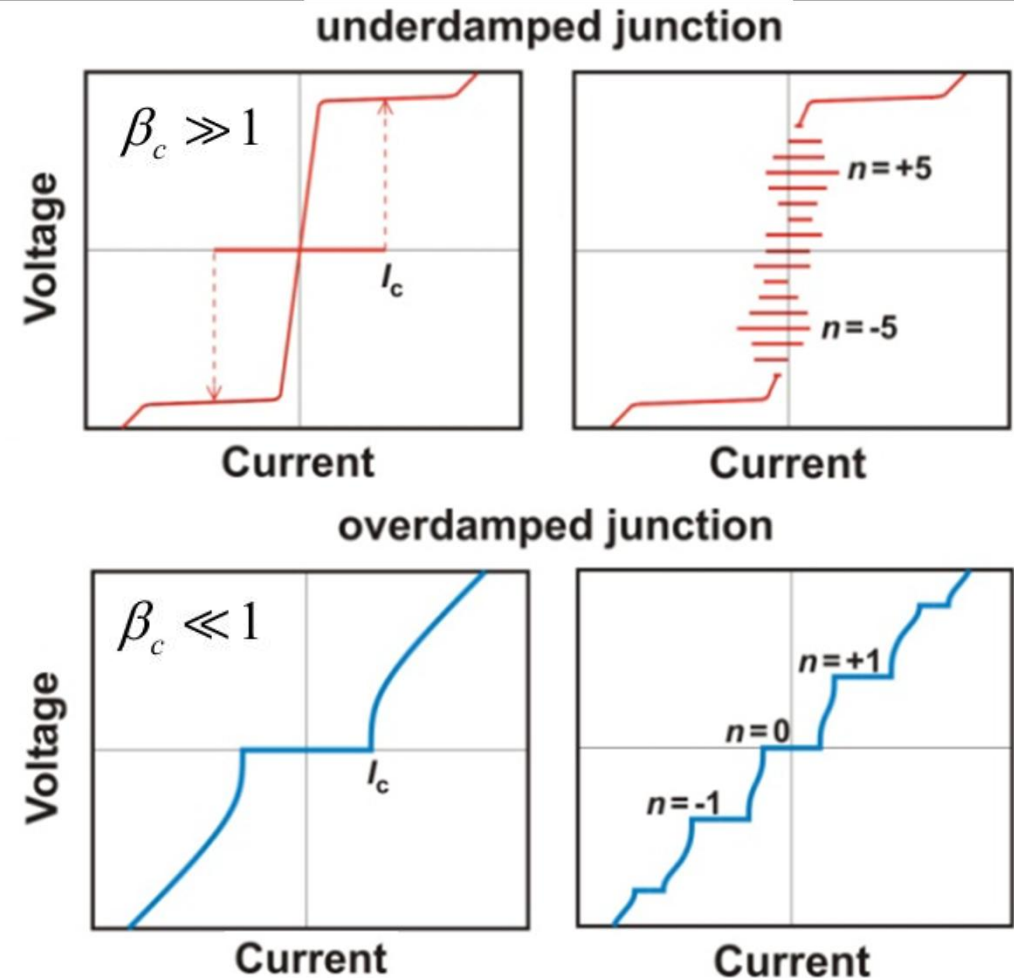
$$\Omega = f / f_c = hf / 2eI_c R_n$$

$$f_c = 2eI_c R_n / h$$

$$\beta_c = 2eI_c R_n^2 C / \hbar \sim \text{Stewart-McCumber parameter}^{[1,2]}$$

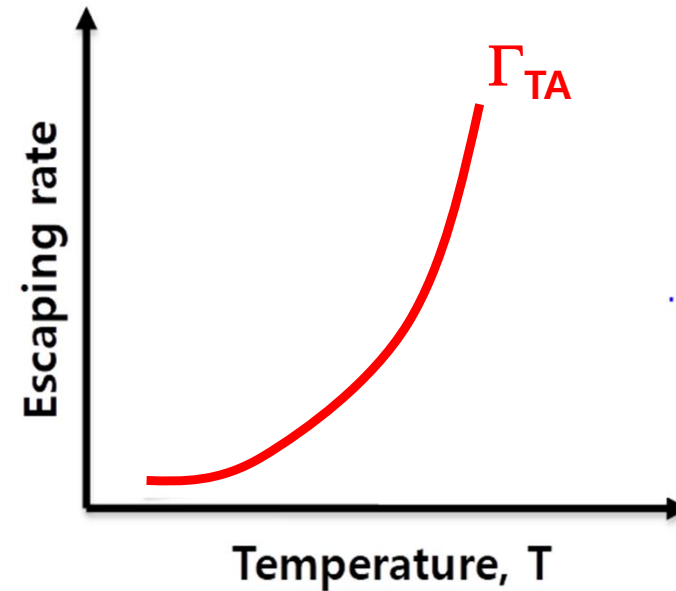
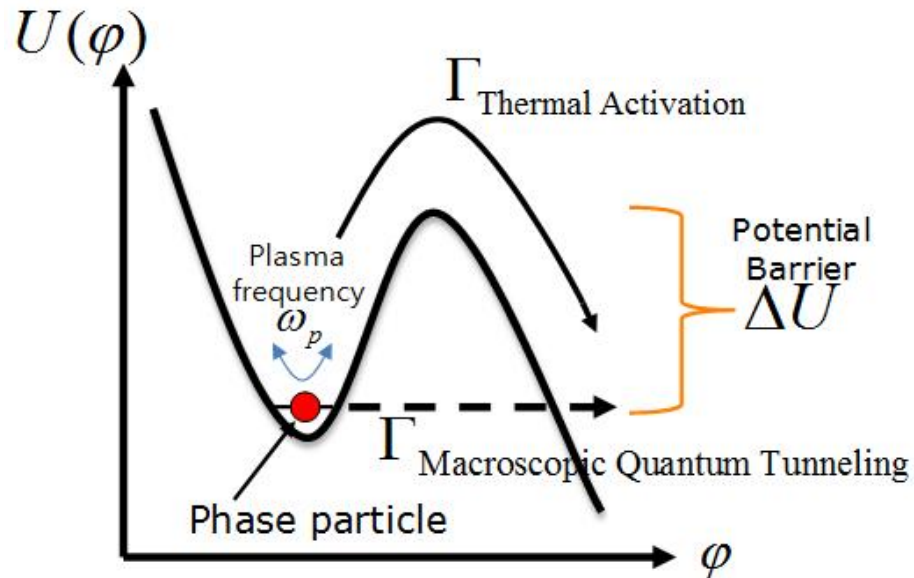
$\beta_c \gg 1$: underdamped JJ with hysteretic I - V curve ~ SIS JJ

$\beta_c \ll 1$: overdamped JJ with nonhysteretic I - V curve ~ SNS JJ



[1] W. C. Stewart, Appl. Phys. Lett. 12, 277 (1968). Current-voltage characteristics of Josephson junctions.

[2] D. E. McCumber, J. Appl. Phys. 39, 3113 (1968). Effect of ac impedance on dc voltage-current characteristics of superconductor weak-link junctions



TA Escaping rate:

$$\Gamma_{TA} = a_t(\omega_p/2\pi)\exp(-\Delta U/k_B T_{esc}) \sim \text{thermal activation mechanism}^{[1]}$$

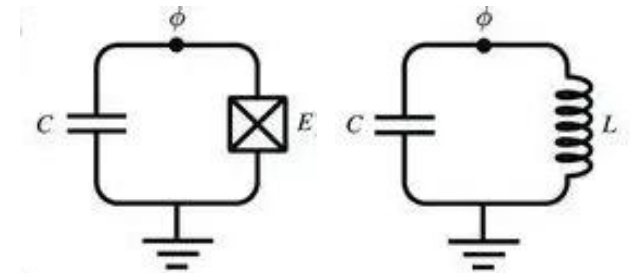
$$\omega_p = \omega_{p0}(1 - \gamma^2)^{1/4}$$

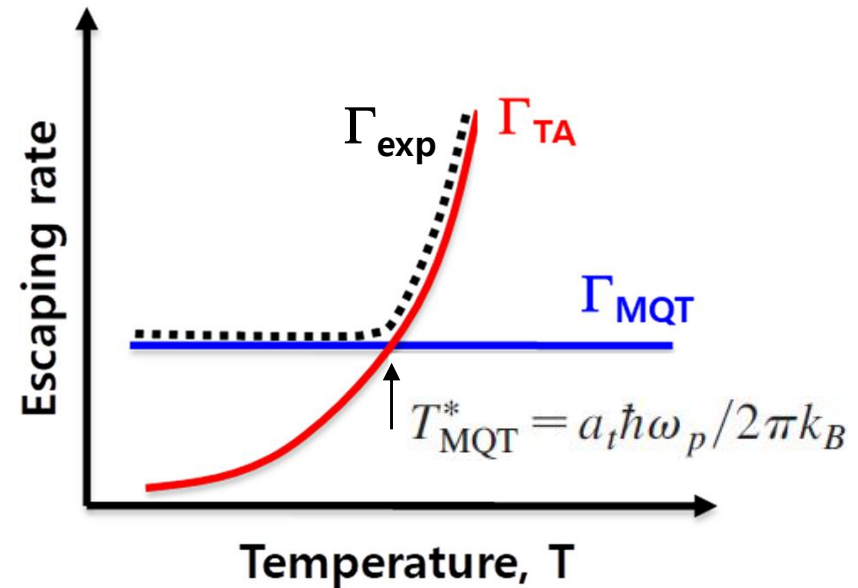
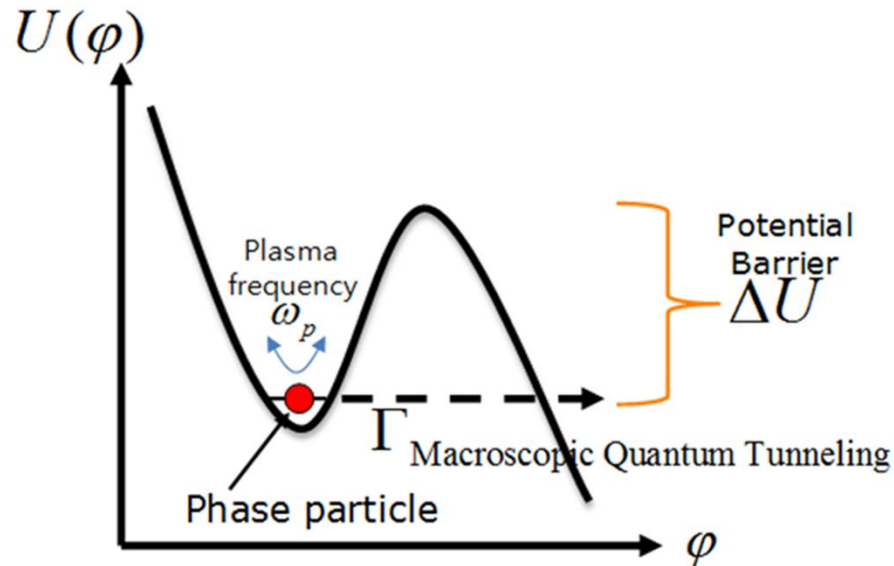
$$\omega_{p0} = (2eI_{c0}/\hbar C)^{1/2} \sim \text{Josephson plasma frequency}$$

$$\Delta U(\phi) = 2E_{J0}[(1 - \gamma^2)^{1/2} - \gamma \cos^{-1} \gamma] \text{ with } \gamma = I/I_{c0},$$

$$a_t = (1 + 1/4Q^2)^{1/2} - 1/2Q$$

$$Q = Q_0 [1 - \gamma^2]^{1/4}, \quad Q_0 = 4I_{c0}/\pi I_{r0} = R_n C \omega_{p0}, \quad \beta_c = Q_0^2 = 2eI_{c0} R_n^2 C / \hbar$$





MQT Escaping rate:

$$\Gamma_{MQT} = 12\omega_p (3\Delta U / \hbar\omega_p)^{1/2} \exp[-7.2(1 + 0.87/Q)\Delta U / \hbar\omega_p] \sim \text{MQT model}^{[1-2]}$$

“Tunneling rate is suppressed in a macroscopic and dissipative system”^[1]

$$\omega_p = \omega_{p0}(1 - \gamma^2)^{1/4}$$

$$\omega_{p0} = (2eI_{c0}/\hbar C)^{1/2} \sim \text{Josephson plasma frequency}$$

$$\Delta U(\varphi) = 2E_{J0}[(1 - \gamma^2)^{1/2} - \gamma \cos^{-1}\gamma] \text{ with } \gamma = I/I_{c0},$$

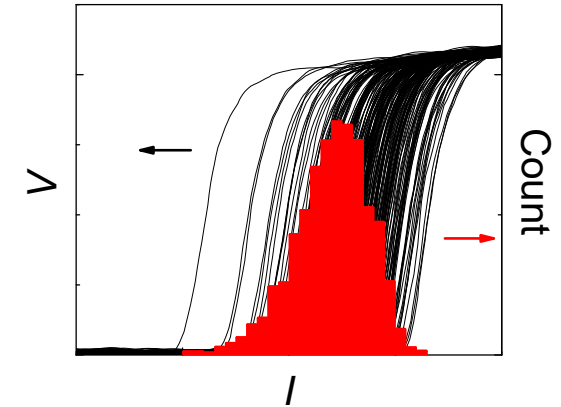
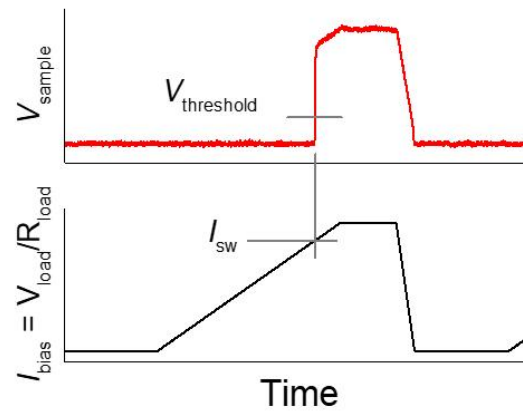
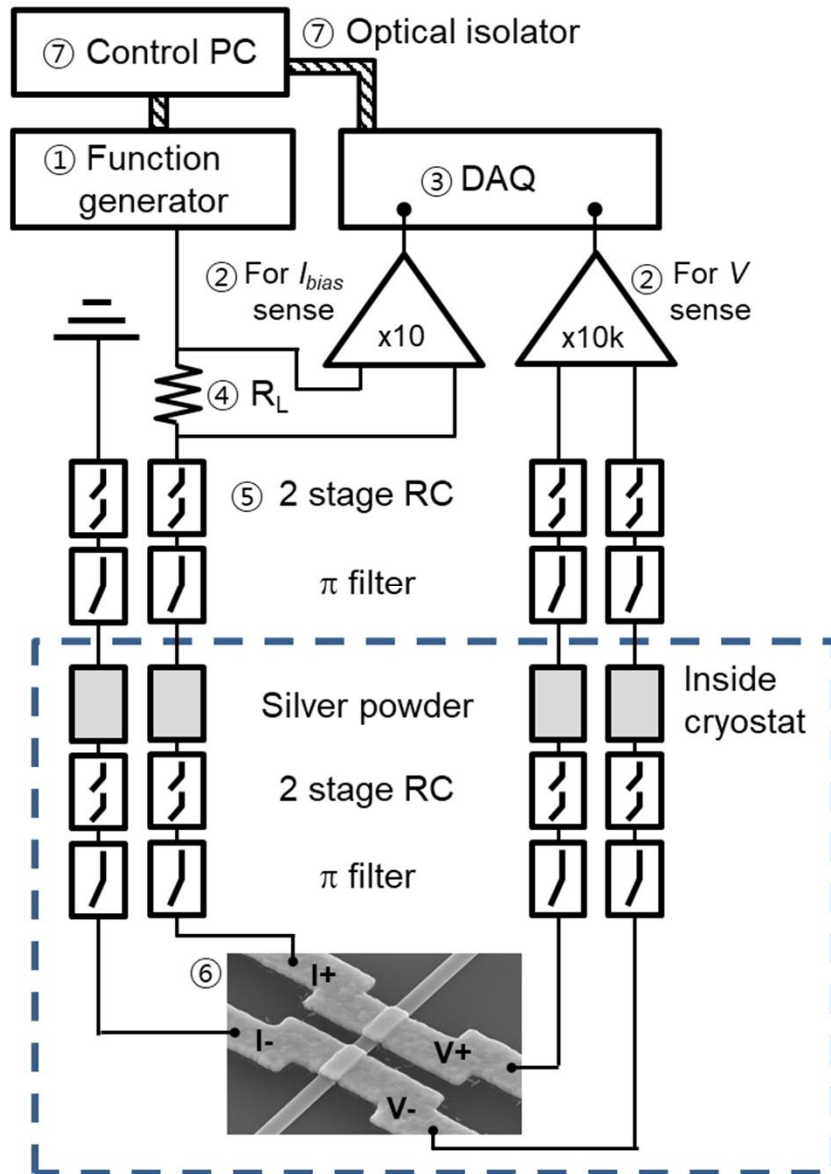
$$a_t = (1 + 1/4Q^2)^{1/2} - 1/2Q$$

$$Q = Q_0 [1 - \gamma^2]^{1/4}, \quad Q_0 = 4I_{c0}/\pi I_{r0}, \quad \beta_c = Q_0^2$$

No temperature dependence!

[1] A. O. Caldeira, A. J. Leggett, Annals of Physics 149, 374 (1983). Quantum tunneling in a dissipative system

[2] M. H. Devoret, J. M. Martinis, J. Clarke, Phys. Rev. Lett. 55 1908 (1985). Measurements of MQT out of the zero-voltage state of a current-biased JJ



Phase particle escape rate conversion

- Fulton-Dunkleberger (F-D) relation:

$$\Gamma(k\Delta I) = \frac{dI/dt}{\Delta I} \ln \frac{\sum_{j \geq k} N(I_j)}{\sum_{i \geq k+1} N(I_i)}$$

- i, j, k integer ΔI : bin size, $N(I_i)$: event count in the i^{th} bin

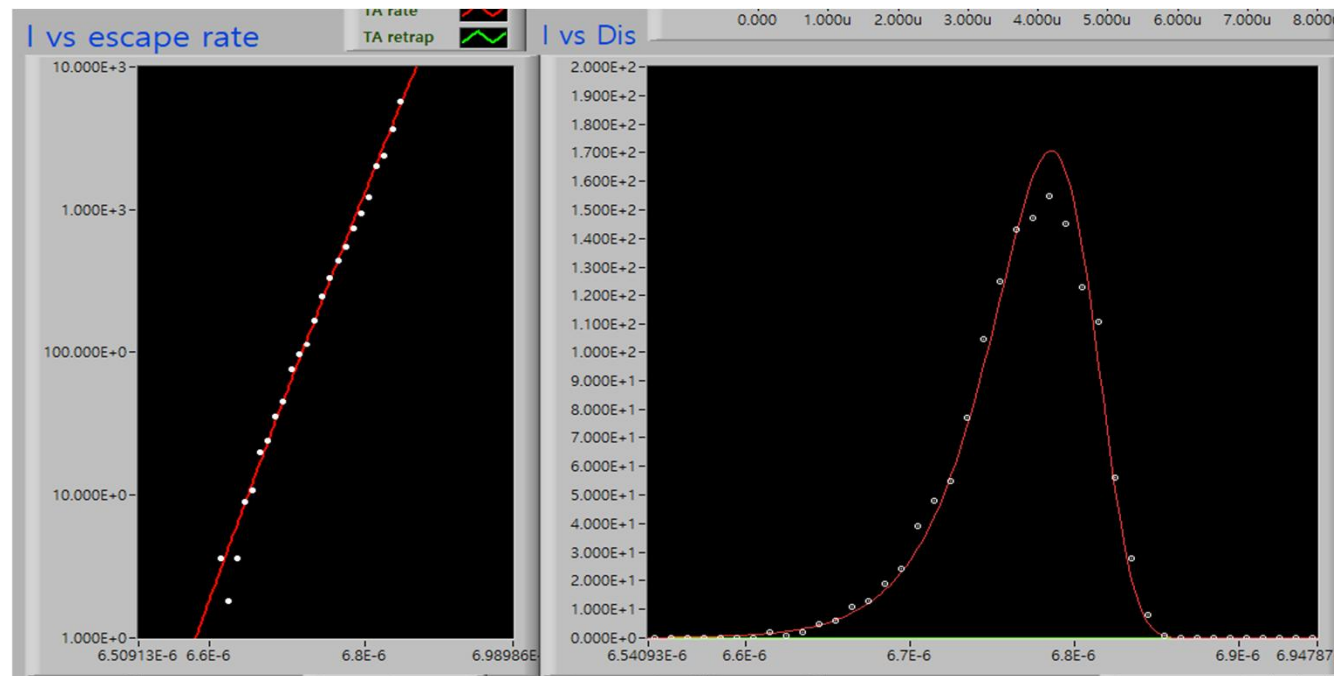
T. A. Fulton, L. N. Dunkleberger, Phys. Rev. B, 9, 4760-4768 (1974)

Switching current probability

- Inverse F–D relation: model $\Gamma(I) \rightarrow$ predicted $P(I)$

$$P(I) = \frac{\Gamma(I)}{dI/dt} \exp \left[-\frac{1}{dI/dt} \int_0^I \Gamma(I') dI' \right]$$

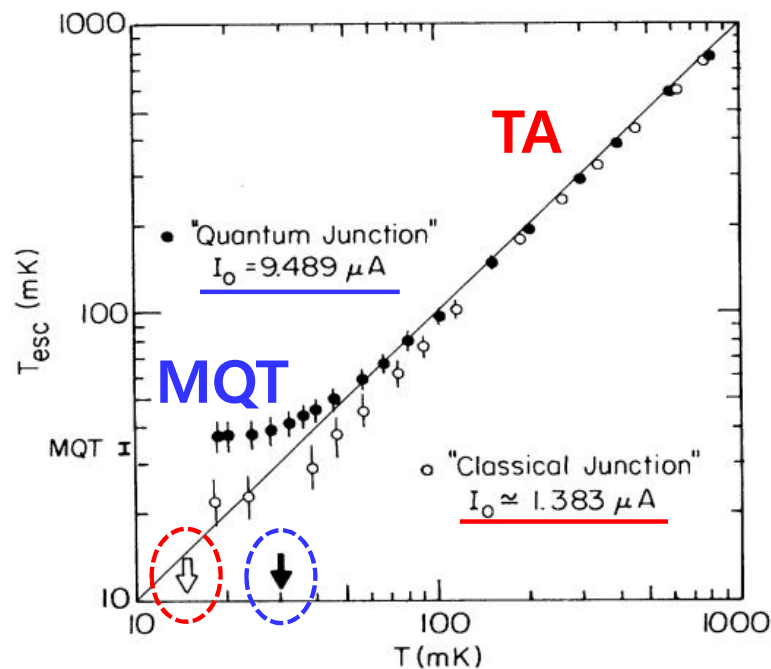
- Scaling $P(I)$ by the area under the measured $N(I)$ curve
- $N(I) = P(I) \int N(I') dI' = P(I) \Delta I N_{\text{total}}$



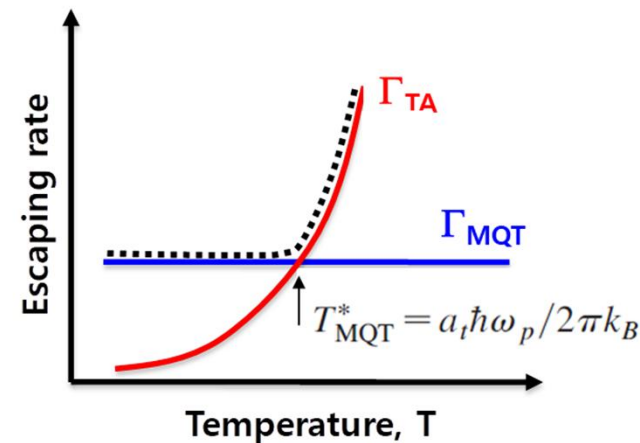
1. Define a search grid over parameter set $\{I_{c0}, I_{r0}, T_{\text{escape}}, dI/dt, C\}$
2. Compute predicted $\Gamma(I)$ and $N(I)$ for each parameter combination
3. Compare to measured $\Gamma(I)$ and $N(I)$ curves
4. Optimize parameters by minimizing χ^2 on the $N(I)$ distribution

Free parameters by model

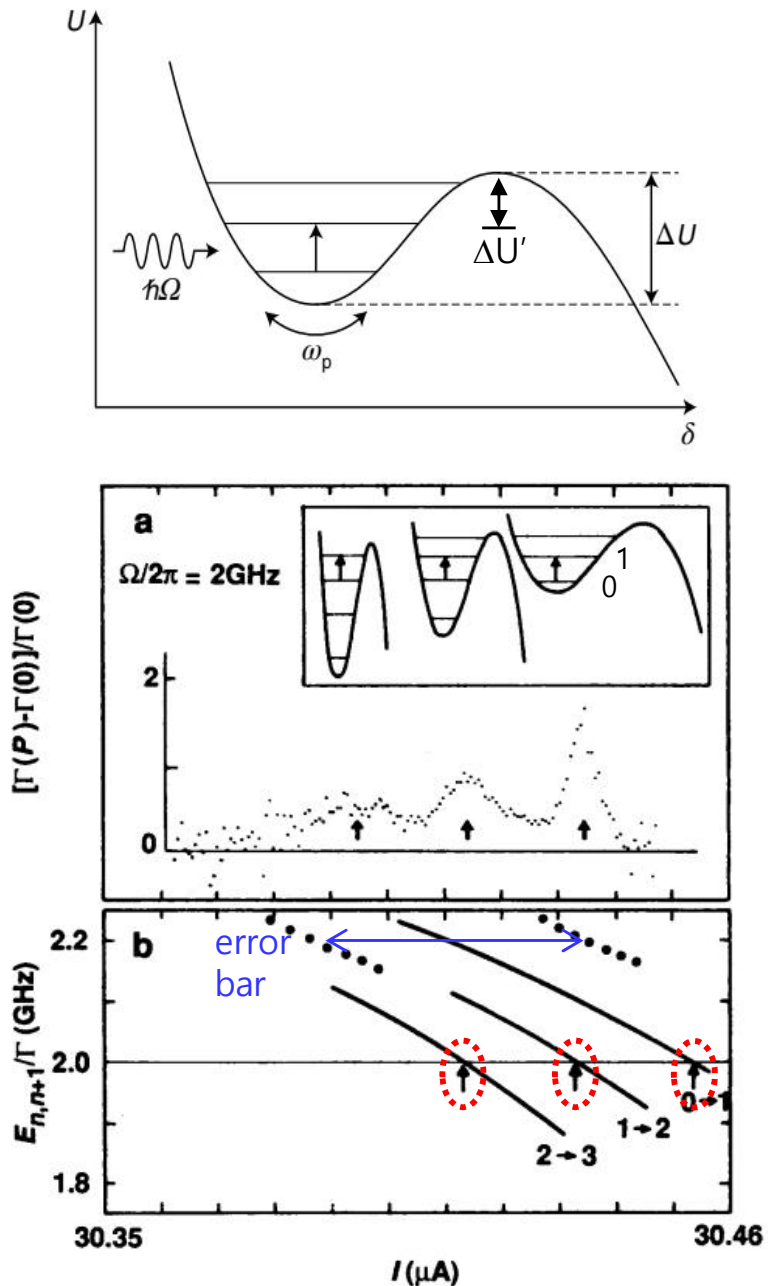
- MQT : C, I_{c0}
- TA : $C, I_{c0}, T_{\text{esc}}$,
- TAPD : $C, I_{c0}, I_{r0}, T_{\text{esc}}$



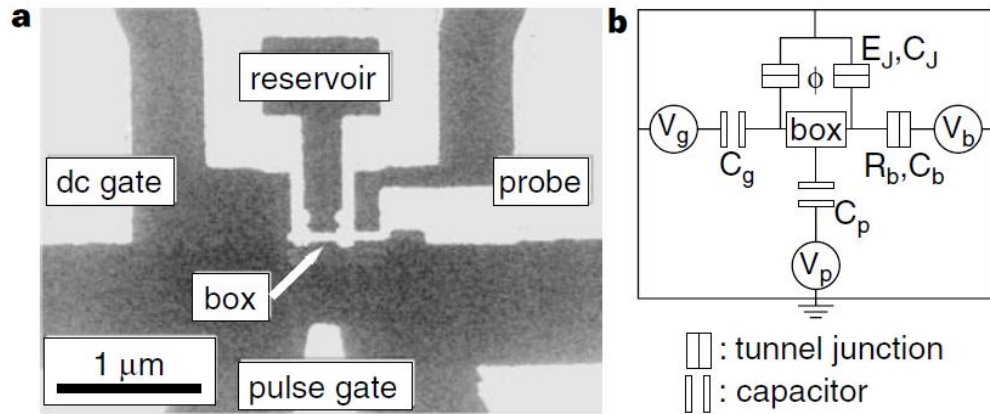
$10 \times 10\text{-}\mu\text{m}^2$ Nb-NbO_x-PbIn tunnel junction



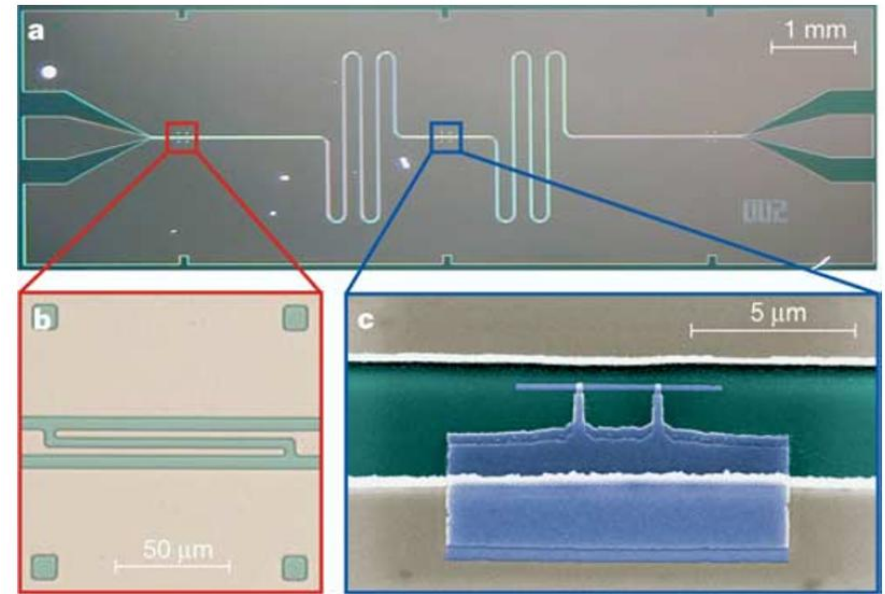
- Crossover temperature from TA to MQT regimes: $T_{\text{MQT}}^* = a_t \hbar \omega_p / 2\pi k_B \propto \sqrt{I_{c0}}$
 $\omega_{p0} = (2eI_{c0}/\hbar C)^{1/2} \sim$ Josephson plasma frequency
- Saturation of the escape temperature, $T_{\text{MQT}}^* = 30 \text{ mK}$, is observed
- Two arrows \sim Two distinct escape temperatures obtained in a single JJ by tuning its critical current with magnetic fields \sim flattening of T_{esc} arises from macroscopic quantum tunneling rather than from any spurious noise source.



- Cubic potential U vs. phase difference δ showing three quantized energy levels.
- Transition from the ground state to the first excited state is induced by a photon of frequency $\omega/2\pi$.
- Reduced barrier height and width at the excited state enhance the tunneling rate Γ .
- Irradiated with microwave ($f_{\text{MW}} = 2.0$ GHz), the JJ exhibits resonant peaks in the $\Gamma(I)$ plot.
- As the bias current increases, the washboard potential becomes more tilted and the quantized energy level spacing within the washboard potential decreases.
- The resonant peak occurs when the f_{MW} coincides with the level spacing between two energy levels.
- The first observation of quantized energy levels for a macroscopic variable, i.e. the Josephson phase difference

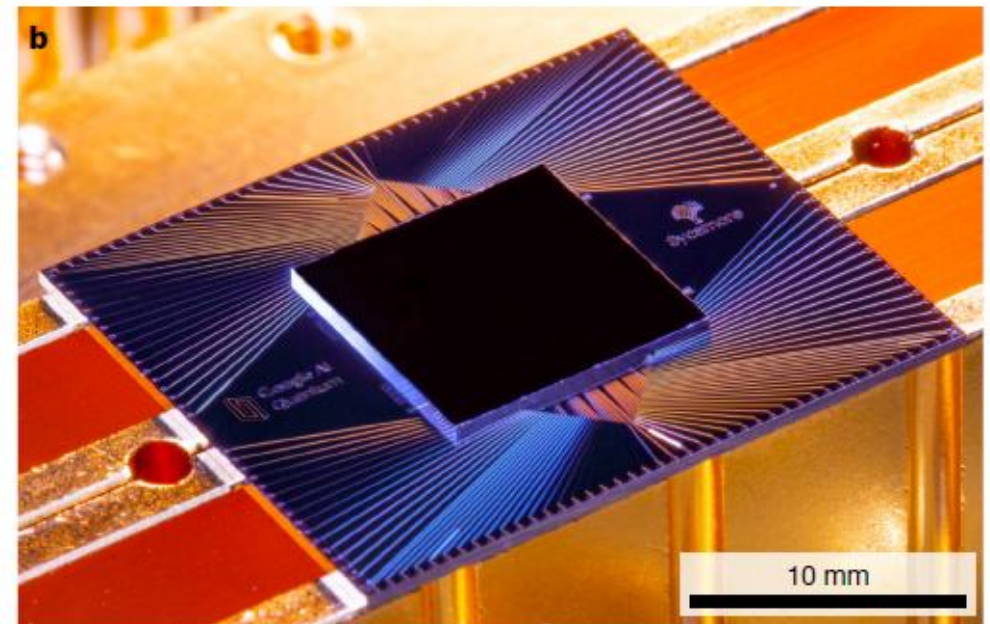


Y. Nakamura, Yu. A. Pashkin, J. S. Tsai, Nature 398, 786 (1999).

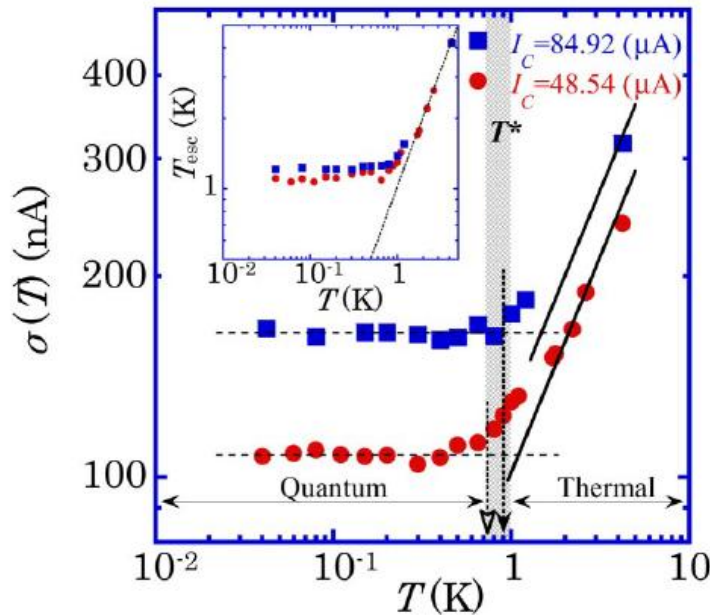
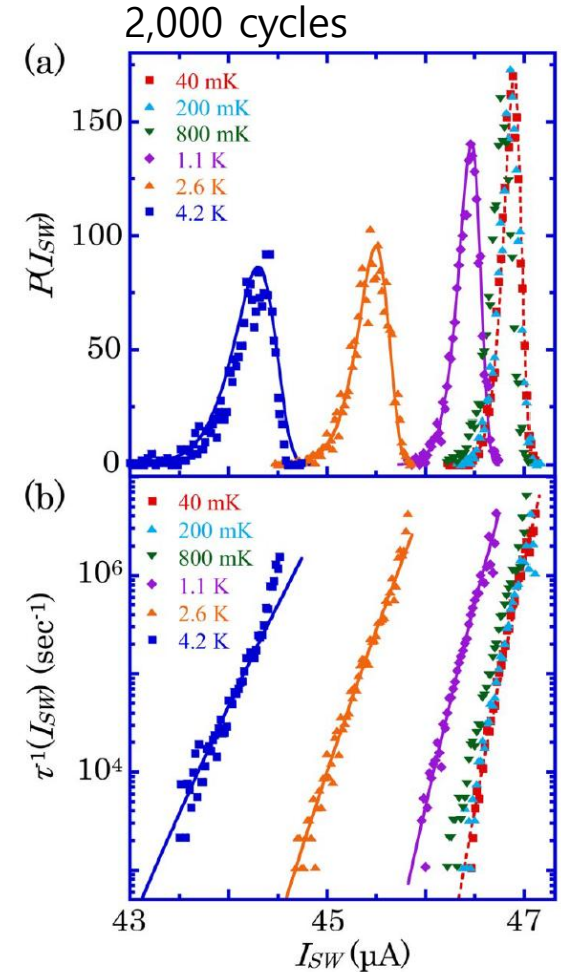
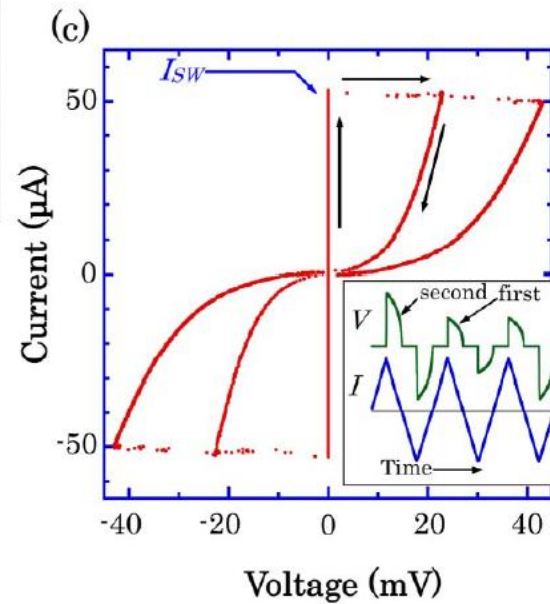
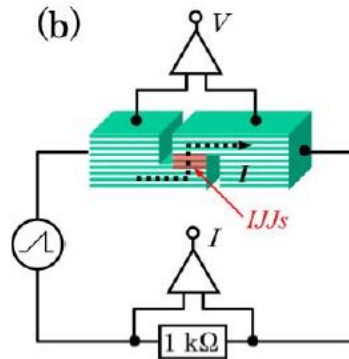
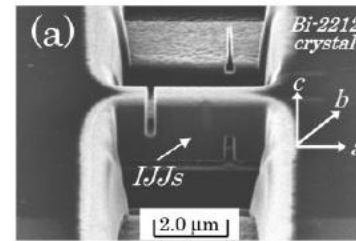
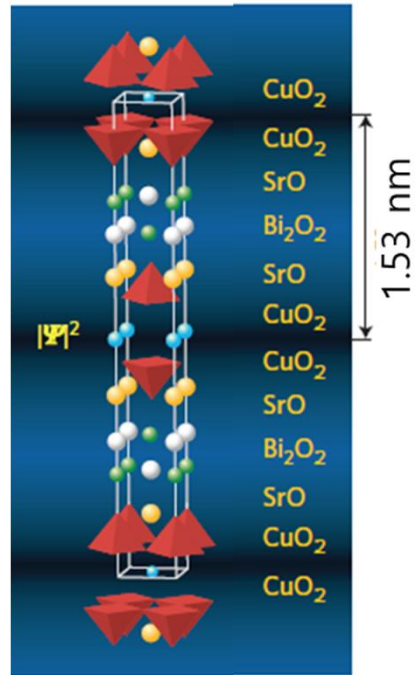


A. Wallraff et al., Nature 431, 162 (2004).

<p>Full-Stack (End-to-End)</p> <p>Google</p> <p>IBM</p> <p>Microsoft</p> <p>amazon</p> <p>Honeywell</p> <p>rigetti</p> <p>Alibaba Group</p> <p>D:WAVE</p> <p>XANADU</p>	<p>Software Applications</p> <p>RIVERLANE, Menten AI, ZAPATA, acWARE, MULTIVERSE, QUANTUM, QBITLOGIC, QRITHM, QULAB</p>
	<p>Cloud Computing agnostiq</p> <p>IQBit, BraneCell, Aliro QUANTUM, BLACK DRAGON</p>
	<p>Quantum Encryption and AI</p> <p>ISARA, agnostiq, Post-Quantum, SPEQTRAL, SHIELD, IDQ, ZY4</p>
	<p>Systems & Firmware</p> <p>IQBit, Q-CTRL, Aliro QUANTUM, qib quantum benchmark, Labber QUANTUM, QM QUANTUM MACHINES, QILIMANJARO, QINDOM, STRANGE WORKS</p>
	<p>Quantum Hardware</p> <p>ColdQuanta, qci, IONQ, PsiQ, EeroQ>, QUANTUM FACTORY, QUANDELA, IQM, Quantum Microwave</p>



F. Arute et al., Nature 574, 505 (2019).

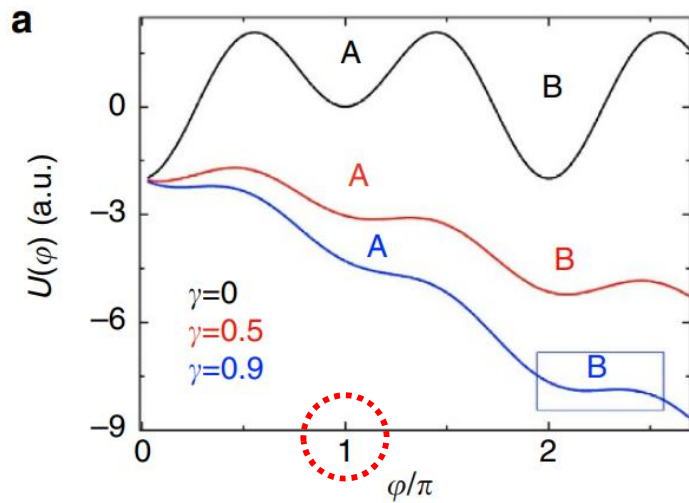


d-wave superconductor

Meanwhile, based on the uncertainties of I_C and C , and the bias dependence of T^* as obtained from the formula $T^* \sim \hbar\omega_p/2\pi k_B$ [12,27] is 0.75–0.80 K for $I_C = 48.54 \mu\text{A}$ and 0.85–1.0 K for $84.92 \mu\text{A}$. These results agree well with the experimental values of T^* 's.

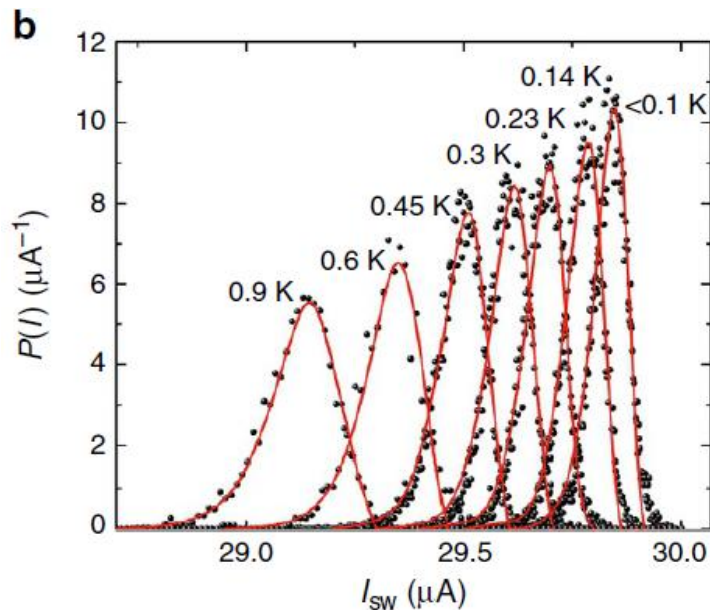
[1] K. Inomata et al., Phys. Rev. Lett. 95, 107005 (2005). "MQT in a d-wave high-Tc Bi-2212 superconductor"

[2] X. Y. Jin et al., Phys. Rev. Lett. 96, 177003 (2006). "Enhanced MQT in Bi-2212 IJJ stacks"



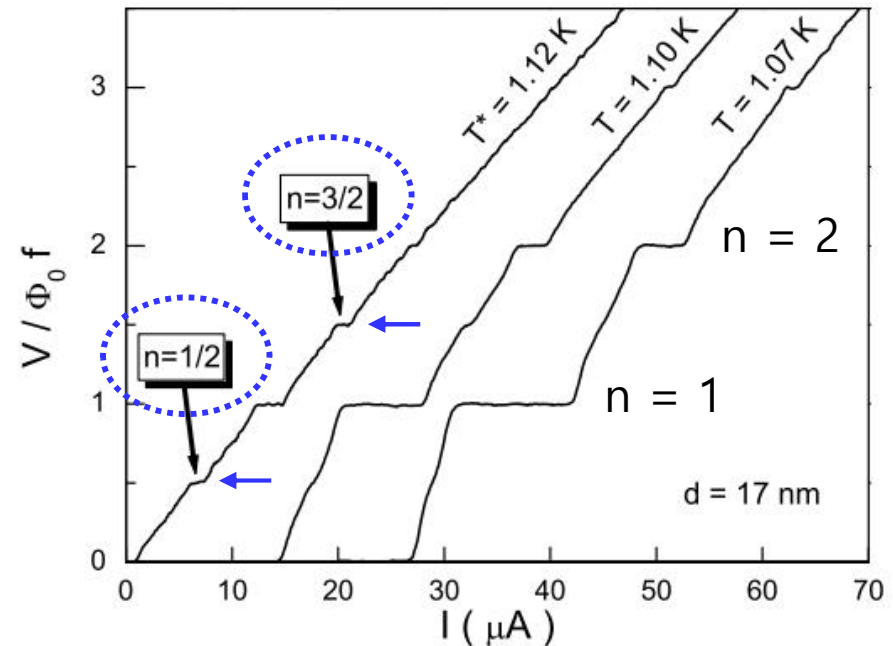
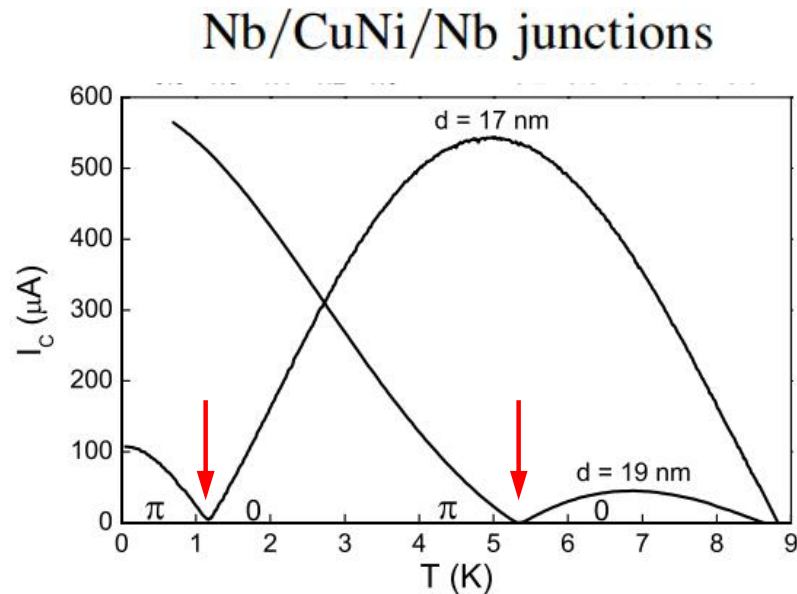
- NbN-GdN-NbN (SFS) junction
- GdN ~ ferromagnetic insulator resulting in a spin filter effect
- The **current-phase relation** is modified to form a **double-well potential**

The second harmonic component in the CPR, $I = I_1 \sin\varphi + I_2 \sin 2\varphi$, leads to a modified washboard potential $U(\varphi) = -E_1(\cos\varphi + \frac{g}{2}\cos 2\varphi + \frac{I_{c0}}{I_1}\gamma\varphi)$, $E_1 = \hbar I_1/2e$, which may assume the form of a double well for values of $g = I_2/I_1$ larger



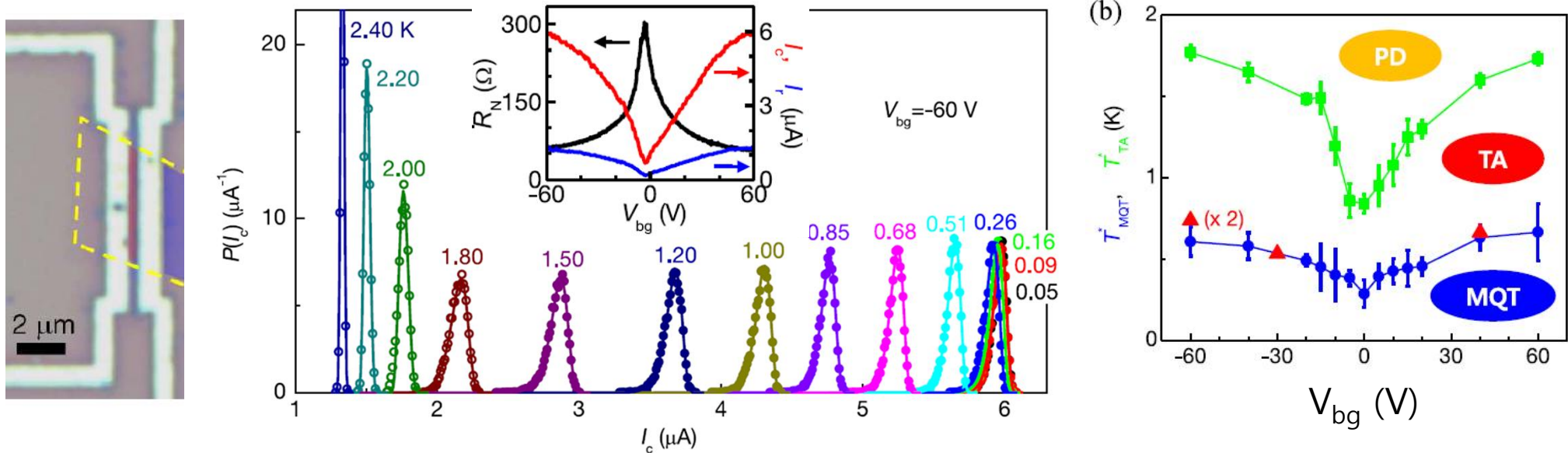
- Observation of MQT below 100 mK
- **Unimodal switching current distributions** (SCDs) instead of bimodal SCD expectation ~ #Fully caused by π -periodic term?

Half-Integer Shapiro Steps at the $0-\pi$ Crossover of a Ferromagnetic Josephson Junction

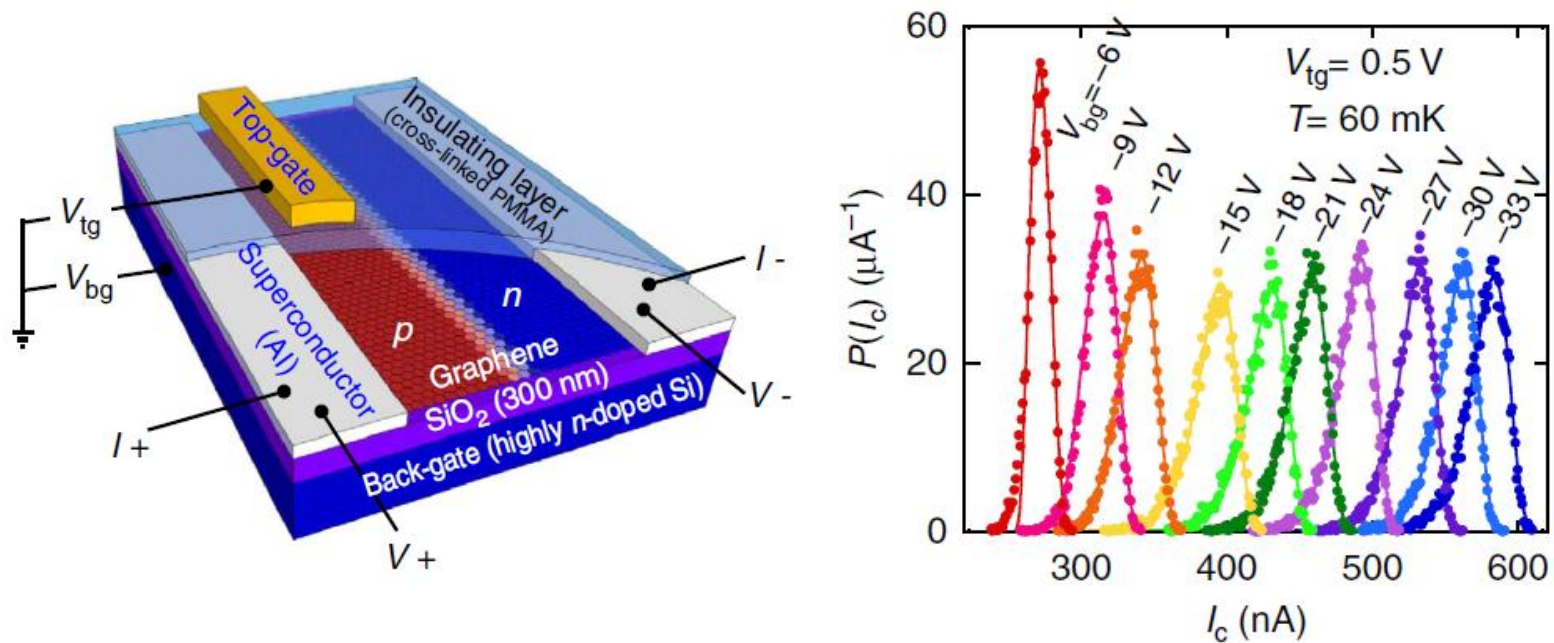


The second harmonic term in the CPR, $I_c = I_1 \sin(\phi) + I_2 \sin(2\phi)$ results in the **half-integer Shapiro steps**

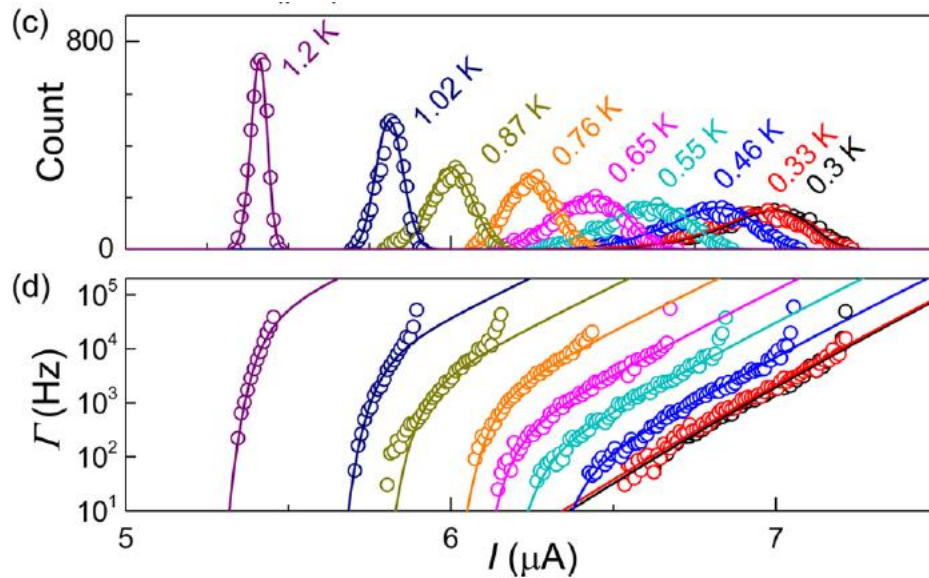
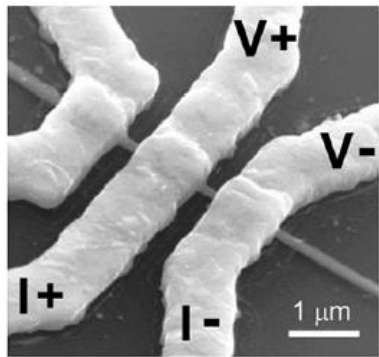
Higher temperature? Full scan of microwave power?



[1] G.-H. Lee, YJD et al., Phys. Rev. Lett. 107, 146605 (2011). "Electrically tunable MQT in a graphene-based JJ"

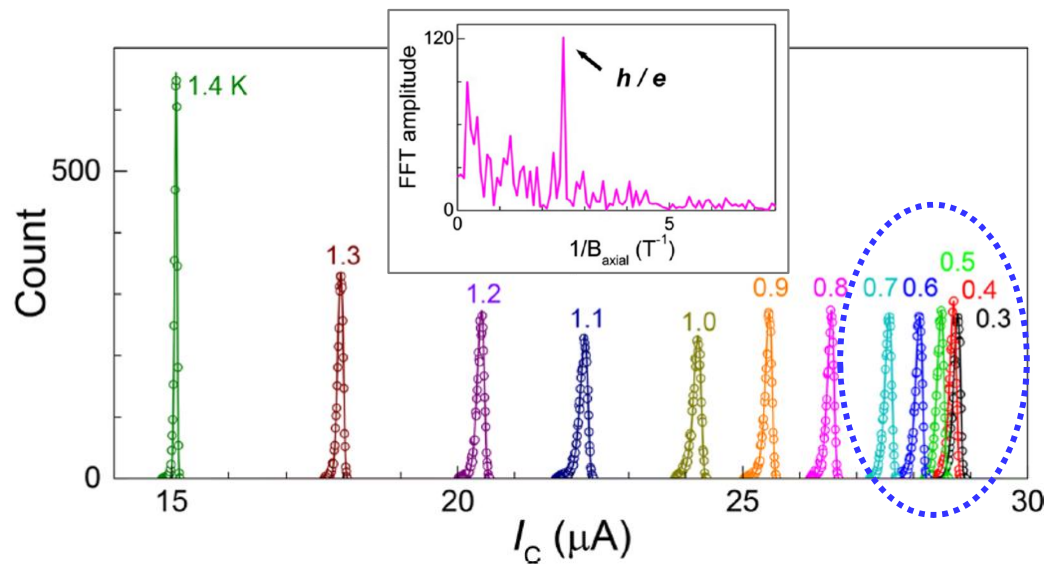
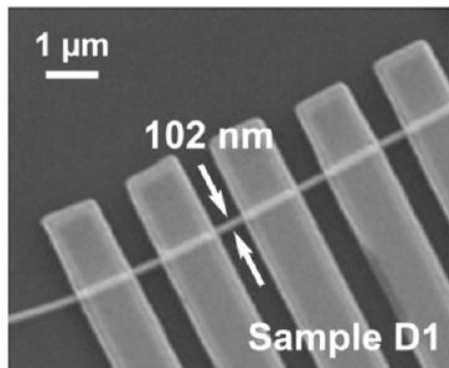


[2] J.-H. Choi, YJD et al., Nat. Comm. 4, 2525 (2013). "Complete gate control of supercurrent in graphene p-n junctions"



- PbIn – PbS – PbIn JJ's [1]
- No MQT down to $T = 0.3$ K

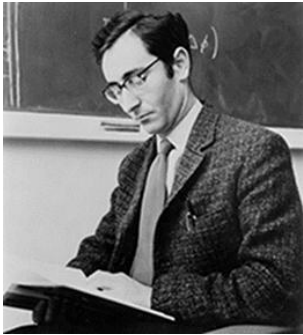
[1] B.-K. Kim, YJD et al., ACS Nano 11, 221 (2017). "Strong superconducting proximity effects in PbS semiconductor nanowires"



- Al – β -Ag₂Se – Al JJ's [2,3]
- MQT below $T^* = 0.7$ K

[2] J. Kim, YJD et al., ACS Nano 10, 3936 (2016). "Quantum electronic transport of topological surface states in β -Ag₂Se nanowire"

[3] J. Kim, YJD et al., Nano Lett. 17, 6997 (2017). "MQT in superconducting junctions of β -Ag₂Se topological insulator nanowire"



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“for the discovery of macroscopic quantum mechanical tunnelling and energy quantisation in an electric circuit”

