Topological Superconductivity : Concepts, Platforms, and Current Frontiers

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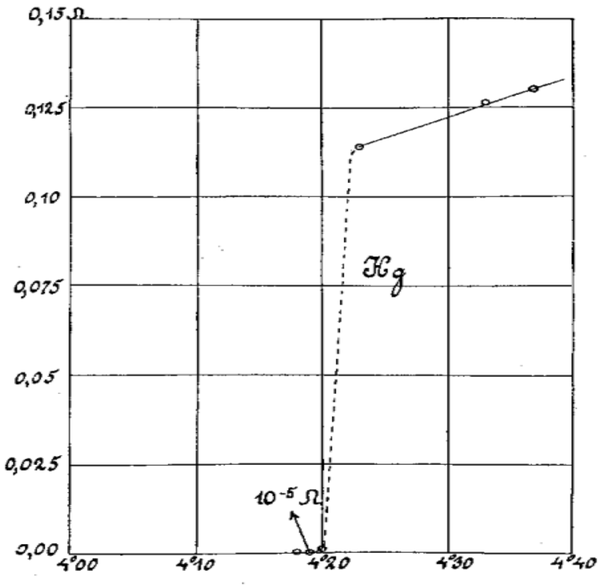
2026 05 28

중시계스쿨

Part 1 : Theoretical Background

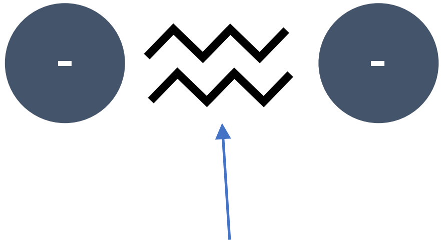
Superconductivity



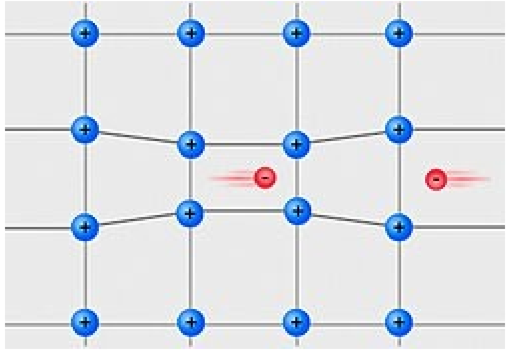
1911 by Heike Kamerlingh Onnes

Cooper Pair

Electron Electron



Pairing Glue



'Phonon-mediated pairing'

Mean Field Approach

General Hamiltonian under attractive interaction between electrons

$$H = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\mathbf{k}'} \sum_{\sigma\tau\sigma'\tau'} V_{\sigma\tau\sigma'\tau'}(\mathbf{k},\mathbf{k}') c^\dagger_{\mathbf{k}\sigma} c^\dagger_{-\mathbf{k}\tau} c_{-\mathbf{k}'\tau'} c_{\mathbf{k}'\sigma'}$$

$$\xi_k = \epsilon_k - \mu$$

$$\epsilon_k = \text{bare band energy}$$

$$\mu = \text{chemical potential}$$

BCS Mean Field Approach

$$c^\dagger c^\dagger c c \approx c^\dagger c^\dagger \langle cc \rangle + \langle c^\dagger c^\dagger \rangle cc - \langle c^\dagger c^\dagger \rangle \langle cc \rangle$$

$$H_{\mathrm{MF}} = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\sigma,\tau} \Delta_{\sigma\tau}(\mathbf{k}) c^\dagger_{\mathbf{k}\sigma} c^\dagger_{-\mathbf{k}\tau} + \sum_{\mathbf{k},\sigma,\tau} \Delta^*_{\sigma\tau}(\mathbf{k}) c_{-\mathbf{k}\tau} c_{\mathbf{k}\sigma}$$

Order Parameter $\Delta_{\sigma\tau}(\mathbf{k}) = \sum_{\mathbf{k}'} \sum_{\sigma'\tau'} V_{\sigma\tau\sigma'\tau'}(\mathbf{k},\mathbf{k}') \langle c_{-\mathbf{k}'\tau'} c_{\mathbf{k}'\sigma'} \rangle$

Order Parameter

Order Parameter $\Delta_{\sigma\tau}(\mathbf{k}) = \sum_{\mathbf{k}'} \sum_{\sigma'\tau'} V_{\sigma\tau\sigma'\tau'}(\mathbf{k},\mathbf{k}') \langle c_{-\mathbf{k}'\tau'} c_{\mathbf{k}'\sigma'} \rangle$

$$\hat{\Delta}(\mathbf{k}) = \begin{pmatrix} \Delta_{\uparrow\uparrow}(\mathbf{k}) & \Delta_{\uparrow\downarrow}(\mathbf{k}) \\ \Delta_{\downarrow\uparrow}(\mathbf{k}) & \Delta_{\downarrow\downarrow}(\mathbf{k}) \end{pmatrix}$$

Any general 2x2 matrix can be expressed with Pauli matrices

$$\hat{\Delta}(\mathbf{k}) = [\psi(\mathbf{k})\boldsymbol{\tau}_0 + \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\tau}]\, i\boldsymbol{\tau}_y$$

$$\hat{\Delta}(\mathbf{k}) = \begin{pmatrix} -d_x + id_y & \psi + d_z \\ -\psi + d_z & d_x + id_y \end{pmatrix}$$

The antisymmetry requirement of particle exchange yields

$$\hat{\Delta}(\mathbf{k}) = -\hat{\Delta}^T(-\mathbf{k}) \Longrightarrow \psi(\mathbf{k}) = \psi(-\mathbf{k}) \qquad \mathbf{d}(\mathbf{k}) = -\mathbf{d}(-\mathbf{k})$$

Spin Configuration

$$\psi(\mathbf{k}) = \psi(-\mathbf{k}) \qquad \mathbf{d}(\mathbf{k}) = -\mathbf{d}(-\mathbf{k})$$

s-wave

$$\psi(\mathbf{k}) = \Delta_s, \qquad \mathbf{d}(\mathbf{k}) = 0$$

$$\hat{\Delta}_s(\mathbf{k}) = \Delta_s i\sigma_y = \Delta_s \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\Delta_{\uparrow\downarrow} = +\Delta_s,$$
$$\Delta_{\downarrow\uparrow} = -\Delta_s,$$
$$\Delta_{\uparrow\uparrow} = \Delta_{\downarrow\downarrow} = 0$$

Corresponds to spin singlet $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Spin Configuration

$$\psi(\mathbf{k}) = \psi(-\mathbf{k}) \qquad \mathbf{d}(\mathbf{k}) = -\mathbf{d}(-\mathbf{k})$$

p-wave

$$\mathbf{d}(\mathbf{k}) = (0, 0, \Delta_p k_x)$$

p_x-wave

$$\hat{\Delta}_p(\mathbf{k}) = \Delta_p k_x \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\Delta_{\uparrow\downarrow} = \Delta_{\downarrow\uparrow}$$

$$\Delta_{\uparrow\uparrow} = \Delta_{\downarrow\downarrow} = 0$$

Corresponds to spin triplet $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

Similarly, using slightly different $\mathbf{d}(\mathbf{k})$

$$\hat{\Delta}_p(\mathbf{k}) = \begin{pmatrix} \Delta_p k_x & 0 \\ 0 & 0 \end{pmatrix}$$

$\Delta_{\uparrow\uparrow} \neq 0$ $\quad |\uparrow\uparrow\rangle$ $\quad$ spin triplet

Bogoliubov Quasiparticles

To solve this $$H_{\mathrm{MF}} = \sum_{\mathbf{k},\sigma} \xi_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k},\sigma,\tau} \Delta_{\sigma\tau}(\mathbf{k}) c^{\dagger}_{\mathbf{k}\sigma} c^{\dagger}_{-\mathbf{k}\tau} + \sum_{\mathbf{k},\sigma,\tau} \Delta^{*}_{\sigma\tau}(\mathbf{k}) c_{-\mathbf{k}\tau} c_{\mathbf{k}\sigma}$$

Nambu Spinor

$$\Psi_{\mathbf{k}} = \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c_{\mathbf{k}\downarrow} \\ c^{\dagger}_{-\mathbf{k}\uparrow} \\ c^{\dagger}_{-\mathbf{k}\downarrow} \end{pmatrix}$$

$$H_{\mathrm{MF}} = \frac{1}{2} \sum_{\mathbf{k}} \Psi^{\dagger}_{\mathbf{k}} \mathcal{H}_{\mathrm{BdG}}(\mathbf{k}) \Psi_{\mathbf{k}}$$

$$\mathcal{H}_{\mathrm{BdG}}(\mathbf{k}) = \begin{pmatrix} \xi_{\mathbf{k}} \sigma_0 & \hat{\Delta}(\mathbf{k}) \\ \hat{\Delta}^{\dagger}(\mathbf{k}) & -\xi_{-\mathbf{k}} \sigma_0 \end{pmatrix}$$

Eigenvalue Equation

$$\mathcal{H}_{\mathrm{BdG}}(\mathbf{k}) |\alpha_{\mathbf{k}}\rangle = E_{\mathbf{k}} |\alpha_{\mathbf{k}}\rangle$$

Eigenvector : Bogoliubov quasiparticle

Bogoliubov Quasiparticles

$$\mathcal{H}_{\mathrm{BdG}}(\mathbf{k})|\alpha_{\mathbf{k}}\rangle = E_{\mathbf{k}}|\alpha_{\mathbf{k}}\rangle$$

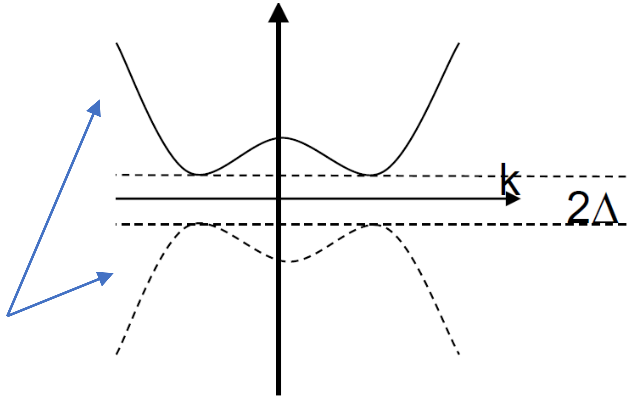
Solution

$$\pm E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + |\hat{\Delta}(\mathbf{k})|^2} \qquad |\hat{\Delta}(\mathbf{k})|^2 = \frac{1}{2}\mathrm{Tr}\hat{\Delta}^\dagger(\mathbf{k})\hat{\Delta}(\mathbf{k})$$

At Fermi surface $\xi_{\mathbf{k}} = 0$

$$E_{\mathbf{k}} = \pm|\hat{\Delta}(\mathbf{k})|$$

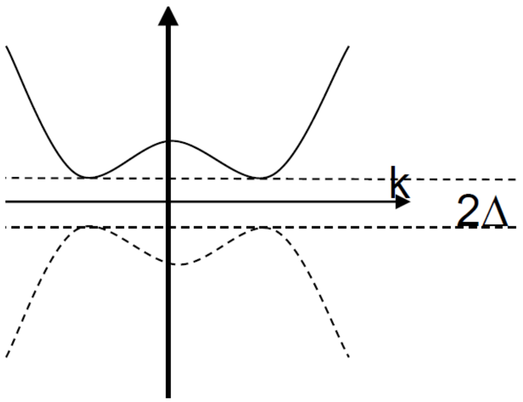
Particle-hole symmetry always present



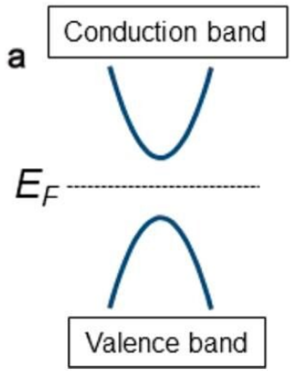
Order Parameter Δ determines gap of the quasiparticle excitation
"Gap Function"

What is Topological Superconductivity?

Gapped Superconductor



Gapped Insulator



- ‘Topology’ can be defined in gapped **superconductor**
- Transition to different topology

  involves closing the gap
- This creates Edge mode (**Majorana**) at the boundary

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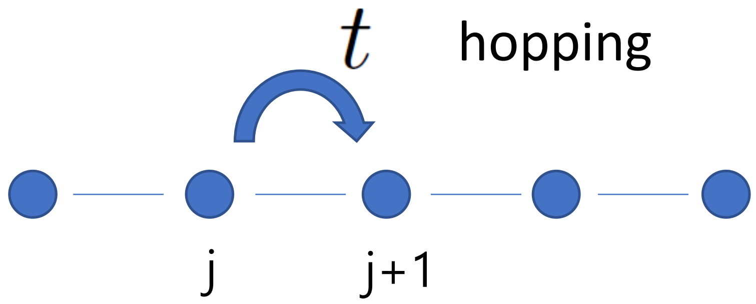
Toy Model 1 : 1D Majorana Chain



μ Chemical potential

Δ Superconducting pairing

Spinless (e.g. all spin up) 1D chain

$$\hat{H} = \sum_j \left[t(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) - \mu c_j^\dagger c_j + \Delta(c_{j+1}^\dagger c_j^\dagger + c_j c_{j+1}) \right]$$

Using Fourier transform $$c_k = \sum_j e^{-ikj} c_j$$

Hamiltonian in k-space

$$\hat{H} = \sum_k \left[(2t\cos k - \mu) c_k^\dagger c_k + i\Delta \sin k \, c_k c_{-k} - i\Delta \sin k \, c_{-k}^\dagger c_k^\dagger \right]$$

Toy Model 1 : 1D Majorana Chain

$$\hat{H} = \sum_k \left[(2t\cos k - \mu)c_k^\dagger c_k + i\Delta \sin k\, c_k c_{-k} - i\Delta \sin k\, c_{-k}^\dagger c_k^\dagger\right]$$

Note that $\Delta_k \propto i\Delta \sin k$ ➡ **p-wave superconductor** (1D p_x-wave)

Using Nambu spinor $\Psi_k = \begin{pmatrix} c_k \\ c_{-k}^\dagger \end{pmatrix}$

$$\hat{H} = \frac{1}{2}\sum_k \Psi_k^\dagger H_{\text{BdG}}(k)\Psi_k$$

$$H_{\text{BdG}}(k) = \mathbf{d}(k)\cdot\boldsymbol{\tau}$$

where $\mathbf{d}(k) = \begin{pmatrix} 0 \\ 2\Delta\sin k \\ 2t\cos k - \mu \end{pmatrix}$

$E_\pm(k) = \pm|\mathbf{d}(k)|$ ➡

$$E_\pm(k) = \pm\sqrt{(2t\cos k - \mu)^2 + 4\Delta^2 \sin^2 k}$$

Toy Model 1 : 1D Majorana Chain

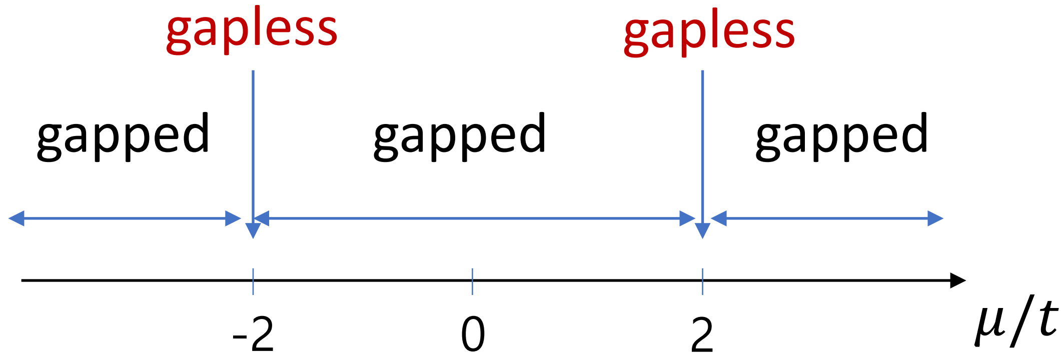
$$E_{\pm}(k) = \pm\sqrt{(2t\cos k - \mu)^2 + 4\Delta^2 \sin^2 k}$$

The gap closes when

$$2\Delta \sin k = 0$$

$$2t\cos k - \mu = 0$$

➡ $\mu = \pm 2t$



Toy Model 1 : 1D Majorana Chain

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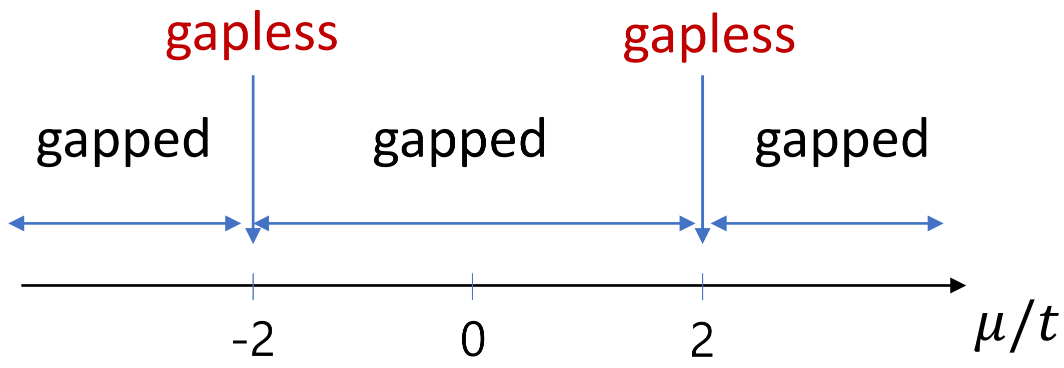
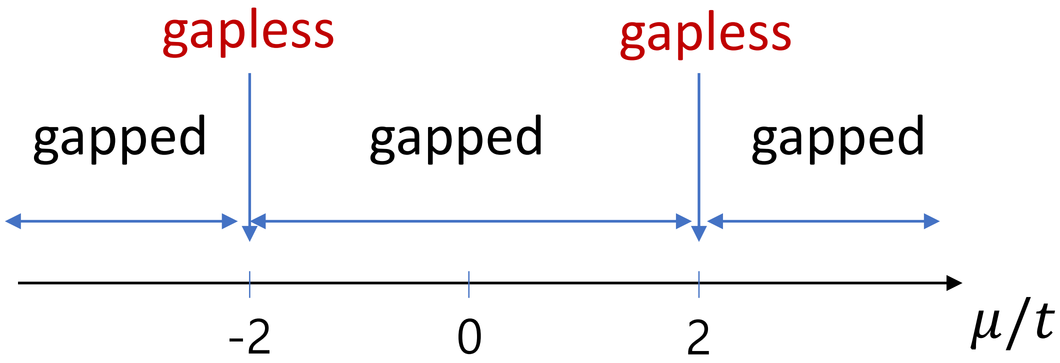
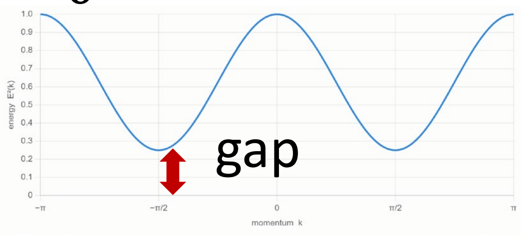


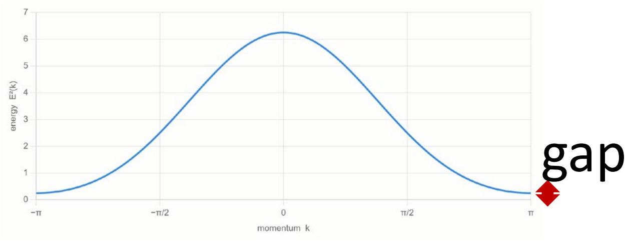
Toy Model 1 : 1D Majorana Chain

$$\frac{\Delta}{t} = 0.5$$

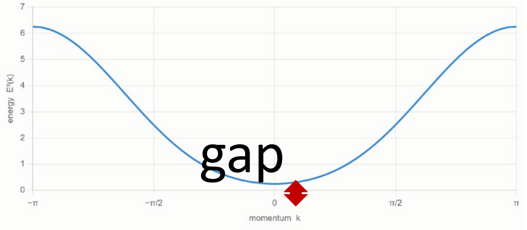


$$\frac{\mu}{t} = 0$$

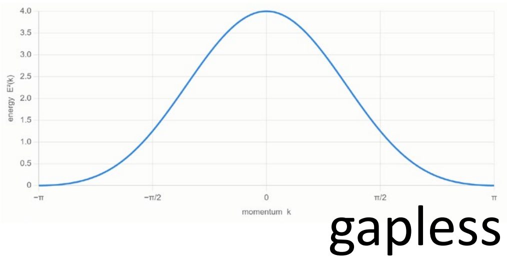


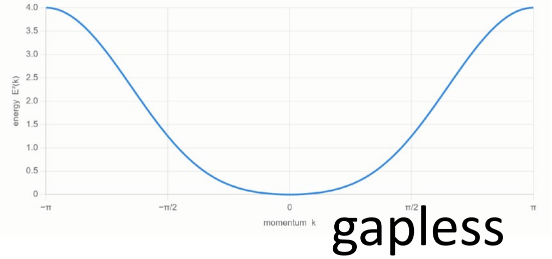


$$\frac{\mu}{t} = -3$$



$$\frac{\mu}{t} = 3$$

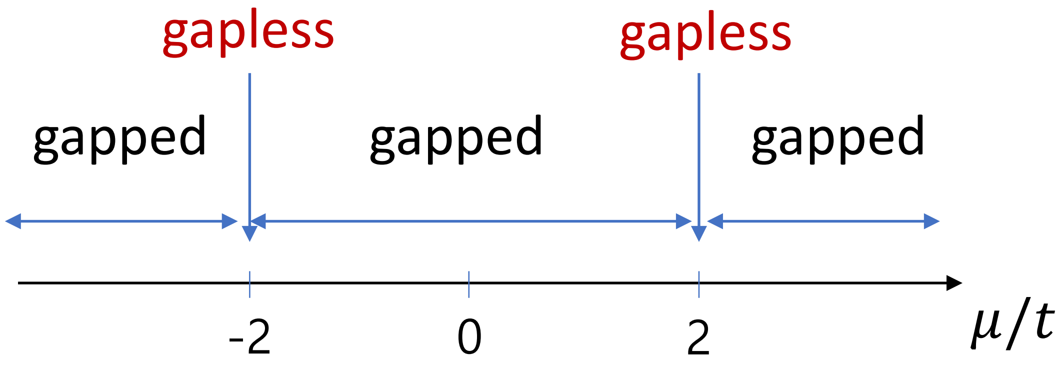




$$\frac{\mu}{t} = -2$$

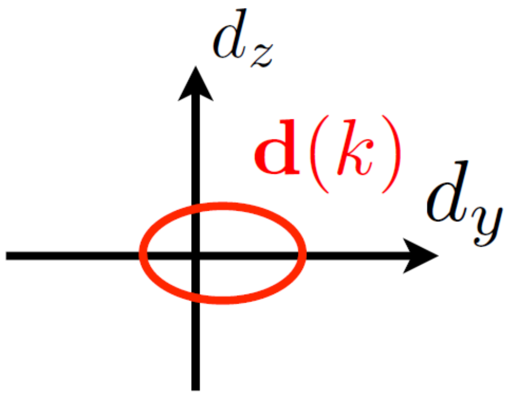
$$\frac{\mu}{t} = 2$$

Toy Model 1 : 1D Majorana Chain



Phase A

$$|\mu| < 2|t|$$



topological

$$\mathbf{d}(k) = \begin{pmatrix} 0 \\ 2\Delta \sin k \\ 2t \cos k - \mu \end{pmatrix}$$

Surrounds origin

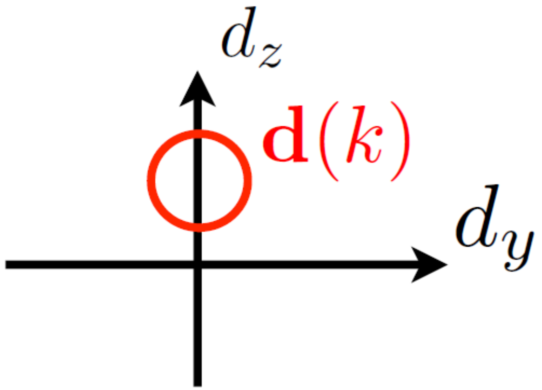
➡ cannot remove winding without crossing d(k) = 0

➡ gap must close

➡ different topological phase

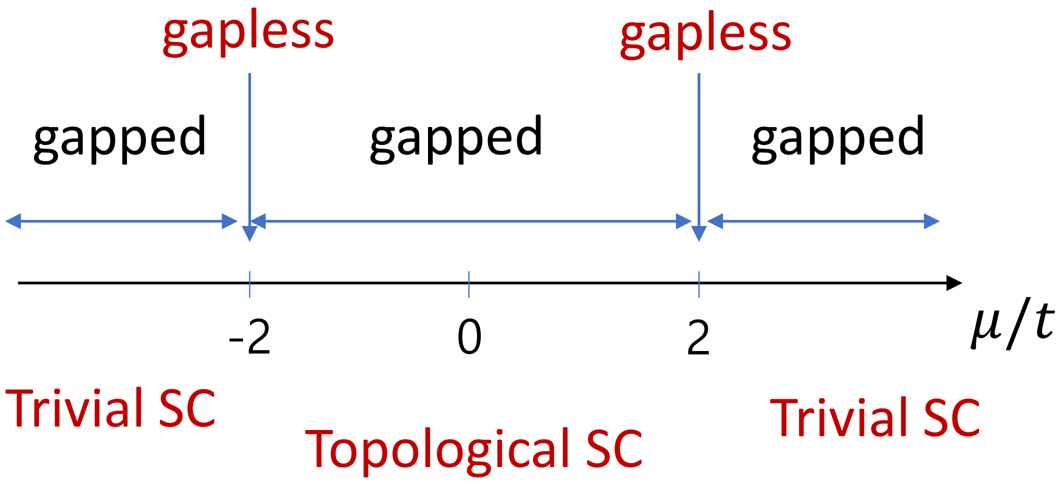
Phase B

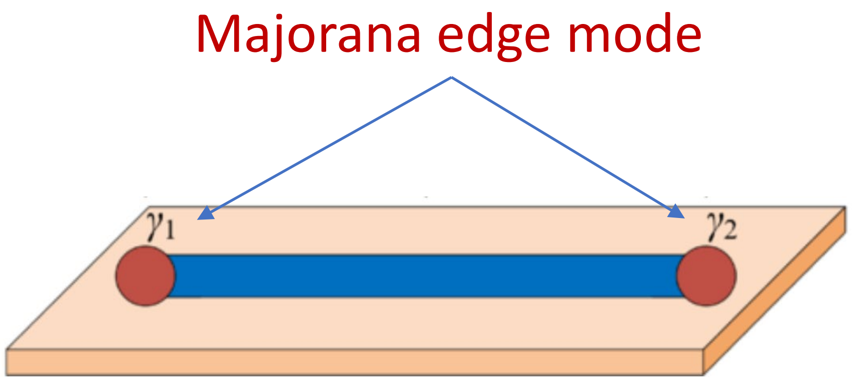
$$|\mu| > 2|t|$$



trivial

Toy Model 1 : 1D Majorana Chain





- 'Topology' can be defined in gapped **superconductor**
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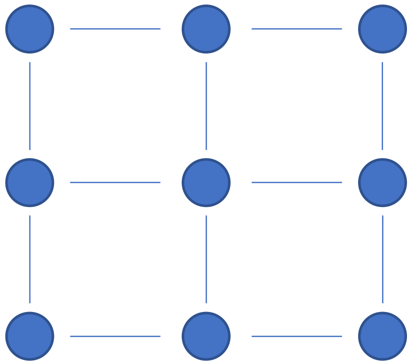
Toy Model 2 : 2D Chiral p-wave Superconductor

2D square lattice



t hopping

μ Chemical potential

From tight binding

$$\varepsilon_{\mathbf{k}} = 2t(\cos k_x + \cos k_y) - \mu$$

Assume $p_x + ip_y$ wave superconducting order parameter

$$d_z(\mathbf{k}) = \Delta(\sin k_x + i \sin k_y)$$

$$H_{\text{BdG}} = \mathbf{h}(\mathbf{k}) \cdot \vec{\sigma} \qquad \mathbf{h}(\mathbf{k}) = \begin{pmatrix} \Delta \sin k_x \\ \Delta \sin k_y \\ 2t(\cos k_x + \cos k_y) - \mu \end{pmatrix}$$

Toy Model 2 : 2D Chiral p-wave Superconductor

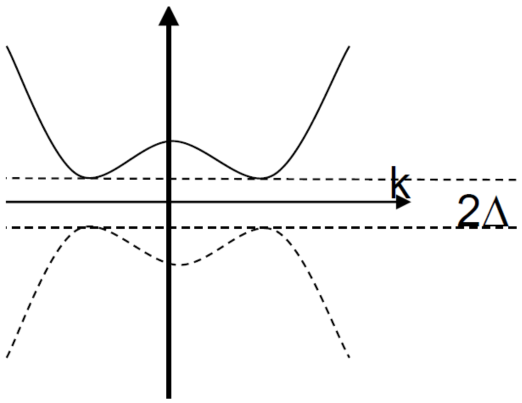
Energy $$E_\pm(\mathbf{k}) = \pm|\mathbf{h}(\mathbf{k})| = \pm\sqrt{\Delta^2(\sin^2 k_x + \sin^2 k_y) + [2t(\cos k_x + \cos k_y) - \mu]^2}$$

$$\text{Phase A: } 4|t| > |\mu| \quad \text{and} \quad \text{Phase B: } 4|t| < |\mu|$$

Trivial

Topological

Summary on Theory



1D p-wave SC

p_x+ip_y wave SC

- ‘Topology’ can be defined in gapped **superconductor**
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  involves closing the gap
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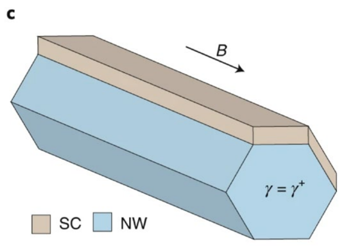
Note that spin needs to be triplet

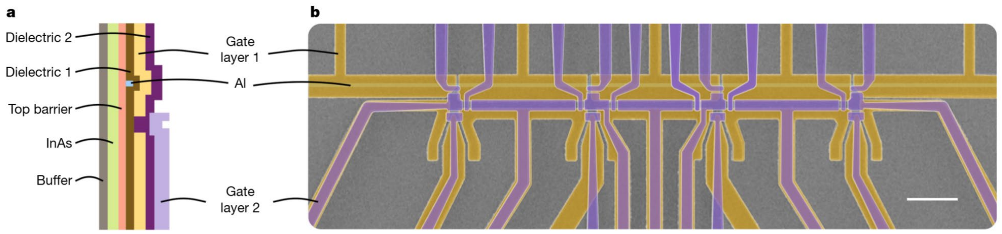
Part 2 : Experimental Platforms

Experiments on 1D p-wave SC

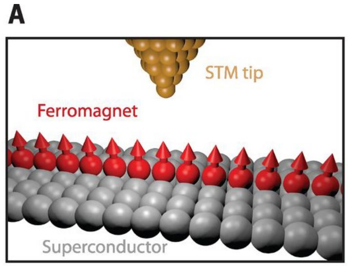
Nanowire + SC



2DEG + SC

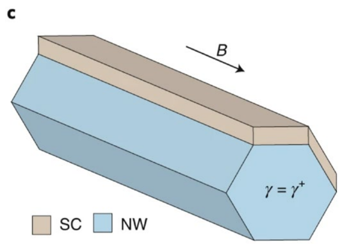


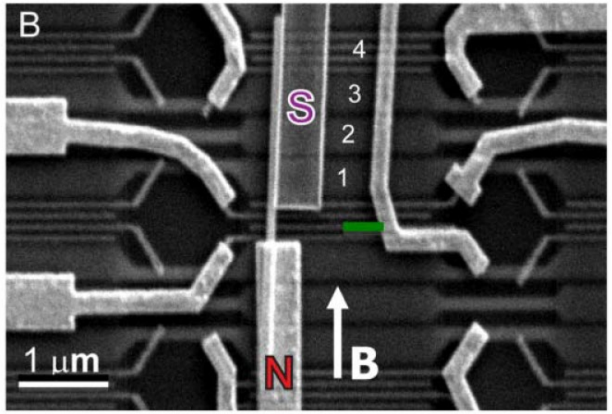
Ferromagnetic chain + SC



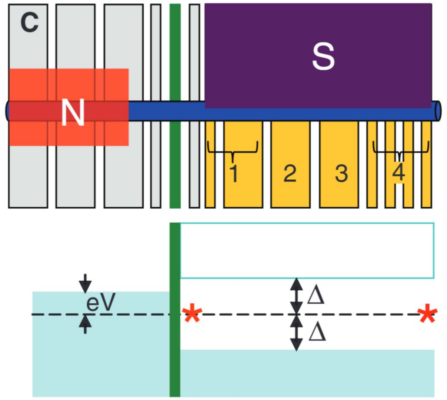
“Engineered” topological superconductor

Nanowire + SC

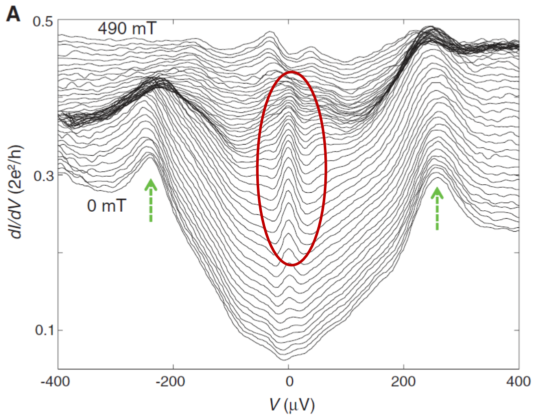


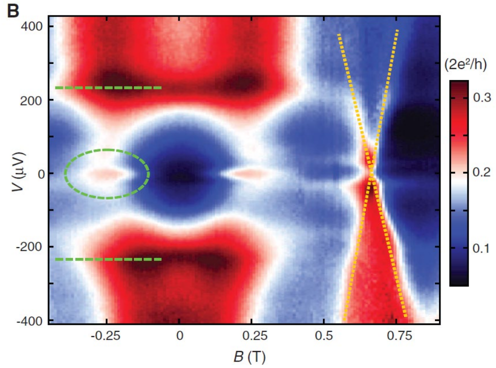


InSb nanowire
+ NbTiN superconductor



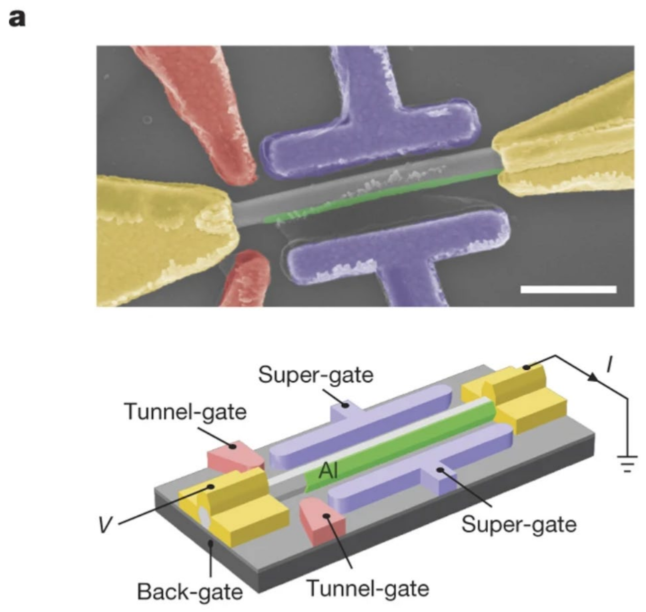
Signal : tunneling conductance at zero bias due to Majorana edge mode

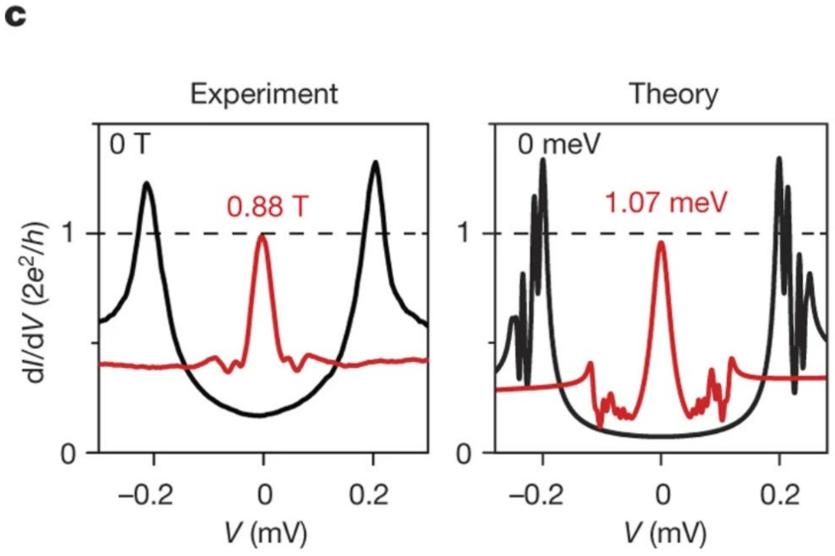


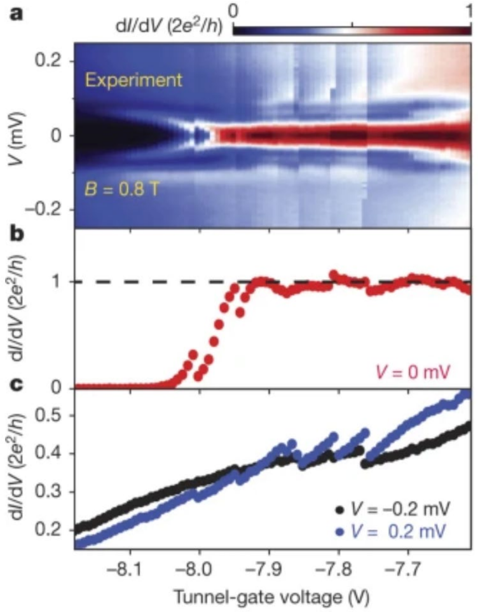


V. Mourik et al. *Science 336 1003* (2012)

Nanowire + SC





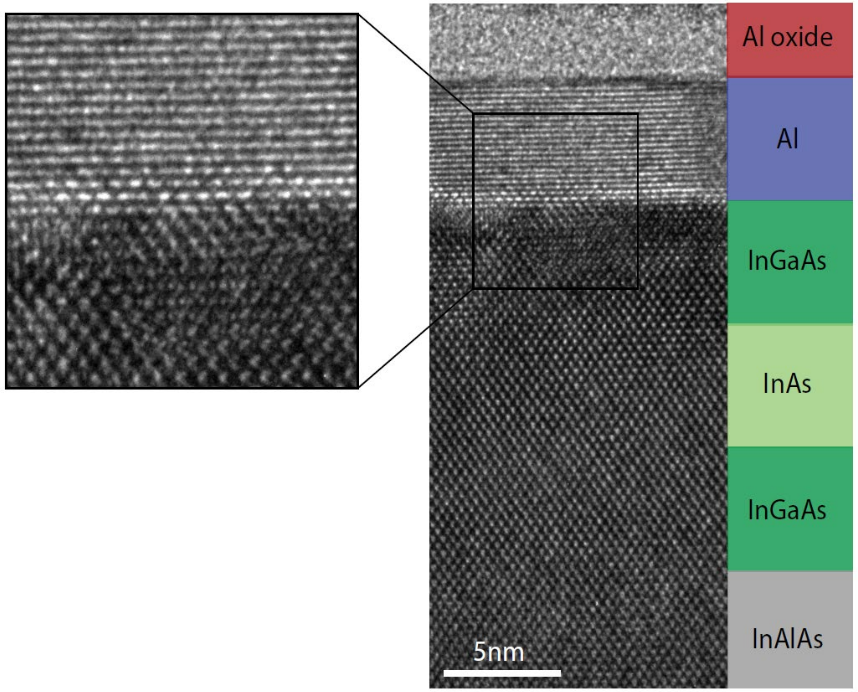


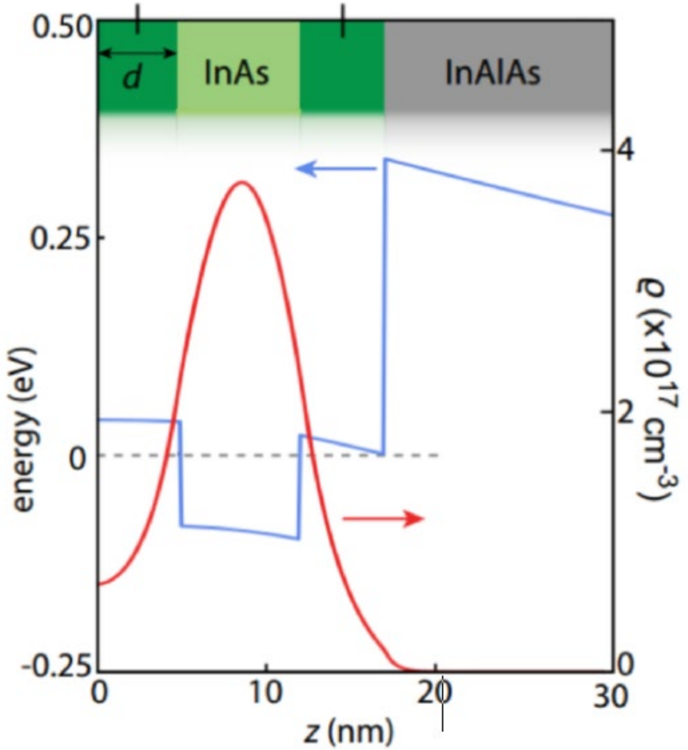
Retracted! Data cropped, calibration wrong

"When the data are replotted over the full parameter range, including ranges that were not made available earlier, points are outside the 2-sigma error bars. We can therefore no longer claim the observation of a quantized Majorana conductance, and wish to retract this Letter."

H. Zhang et al. *Nature 556 74* (2018)

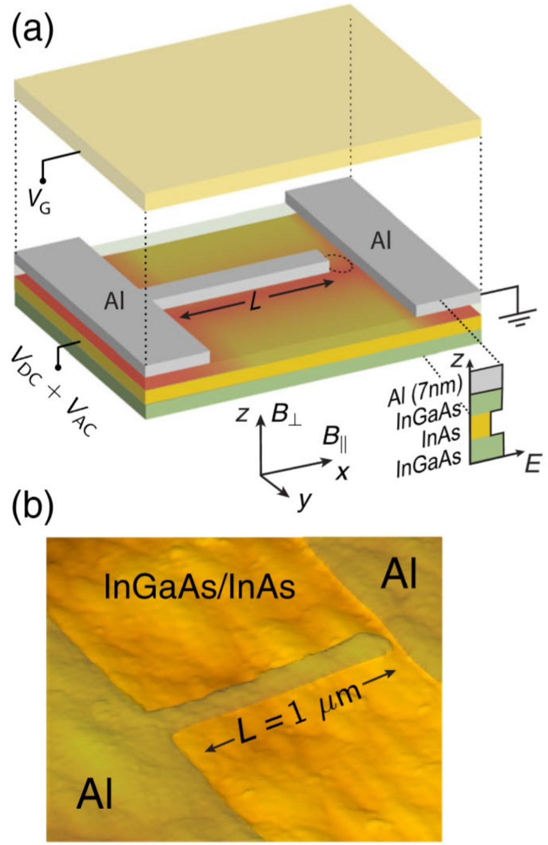
2DEG + SC

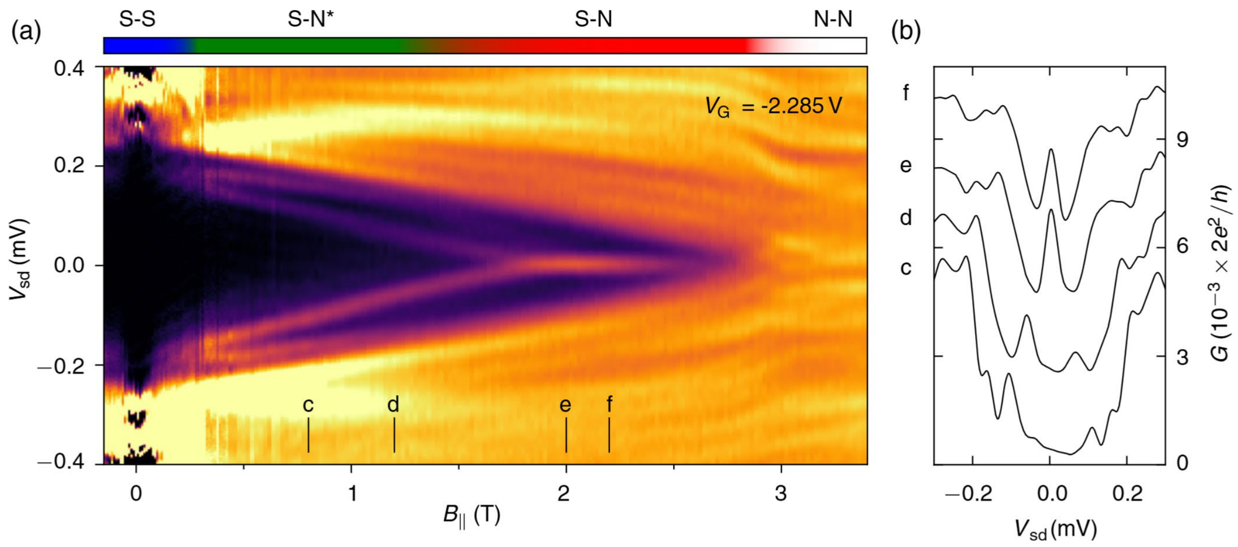




J. Shabani et al. *PRB 93 155402* (2016)

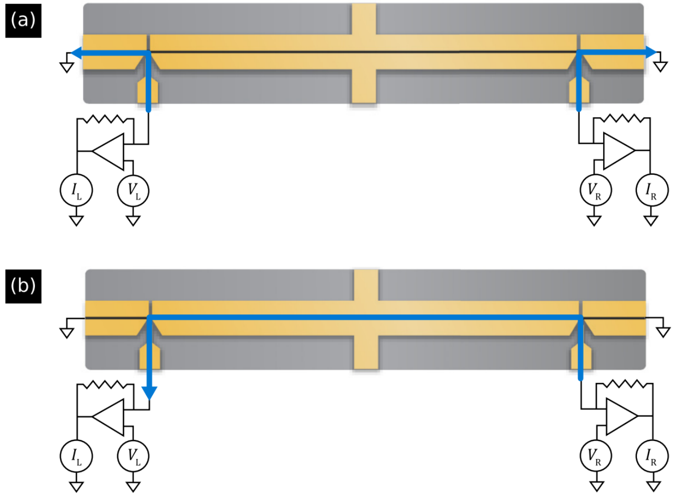
2DEG + SC

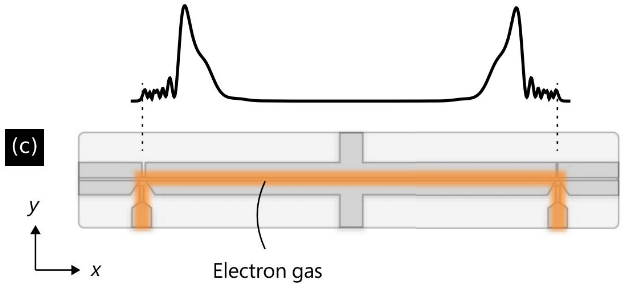


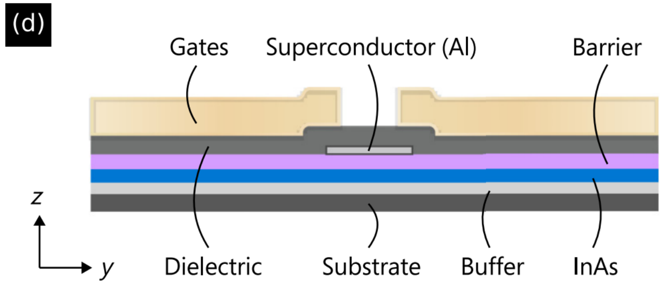


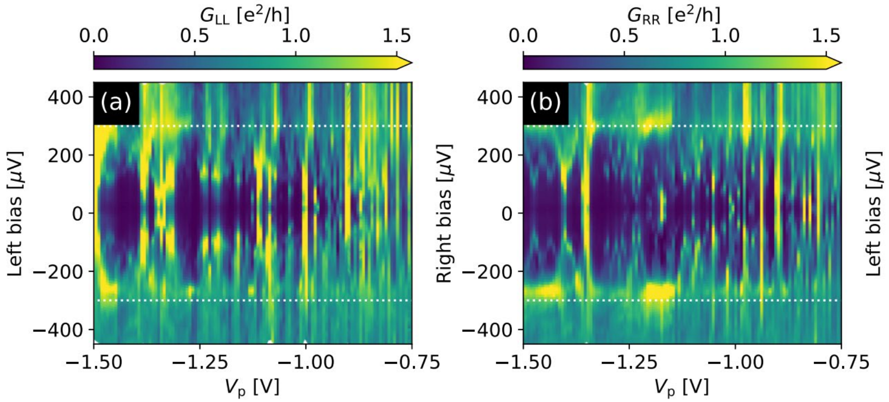
H. Suominen et al. *PRL 119 176805* (2017)

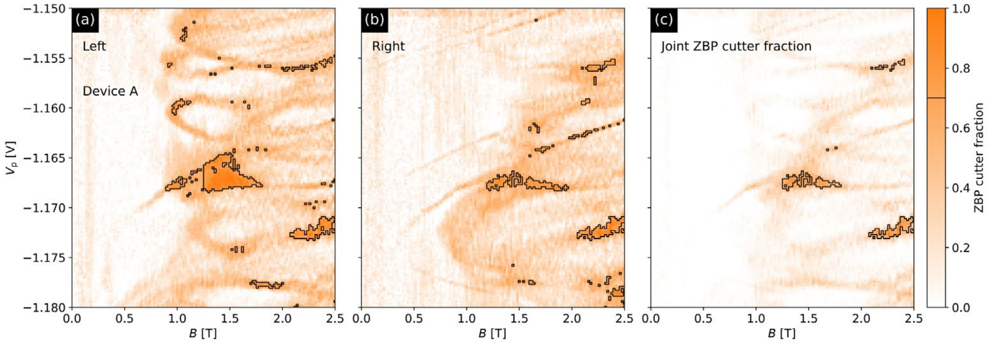
2DEG + SC





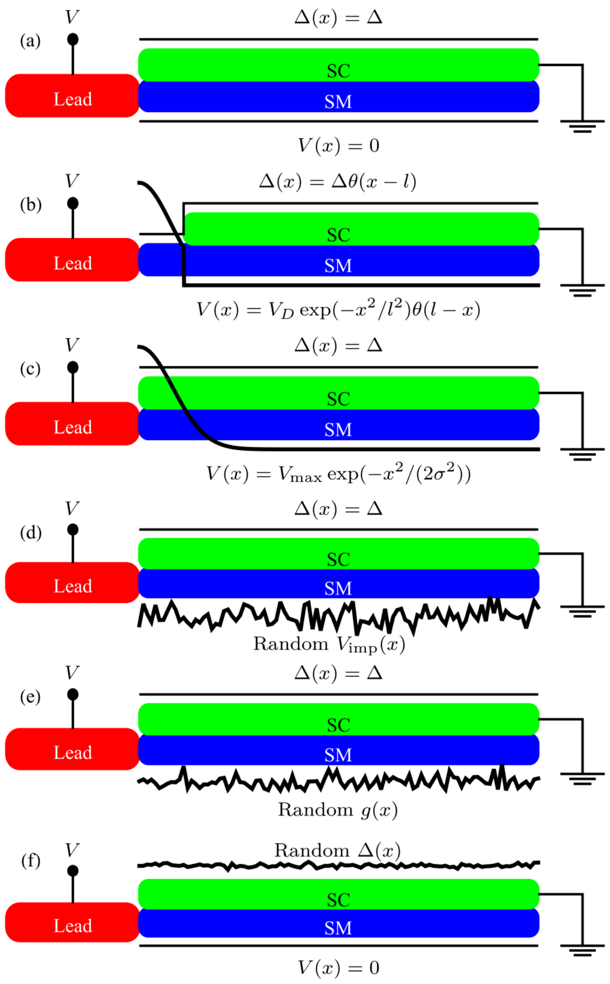






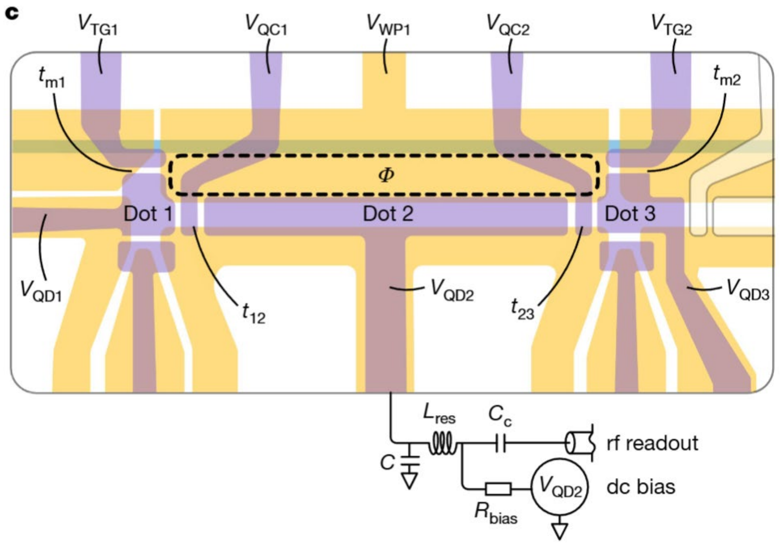
Microsoft Quantum *PRB 107 245423* (2023)

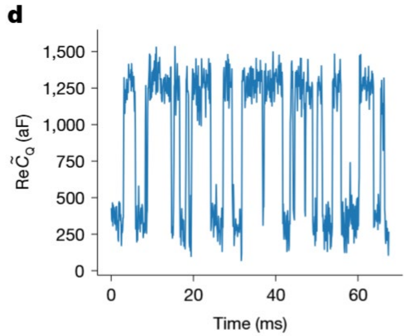
General Problems : Disorder Issue

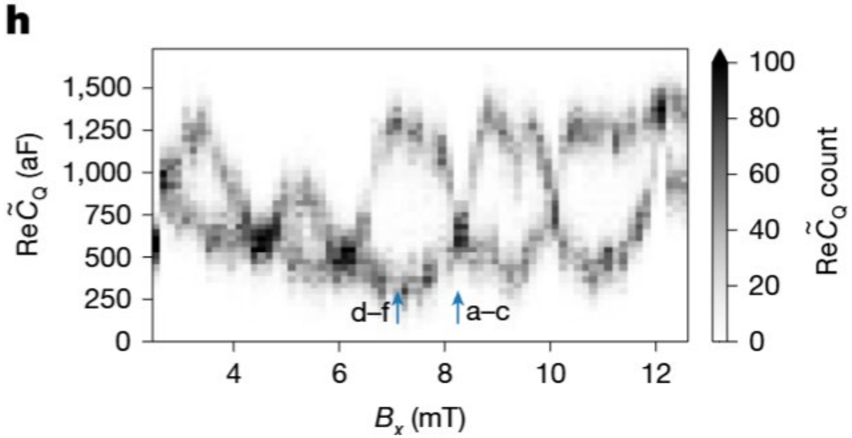


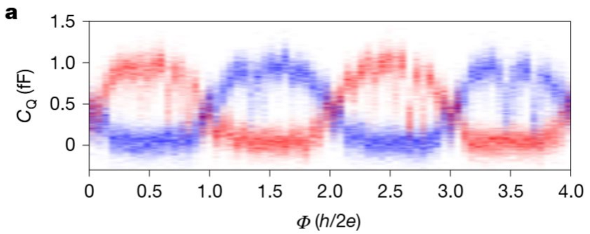
Haining Pan and S. Das Sarma *PRR 2 013377* (2020)

2DEG + SC : Recent Progress



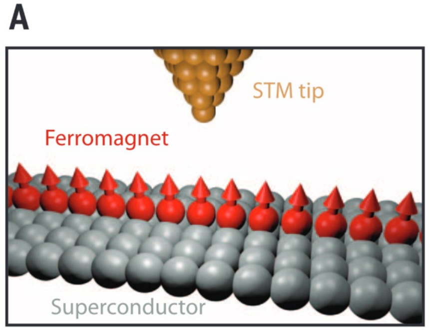


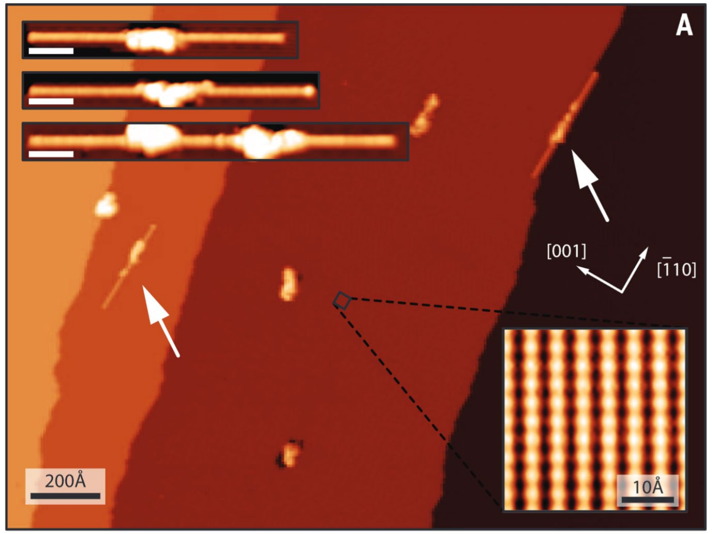


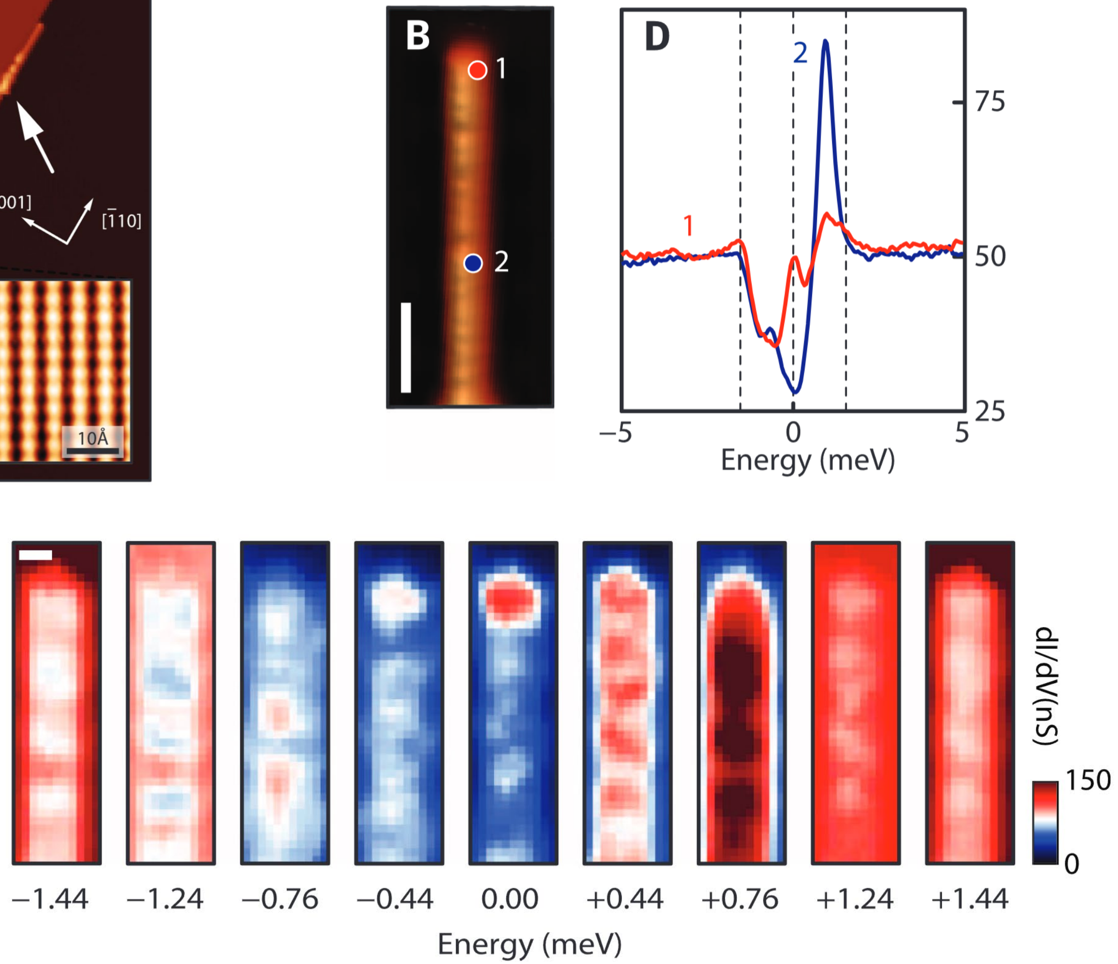


Microsoft Quantum *Nature 638 651* (2025)

Ferromagnetic Chain + SC

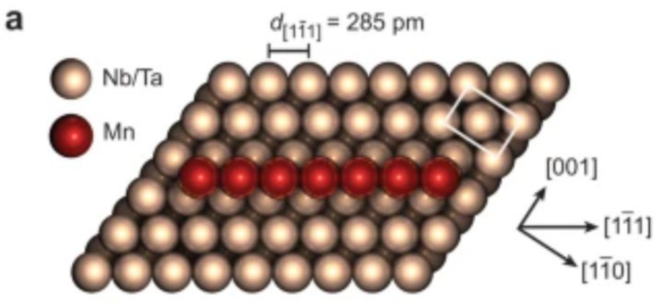


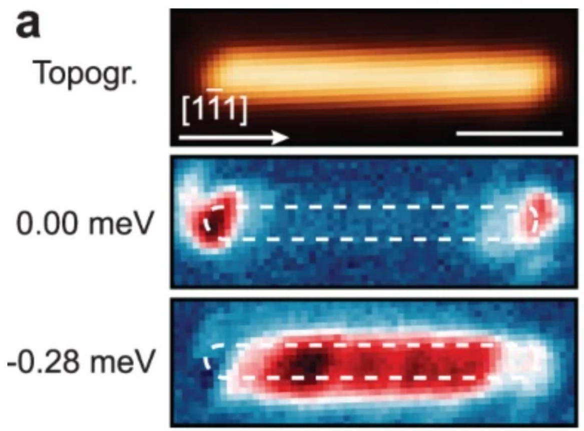


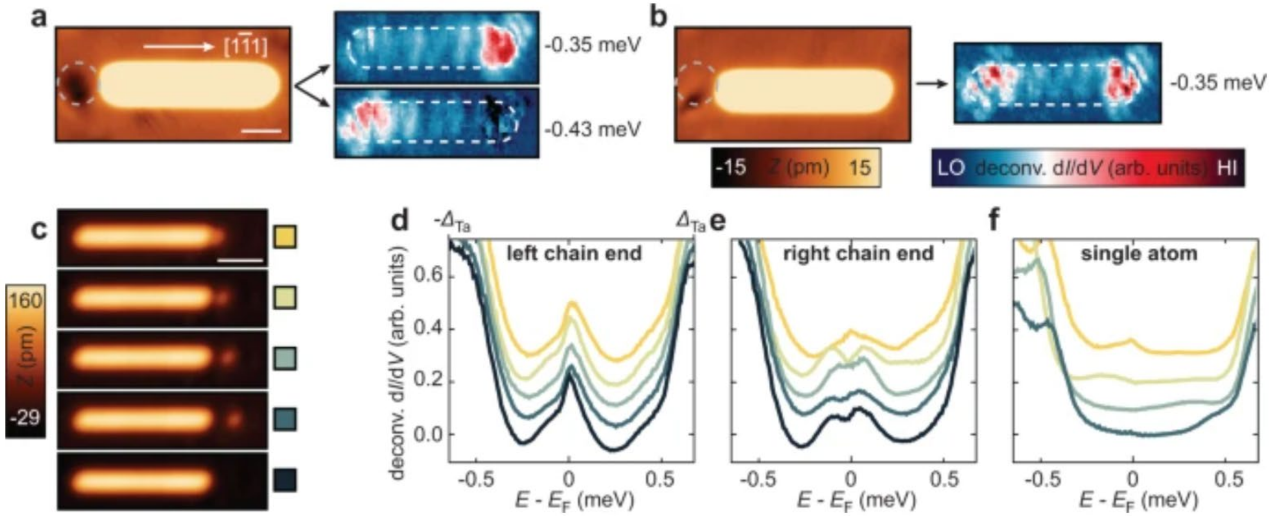


S Nadj-Perge et al. *Science 346 602* (2014)

Ferromagnetic Chain + SC



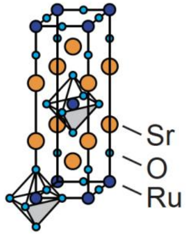




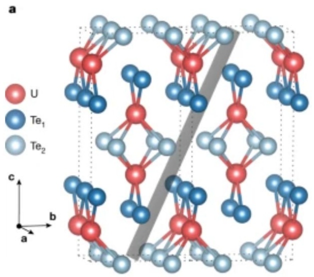
L Schneider et al. *Nat. Comm. 14 2742* (2023)

Experiments on p_x+ip_y wave SC

Sr2RuO4

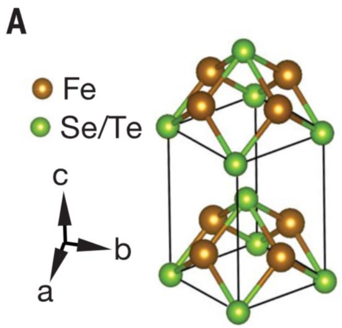


UTe2

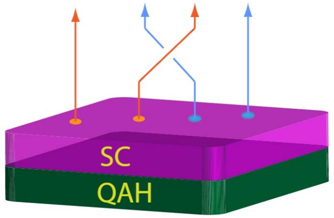


Intrinsic

$FeTe_{1-x}Se_x$

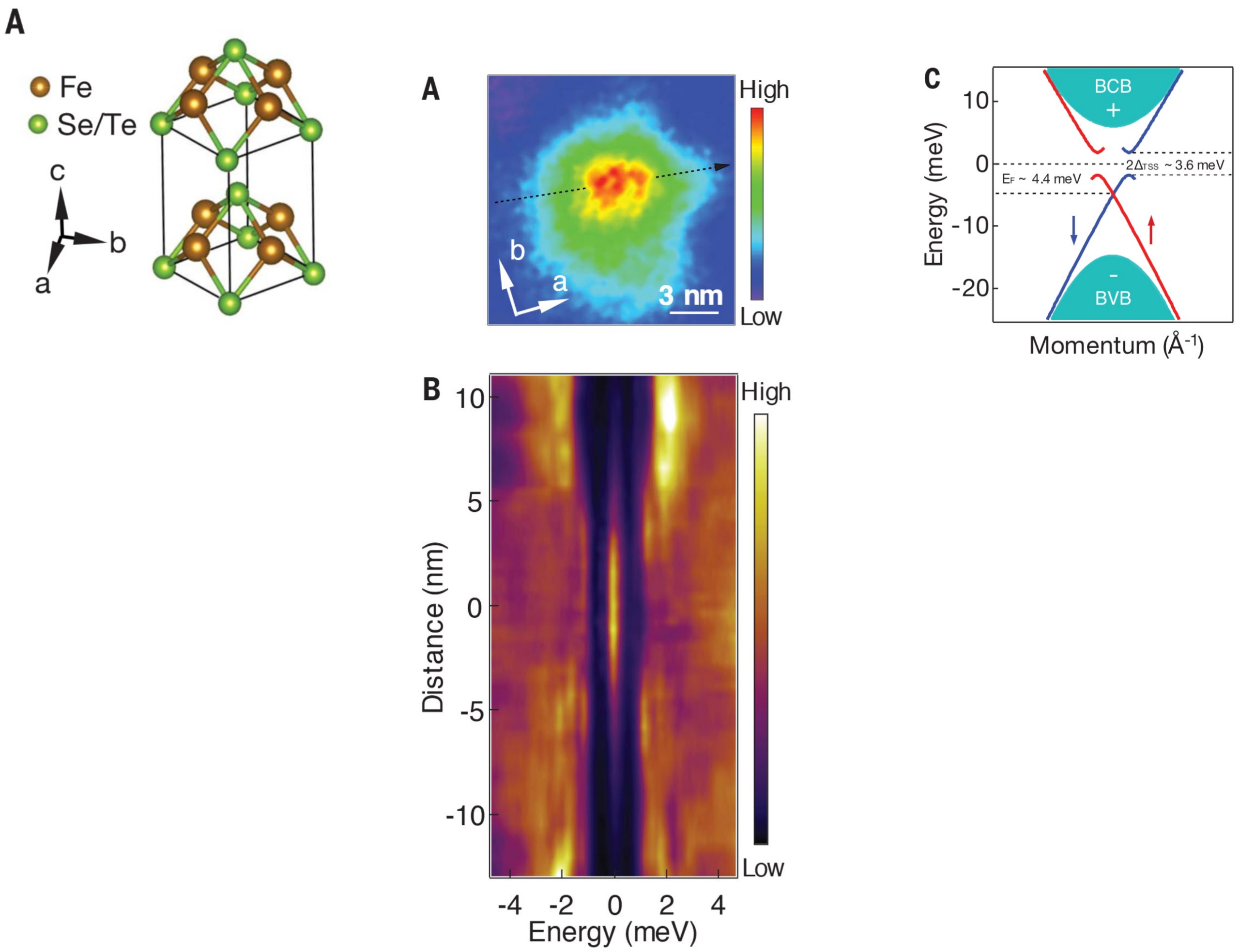


QAH + SC

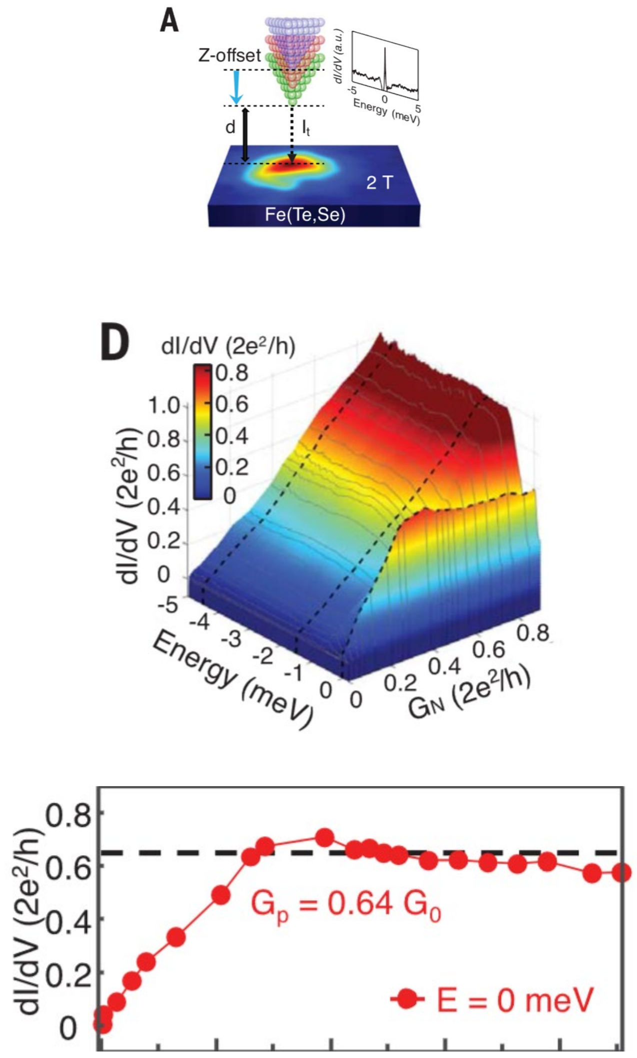


Relies on Proximity Effect

$FeTe_{1-x}Se_x$

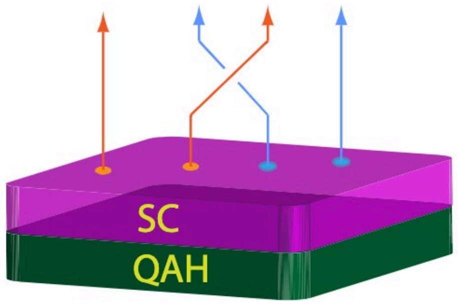


D. Wang et al. *Science 362 333* (2018)

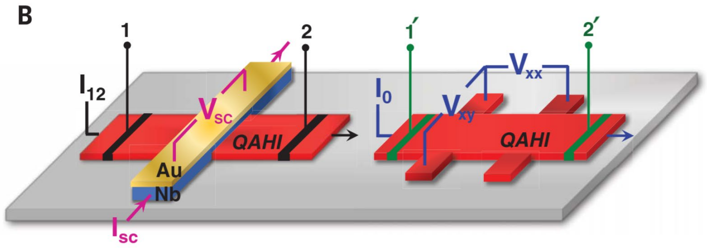


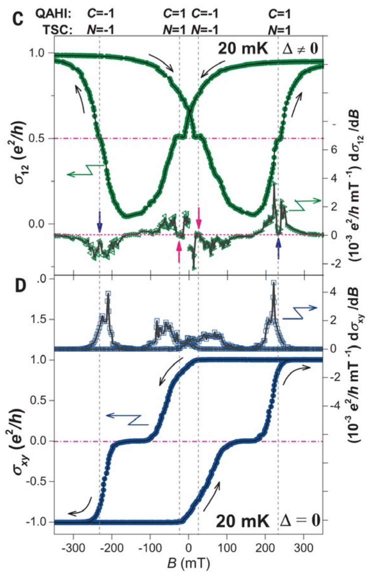
S. Zhu et al. *Science 367 189* (2020)

QAH + SC



X. Qi, T. Hughes, S. Zhang *PRB* (2010)





Q. He et al. *Science* (2017)

A mechanism of $\frac{1}{2}\frac{e^2}{h}$ conductance plateau without 1D chiral Majorana fermions

Wenjie Ji[1] and Xiao-Gang Wen[1]

[1] *Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

W. Ji and X. Wen et al. *PRL* (2018)

Retracted! Raw data into question

...with possibly wrong analysis

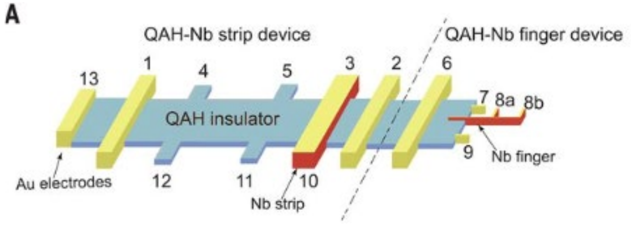
Part 3 : Graphene Based Platforms

Proximitizing Quantum Hall Edge with Superconductor

Induced superconductivity in high-mobility two-dimensional electron gas in gallium arsenide heterostructures

Zhong Wan, Aleksandr Kazakov, Michael J. Manfra, Loren N. Pfeiffer, Ken W. West & Leonid P. Rokhinson

Nature Communications **6**, Article number: 7426 (2015) | Cite this article



W. Ji and X. Wen *PRL* (2018)
M. Kayyalha et al. *Science* (2020)

Induced superconducting correlations in a quantum anomalous Hall insulator

Anjana Uday, Gertjan Lippertz, Kristof Moors, Henry F. Legg, Rikkie Joris, Andrea Bliesener, Lino M. C. Pereira, A. A. Taskin & Yoichi Ando

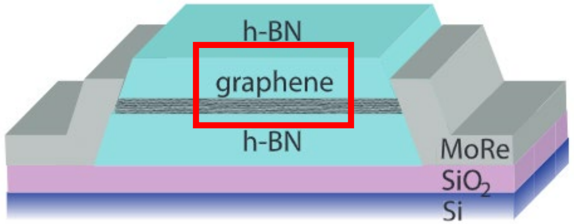
Nature Physics **20**, 1589–1595 (2024) | Cite this article

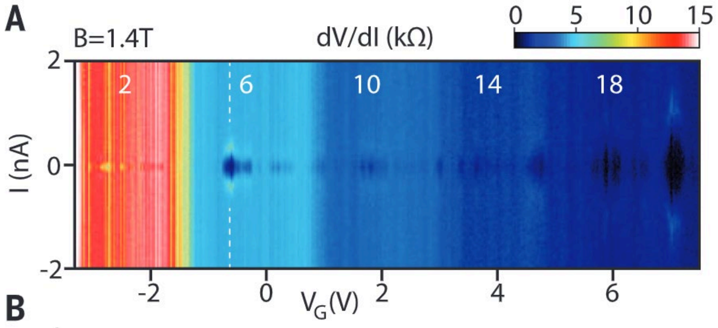
Supercurrent in the quantum Hall regime

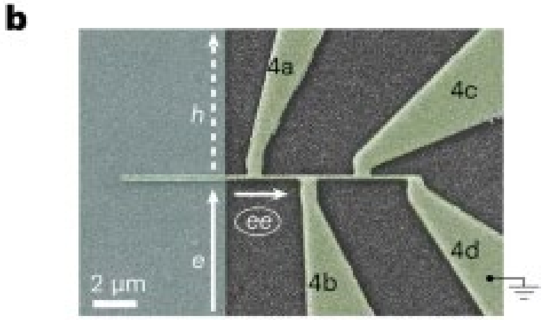
F. AMET, C. T. KE, I. V. BORZENETS, J. WANG, K. WATANABE, T. TANIGUCHI, R. S. DEACON, M. YAMAMOTO, Y. BOMZE, [...], AND G. FINKELSTEIN +1 authors Authors

Info & Affiliations

SCIENCE • 20 May 2016 • Vol 352, Issue 6288 • pp. 966-969 • DOI: 10.1126/science.aad6203







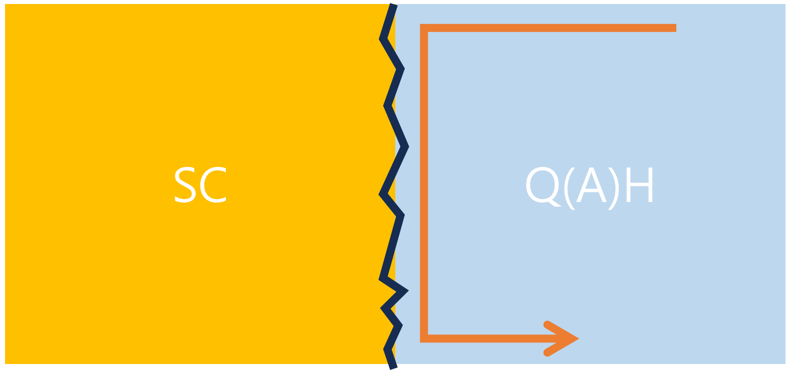
G. Lee et al. *Nat. Phys.* (2017)
Ö. Gül et al. *PRX* (2022)
H. Vignaud et al. *Nature* (2023)
J. Barrier et al. *Nature* (2024)
...

Experimental Challenges : Disorder

Alloy SC – NbN, MoGe, …
: Dirty, Vortex, …

SC – Nb, …
: Cleaner

Lateral Junction



QH – graphene : high magnetic field

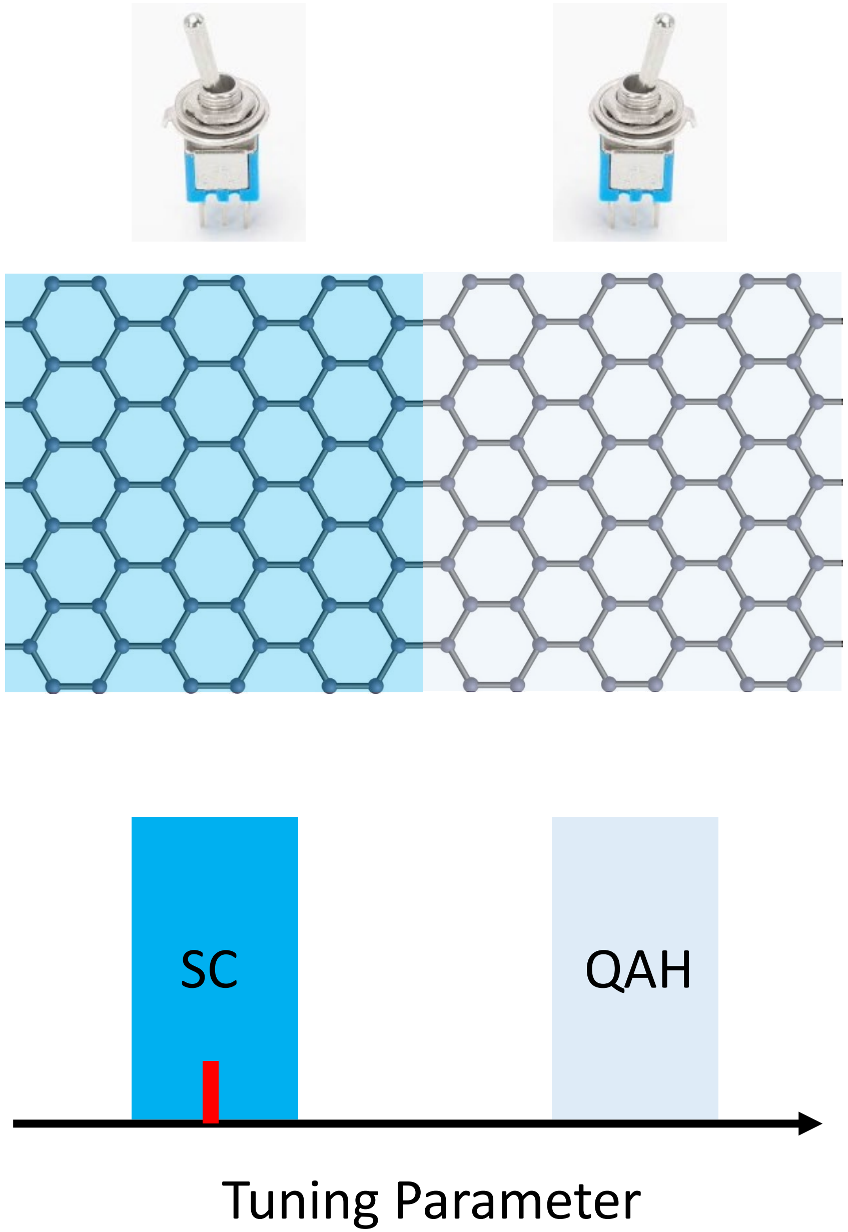
QAH – doped TI : intrinsic disorders from dopants

Between bulk SC and 2D surface – interfacial disorder

Cleaner platform required to achieve topological superconductivity

V. Kurilovic, L. Glazman *PRX* (2023)
V. Kurilovic, L. Glazman *Nat. Comm.* (2023)
Y. Tang, K. Christina, J. Alicea *PRB* (2022)

Quest: Search for Material with Tunable SC and QAH



Crystalline with minimal defect

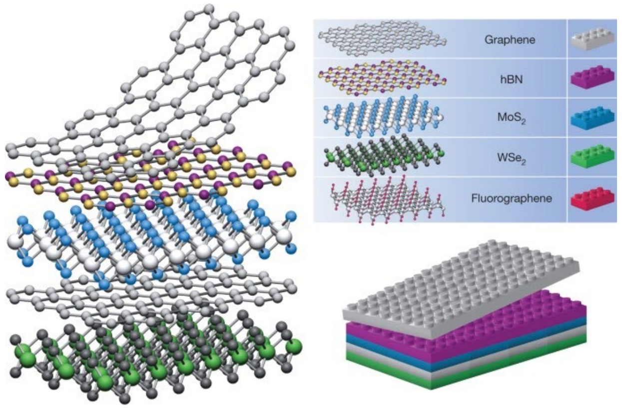
Readily tunable

Superconductivity

QAH – no magnetic field

2D van der Waals Materials

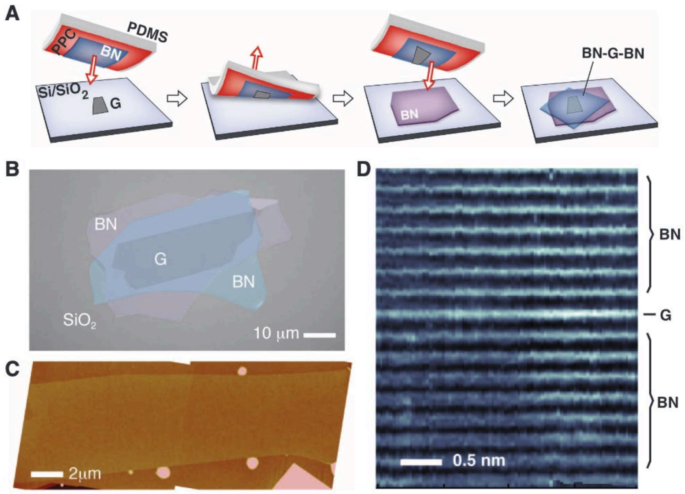
Van der Waals Heterostructures



Geim Group

2010

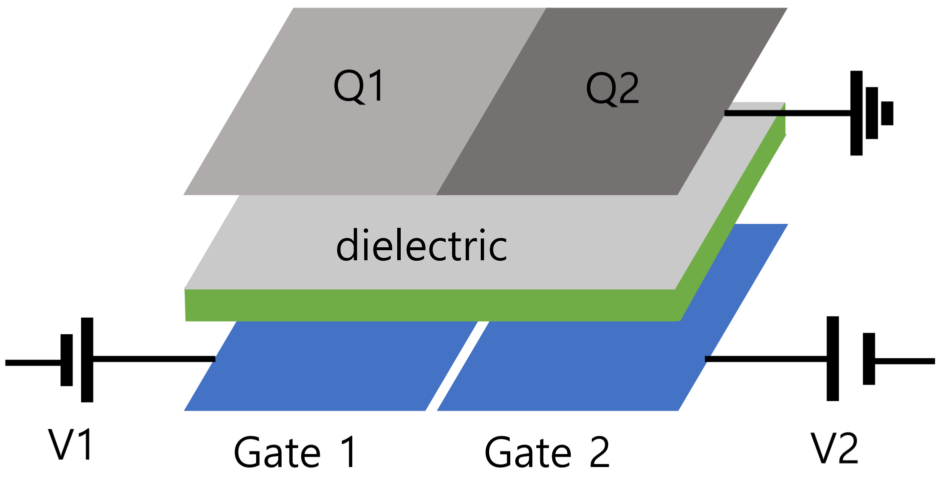
Crystalline with minimal defect



L. Wang et al. *Science* (2013)

Readily tunable

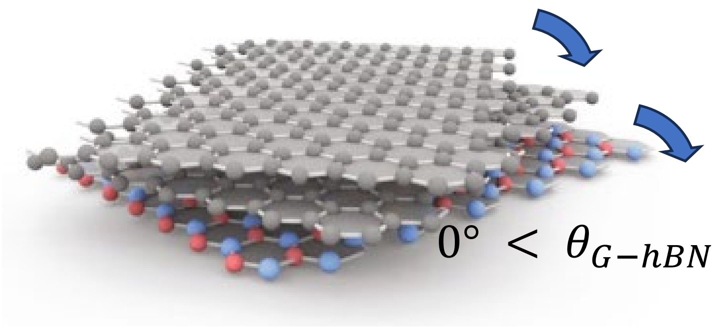
Carrier density



$Q = CV$

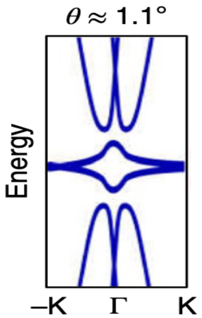
Twisted Bilayer Graphene? SC and QAH not Together!

$\theta_{G-G} \sim 1.1°$

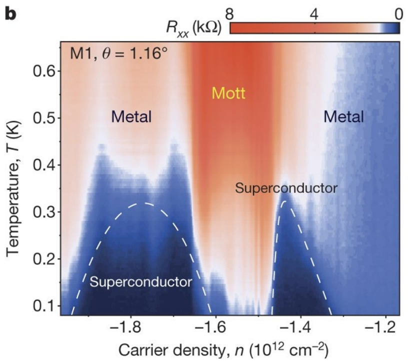


$0° < \theta_{G-hBN} < 30°$

U > K.E.

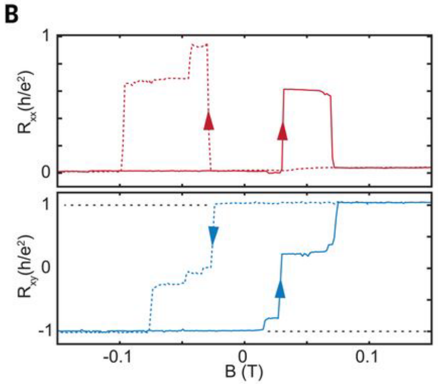


Superconductivity



Y. Cao et al. *Nature* (2018)

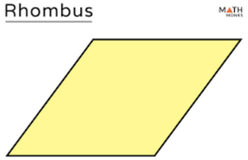
Quantum Anomalous Hall

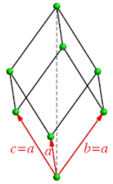


M. Serlin et al. *Science* (2020)

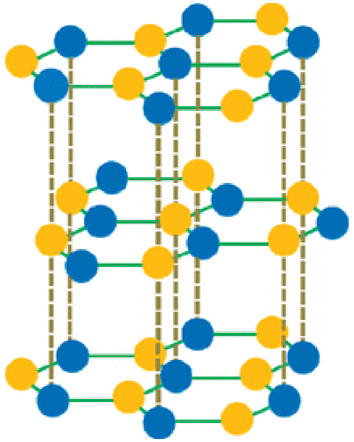
	Observations since 2018 (At the magic angle around 1.1°)
hBN un-aligned	Superconductivity No Quantized Anomalous Hall (Anomalous Hall if lucky)
hBN aligned	Anomalous Hall (Quantized Anomalous Hall if lucky) Superconductivity is absent

Rhombohedral Multilayer Graphene





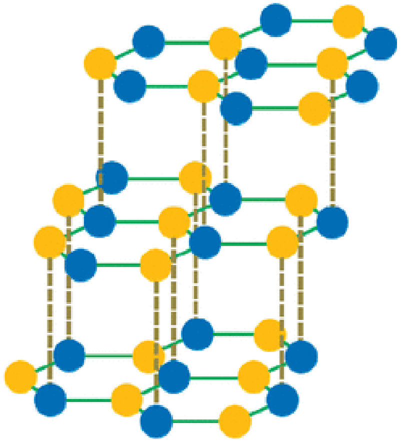
Bernal Stacking Order



ABA

Stable

Rhombohedral Stacking Order



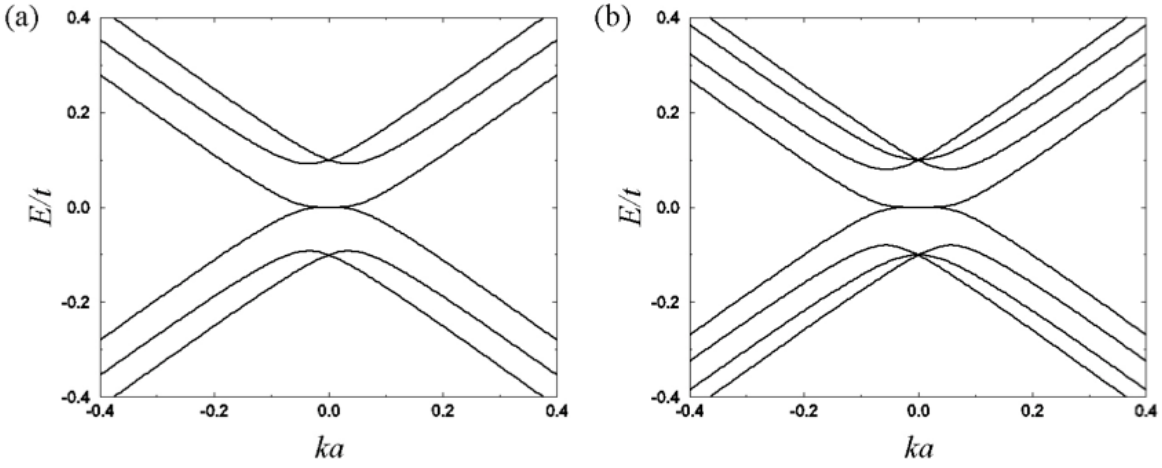
ABC

Meta-Stable

C. Lui et al. *Nano Lett.* (2010)

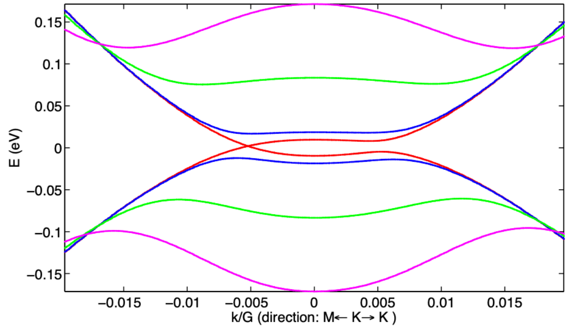
Band Structure

3-layer 4-layer



H. Min et al. *Prog. of Theo. Phys. Suppl.* (2008)

3-layer with D field



F. Zhang et al. *PRB* (2010)

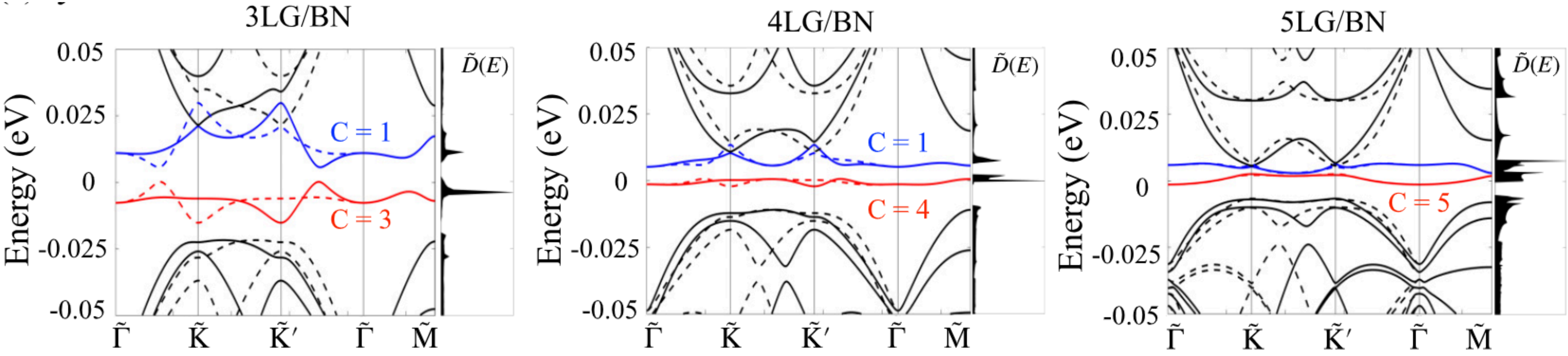
Rhombohedral Multilayer Graphene

Berry curvature $\Omega(k) = i\langle\nabla_k u(k)|\times|\nabla_k u(k)\rangle$

Anomalous Velocity $v = \frac{1}{\hbar}\nabla_k \varepsilon_k - \frac{e}{\hbar} E \times \Omega(k)$

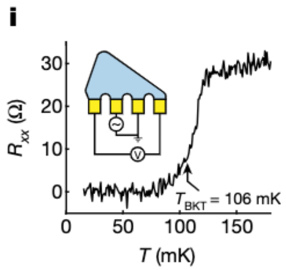
Chern number

$$C = \frac{1}{2\pi}\int_{BZ} d^2k\, \Omega(k)$$



Y. Park et al. *PRB* (2023)

SC in rhombohedral 3-layer graphene

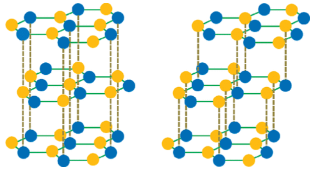


H. Zhou et al. *Nature* (2021)

Strong correlation and Topological Band :

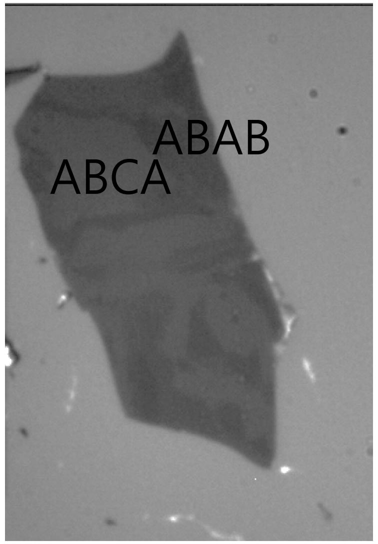
Promising ground for SC and QAH

Seeing the Stacking Order

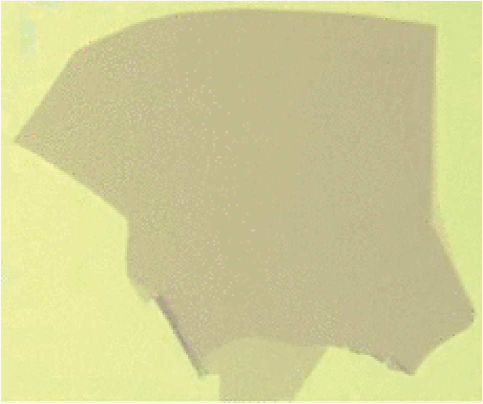


IR optical (~1sec)

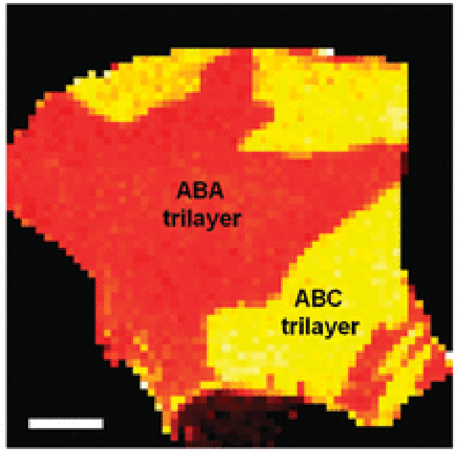
Z. Feng et al. arXiv:2408.09814

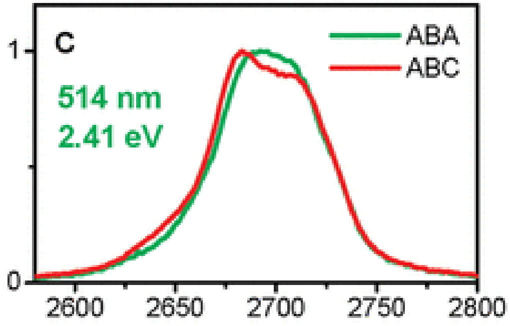


Optical



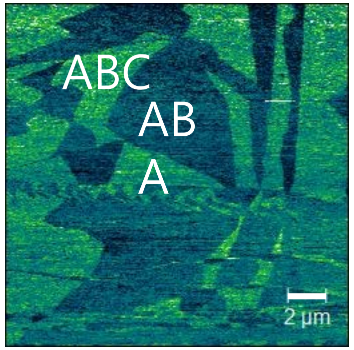
Raman Spectroscopy (>30min)



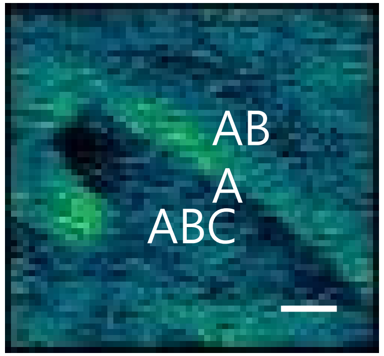


C. Lui et al. *Nano Lett.* (2010)

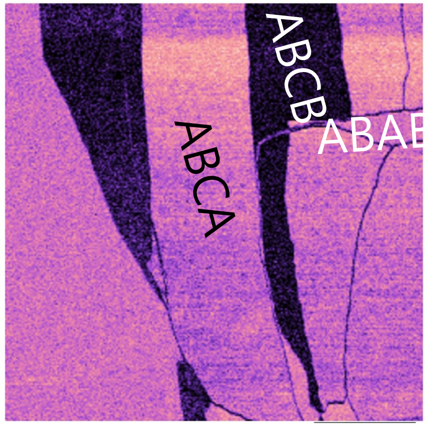
AFM based Mapping (~5min)



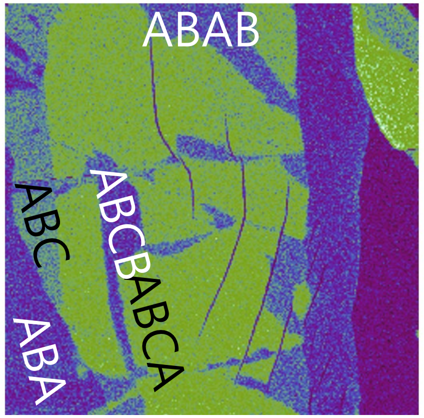
CAFM



KPFM



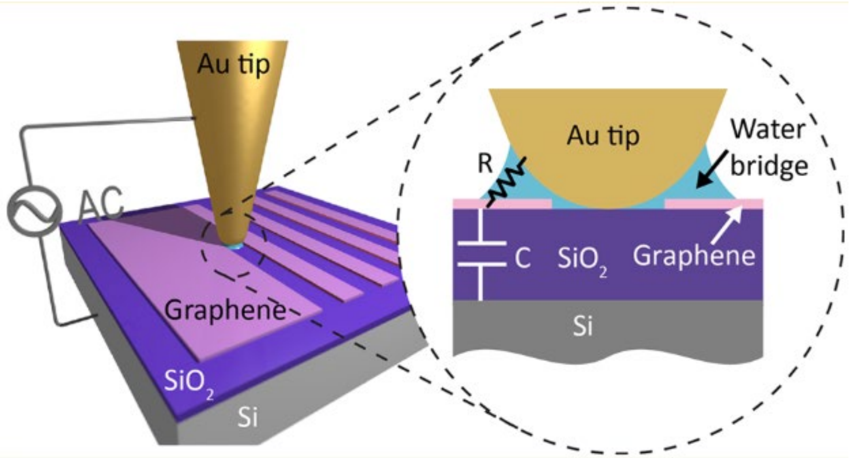
sMIM



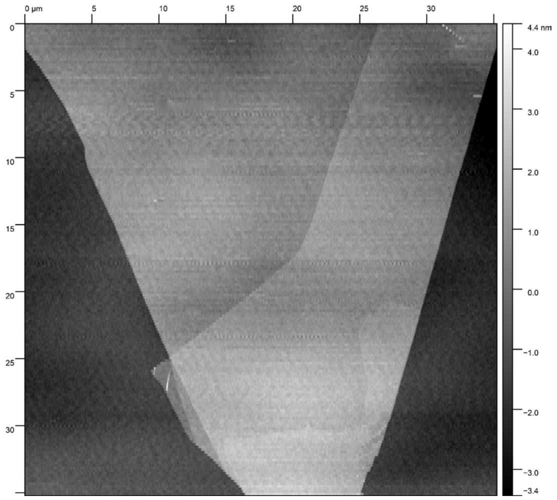
Photothermal AFM-IR

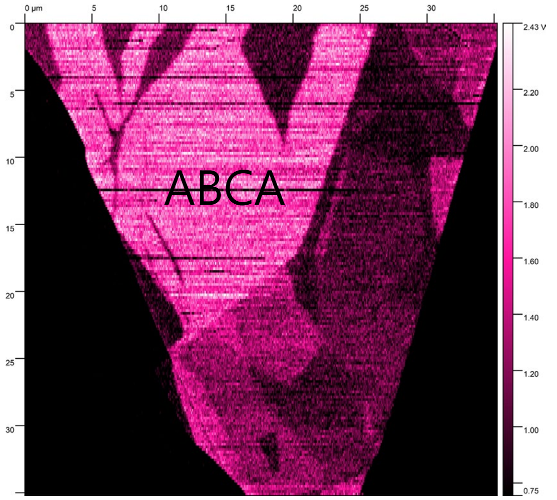
Sample Fabrication

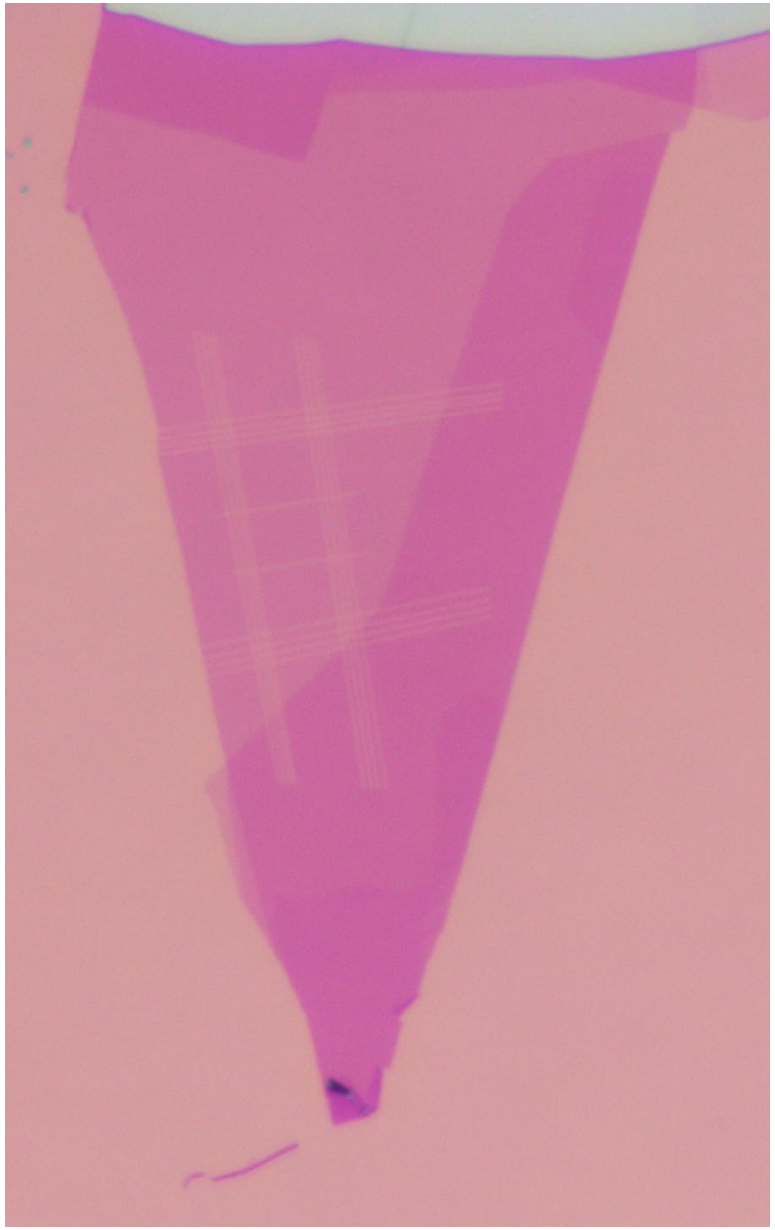
Anodic Oxidation with AFM



H. Li et al. *Nano Lett.* (2018)

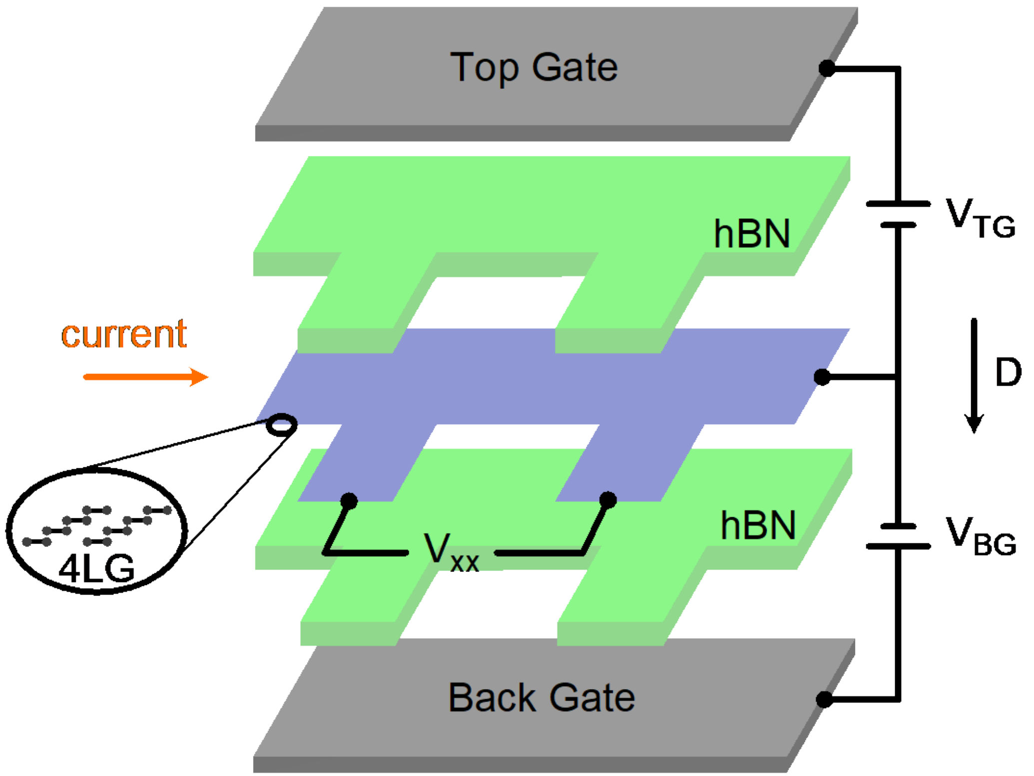




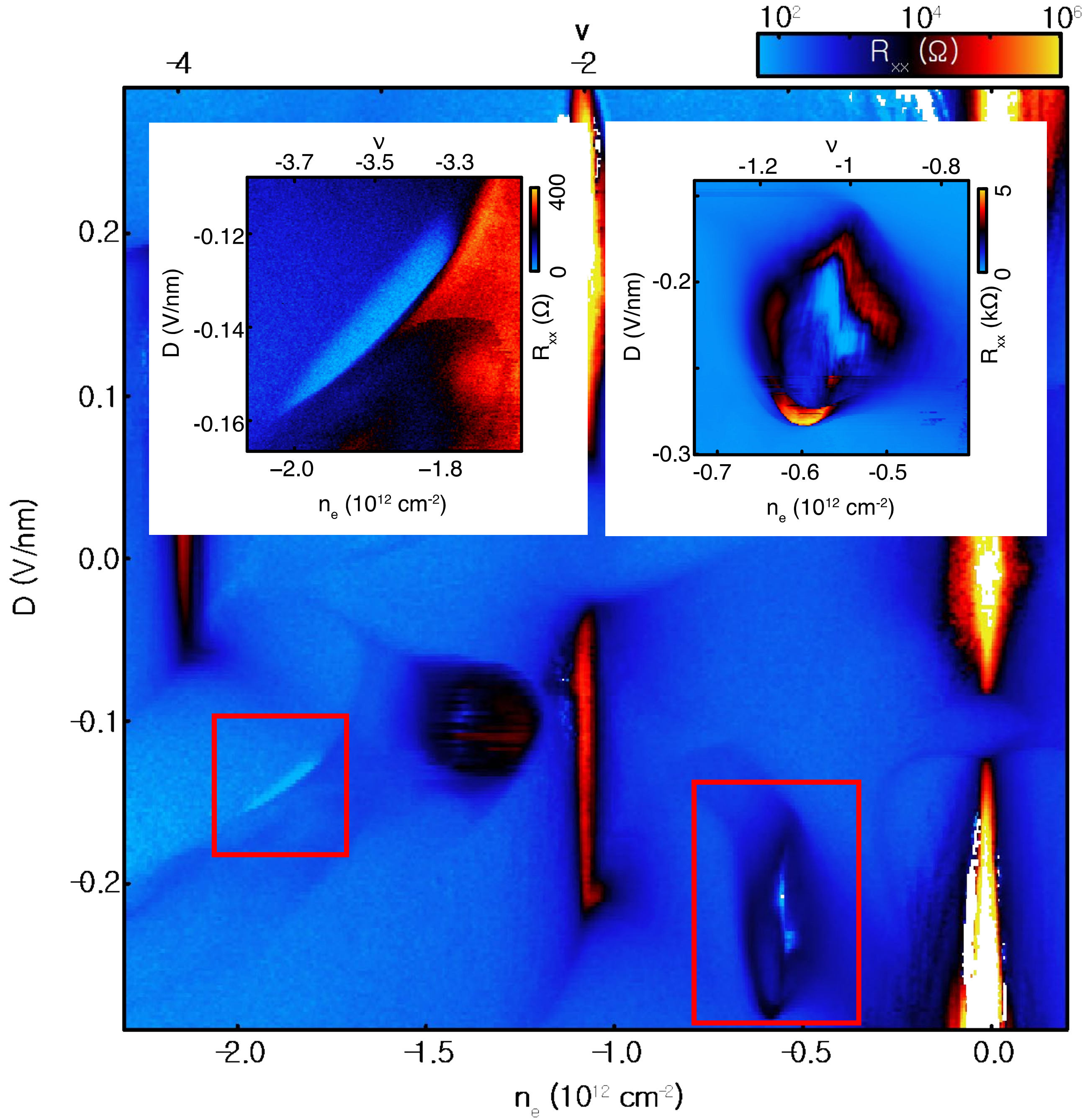


Transport Measurement

YC et al. *Nature* (2025)

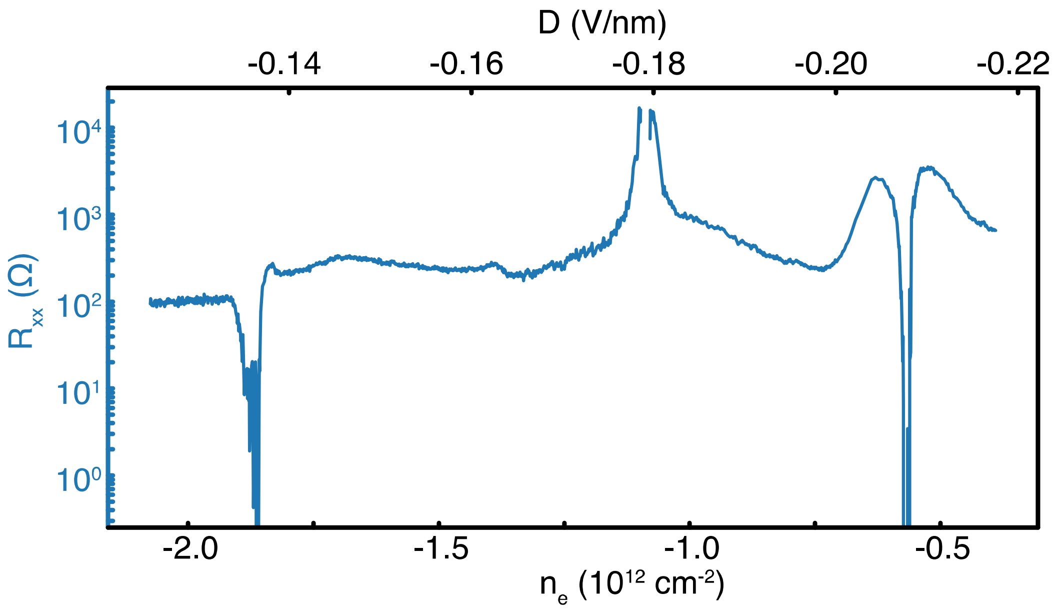


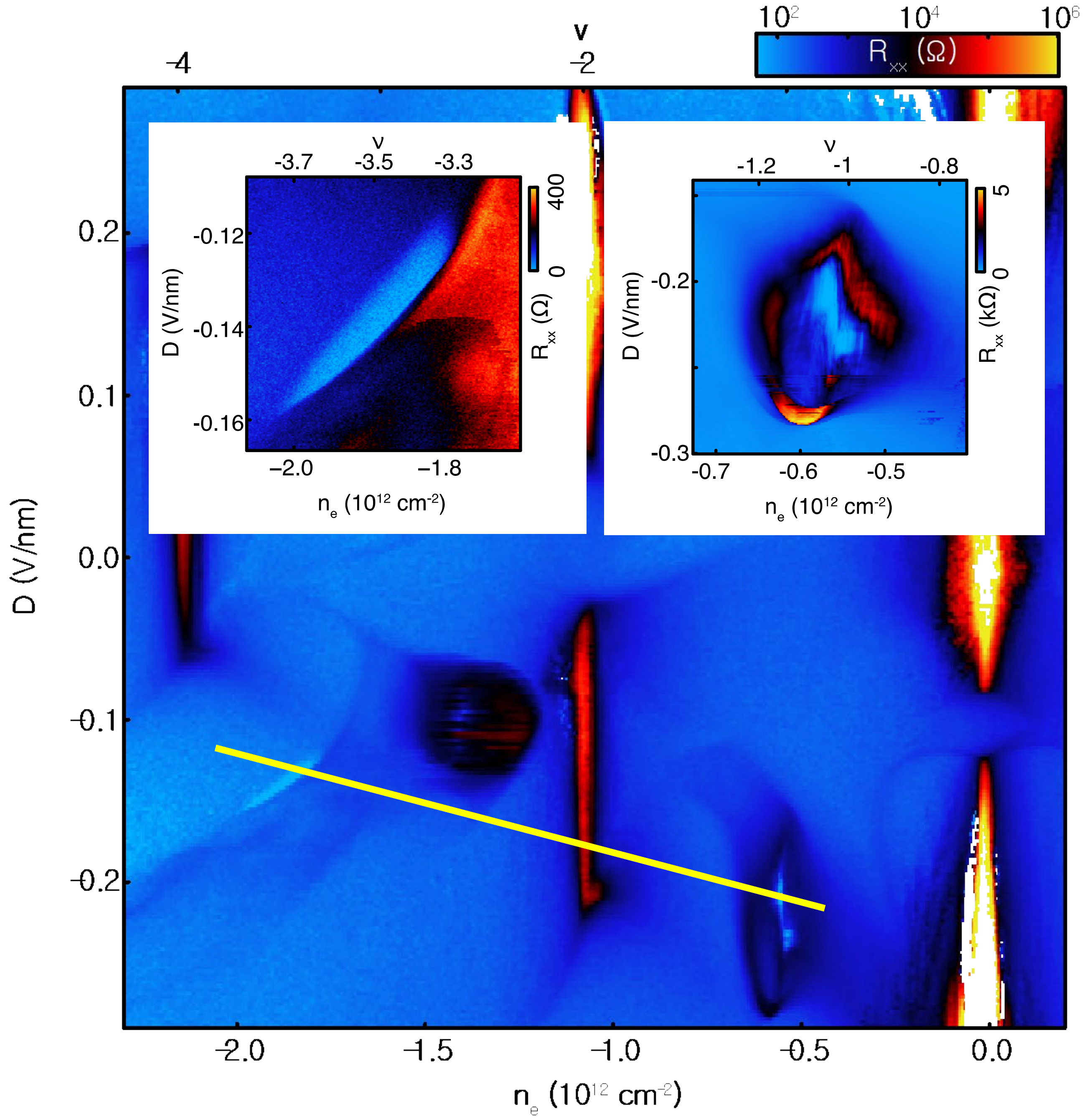
Tetralayer (4 Layer Thick)
Aligned with hBN



Rxx = 0 States

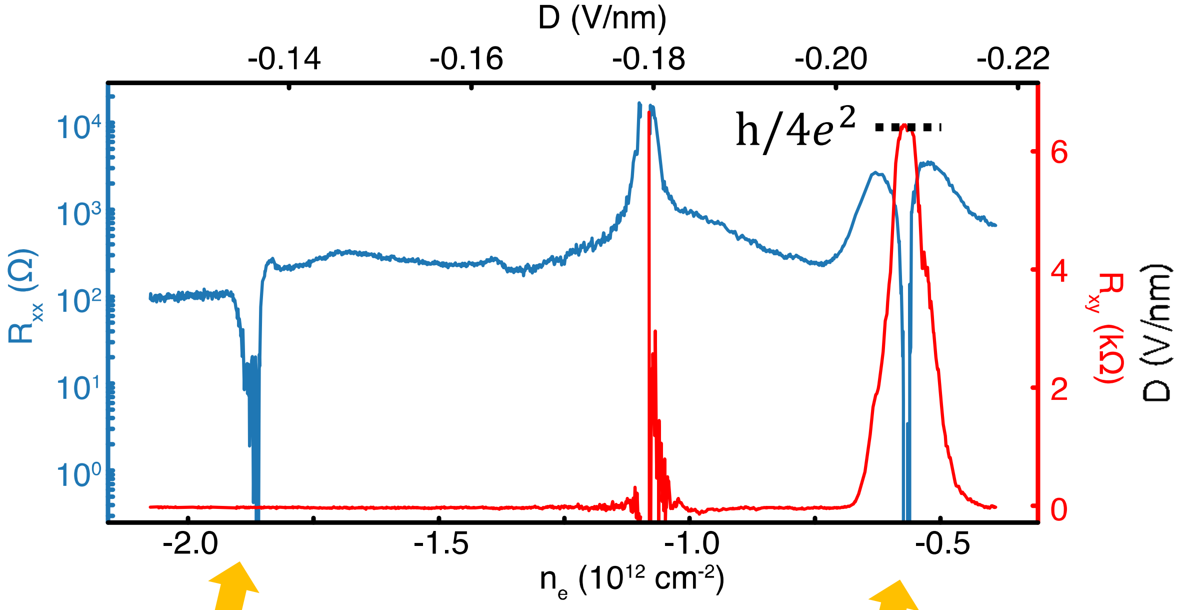
YC et al. Nature (2025)





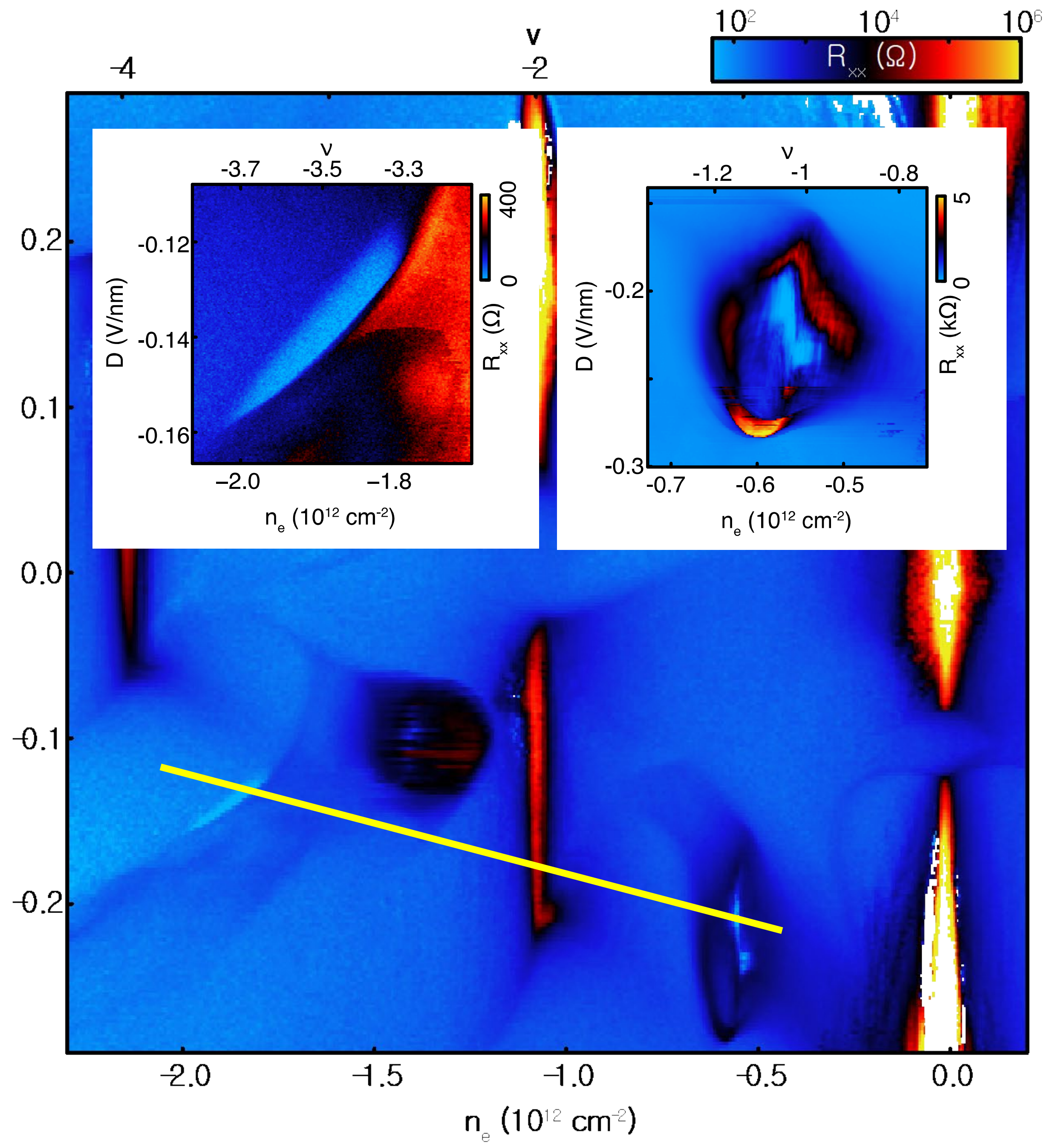
Rxx = 0 States

YC et al. *Nature* (2025)



Superconductor

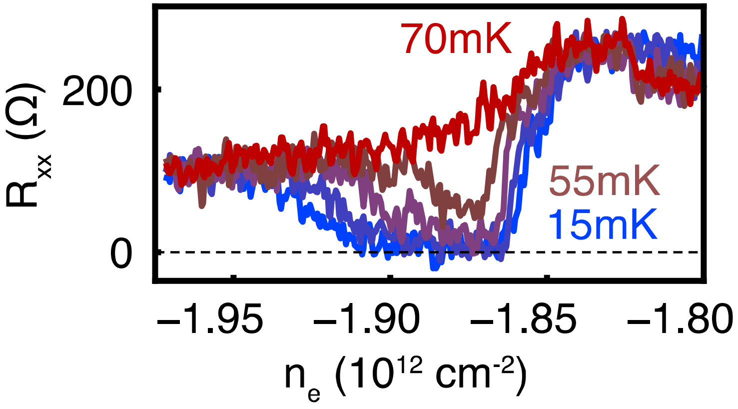
Quantum Anomalous Hall (|C| = 4)

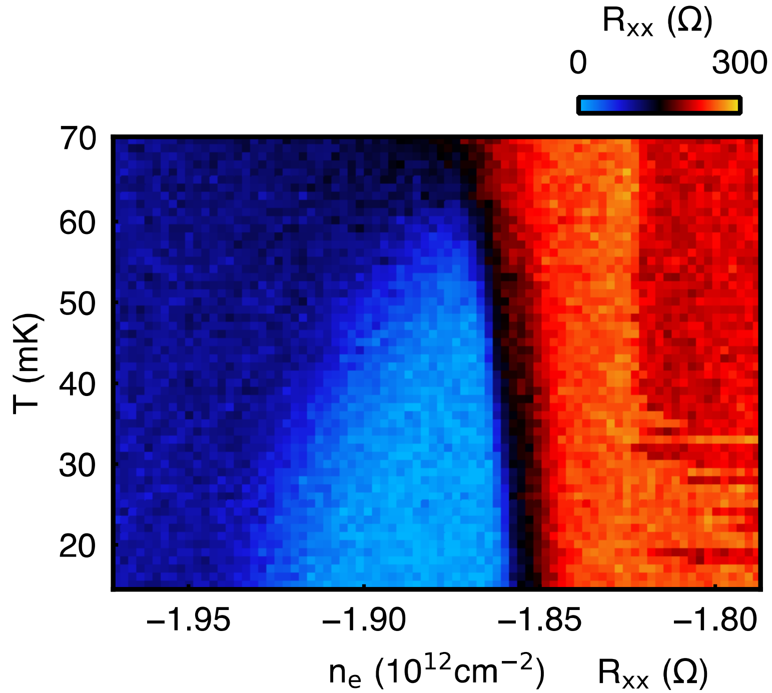


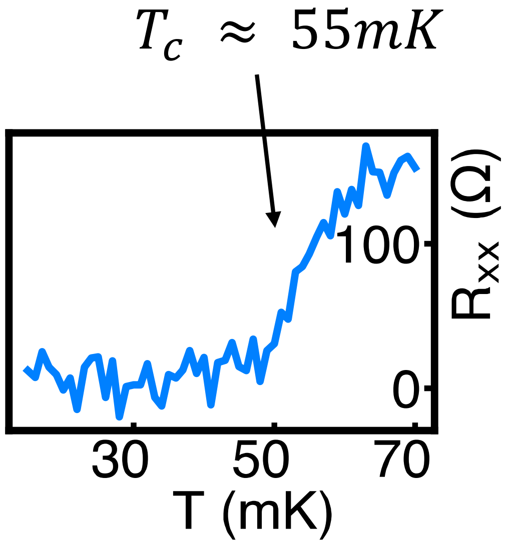
YC et al. *Nature* (2025)

Superconductor

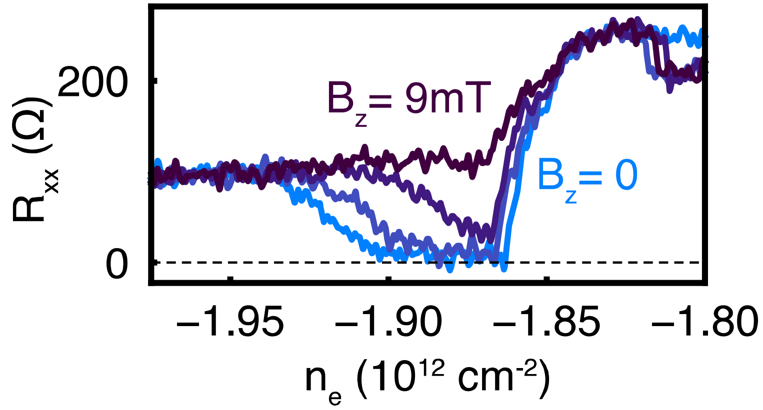
Temperature Dependence

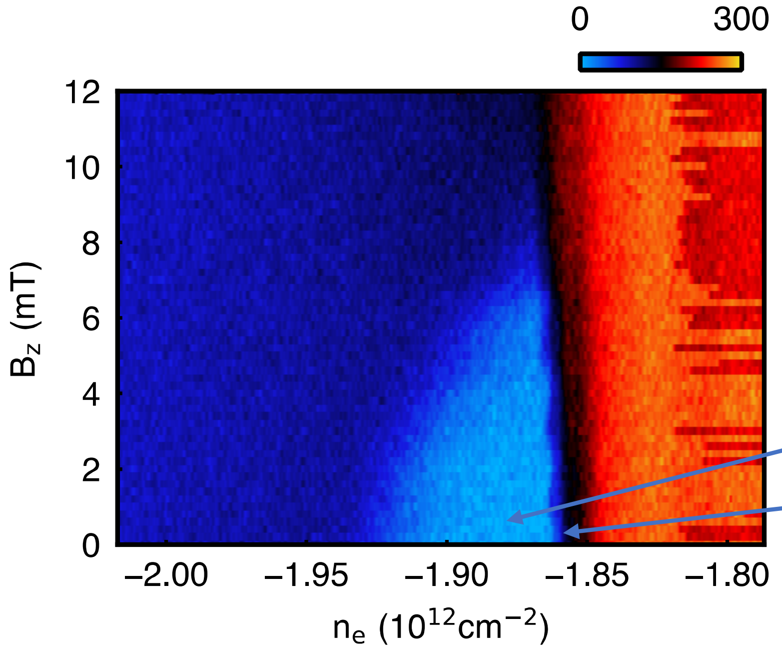




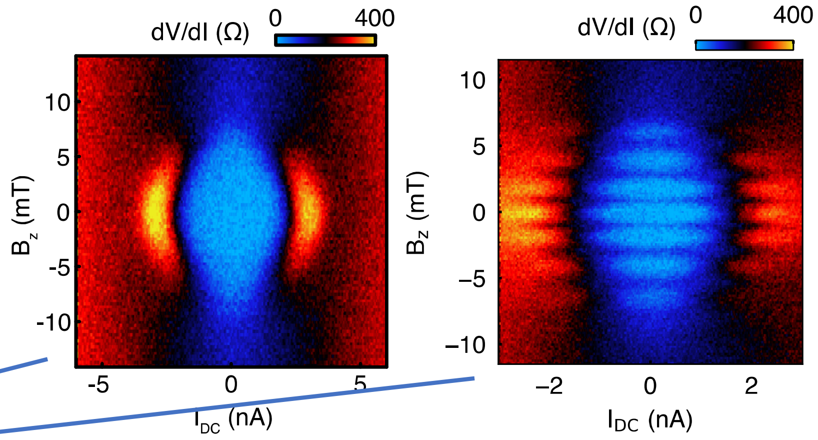


Magnetic Field Dependence

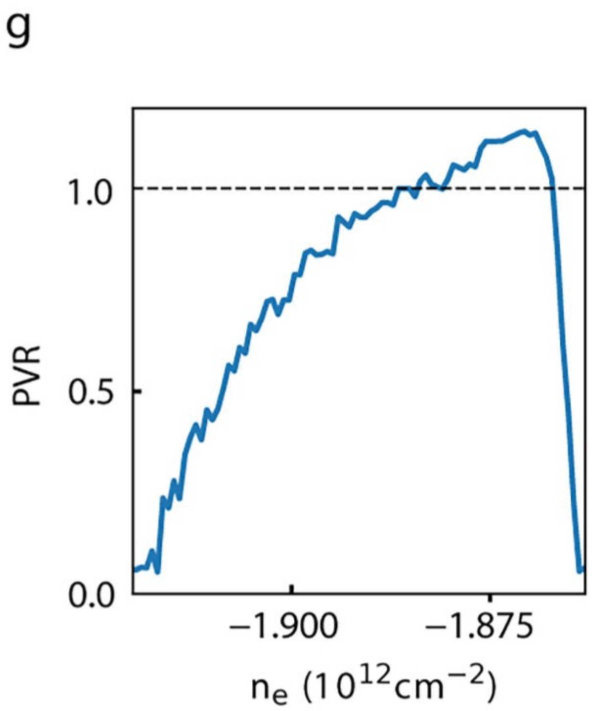




Phase Coherence



Superconductor



Spin singlet

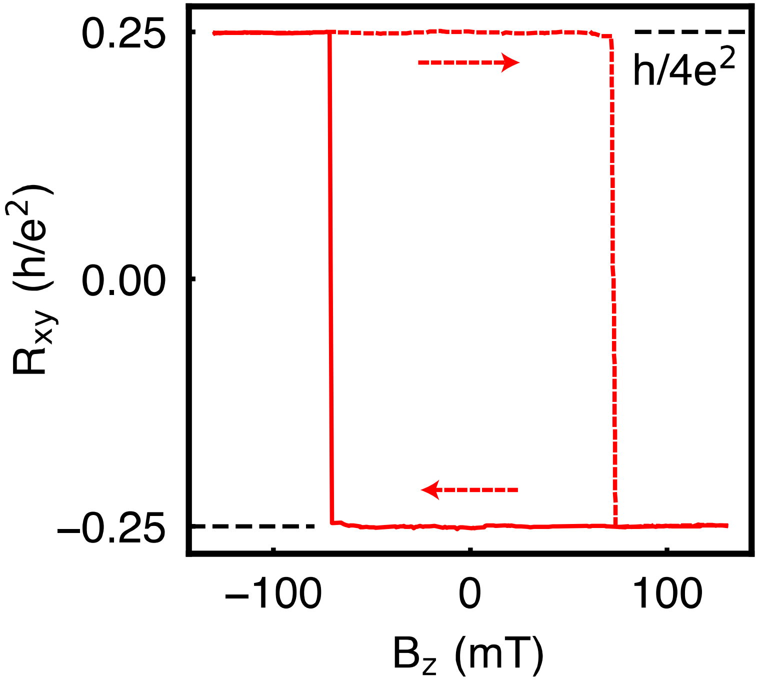
Likely s-wave superconductor

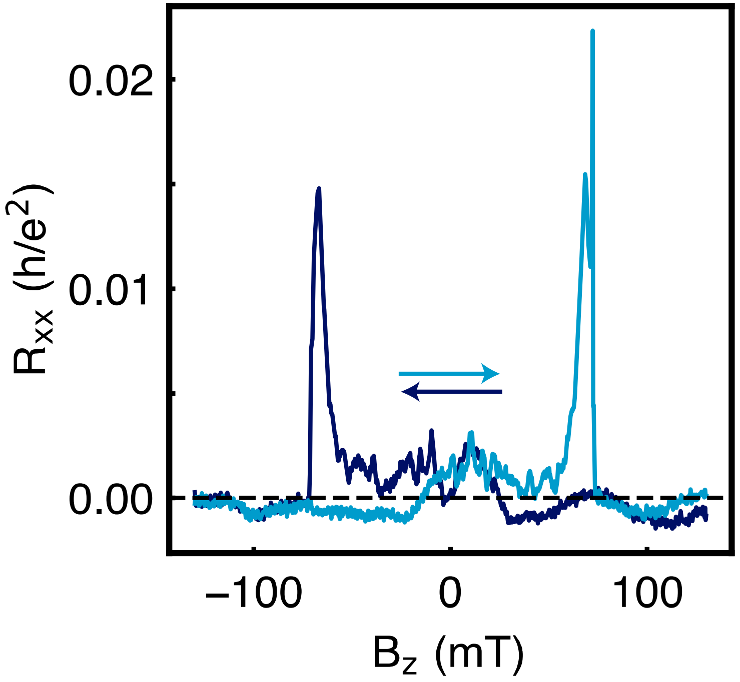
YC et al. *Nature* (2025)

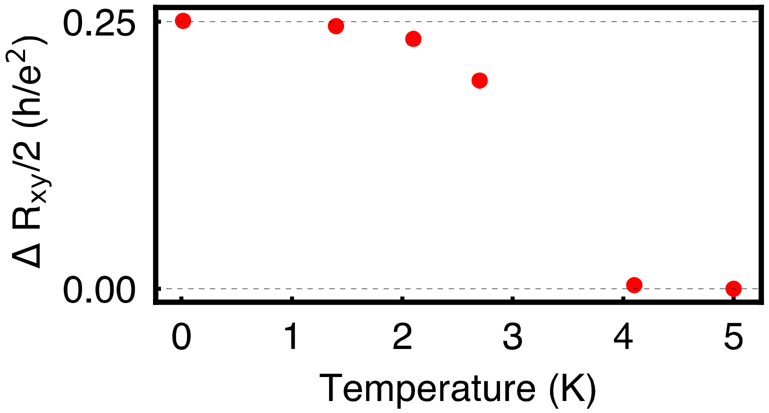
Quantum Anomalous Hall (QAH)

|C| = 4

Quantization persists to ~2K



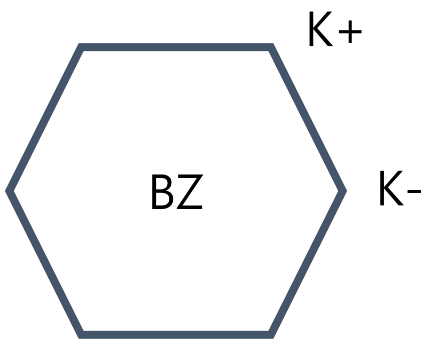


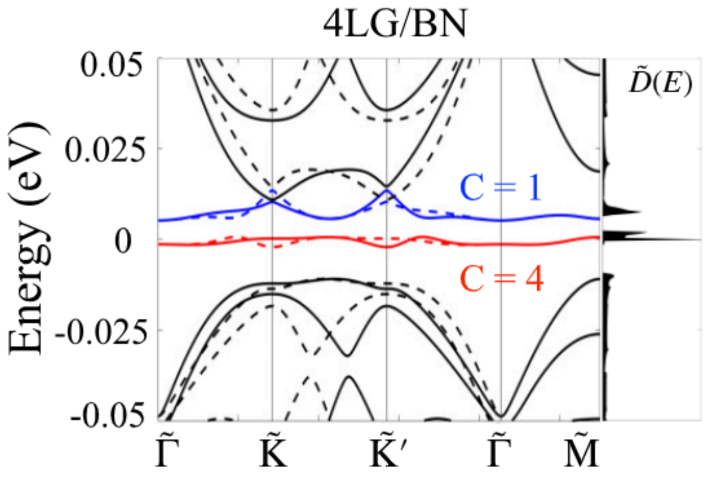


$R_{xx} = 0\ \Omega$

$R_{xy} = h/4e^2$

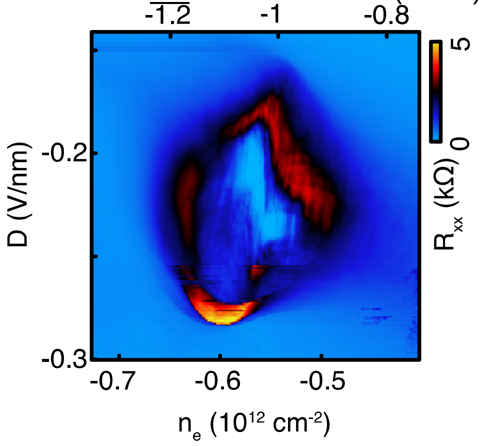
Quantum Anomalous Hall (QAH)



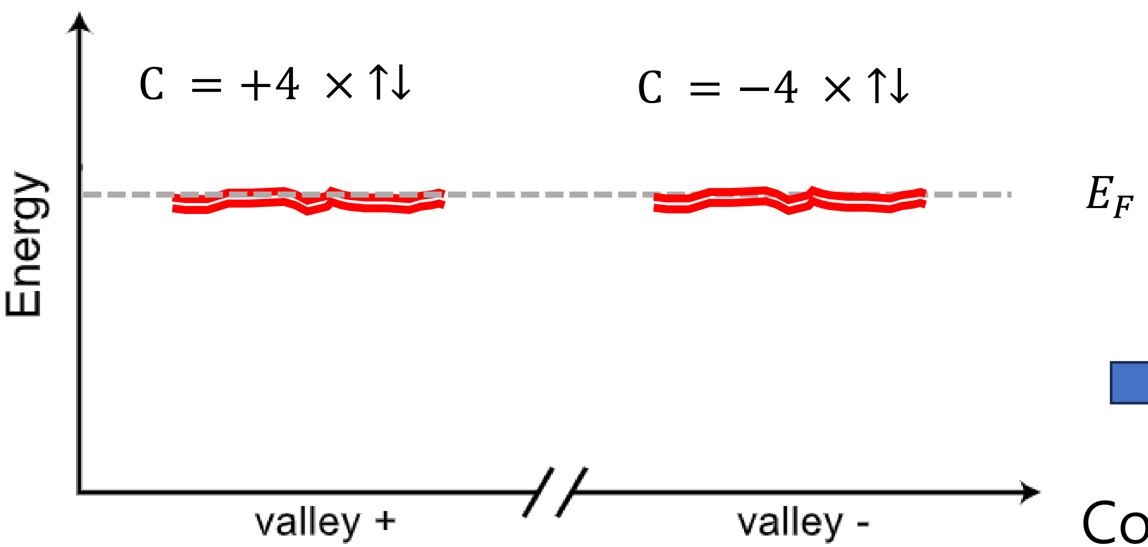


Y. Park et al. *PRB* (2023)

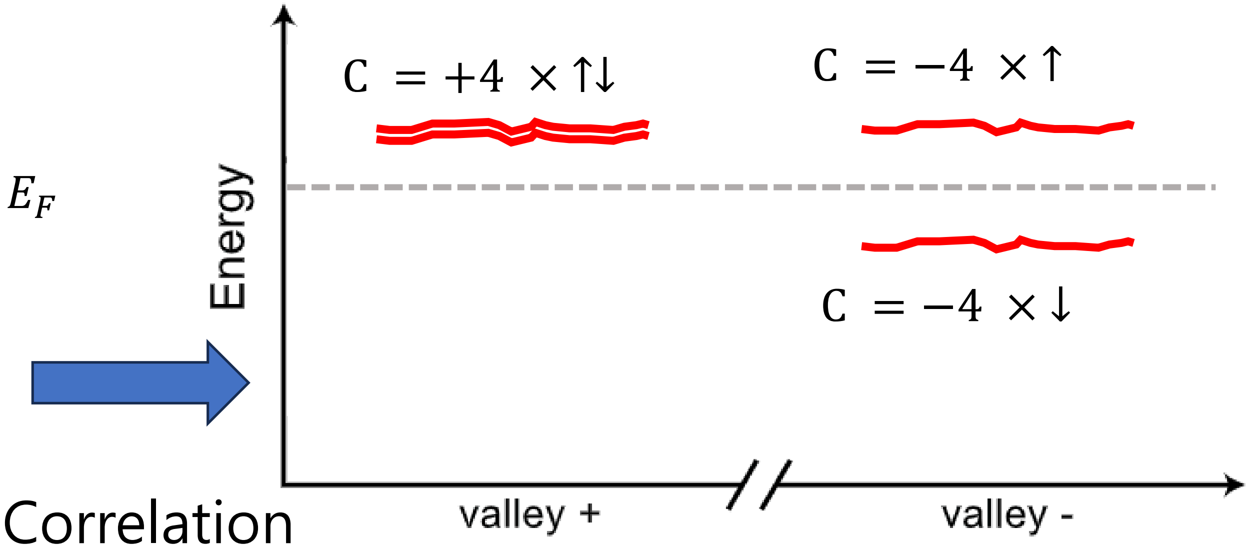
YC et al. *Nature* (2025)



$\nu = -1$



$\nu = -1$ $C_{net} = -4$

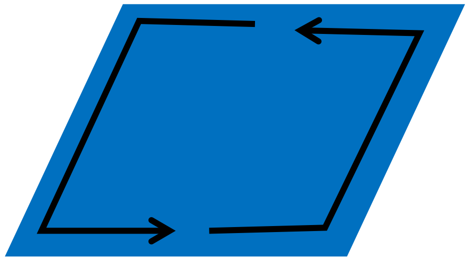


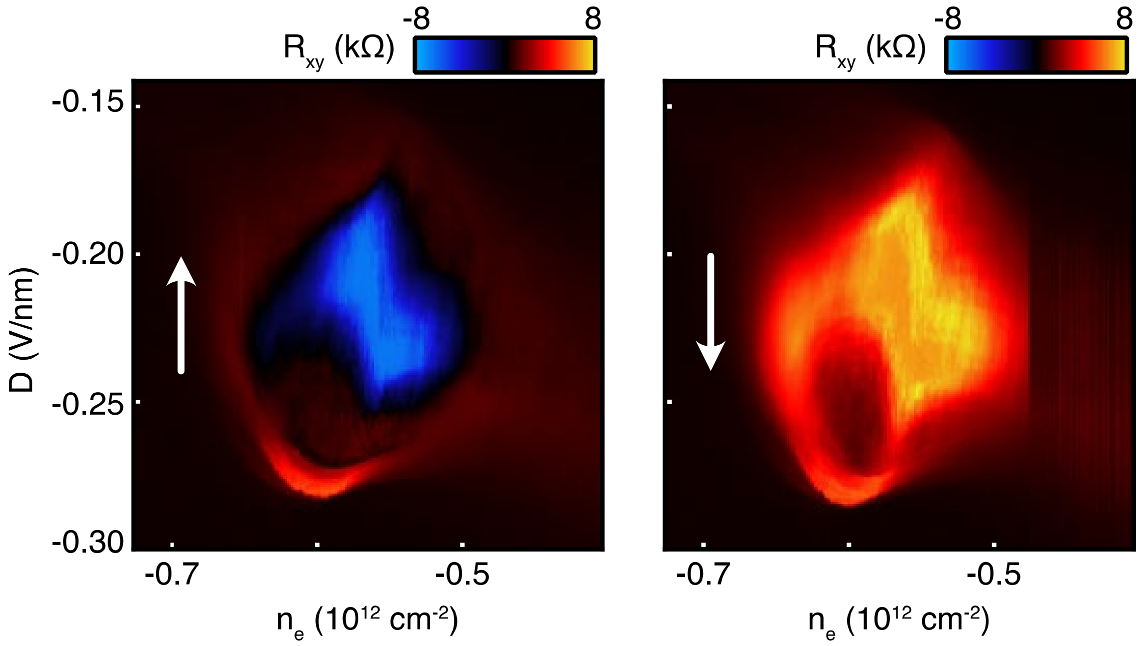
Correlation

Breaks Time-reversal Symmetry

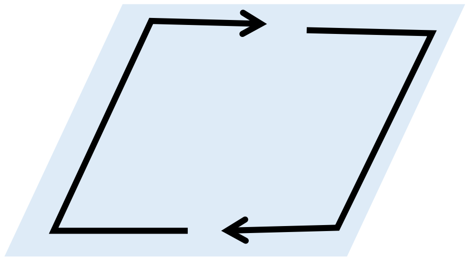
YC et al. *Nature* (2025)

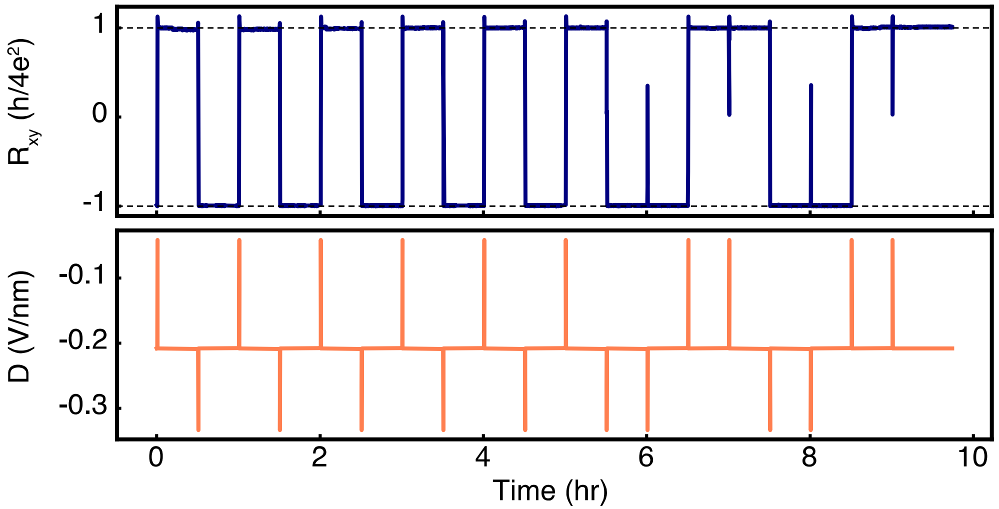
Electrical Switching of QAH Chirality



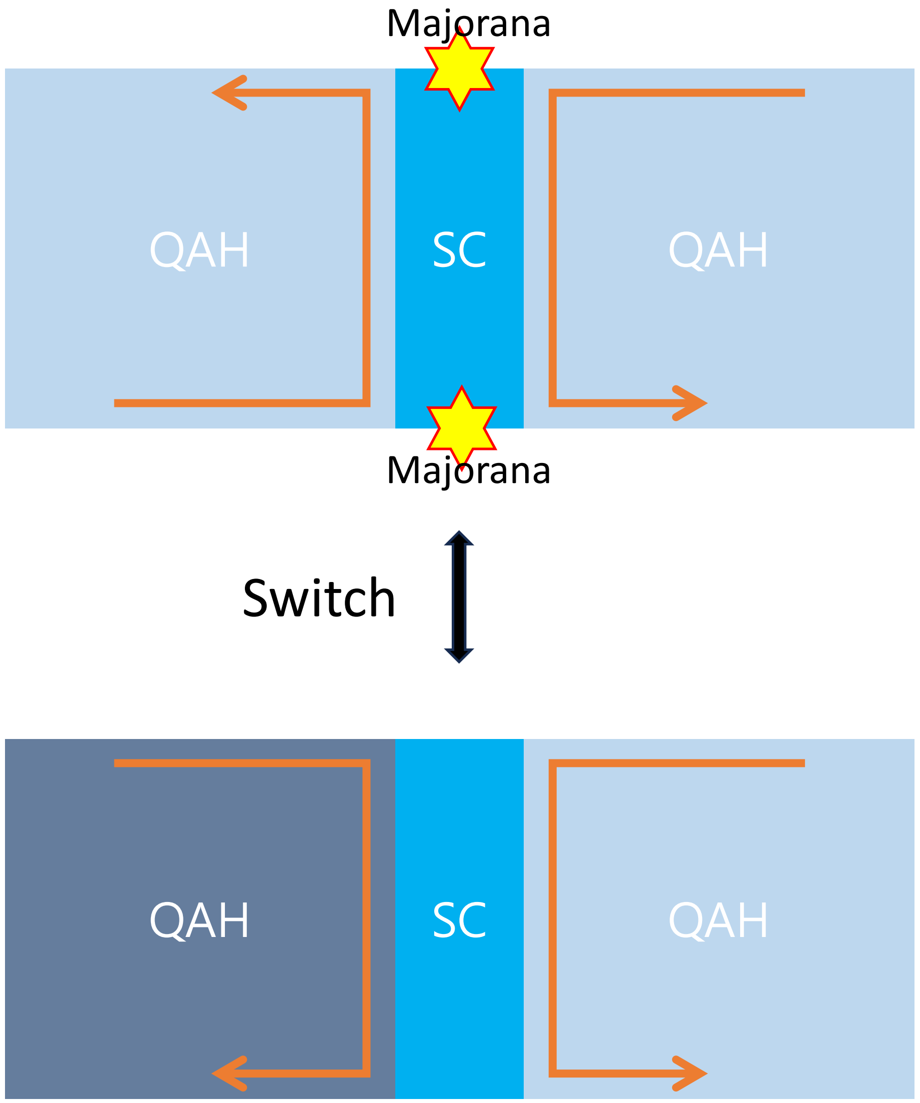


Bz < 1mT

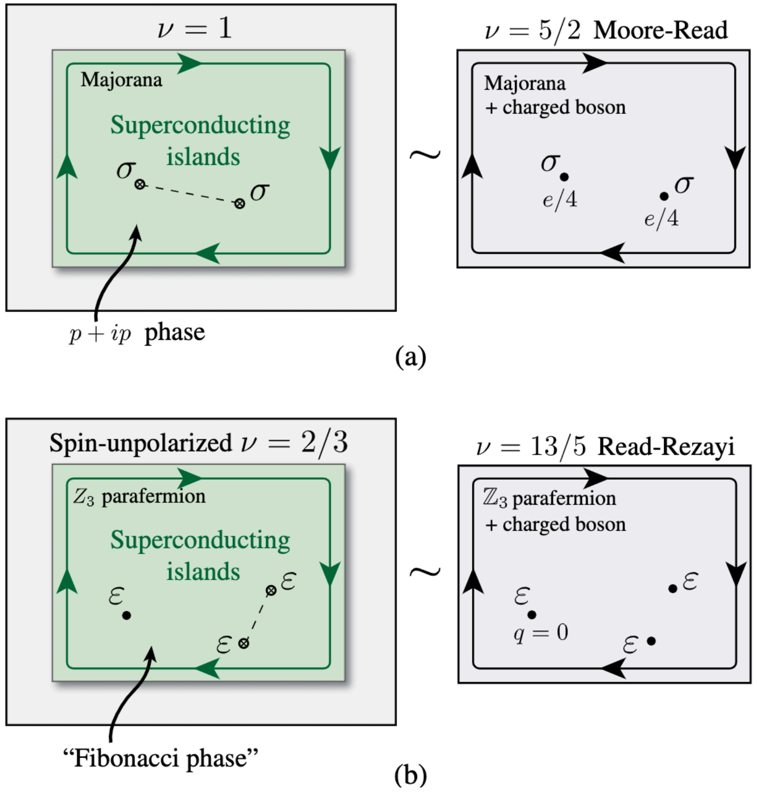




Re-configurable Junctions



Parafermions : SC + FQ(A)H

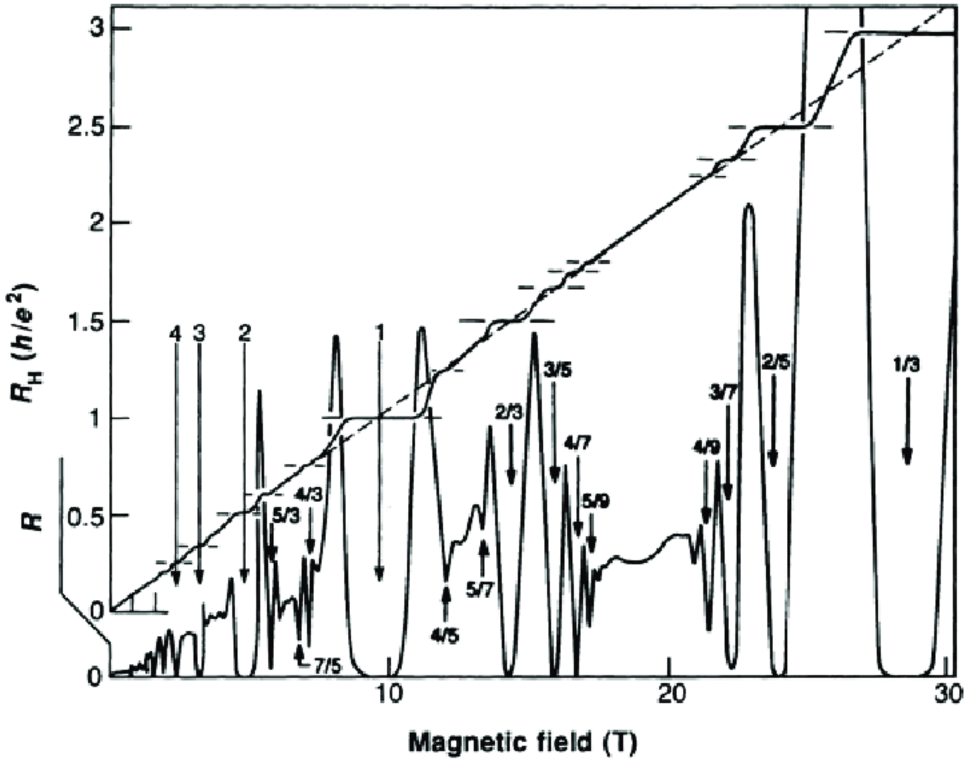


R. Mong et al. *PRX* (2014)

SC +Q(A)H
: Majorana

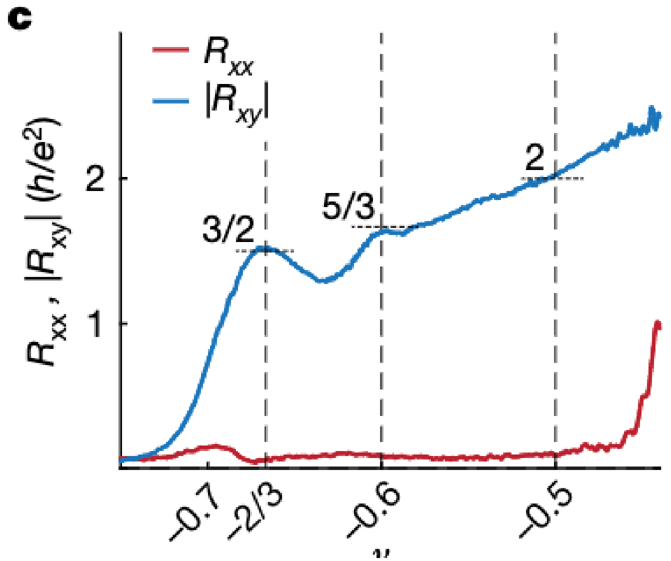
SC +FQ(A)H
: Parafermion

Fractional Quantum Hall effect



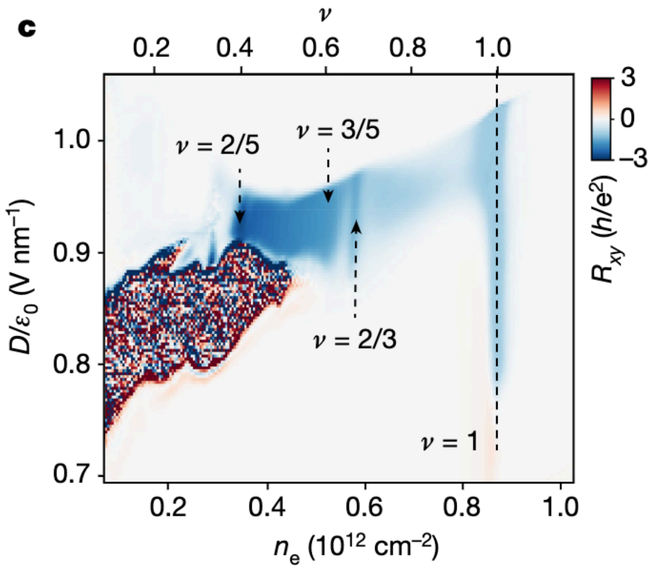
Fractional Quantum Anomalous Hall

FQAH in twisted MoTe2



J. Cai et al. *Nature* (2023)
Y. Zeng et al. *Nature* (2023)
H. Park et al. *Nature* (2023)

FQAH in 5-layer rhombohedral graphene



Z. Lu et al. *Nature* (2024)

Capacitance Measurement

Measure

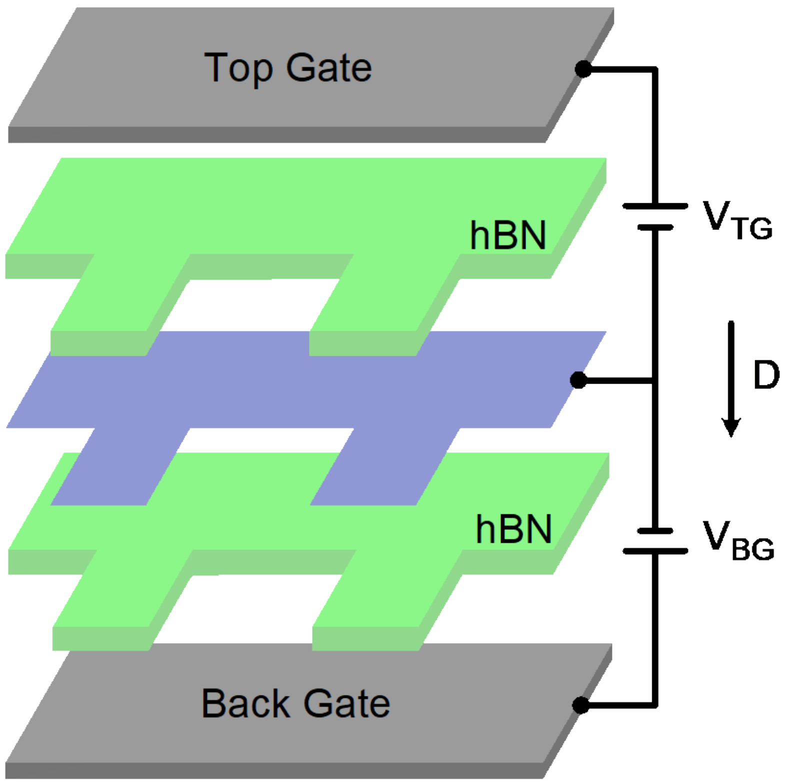
$$C_P = \frac{\delta Q_t}{\delta V_b}$$

$$C_P \sim \kappa = \frac{d\mu}{dn}$$

: Inverse Compressibility

J. Eisenstein et al. *PRB* (1994)

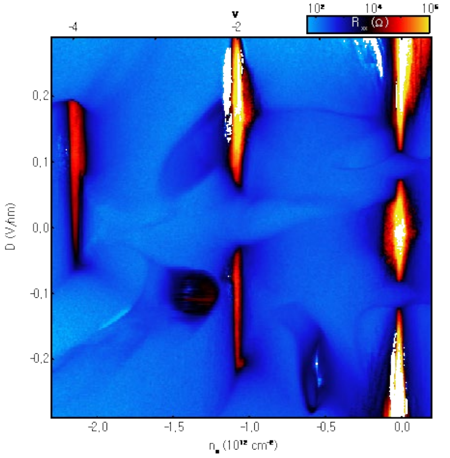
A. Zibrov et al. *Nature* (2017)



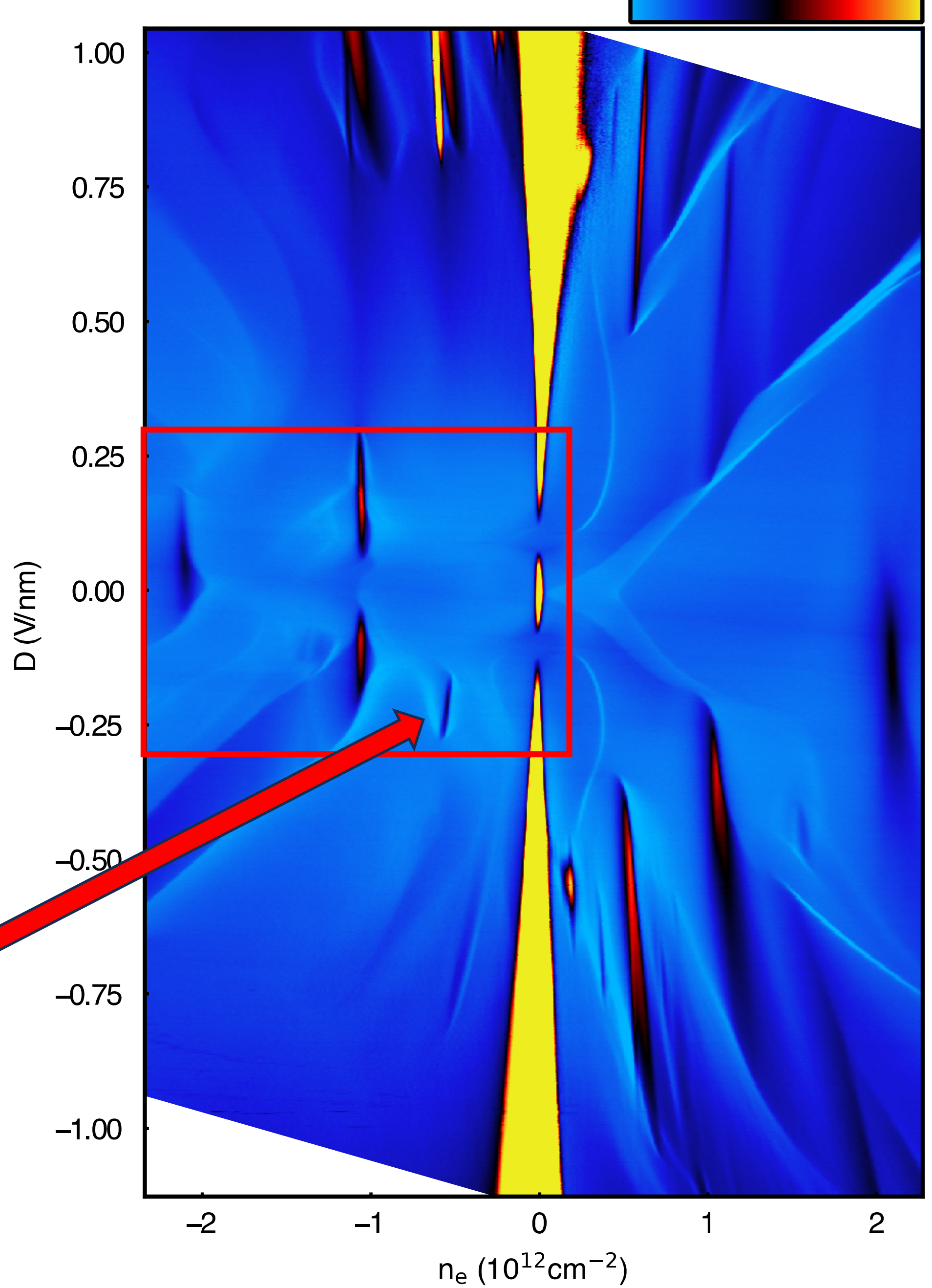
Capacitance

YC et al. *Nature* (2025)

Transport



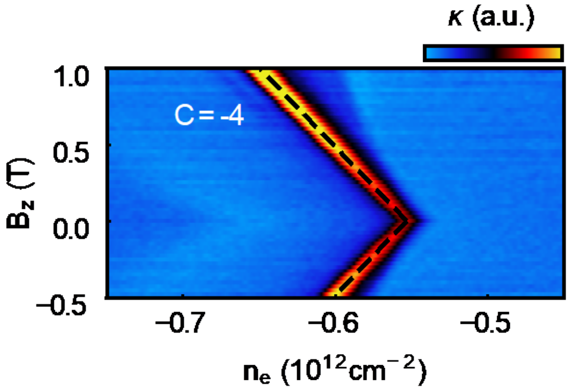
Inverse Compressibility



QAH

$\nu = -1$

$C = -4$



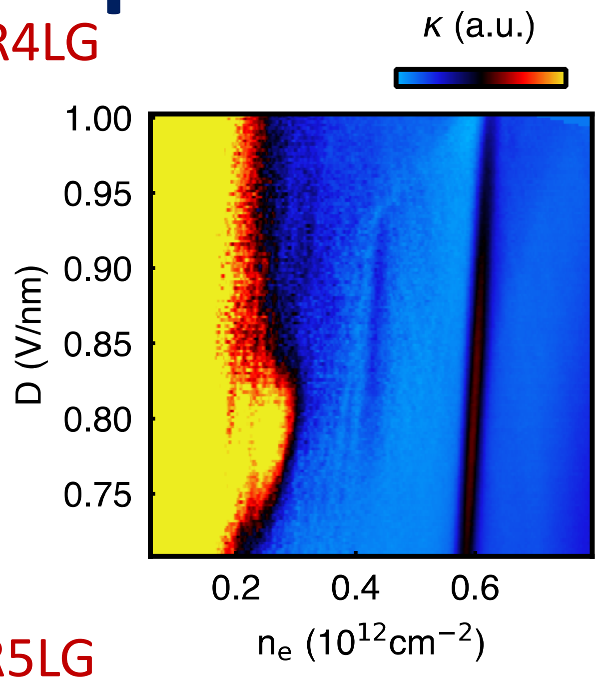
Streda Formula

$$C = \Phi_0 \frac{dn}{dB}$$

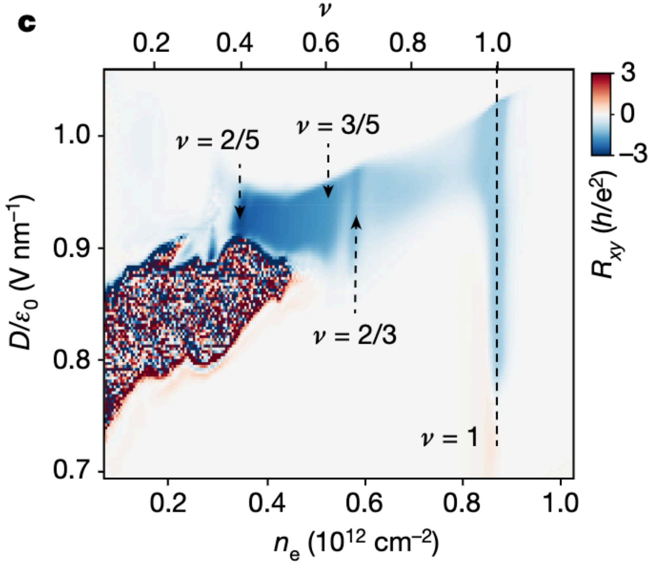
YC et al. *Nature* (2025)

Capacitance

R4LG

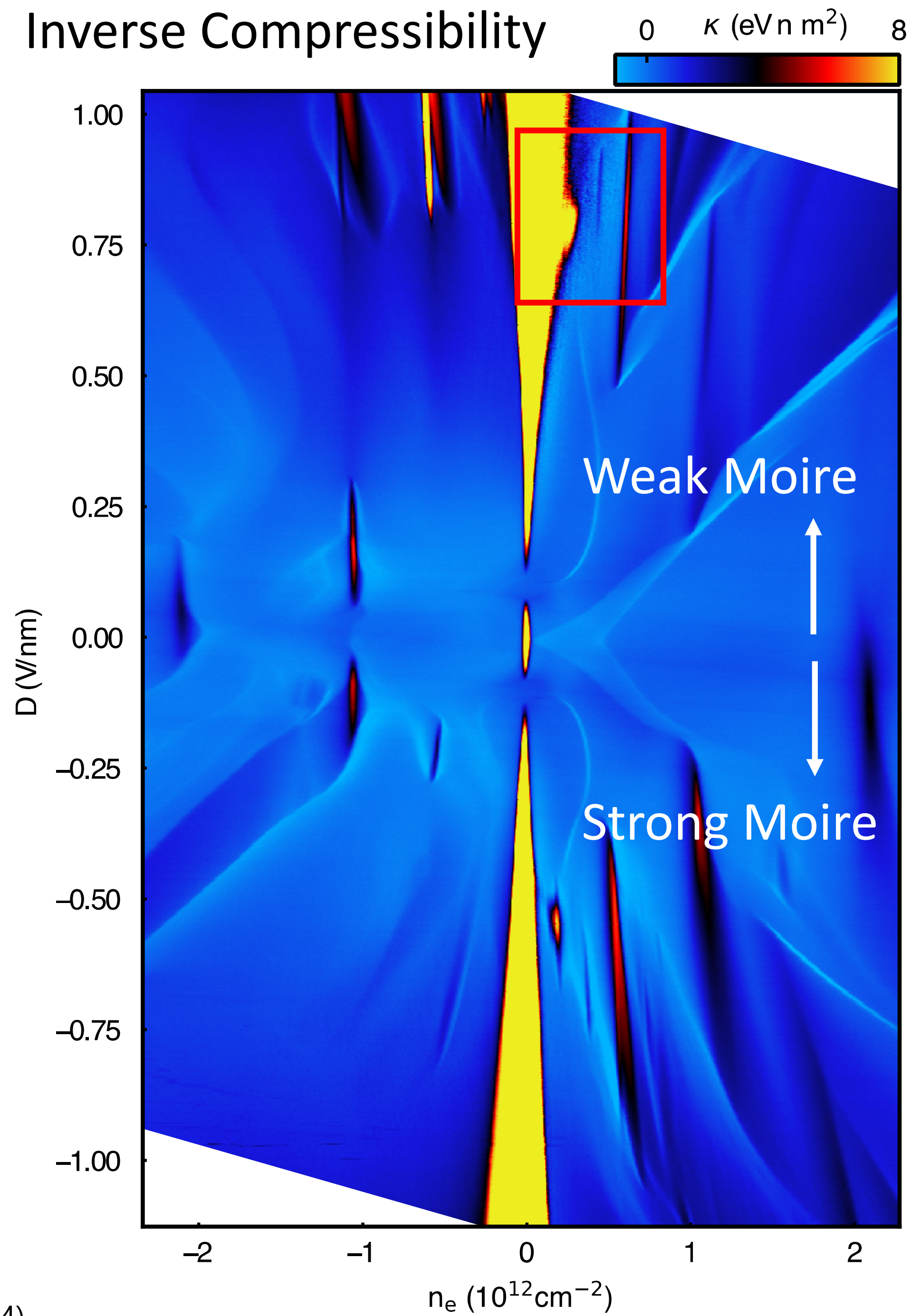


R5LG



Z. Lu et al. *Nature* (2024)

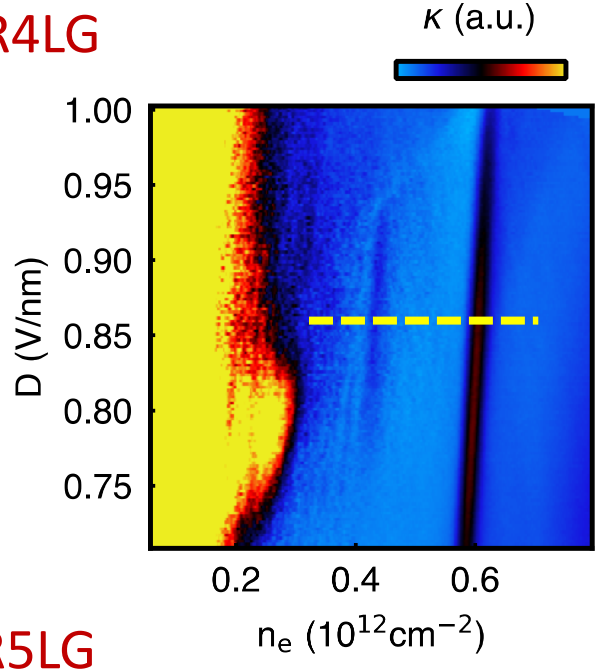
Inverse Compressibility

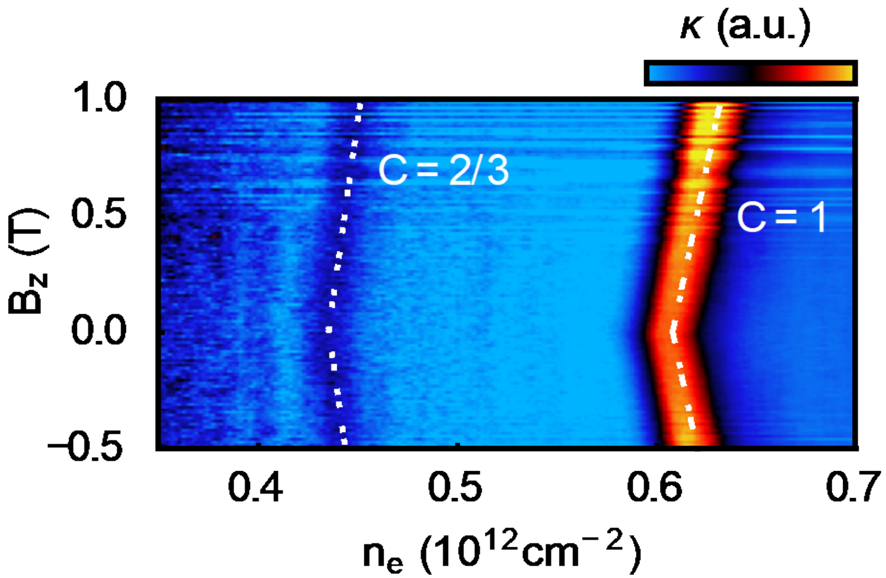


YC et al. *Nature* (2025)

Fractional Chern Insulator

R4LG

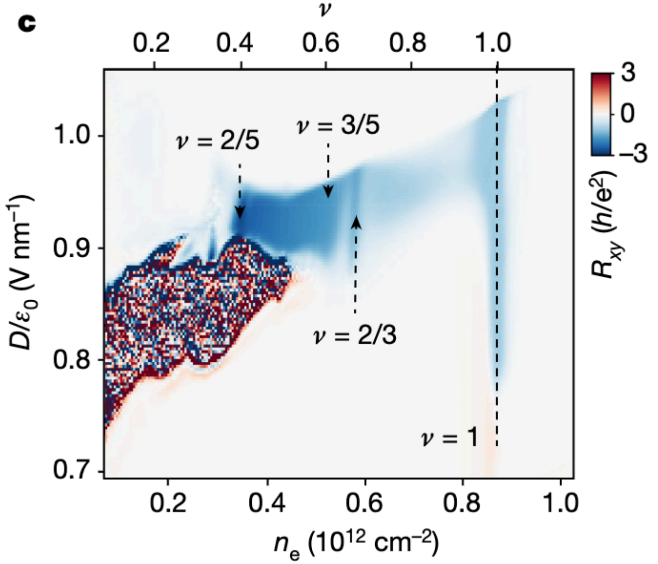




Streda Formula

$$C = \Phi_0 \frac{dn}{dB}$$

R5LG

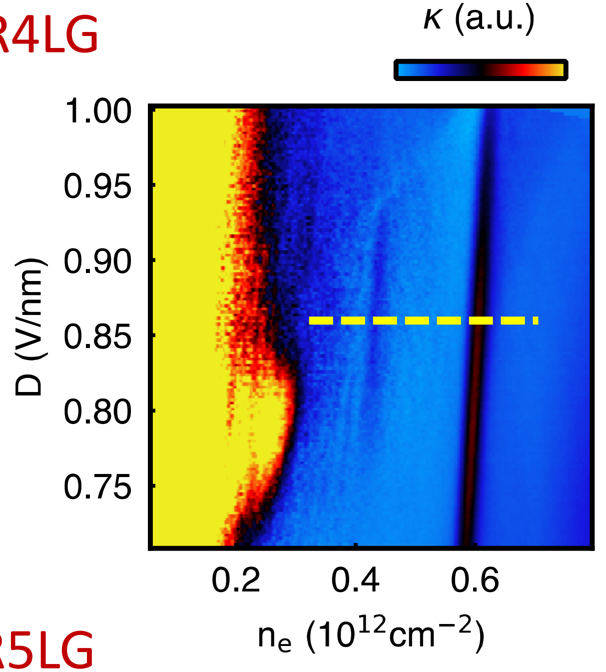


Z. Lu et al. *Nature* (2024)

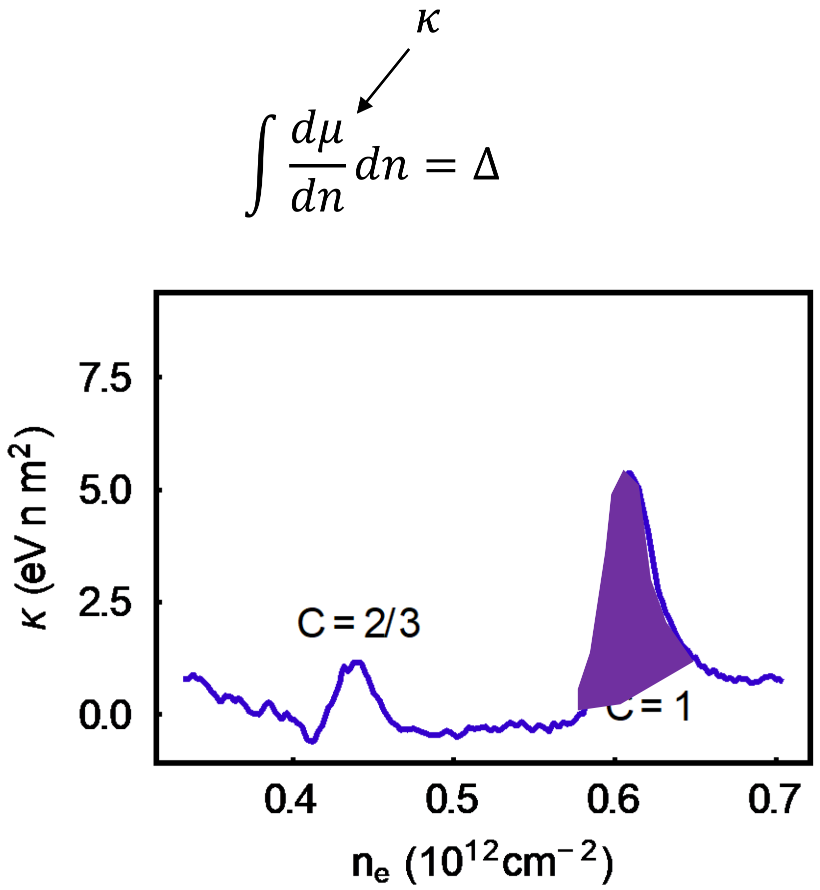
YC et al. *Nature* (2025)

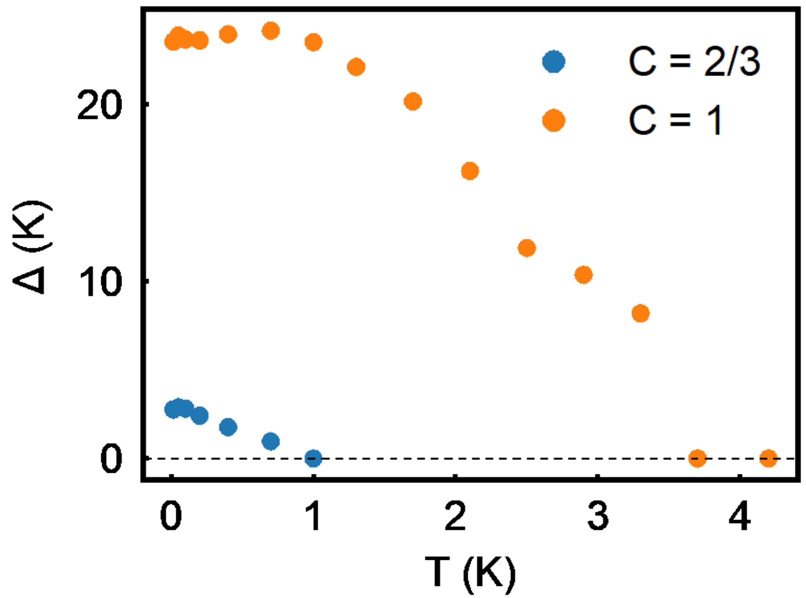
Fractional Chern Insulator

R4LG

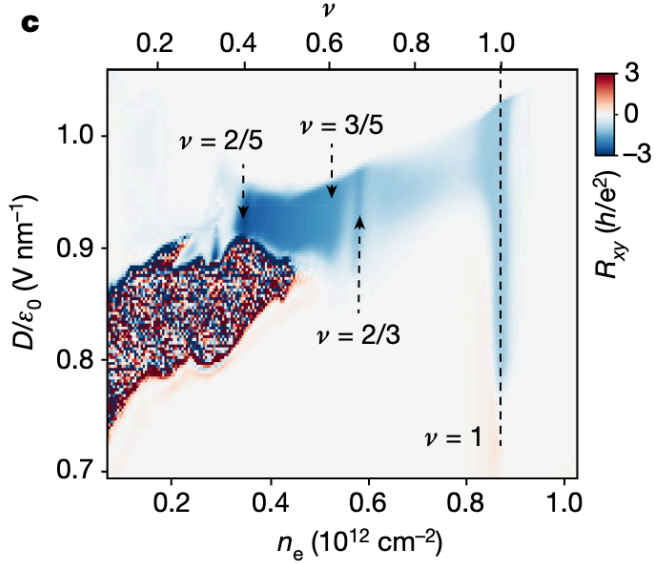


$$\int \frac{d\mu}{dn} dn = \Delta$$



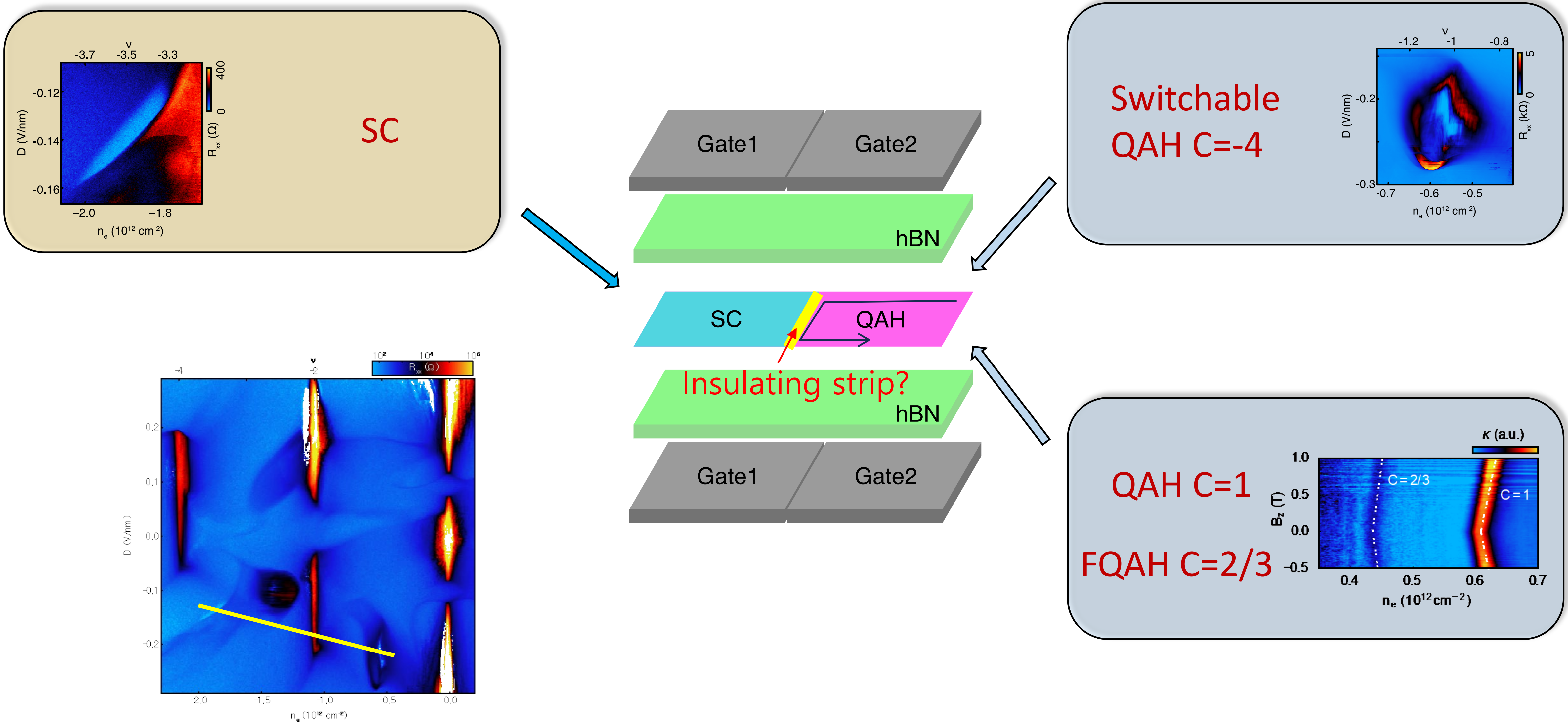


R5LG



Z. Lu et al. *Nature* (2024)

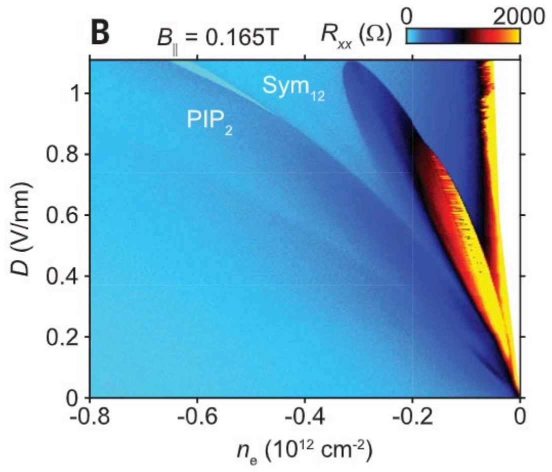
Multiple Phases in Single Material



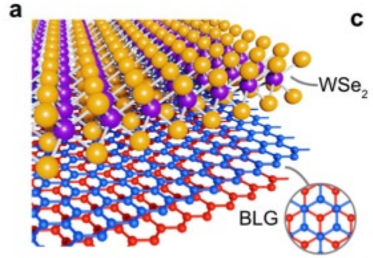
Inducing Spin-Orbit Coupling

Bilayer Graphene

$T_c \approx 25mK$

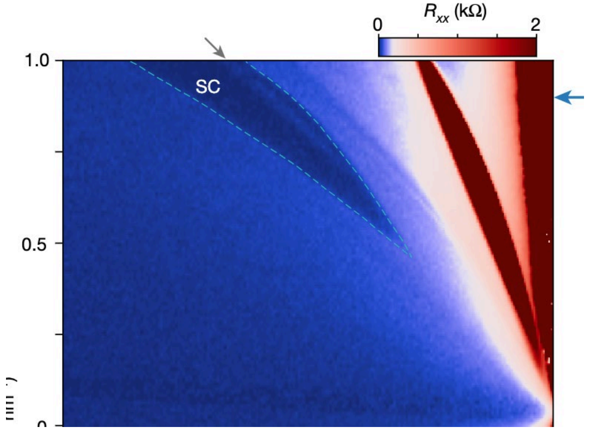


H. Zhou et al. *Science* (2022)



With WSe2

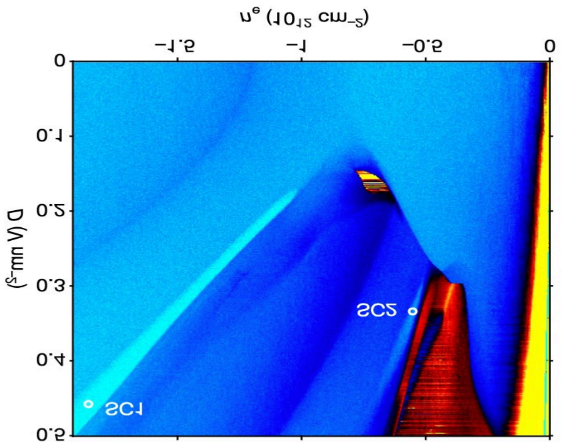
$T_c \approx 260mK$



Y. Zhang et al. *Nature* (2023)

ABC Trilayer Graphene

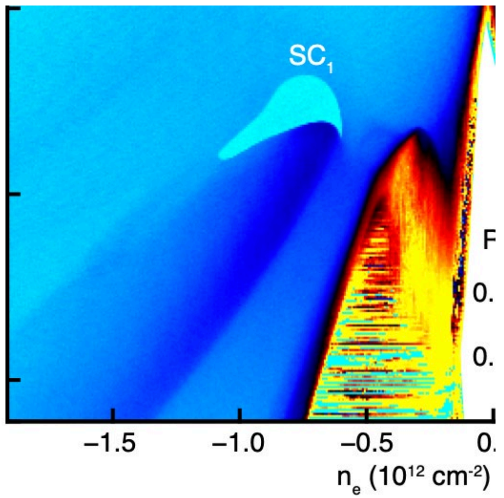
$T_c \approx 100mK$



H. Zhou et al. *Nature* (2021)

With WSe2

$T_c \approx 300mK$

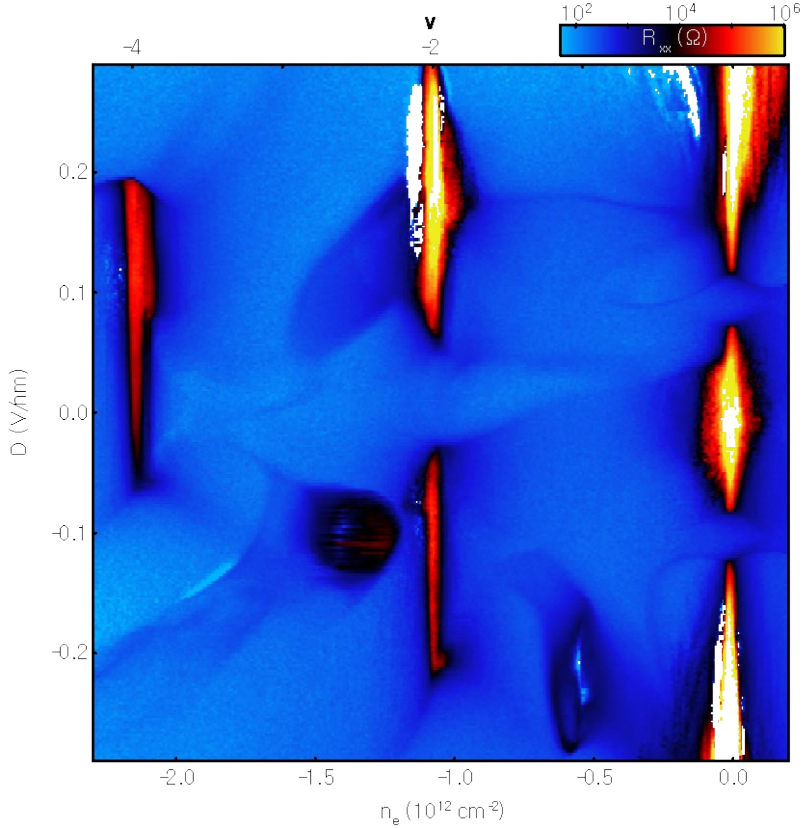


C. Patterson et al. *arXiv:2408.10190*

Inducing Spin-Orbit Coupling

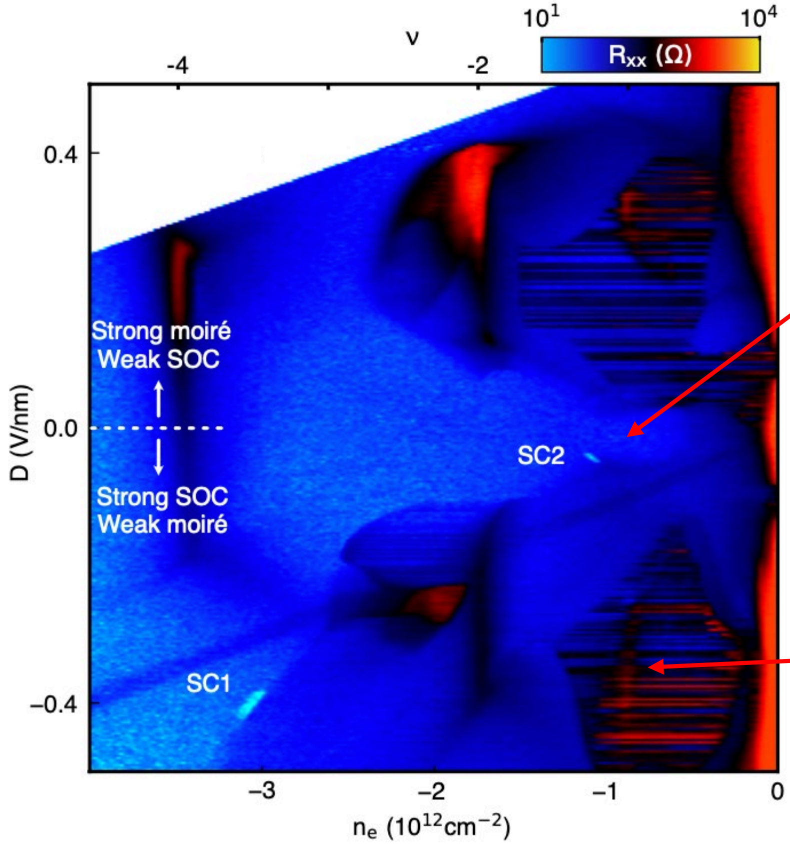
YC et al. *Nature* (2025)

ABCA Graphene

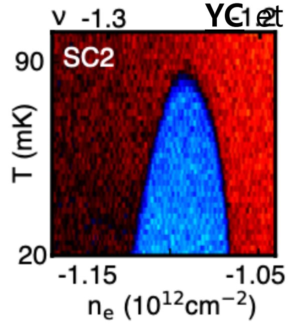


$T_c \approx 55mK$

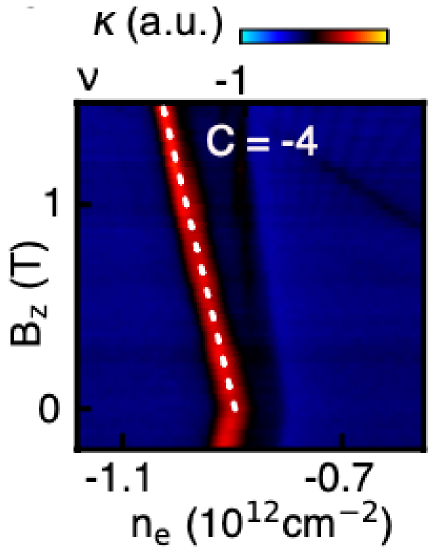
With WS2



$T_{c,1} \approx 35mK$ $T_{c,2} \approx 85mK$

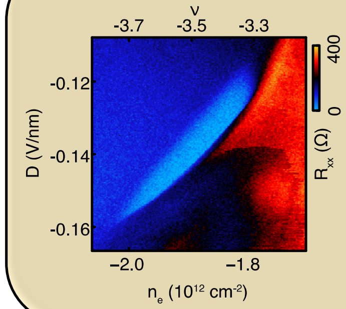


New SC pocket!

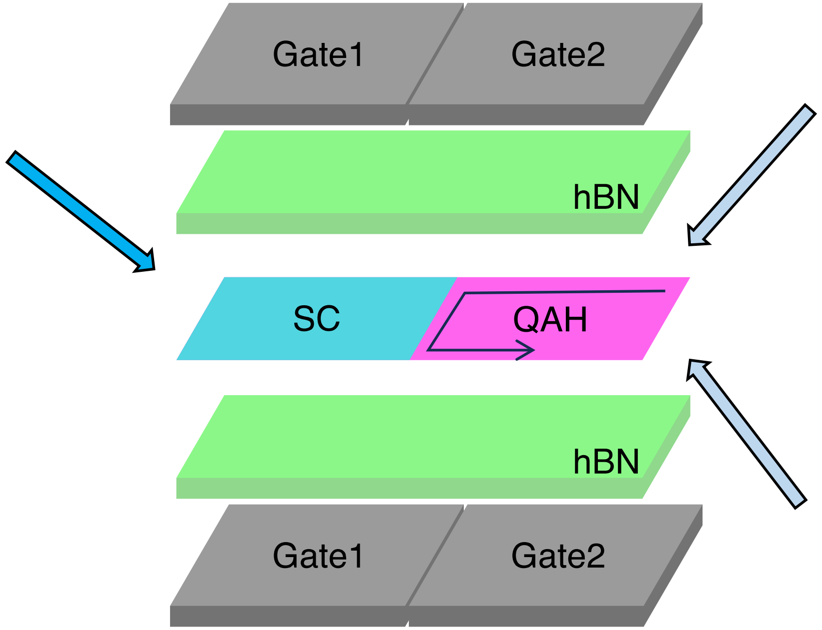


Chern Insulator

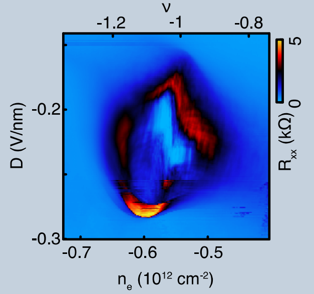
Multiple Phases in Single Material



SC

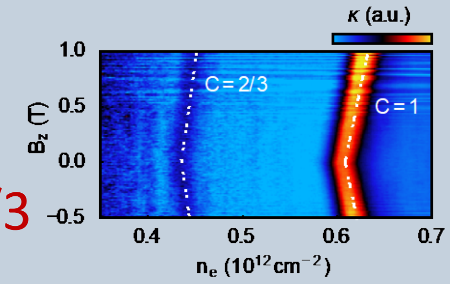


Switchable
QAH C=-4



QAH C=1

FQAH C=2/3

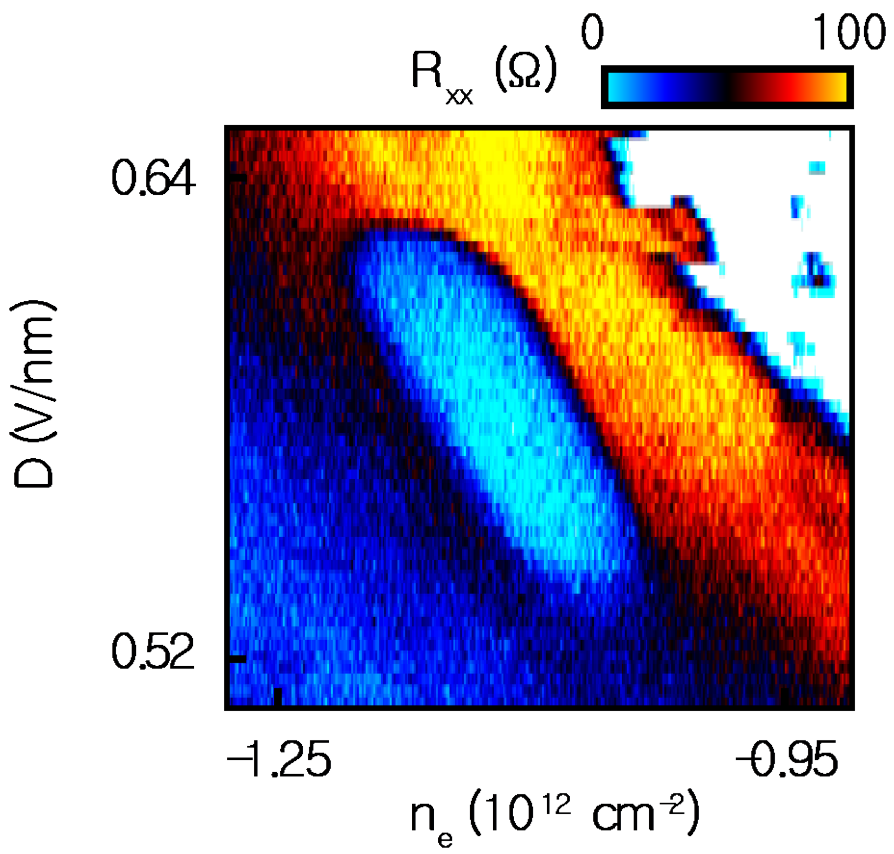


Optional : Spin-Orbit Coupling

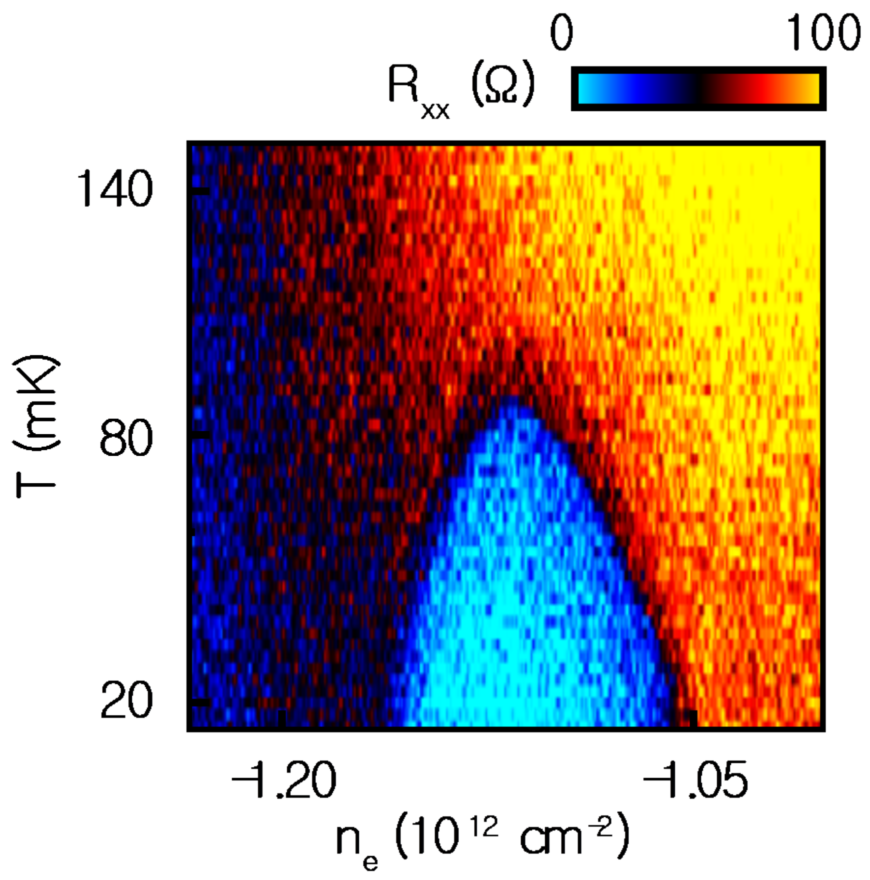
YC et al. *Nature* (2025)

Spin Polarized Superconductor?

Another Superconducting Pocket



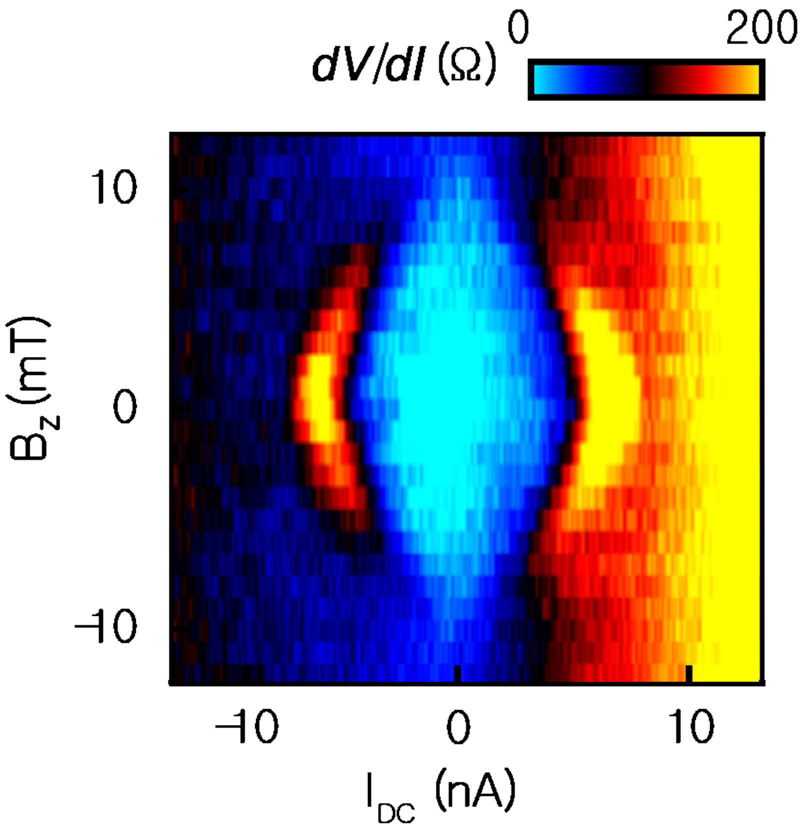
$T_c \approx 85mK$



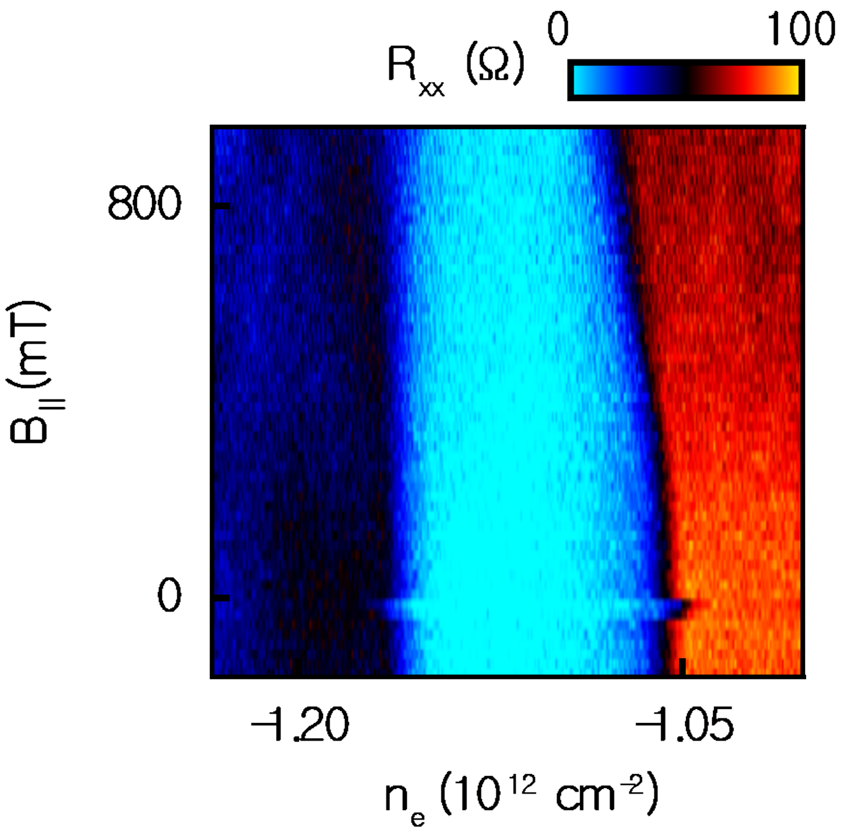
YC et al. *Nature* (2025)

Spin Polarized Superconductor?

$B_{c\perp} \approx 10mT$



$B_{c\parallel} > 1T$

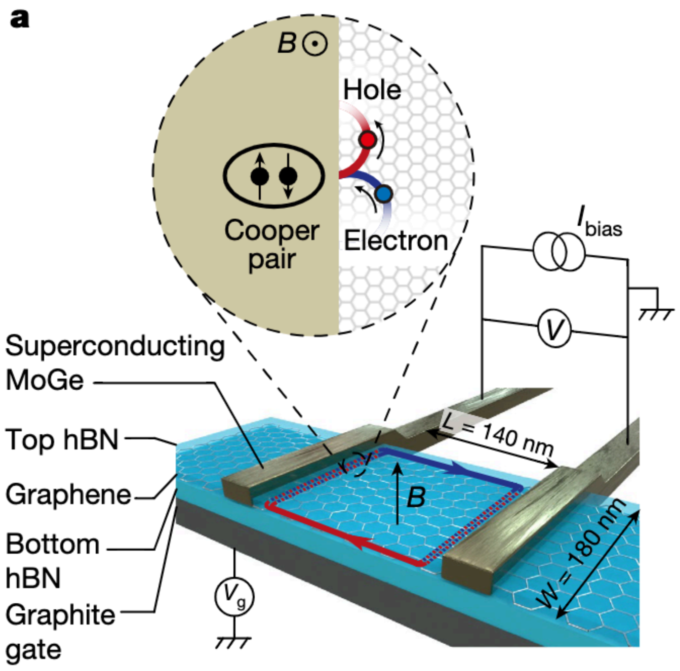


$B_p = 1.86 \times T_c \approx 160mT$

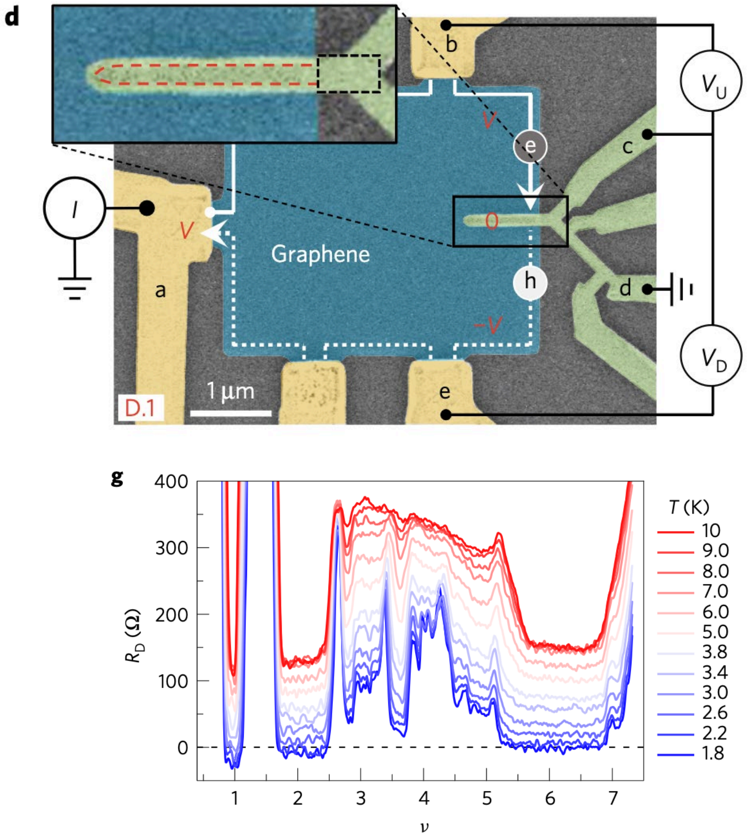
Negligible spin-orbit coupling
Likely Spin polarized

Future Directions

Revisit SC + Q(A)H Experiments in a low-disorder limit

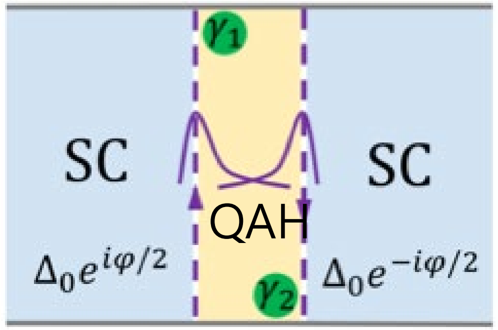


H. Vignaud et al. *Nature* (2023)



G. Lee et al. *Nat. Phys.* (2017)

Majorana modes, Parafermions, Braiding, …



Z.Sun et al. *PRL* (2024)