Superconducting quantum circuits with Josephson junctions and their enabling technologies

구 자 승

양자기술연구소

한국표준과학연구원 (KRISS)

KRISS
한국표준과학연구원

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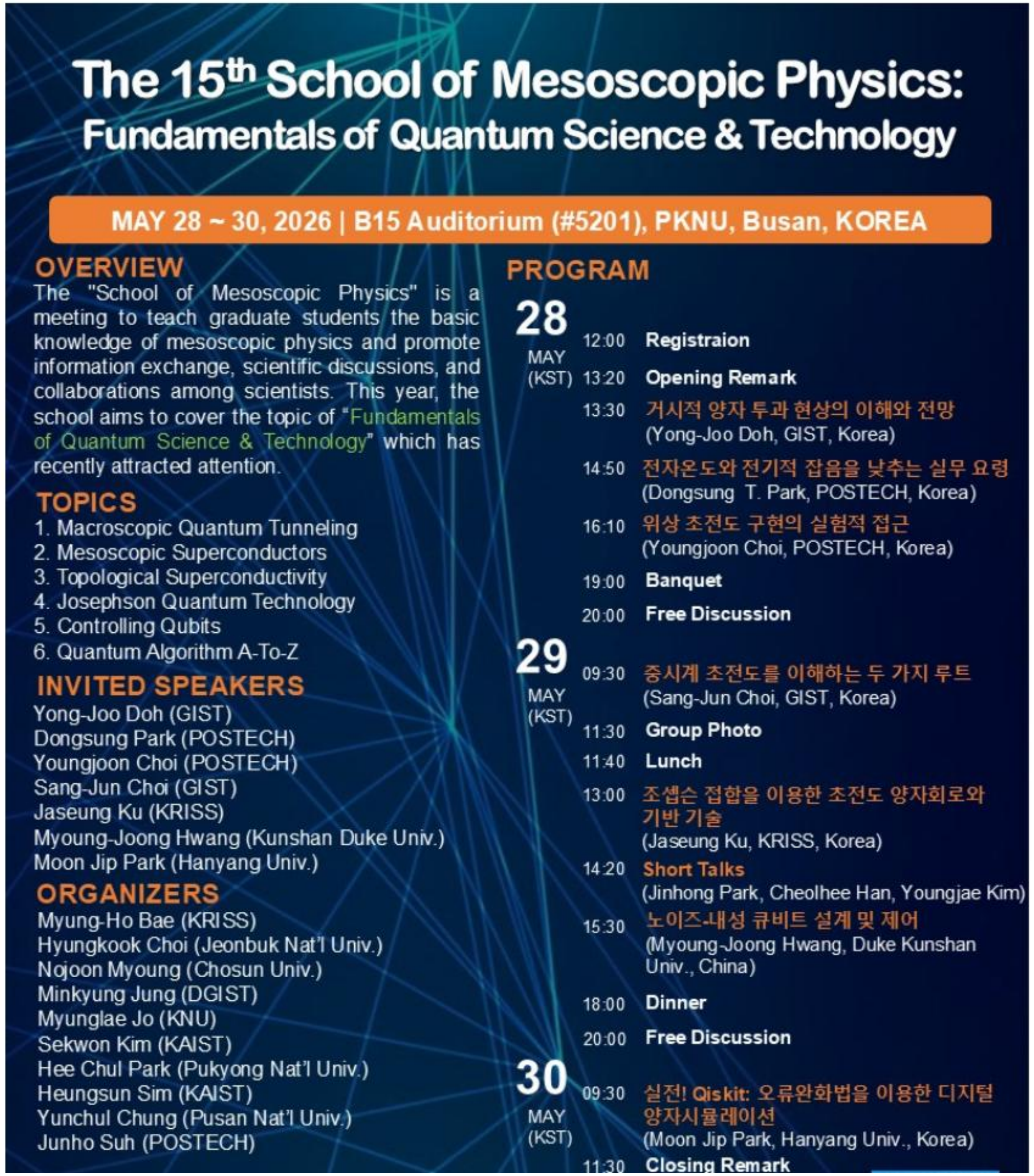
Outline



The 15th School of Mesoscopic Physics: Fundamentals of Quantum Science & Technology

MAY 28 ~ 30, 2026 | B15 Auditorium (#5201), PKNU, Busan, KOREA

OVERVIEW

The "School of Mesoscopic Physics" is a meeting to teach graduate students the basic knowledge of mesoscopic physics and promote information exchange, scientific discussions, and collaborations among scientists. This year, the school aims to cover the topic of "Fundamentals of Quantum Science & Technology" which has recently attracted attention.

TOPICS

1. Macroscopic Quantum Tunneling
2. Mesoscopic Superconductors
3. Topological Superconductivity
4. Josephson Quantum Technology
5. Controlling Qubits
6. Quantum Algorithm A-To-Z

INVITED SPEAKERS

Yong-Joo Doh (GIST)
Dongsung Park (POSTECH)
Youngjoon Choi (POSTECH)
Sang-Jun Choi (GIST)
Jaseung Ku (KRISS)
Myoung-Joong Hwang (Kunshan Duke Univ.)
Moon Jip Park (Hanyang Univ.)

ORGANIZERS

Myung-Ho Bae (KRISS)
Hyungkook Choi (Jeonbuk Nat'l Univ.)
Nojoon Myoung (Chosun Univ.)
Minkyung Jung (DGIST)
Myunglae Jo (KNU)
Sekwon Kim (KAIST)
Hee Chul Park (Pukyong Nat'l Univ.)
Heungsun Sim (KAIST)
Yunchul Chung (Pusan Nat'l Univ.)
Junho Suh (POSTECH)

PROGRAM

28 MAY (KST)	12:00	**Registraion**
	13:20	**Opening Remark**
	13:30	거시적 양자 투과 현상의 이해와 전망 (Yong-Joo Doh, GIST, Korea)
	14:50	전자온도와 전기적 잡음을 낮추는 실무 요령 (Dongsung T. Park, POSTECH, Korea)
	16:10	위상 초전도 구현의 실험적 접근 (Youngjoon Choi, POSTECH, Korea)
	19:00	**Banquet**
	20:00	**Free Discussion**
29 MAY (KST)	09:30	중시계 초전도를 이해하는 두 가지 루트 (Sang-Jun Choi, GIST, Korea)
	11:30	**Group Photo**
	11:40	**Lunch**
	13:00	조셉슨 접합을 이용한 초전도 양자회로와 기반 기술 (Jaseung Ku, KRISS, Korea)
	14:20	**Short Talks** (Jinhong Park, Cheolhee Han, Youngjae Kim)
	15:30	노이즈-내성 큐비트 설계 및 제어 (Myoung-Joong Hwang, Duke Kunshan Univ., China)
	18:00	**Dinner**
	20:00	**Free Discussion**
30 MAY (KST)	09:30	실전! Qiskit: 오류완화법을 이용한 디지털 양자시뮬레이션 (Moon Jip Park, Hanyang Univ., Korea)
	11:30	**Closing Remark**

- ✓ Josephson Junction
- ✓ SQUID
- ✓ Superconducting qubit
- ✓ Quantum-limited amplifier
- ✓ SFQ circuit

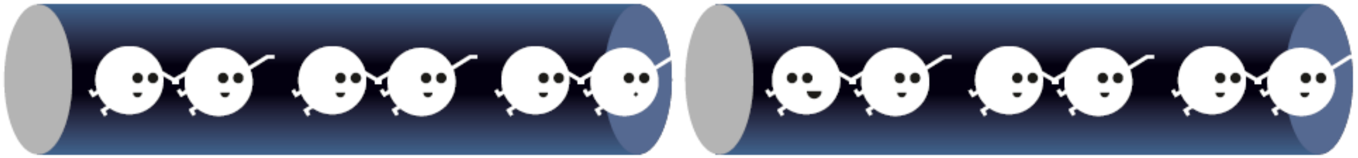
Nobel prize in physics 2025

The Nobel Prize in Physics 2025 was awarded jointly to John Clarke, Michel H. Devoret and John M. Martinis "for the discovery of macroscopic quantum mechanical tunnelling and energy quantisation in an electric circuit"

https://www.nobelprize.org/

“Macroscopic object” in Josephson junction

In a normal conductor, the electrons jostle with each other and with the material.



When a material becomes a superconductor, the electrons join up as pairs, *Cooper pairs*, and form a current where there is no resistance. The gap in the illustration marks the Josephson junction.

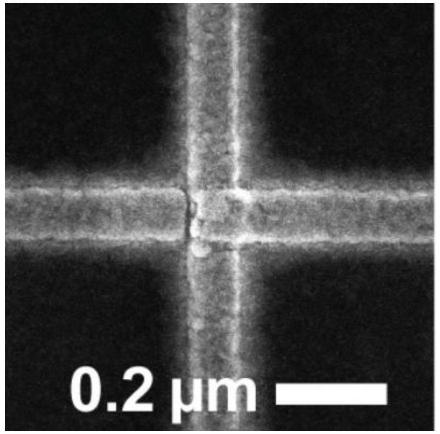
Cooper pairs can behave as if they were all a single particle that fills the entire electrical circuit. Quantum mechanics describes this collective state using a shared *wave function*. The properties of this wave function play the leading role in the laureates' experiment.

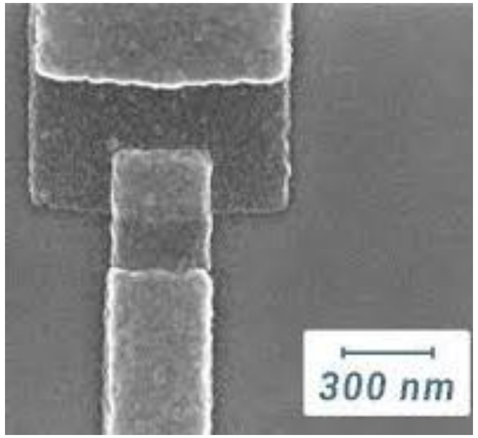
- Normal metal
- Josephson junction (superconductor/insulator/superconductor)
- Collection of Copper-pairs can behave as if they were *a single particle*.

https://www.nobelprize.org/

Josephson Tunnel Junction

Brian Josephson, Nobel prize 1973





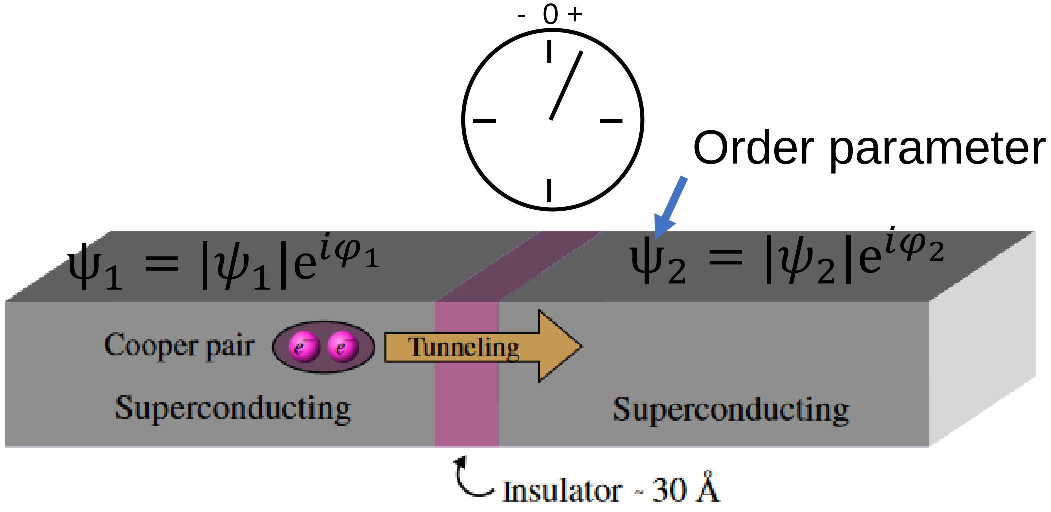
- Josephson tunnel junction (SIS-junction)
- Two superconductors with an insulator(1-2nm), e.g., $Al/Al_2O_3/Al$

- Josephson effect : Supercurrent flows!

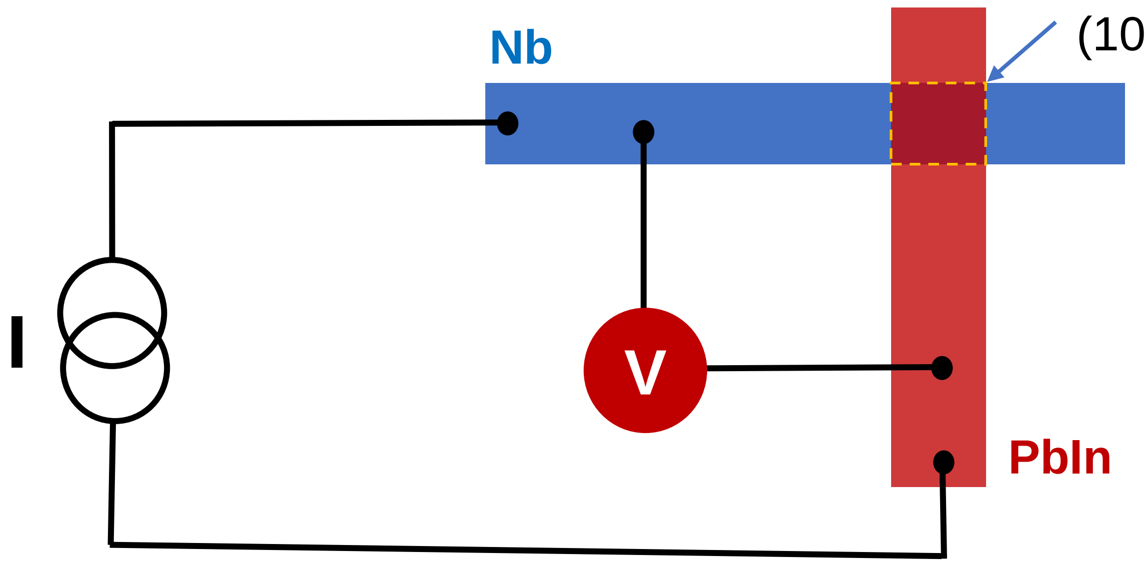
$$I = I_c \sin(\varphi)\,, \quad \frac{d(\varphi)}{dt} = \left(\frac{2e}{\hbar}\right) V$$

$\varphi = \varphi_1 - \varphi_2$, I_c: critical current

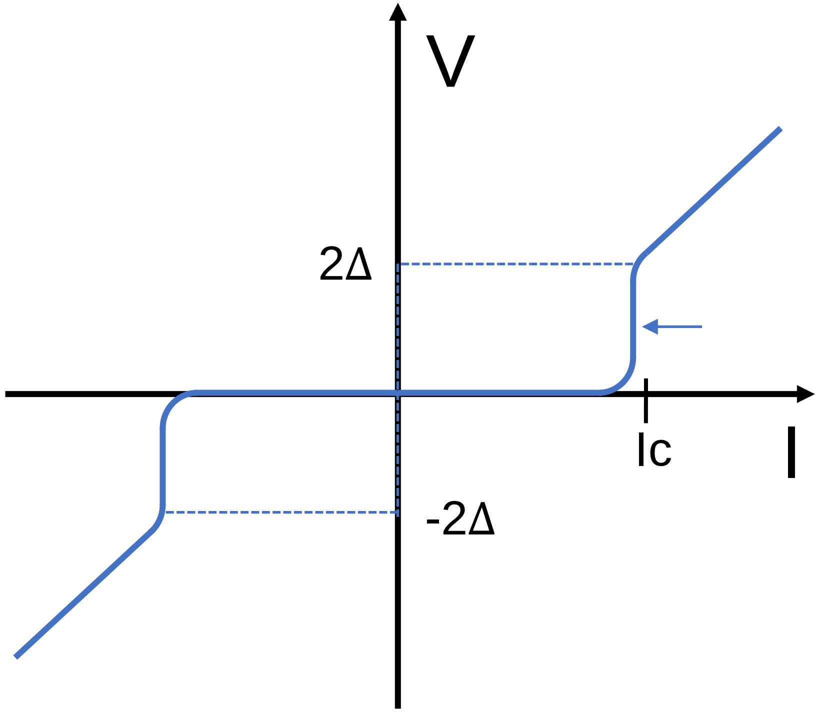
- Nonlinear inductance as a circuit element



Josephson junction sample and VI measurement



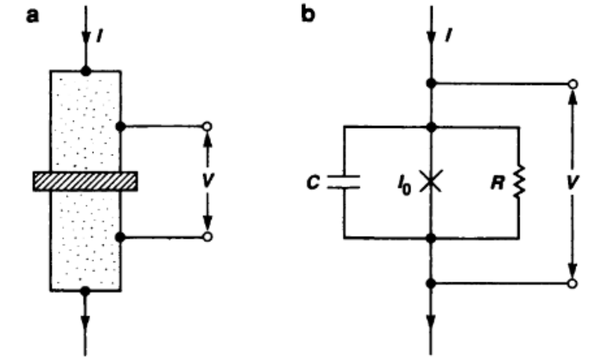
(Josephson junction)
Nb-NbOx-PbIn
(10x10 um^2)

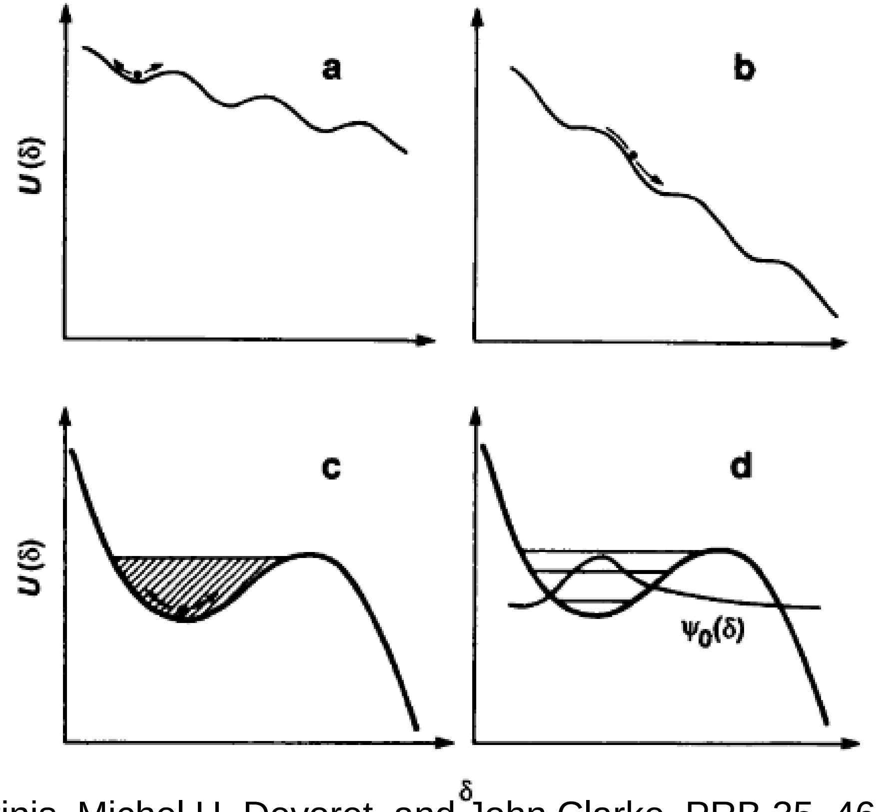


- Current-biased Josephson junction (JJ)
- Apply current and measure voltage across JJ.

- V-I measurement
- For I < Ic, V remains zero-voltage state.
- Near Ic, V switches to finite-voltage state.

Josephson junction dynamics: 1D washboard potential





J. Martinis, Michel H. Devoret, and John Clarke, PRB 35, 4682 (1987)

RCSJ (Resistively and Capacitively Shunted Junction) model (circuit-model)

Dynamics governed by equation of motion of phase particle

$$I(t) = I_0 sin(\varphi) + \frac{V}{R} + C\frac{dV}{dt}$$

$$\frac{I}{I_0} = \sin(\varphi) + \frac{1}{Q}\frac{d\varphi}{d\tau} + \frac{d^2\varphi}{d\tau^2} \qquad \tau = \omega_p t,\ \omega_p = \sqrt{2eI_0/\hbar C}$$

- A "phase particle" of mass $(\hbar/2e)^2 C$ moving along δ axis in an effective potential U.

$$U(\delta) = -E_J cos\delta - \left(\frac{\hbar I}{2e}\right)\delta$$

- Slope is proportional to bias current I.

- Particle trapped in a wall → zero-voltage state (a)
- Particle rolling down → finite-voltage state (b)

Josephson inductance and energy

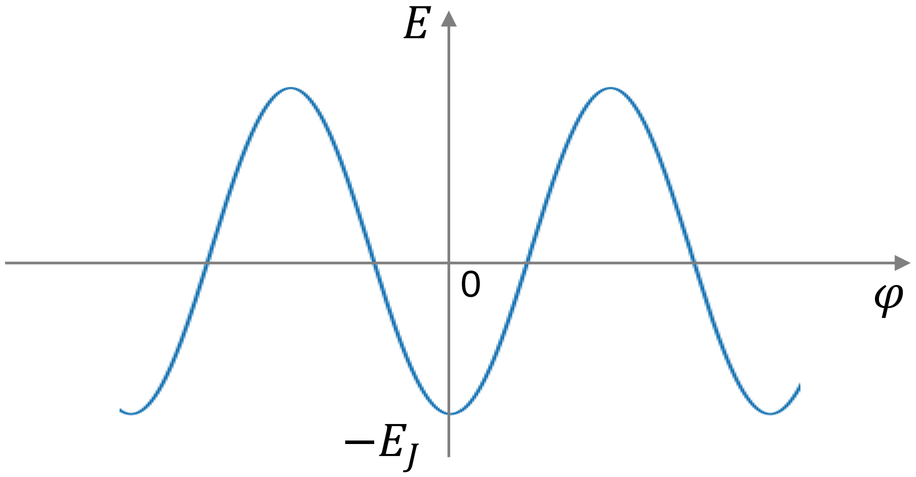
- JJ is a lossless and ***nonlinear inductor.***

$L(\varphi) = \frac{L_J}{\cos(\varphi)}, \quad L_J = \frac{\Phi_0}{2\pi I_c}$: Josephson Inductance

$$L(I) = \frac{\Phi_0}{2\pi}\frac{1}{\sqrt{I_c^2 - I^2}}$$

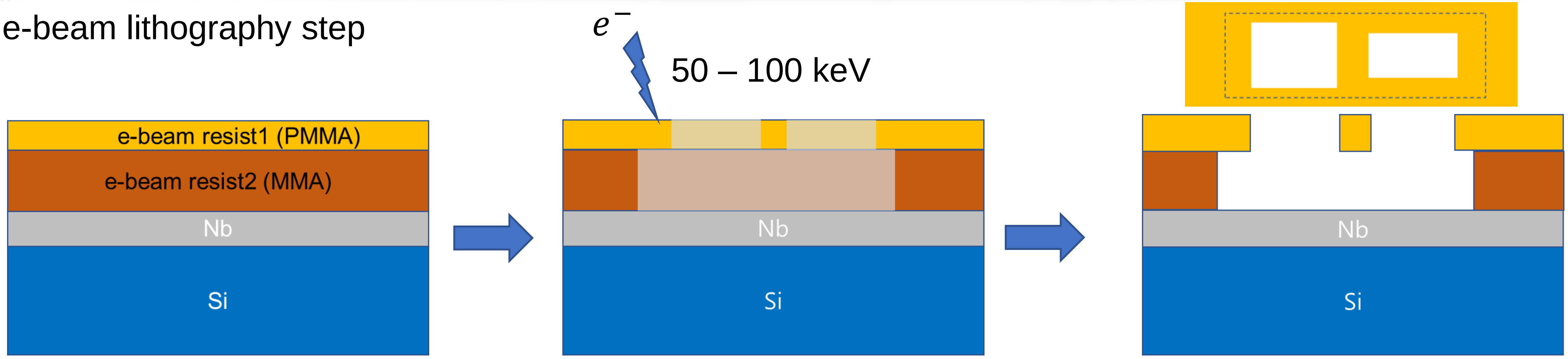
- Energy in JJ

$H = -E_J cos\varphi, \quad \mathrm{E_J} = \hbar I_c/2e$: Josephson energy



Josephson Tunnel Junction Fabrication

- ✓ e-beam lithography step

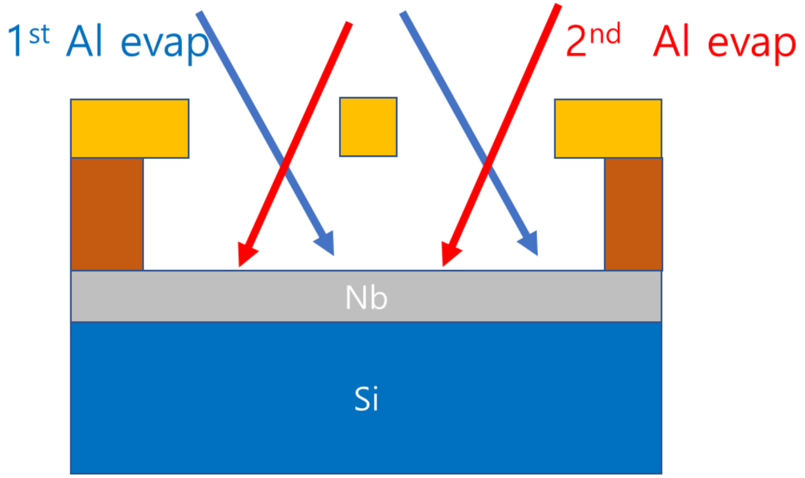


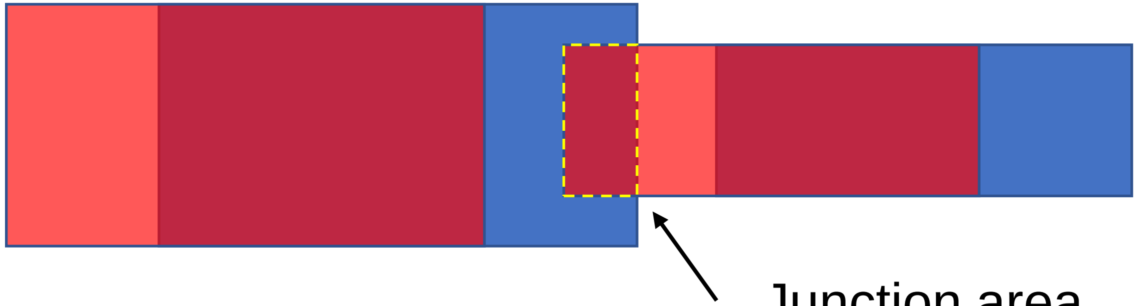
e-beam resist coating

e-beam exposure

develop

- ✓ Al evaporation



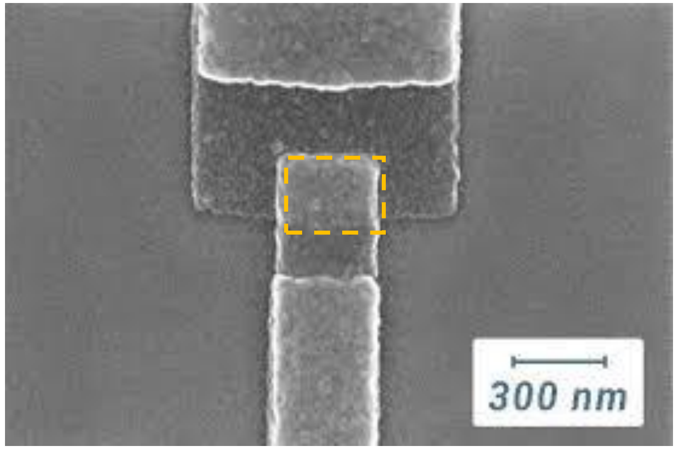


Junction area

* Oxidation between 1st and 2nd Al evaporation

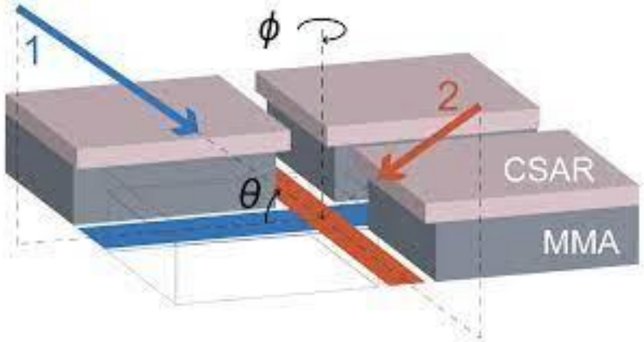
Josephson Junction Fabrication Style

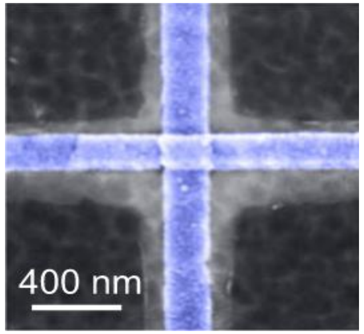
Inline JJ (Dolan-bridge)

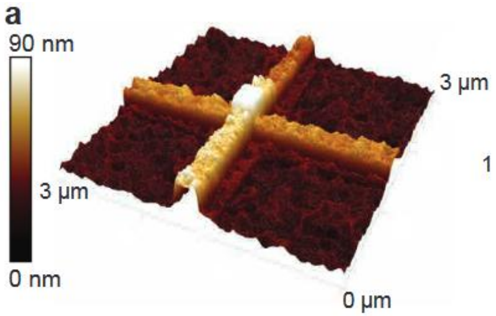


- Need resist bridge
- Submicron JJ
- Most conventional method

Manhattan JJ

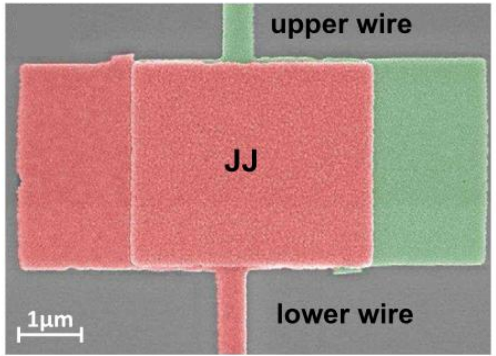


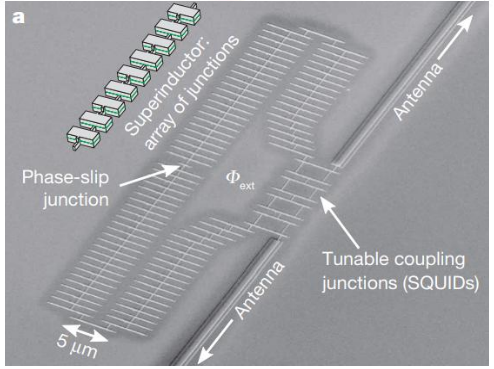




- No bridge required
- Submicron JJ
- Good size control
- Need both tilt and rotation function

Bridgeless JJ



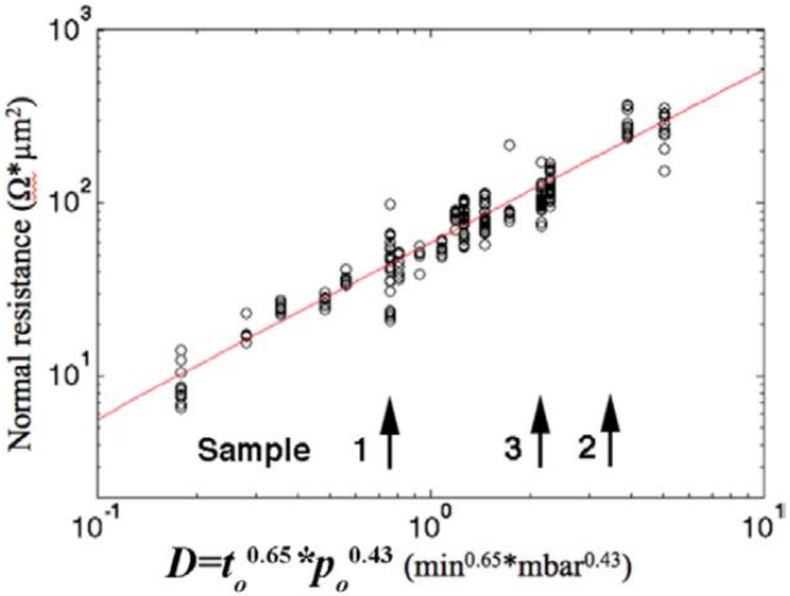


- Different undercut size
- No bridge required
- Can make a large JJ

Controlling JJ parameters in practice

How to control I_c

- $I_c = J_c \times A$
  , where J_c=critical current density, A/m^2
  A=junction area
- J_c set by oxide thickness and controlled by oxygen pressure P and oxidation time t
- $J_c \propto \sim \frac{1}{\sqrt{P * t}}$



[Zeng et al., J. Phys. D: Appl. Phys. 48 (2015) 395308]

Calculating I_c from R_n

- Ambegaokar-Baratoff equation

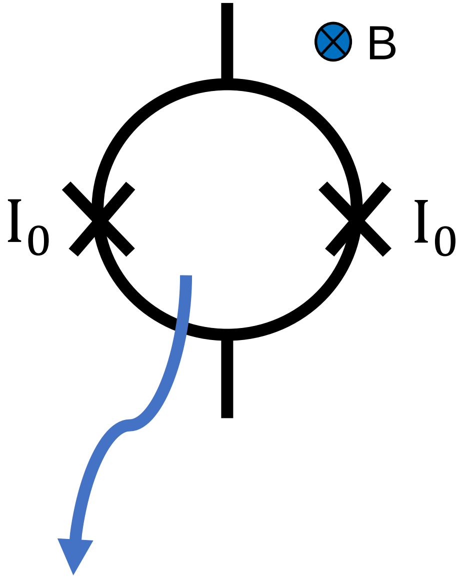
$$I_c = \frac{\pi \Delta}{2 e R_n}$$

I_c: Critical current
Δ : Superconducting gap
R_n: Normal state resistance

Ex) Al ($\Delta \sim 200\ \mu eV$)
For $I_c = 100$ nA,$R_n = 3.1$ kΩ

- Can estimate qubit frequency from R_n

SQUID (Superconducting **QU**antum **I**nterference **D**evice)

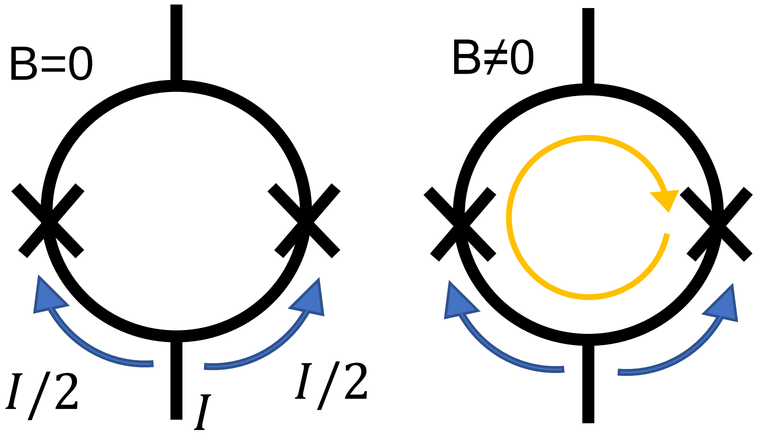


$$\Phi = B * A$$

Area of loop

Magnetic field

Magnetic flux



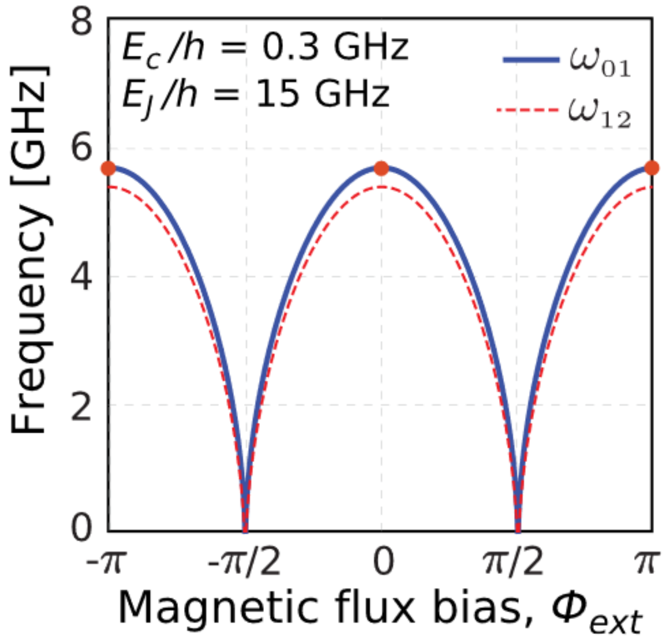
$$I_c = 2I_0$$

$$I_c < 2I_0$$

$$I_c(\Phi) = 2I_0 \left| \cos\left(\frac{\Phi}{\Phi_0}\pi\right) \right|$$

$\Phi_0 = \frac{h}{2e} = 2 \times 10^{-15}$: Flux quantum

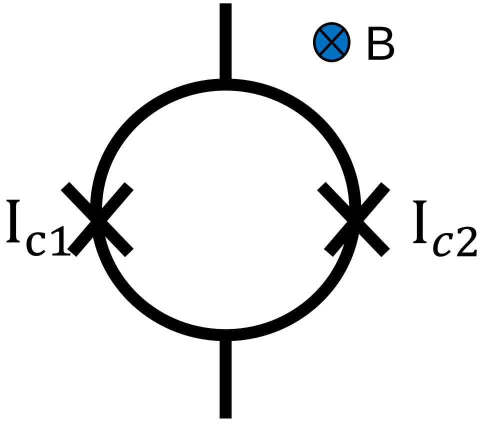
- Usage
  - Effectively modulate Ic by B field
  - Tunable qubit frequency
  - Sensitive magnetic sensor



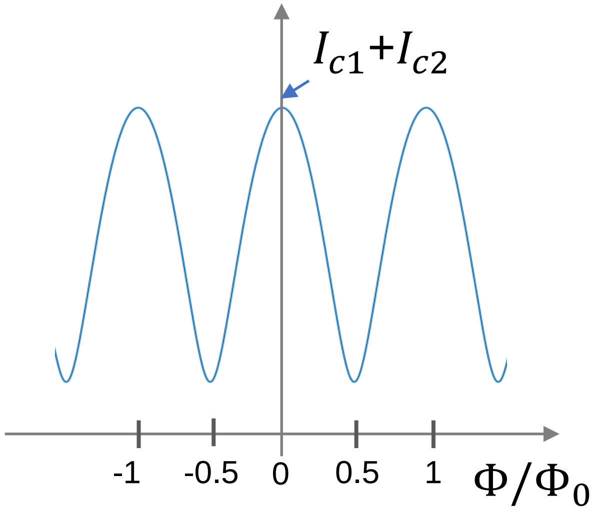
SQUID - Usage

Asymmetric SQUID

- Used in tunable qubit
- Frequency tunability and less flux noise

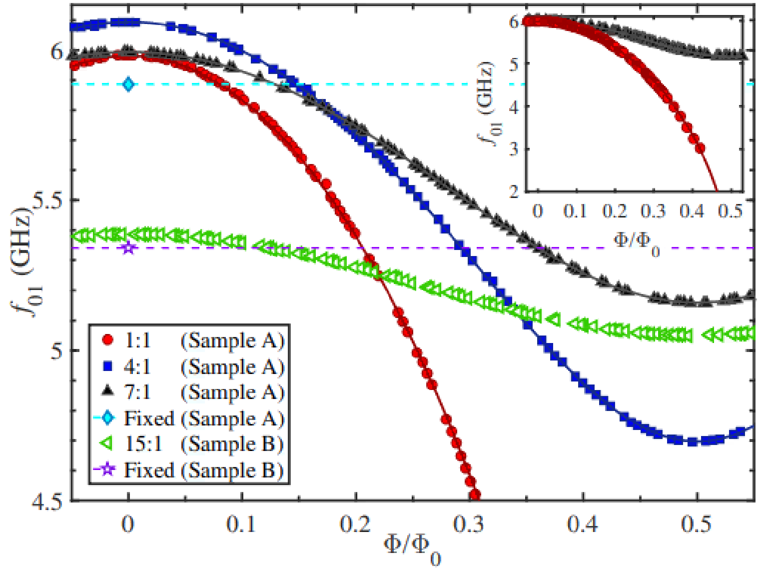


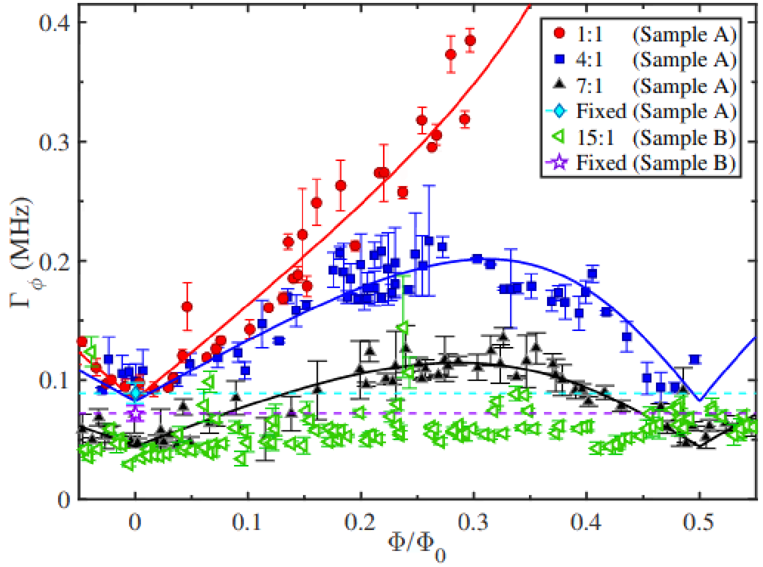
$I_{c1} \neq I_{c2}$



$$I_c(\Phi) = (I_{c1}+I_{c2})\left|\cos\left(\frac{\Phi\pi}{\Phi_0}\right)\right|\sqrt{1+d^2\tan^2\left(\frac{\Phi\pi}{\Phi_0}\right)}$$

$, d=\frac{I_{c1}-I_{c2}}{I_{c1}+I_{c2}}$





[M.Hutchings et al, PRAppl. 8, 044003 (2017)]

History of superconducting qubits

1999 - 2002 — Charge, phase, flux qubit

2004 — circuit-QED (charge qubit + superconducting resonator)

2007 — **2D Transmon**

2009 — Fluxonium

2011 — 3D Transmon

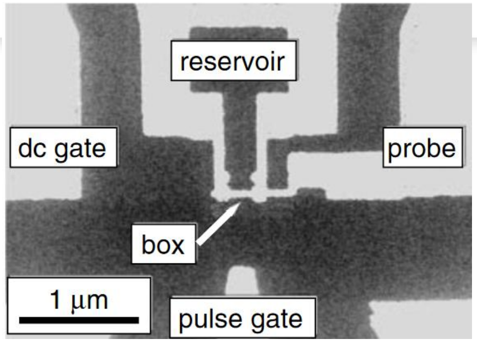
2016 — IBM 5Q cloud

2019 — Google Sycamore, 53Q

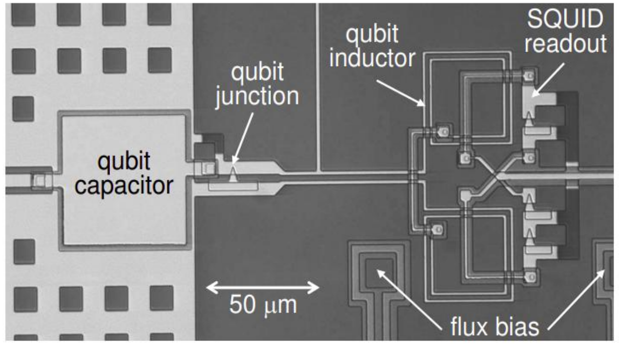
2020 — IBM Hummingbird, 65Q

2023 — IBM Heron, 133Q

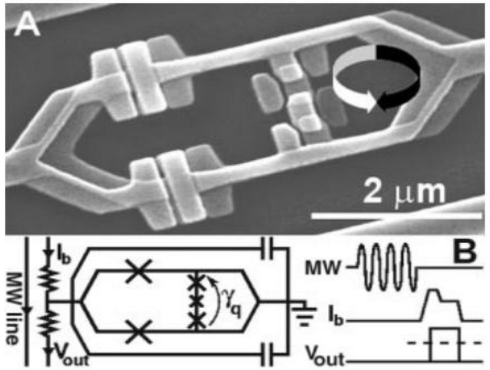
2024 — Goolge Willow, 105Q



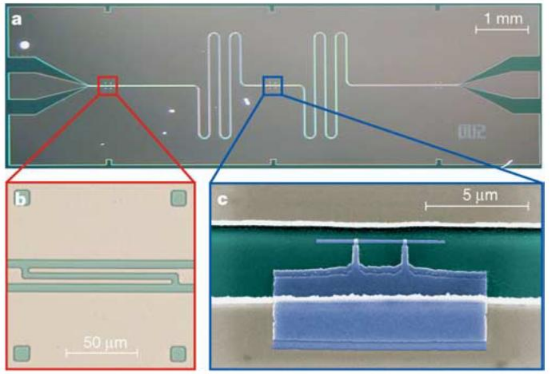
charge qubit(Nakamura)



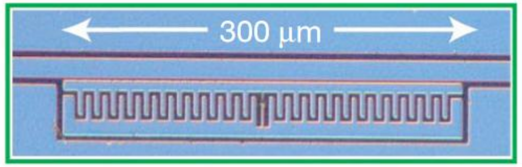
Phase qubit(Martinis)



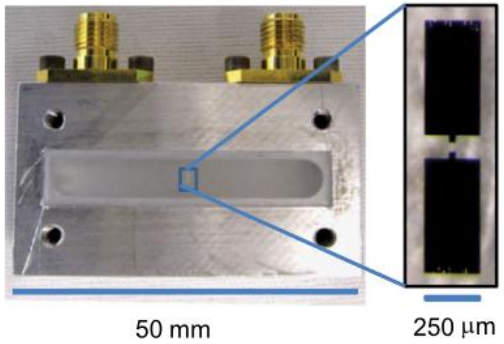
Flux qubit (Mooij)



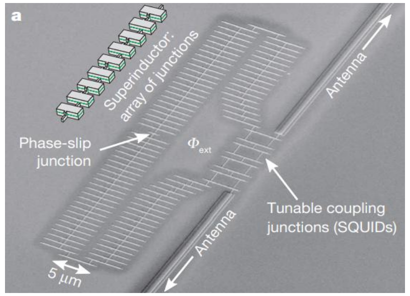
charge qubit
+ superconducting
resonator (Yale)



2D transmon (Yale)

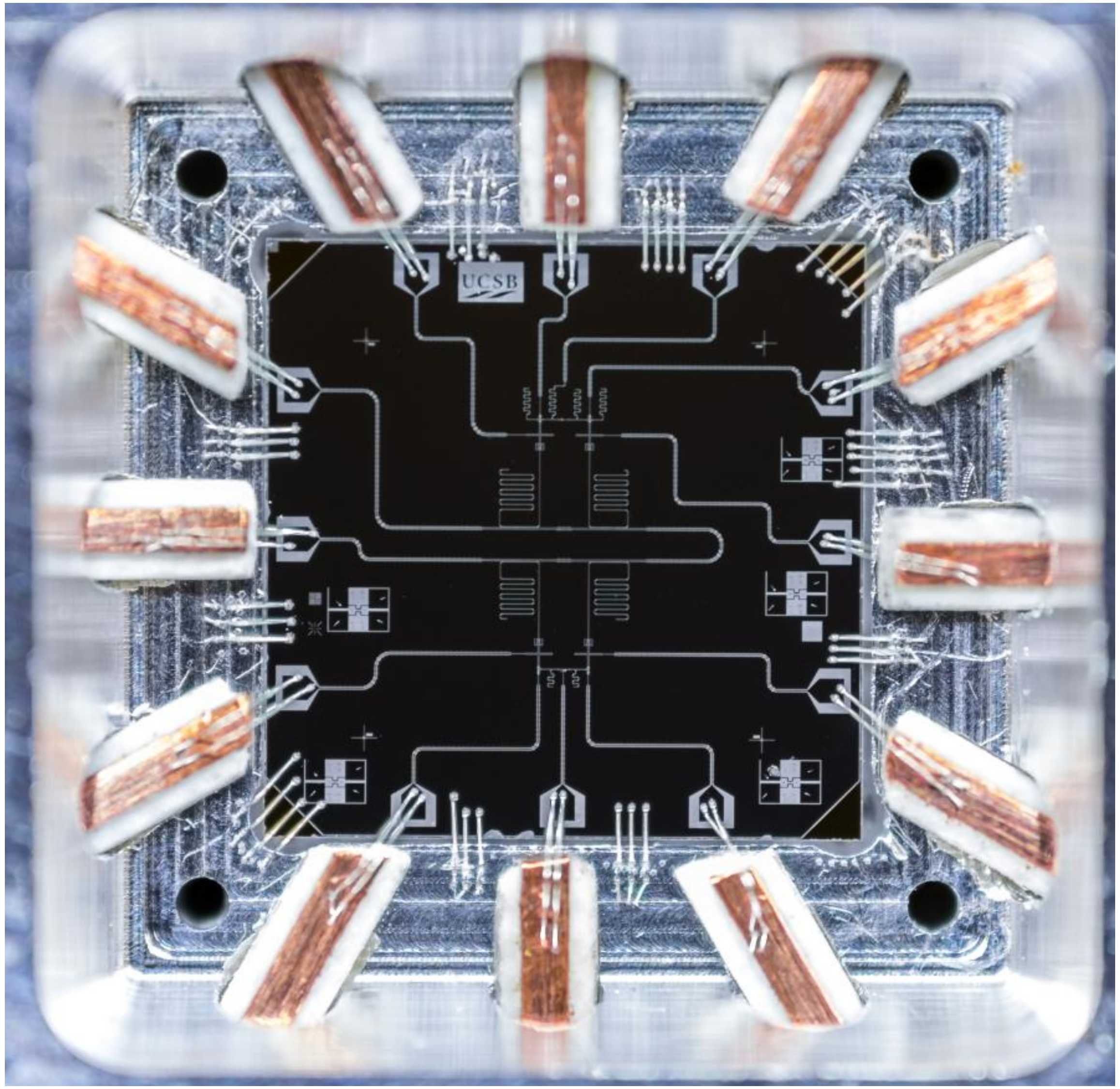


3D transmon (Yale)



Fluxonium (Yale)

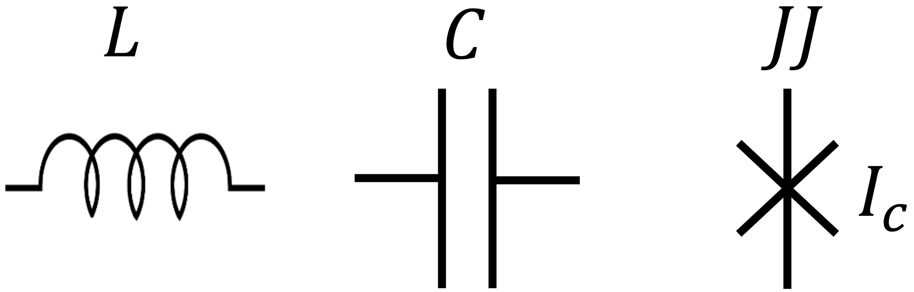
Superconducting circuit



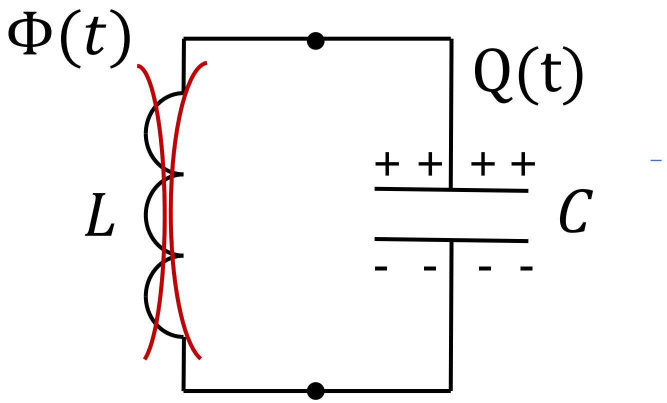
[USCB, 4Q]

Ingredients of superconducting electrical circuits

- Inductor
- capacitor
- **Josephson junction(JJ)**
- SQUID
- Superconducting resonator
- MW transmission line



Classical LC to Quantum LC circuit



$$\hat{H} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

Quantum LC resonator

1. Pick generalized coordinate
2. Find kinetic and potential energy
3. Lagrangian
4. Hamiltonian
5. Quantum operators (canonical quantization)

[M.Devoret, “Quantum fluctuations in electrical circuit”]

LC oscillator vs Mechanical oscillator

$$H = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

Charging energy
(kinetic E)

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

Inductive energy
(potential E)

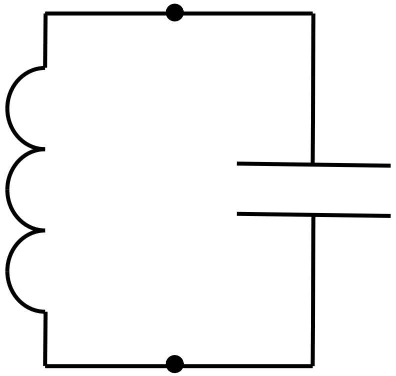
$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2$$

Kinetic energy

$$[\hat{x}, \hat{p}] = i\hbar$$

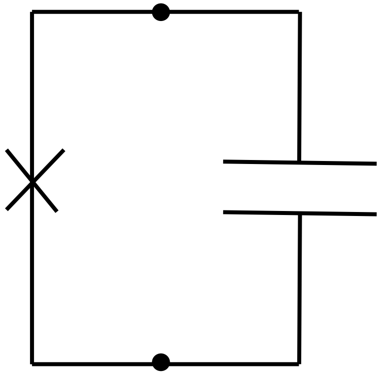
Potential energy

Harmonic oscillator vs Anharmonic oscillator



$$H = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$



$$H = \frac{\hat{Q}^2}{2C} - E_J cos \frac{2\pi\hat{\Phi}}{\Phi_0}$$

$$= 4E_c\hat{n}^2 - E_J cos\hat{\phi}$$

$$[\hat{\phi}, \hat{n}] = i$$

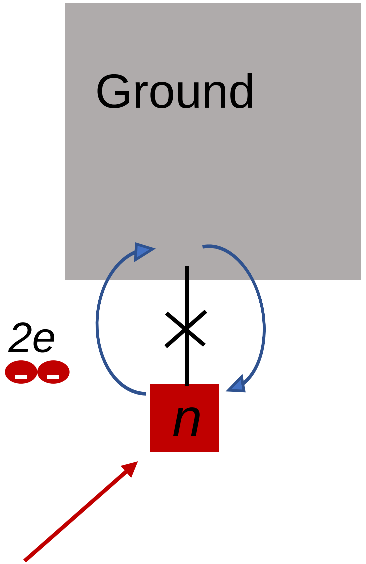
$$\hat{n} = \frac{\hat{Q}}{2\mathrm{e}}, \hat{\phi} = 2\pi\frac{\hat{\Phi}}{\Phi_0}$$

$E_c = \frac{e}{2\mathrm{C}}$: charging energy

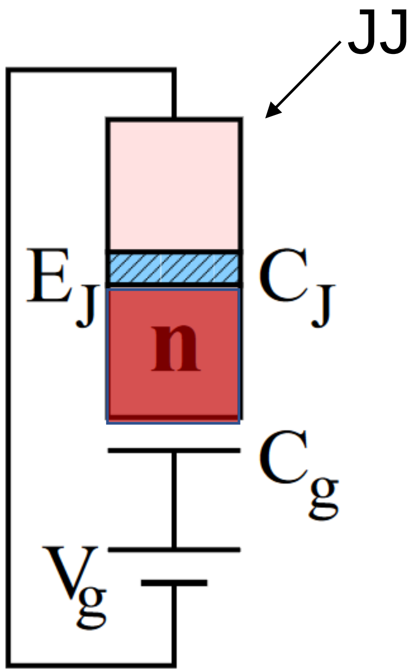
$E_J = \frac{\hbar I_c}{2e}$: Josephson energy

Charge qubit (=Cooper pair box qubit)

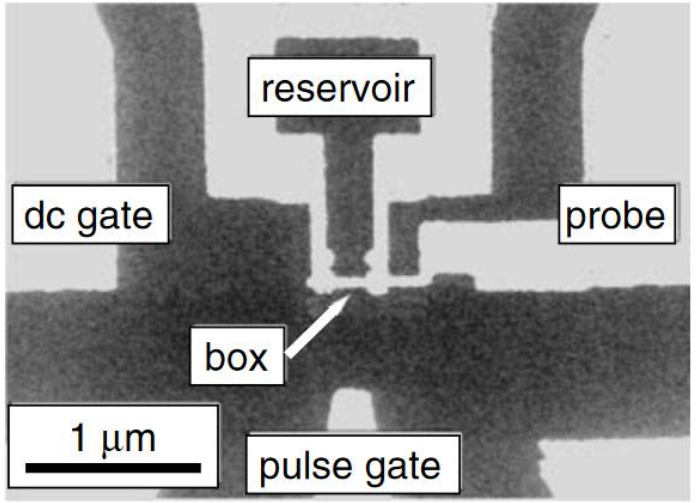
First superconducting qubit

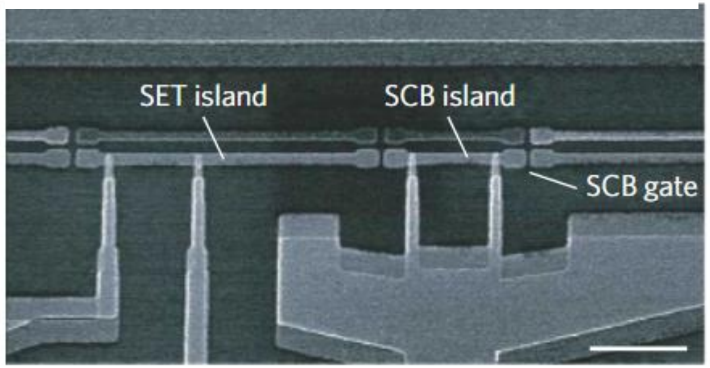


“Cooper-pair box”



$$E_c \gg E_J$$

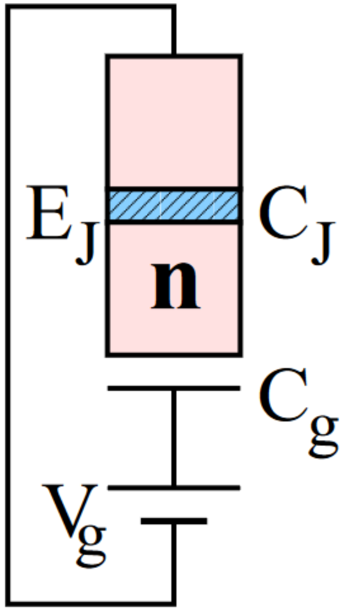




H = Charging energy of Cooper-pair box

+ Josephson energy of Josephson junction

Charge qubit energy levels



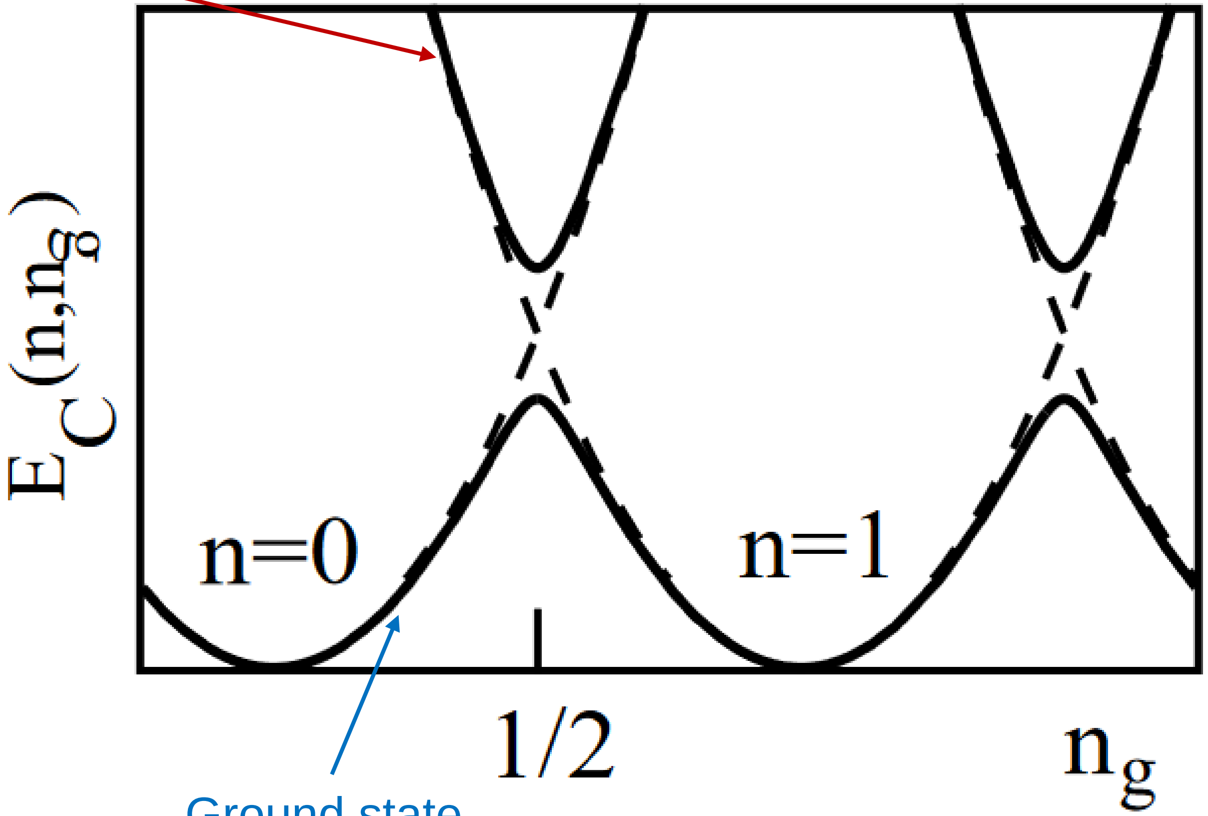
Electrostatic energy

$$U = \frac{1}{2} C_g V_g^2 = 4E_c\left(n - \mathrm{n_g}\right)^2,$$

$$n_g = C_g V_g / 2e,$$

$n =$ # of Cooper-pairs

Total energy vs gate voltage



Excited state energy, $|e\rangle$

Ground state energy, $|g\rangle$

Charge qubit Hamiltonian

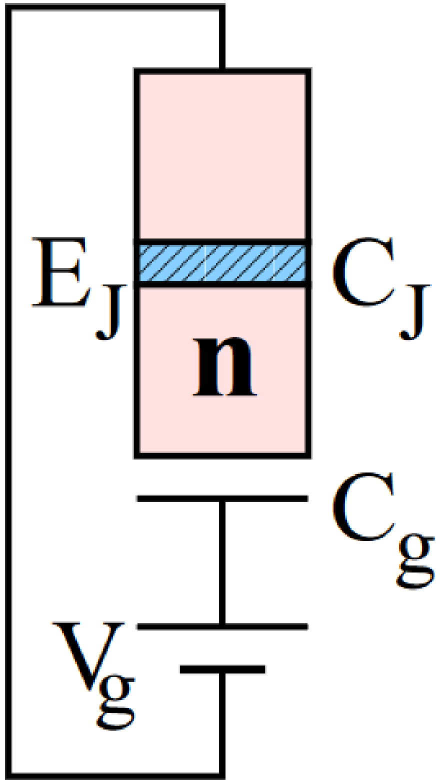
Hamiltonian

$$H = 4E_c(\hat{n} - \mathrm{n_g})^2 - \mathrm{E_J}\cos\hat{\varphi}$$

$$[\hat{n}, \hat{\varphi}] = i$$

$\hat{n}$: charge operator
$\hat{\varphi}$: phase operator
$\mathrm{n_g}$: offset charge number

$E_c = \frac{e^2}{2C_\Sigma}$: Charging energy
$E_J = \frac{\hbar I_c}{2e}$: Josephson energy



* Charge qubit is sensitive to charge noise!

Energy levels of charge qubit

Hamiltonian

$$H = 4E_c\left(\hat{n} - \mathrm{n_g}\right)^2 - \mathrm{E_J}\cos\hat{\varphi}$$

$$\hat{n} = |N\rangle\langle N|,$$

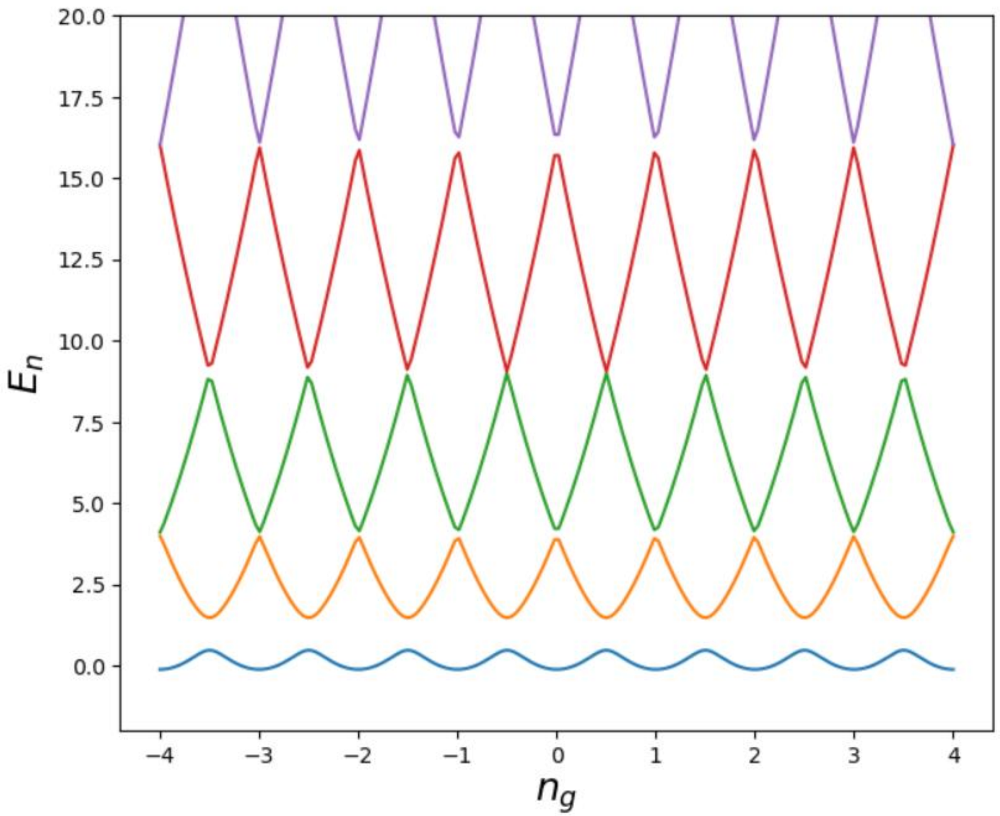
$$\cos\hat{\varphi} = \frac{1}{2}\sum_{-\infty}^{+\infty} |N\rangle\langle N+1| + |N+1\rangle\langle N|$$

$$H = 4E_c\left(|N\rangle\langle N| - \mathrm{n_g}\right)^2 - \frac{\mathrm{E_J}}{2}\sum_{-\infty}^{+\infty} |N\rangle\langle N+1| + |N+1\rangle\langle N|$$

$$H|\psi_n\rangle = E_n H|\psi_n\rangle$$

[M.Devoret, "Quantum fluctuations in electrical circuit"]

Calculate eigenvalues of H matrix using QUTIP



N=10
Ej=Ec=1

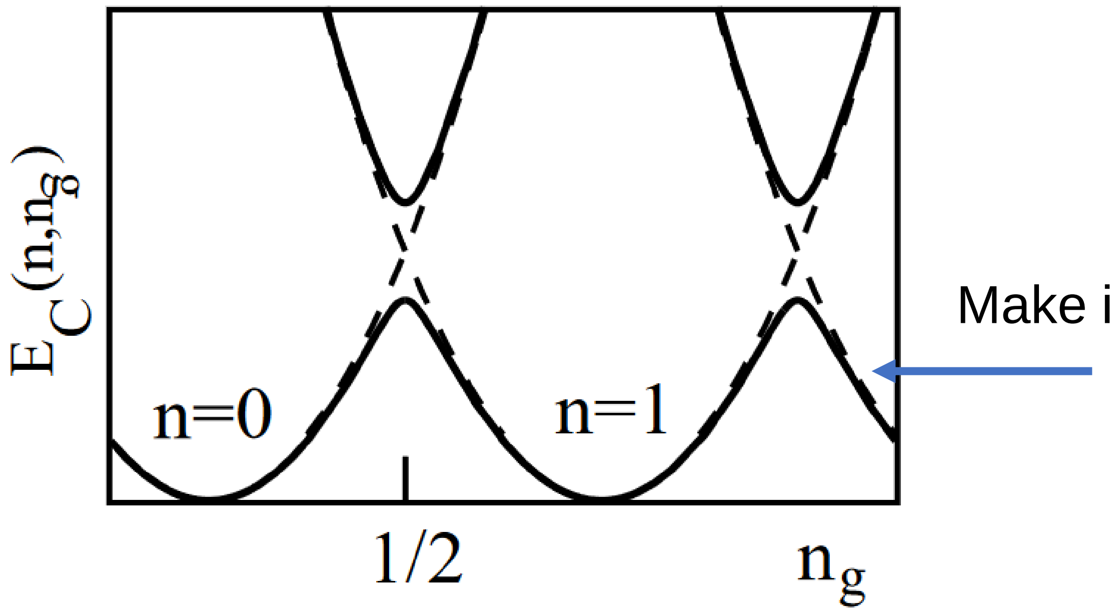
https://nbviewer.org/urls/qutip.org/qutip-tutorials/tutorials-v5/lectures/Lecture-11-Charge-Qubits.ipynb

Transmon qubit

- TRANSMON = Transmission-line shunted plasma oscillation qubit
- Developed in Yale Univ. in 2007

Motivation

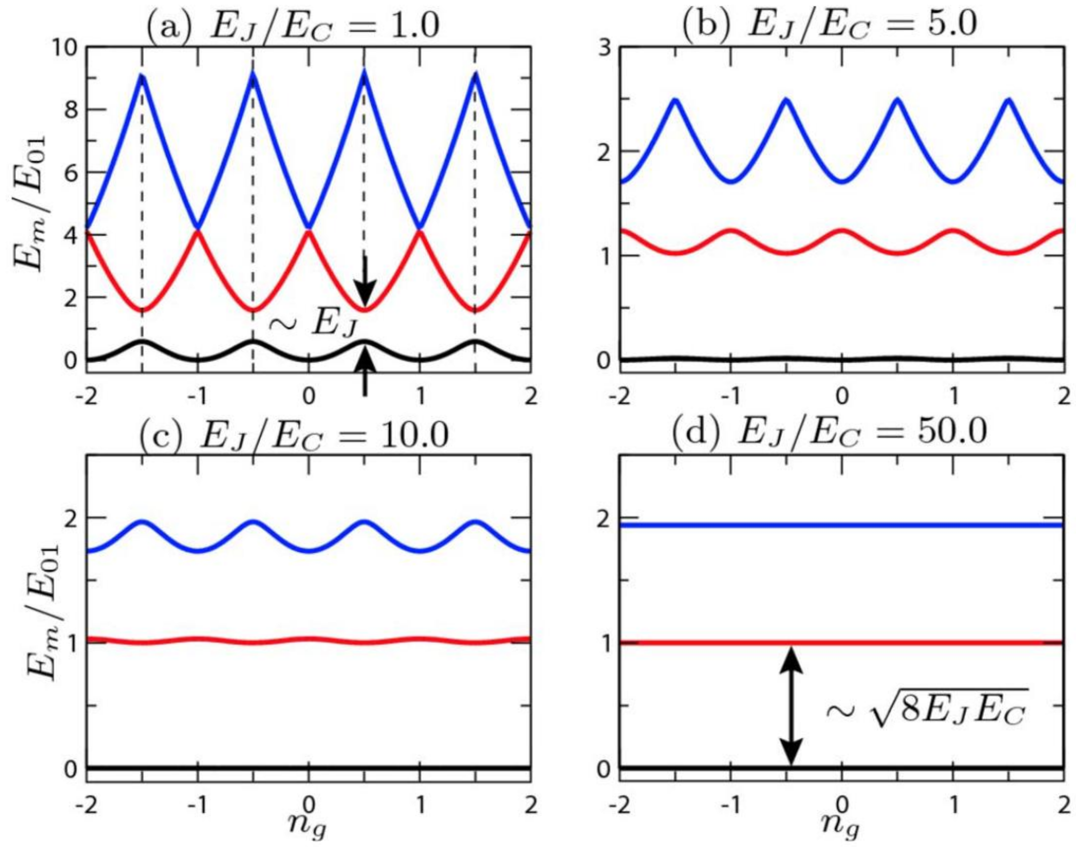
How can we make a qubit less sensitive to charge noise?



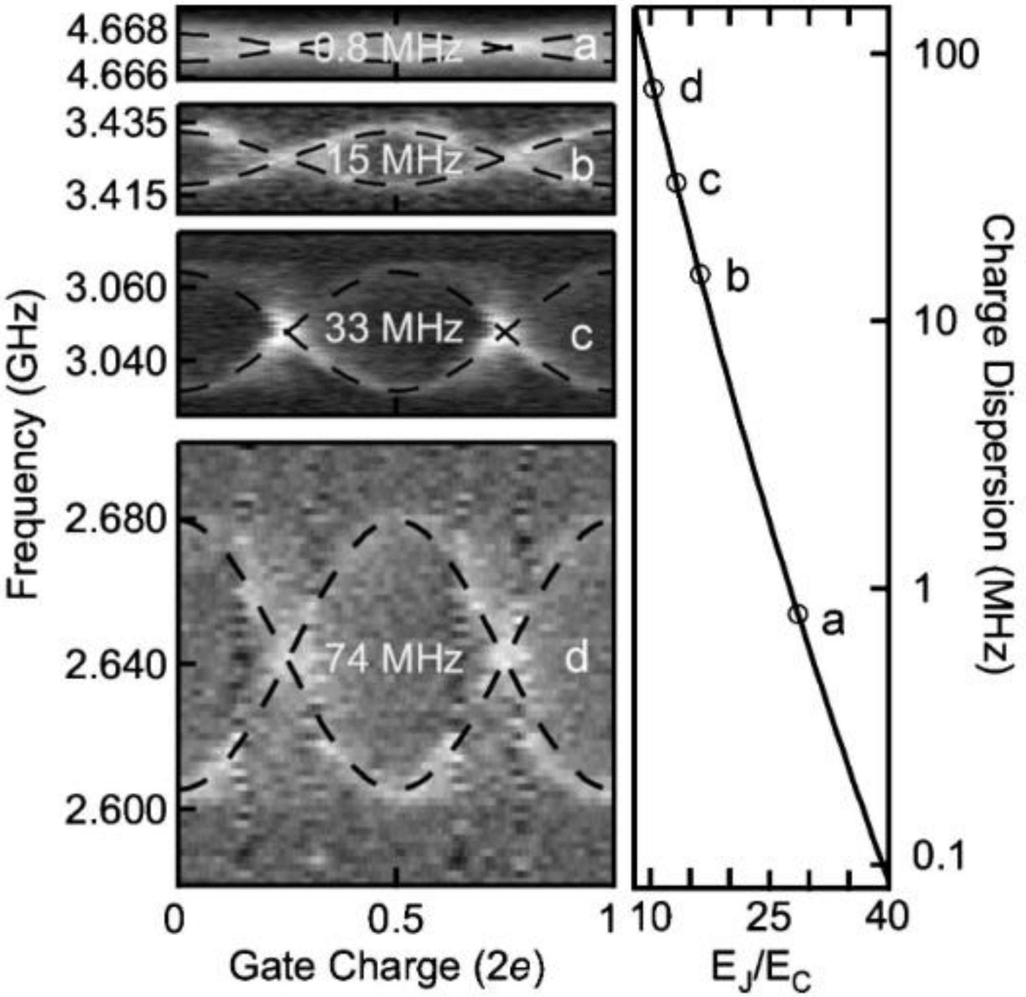
Make it flatter?

Energy Levels vs E_J/E_c

Theory



Experiment



$$f_{01} \approx \sqrt{8E_JE_c}/\hbar$$

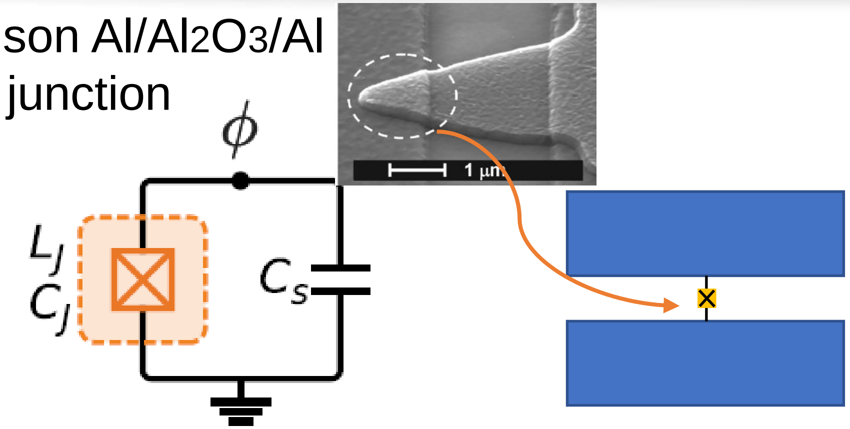
$$\frac{E_J}{E_c} = \ 50{\sim}100$$

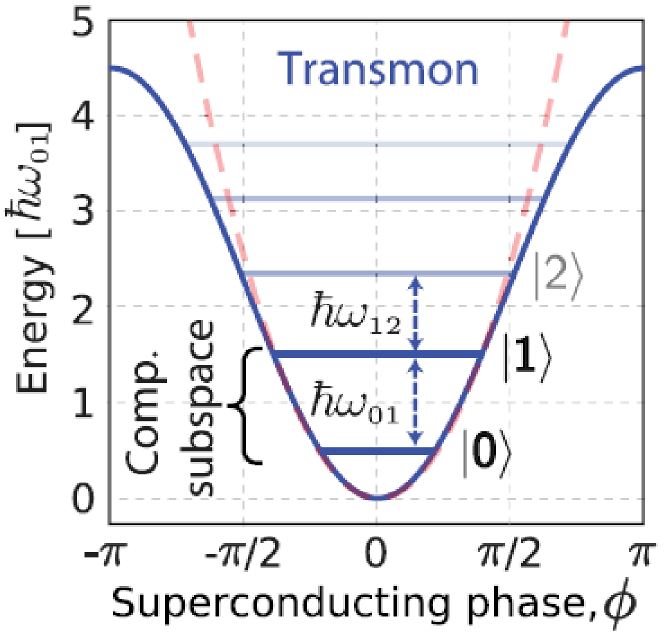
[J. Koch et +al., PRA **76**, 042319 (2007)]

[J.A.Schreider et al., PRB **77**, 180502 (2008)]

Transmon

Josephson Al/Al2O3/Al
Tunnel junction





- $f_{01} \sim 5$ GHz ($=200$mK)
- $f_{12} - f_{01} < 0$

Idea:

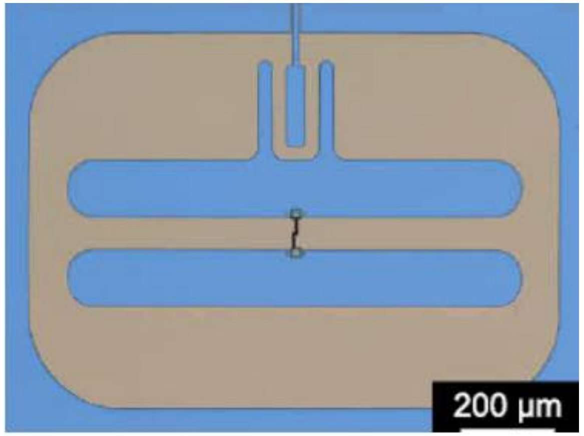
- Increase the ratio E_J/E_c by reducing E_c
- Add *large shunt capacitor*.

- Weakly anharmonic
- Long coherence time due to Suppressed charge noise
- Simple to fabricate!

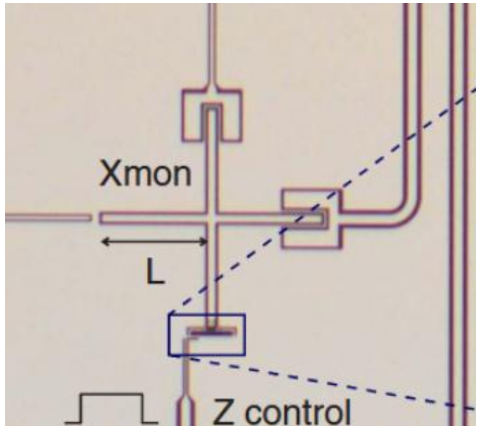
[P. Krantz et al., Appl.Phys.Rev. **6**, 021318 (2019)]

Various “mon”

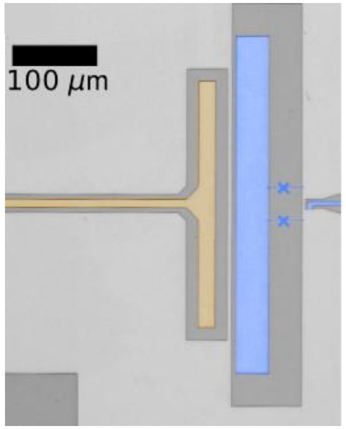
transmon



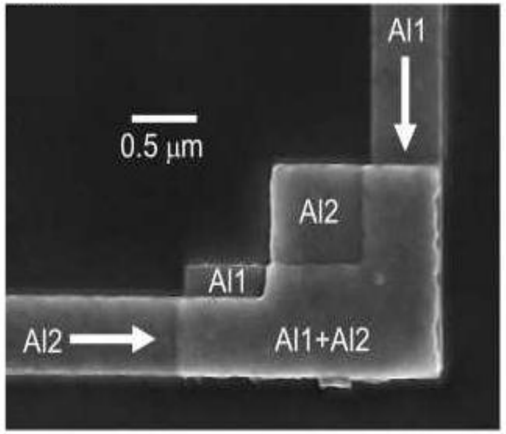
xmon



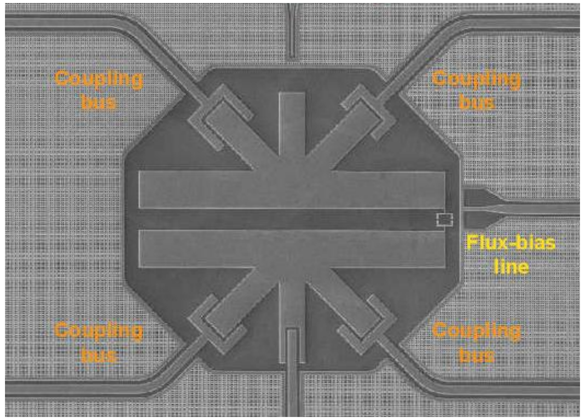
recmon



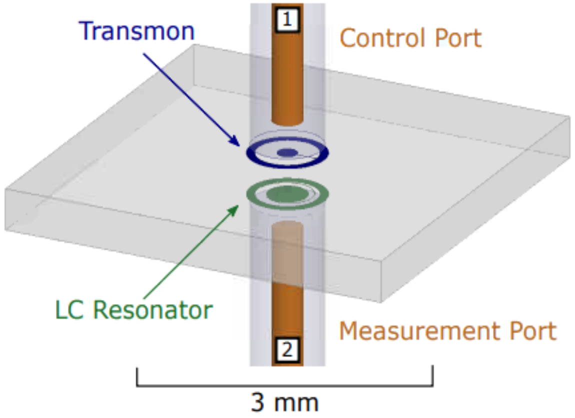
mergemon



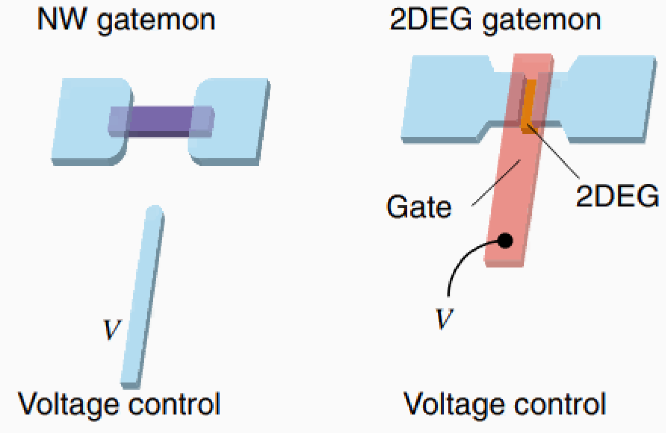
starmon



coaxmon

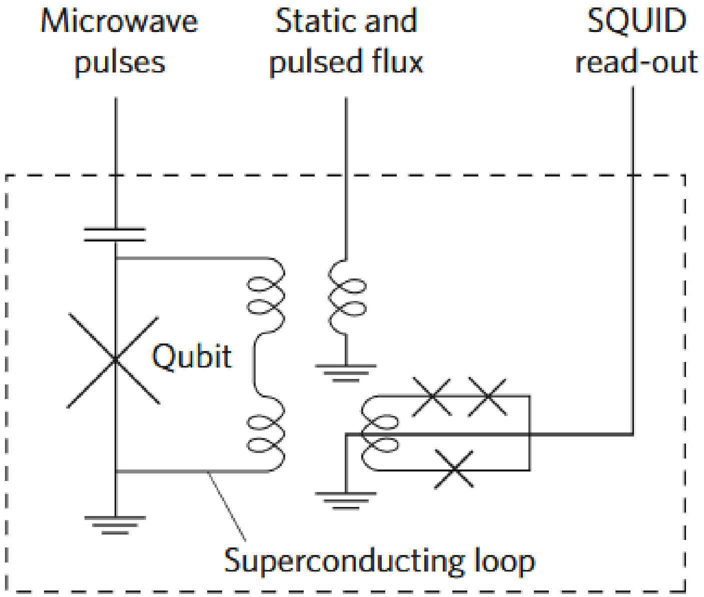


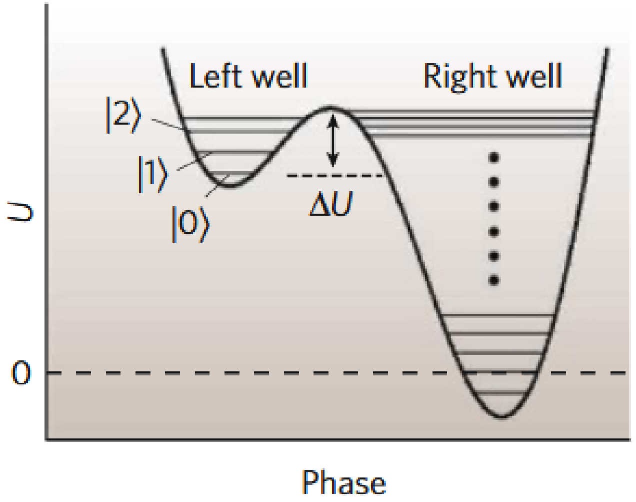
gatemon



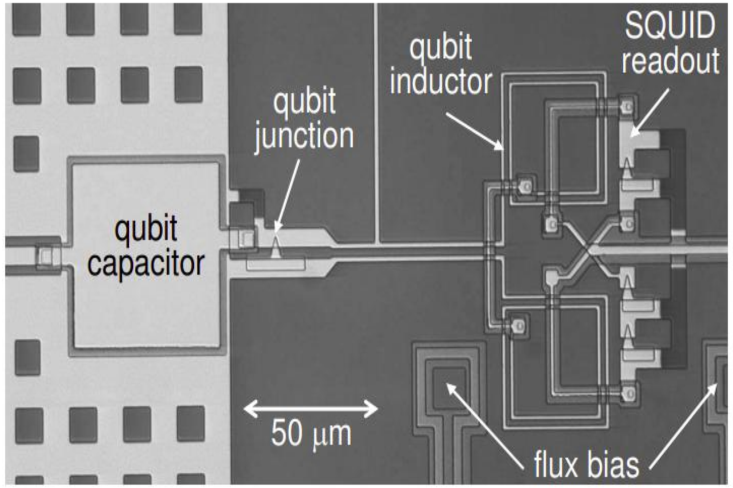
Phase qubit

- Use a current-biased Josephson junction
- Quantized energy levels in a potential well.
- E_J/E_c is large, ~10^6



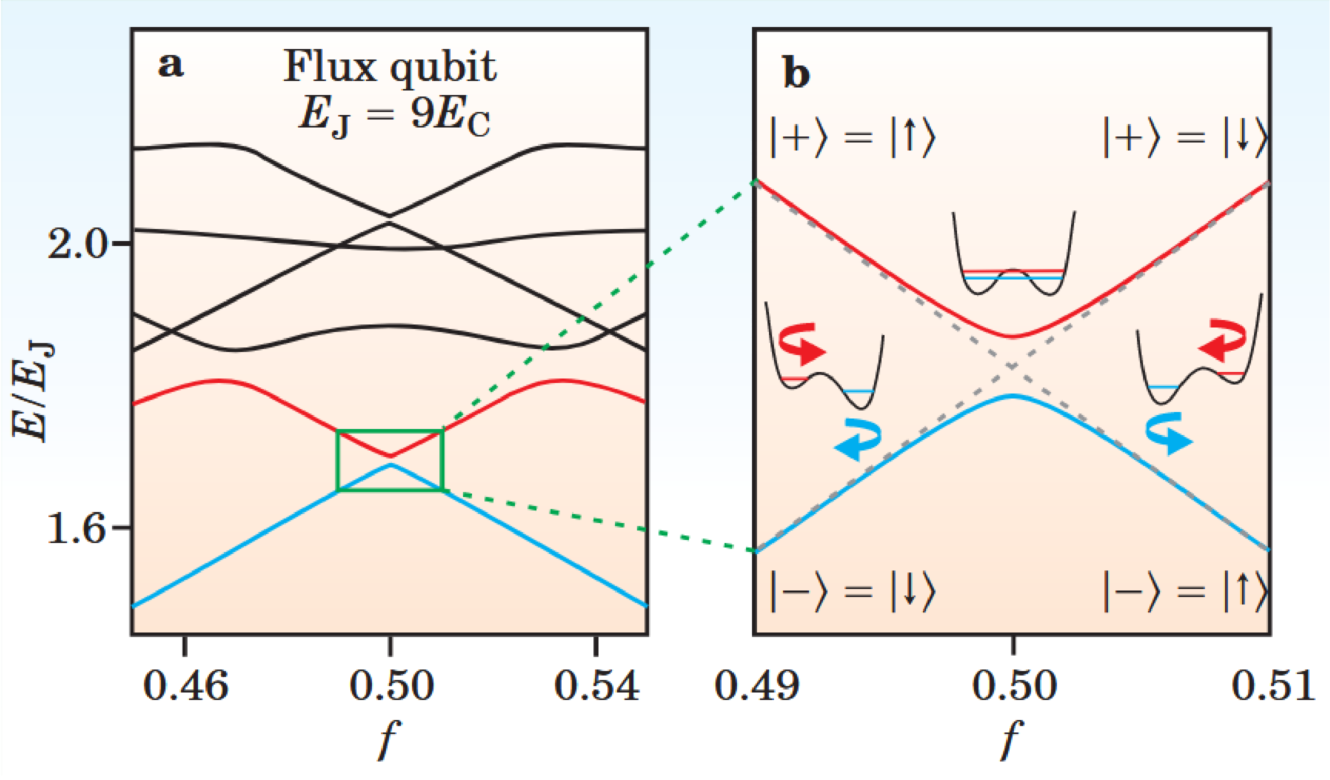


Example circuit

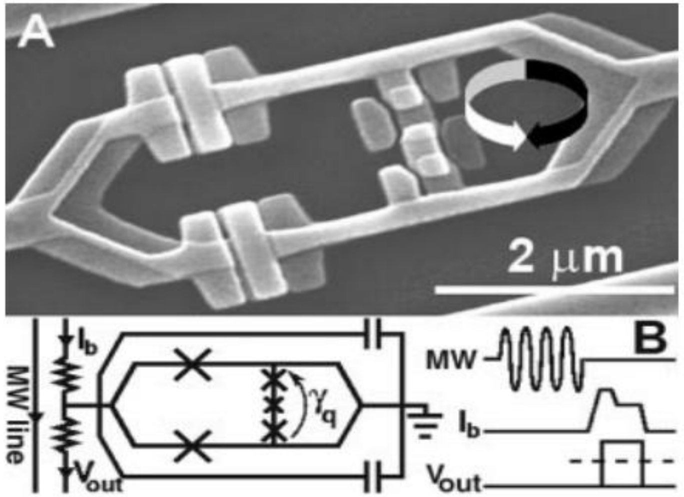


Flux qubit

- A superconducting loop interrupted by a number of Josephson junctions.
- Qubit states are encoded in circulating current states.

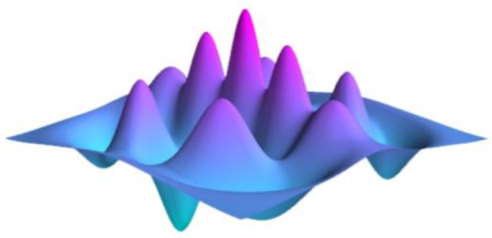


3-JJ flux qubit



[J. Q. You and Franco Nori, Physics Today (2005)]

Simulating qubits

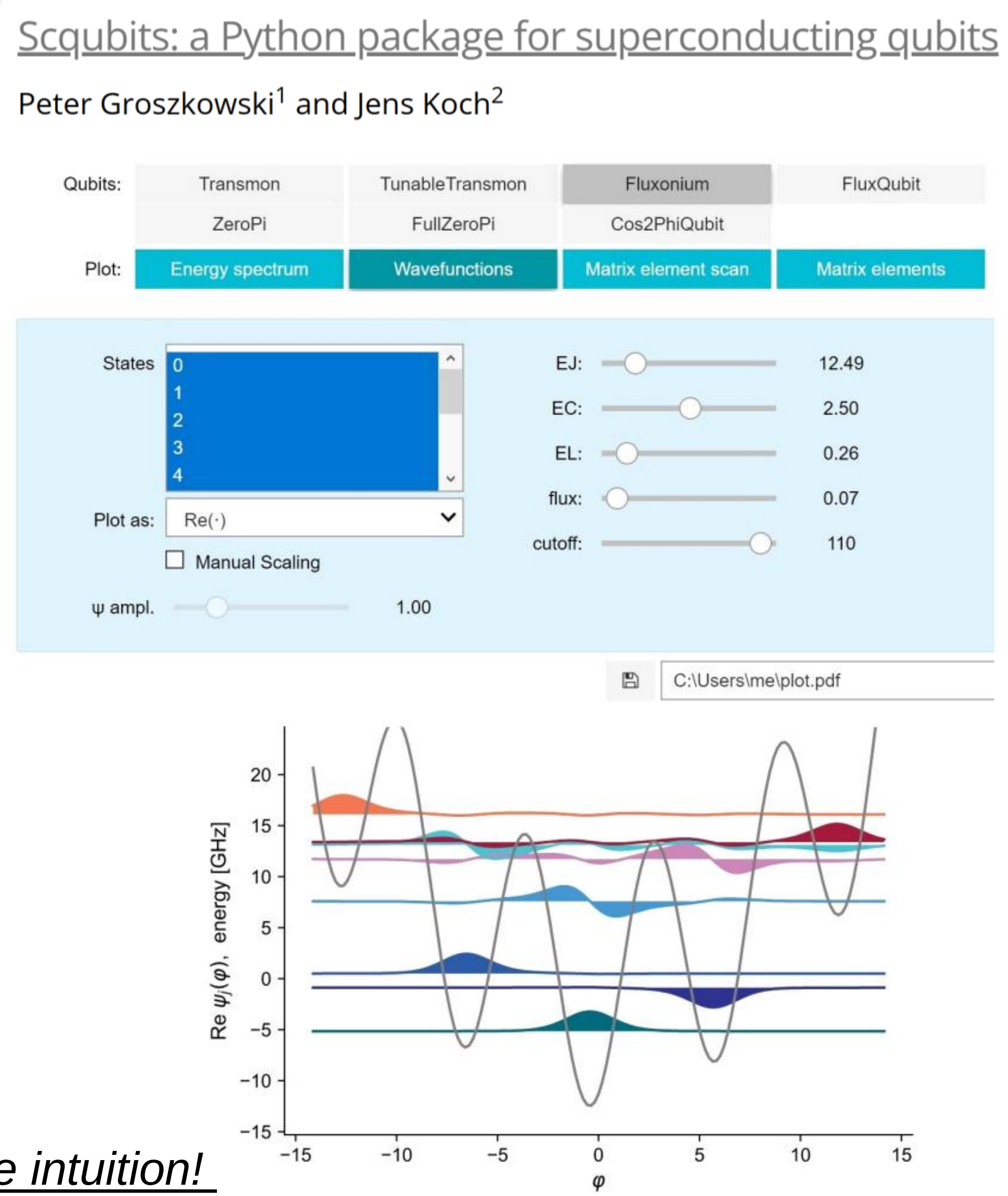


QuTiP

Quantum Toolbox in Python

Get Started with QuTiP v5 | Try QuTiP from your Browser!

- QuTiP is open-source software for simulating the dynamics of open quantum systems.
- For Quantum optics, trapped ions, superconducting circuits, quantum nanomechanical resonators, …



Scqubits: a Python package for superconducting qubits

Peter Groszkowski[1] and Jens Koch[2]

Qubits: Fluxonium

Plot: Wavefunctions

States: 0, 1, 2, 3, 4

Plot as: Re(·)

☐ Manual Scaling

ψ ampl. 1.00

EJ: 12.49

EC: 2.50

EL: 0.26

flux: 0.07

cutoff: 110

C:\Users\me\plot.pdf

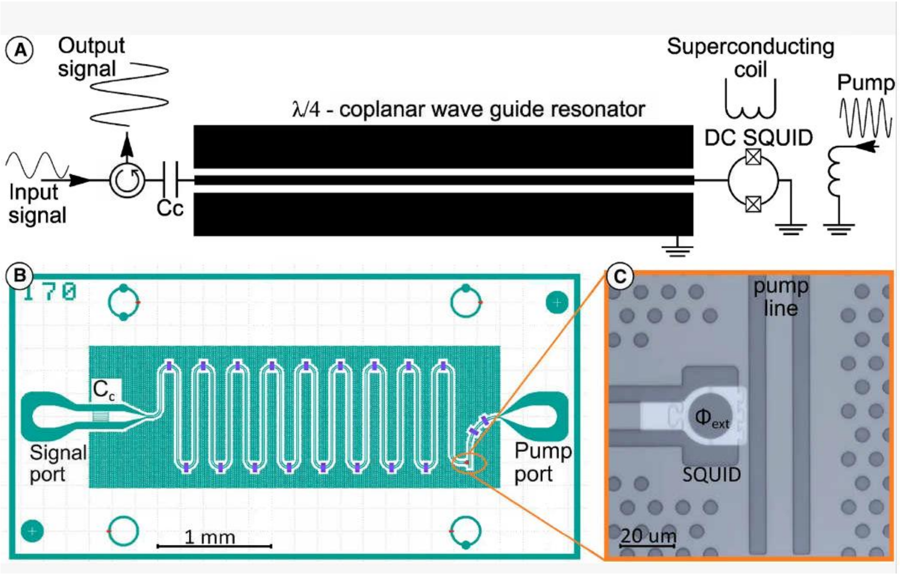
<u>These help to give intuition!</u>

Quantum-limited Josephson Parametric Amplifier

- JJ serves a nonlinear medium of microwave in RF amplifier.
- Only minimum noise can be ideally added to the output of amplifier.
- Need pump signal to amplify signal via wave mixing process

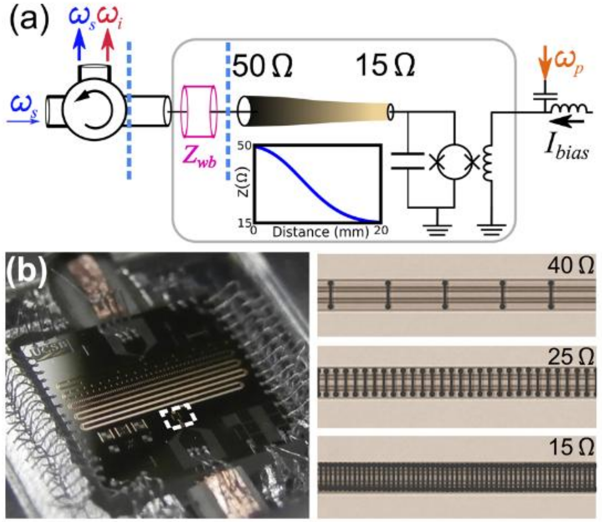
JPA (Josephson Parametric Amplifier)

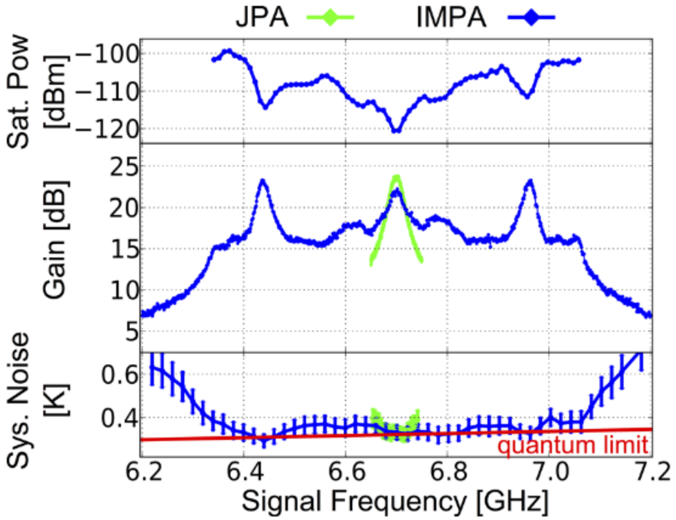
- (Resonator or LC circuit) + SQUID
- High gain > 20 dB
- Narrow bandwidth: ~10 MHz



IMPA (Impedance-matched Parametric Amplifier)

- Wider bandwidth ~400 MHz

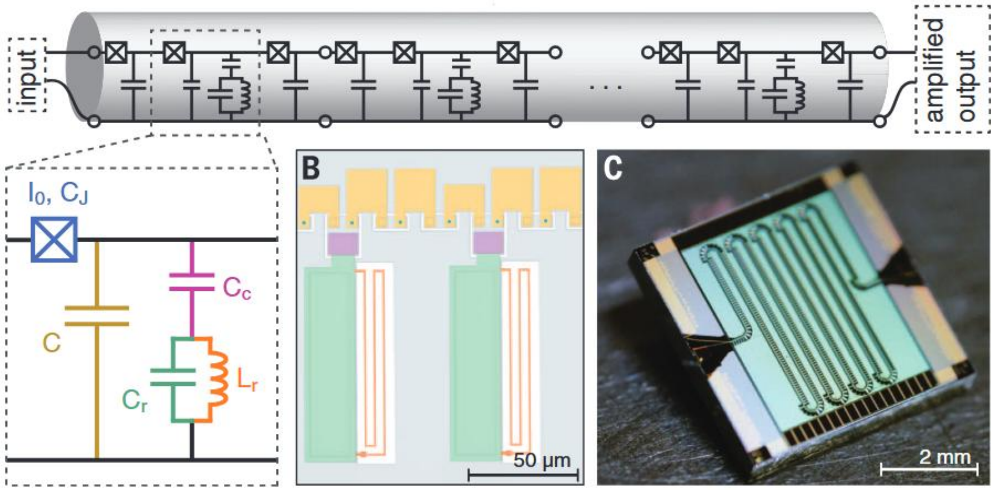


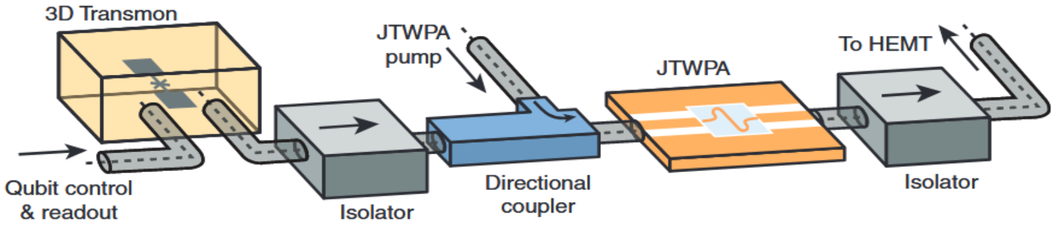


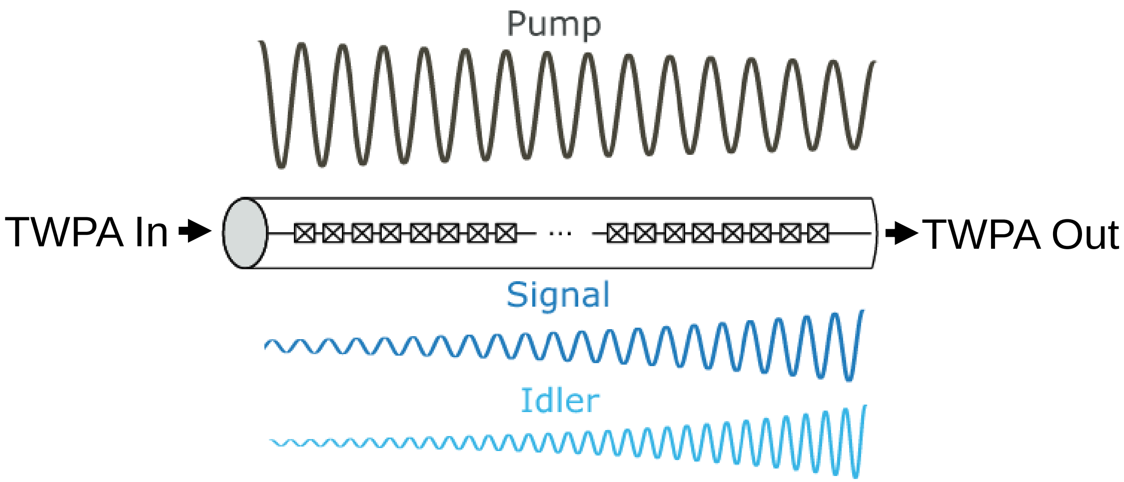
SNAIL (metric Amplifier)

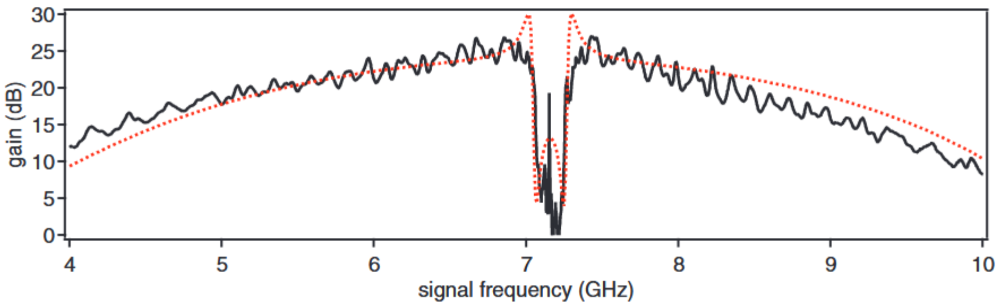
Traveling-wave Parametric Amplifier (TWPA)

- TWPA leverages a nonlinear transmission line formed by thousands of JJ array.
- Critical for high-fidelity qubit readout
- Wide bandwidth ~ 1-2 GHz
- High gain ~ 20 dB
- Gain flatness not good
- Hard to fabricate





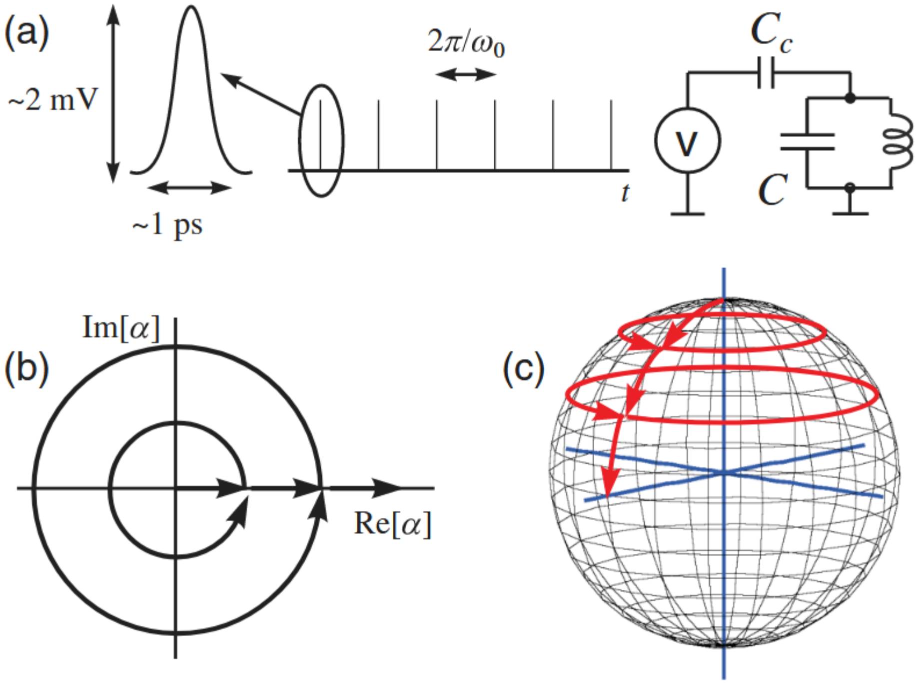




[C.Macklin et al., Science 350, 307 (2015)]

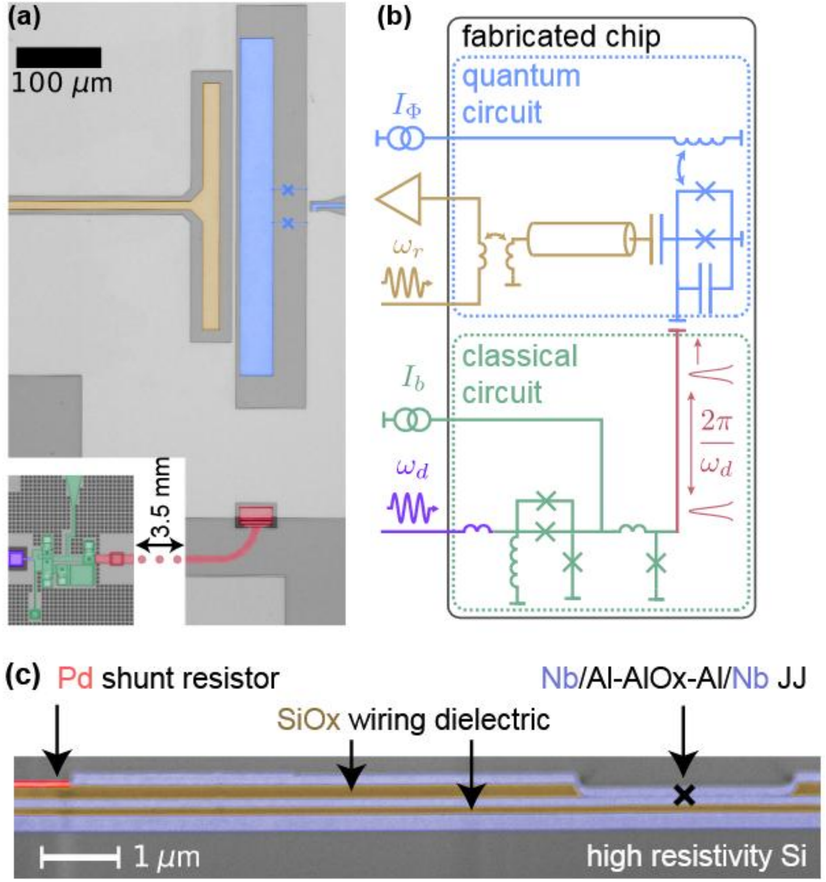
SFQ (Single Flux Quantum) circuit

- Superconducting digital technology
- SFQ pulse is a picosecond voltage pulse from overdamped JJ.



[R.McDermott et al., PRAppl. 2, 014007 (2014)]

- Qubit control demonstrated with SFQ digital logic (SFQ driver)
- Quasiparticle poisoning issue



[E.Leonard et al., PRAppl. 11, 014009 (2019)]

Summary

- Josephson junction is the most critical element in various superconducting circuits.
- SQUID can be used as a tunable inductor in qubit and sensitive sensor.
- JJ serves a lossless nonlinear inductor in various quantum circuits.

  - Superconducting qubits

  - Quantum-limited parametric amplifiers

  - SFQ circuit can be used for qubit control and measurement in energy-efficient way.

Thank you for your attention!