Noise-Resilient Superconducting Qubit Design

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Abstract

This note is an informal companion to the lecture on noise-resilient superconducting qubit design delivered at Mesoscopic School 2026. It summarizes how the energy relaxation and dephasing of a qubit are determined, and applies this framework to modern superconducting qubit designs, including the transmon and fluxonium. The notes were prepared with the assistance of Claude (Anthropic).

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1 Characterization of qubit noise

A qubit is never isolated and couples to an uncontrolled environment of electromagnetic modes, defects, and fluctuating fields. This coupling causes two physically distinct kinds of error, the relaxation and the dephasing. We describe how each noises are modeled, which provides a framework to describe and supress major noise sources for superconducting qubits such as charge noise, flux noise, dielectric loss, and photon shot noise, which will be discussed in this lecture.

1.1 Noise channels

Write the bare qubit Hamiltonian as

$$H_0 = \frac{\hbar\omega_{01}}{2}\,\sigma_z, \tag{1}$$

where ω_{01} is the qubit transition frequency and $\sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0|$. The state of the qubit is a density matrix ρ, conveniently parametrized by the Bloch vector $\vec{r} = (\langle\sigma_x\rangle, \langle\sigma_y\rangle, \langle\sigma_z\rangle)$,

$$\rho = \frac{1}{2}\big(\mathbb{I} + \vec{r}\cdot\vec{\sigma}\big). \tag{2}$$

The longitudinal component $\langle\sigma_z\rangle$ encodes the populations of $|0\rangle$ and $|1\rangle$; the transverse components $\langle\sigma_x\rangle, \langle\sigma_y\rangle$ (equivalently the off-diagonal coherence $\rho_{01} = \langle\sigma_-\rangle$) encode the relative phase of a superposition. Due to the interaction with surroundings, a qubit could suffer two unwanted processes.

- **Energy relaxation (T_1).** The longitudinal component decays as the qubit exchanges energy with the bath, $|1\rangle \to |0\rangle$ (and, at finite temperature, $|0\rangle \to |1\rangle$). This relaxes the populations toward thermal equilibrium at rate $\Gamma_1 = 1/T_1$.
- **Dephasing (T_ϕ).** The transverse component decays as the relative phase of a superposition is randomized, without any exchange of energy. The associated pure-dephasing rate is $\Gamma_\phi = 1/T_\phi$.

Both processes contribute to the decay of the transverse components, which is what an experiment actually measures in a Ramsey or Hahn-echo experiment, leading to the coherence time T_2,

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi}. \tag{3}$$

Namely, coherence is limited both by relaxation and by pure dephasing. The factor of one population (probability) while T_2 governs the decay of an amplitude (coherence), and an amplitude decays at half the rate of the corresponding probability.

1.2 Relaxation time T_1

The qubit couples to its environment because some parameter λ in its Hamiltonian $\hat{H}_q(\lambda)$—gate charge, flux, critical current—is controlled by the environment and fluctuates about its operating value, $\lambda = \lambda_0 + \delta\lambda$. Expanding to first order in the fluctuation,

$$\hat{H}_q(\lambda_0 + \delta\lambda) = \hat{H}_q(\lambda_0) + \left.\frac{\partial \hat{H}_q}{\partial\lambda}\right|_{\lambda_0} \delta\lambda + \cdots, \tag{4}$$

so the qubit couples to the environmental fluctuation through the operator

$$\hat{A} = \frac{\partial \hat{H}_q}{\partial\lambda}. \tag{5}$$

For charge noise it is proportional to the charge operator $\hat{n}$, for flux noise to the phase operator $\hat{\varphi}$. The fluctuation can be treated as classical noise or as quantum noise by upgrading to $\delta\hat{\lambda}$ [1].

If the coupling operator $\hat{A}$ has off-diagonal matrix elements connecting $|0\rangle$ and $|1\rangle$, then noise at the qubit frequency can drive transitions between the qubit eigenstates, causing energy relaxation.

Treating the noise perturbatively, one can relate the up- and down-transition rates to the noise power spectrum [1–3],

$$\Gamma_{1\uparrow} = \frac{1}{\hbar^2}\,\big|\,\langle 0|\hat{A}|1\rangle\,\big|^2\, S_\lambda(-\omega_{01}), \tag{6}$$

$$\Gamma_{1\downarrow} = \frac{1}{\hbar^2}\,\big|\,\langle 0|\hat{A}|1\rangle\,\big|^2\, S_\lambda(+\omega_{01}), \tag{7}$$

where $S_\lambda(\omega)$ is the power spectral density,

$$S_\lambda(\omega) = \int_{-\infty}^{\infty} \mathrm{d}\tau\; \mathrm{e}^{\,\mathrm{i}\omega\tau}\,\big\langle \delta\hat{\lambda}(\tau)\,\delta\hat{\lambda}(0)\big\rangle, \tag{8}$$

of the fluctuation $\delta\hat{\lambda}$, evaluated at the qubit transition frequency. Because $\delta\hat{\lambda}$ is a quantum operator, its spectrum is in general asymmetric, $S_\lambda(+\omega_{01}) \neq S_\lambda(-\omega_{01})$: the positive-frequency part governs the qubit emitting a quantum $\hbar\omega_{01}$ into the environment (relaxation, $|1\rangle \to |0\rangle$), the negative-frequency part its absorption (excitation, $|0\rangle \to |1\rangle$). For an environment in thermal equilibrium the two are related by detailed balance, $S_\lambda(-\omega_{01})/S_\lambda(+\omega_{01}) = \mathrm{e}^{-\hbar\omega_{01}/k_\mathrm{B}T}$, so at low temperature ($k_\mathrm{B}T \ll \hbar\omega_{01}$) absorption is exponentially suppressed and the qubit relaxes toward its ground state.

The total relaxation rate is the sum of the two,

$$\Gamma_1 = \frac{1}{T_1} = \frac{1}{\hbar^2}\,\big|\,\langle 0|\hat{A}|1\rangle\,\big|^2 \big[S_\lambda(+\omega_{01}) + S_\lambda(-\omega_{01})\big]. \tag{9}$$

The relaxation rate $\Gamma_1 = 1/T_1$ is the product of two factors, namely, a qubit matrix element and the environmental noise at the qubit frequency (assuming a low-enough temperature):

$$\Gamma_1 \;\propto\; \Big|\langle 0|\hat{A}|1\rangle\Big|^2 \;\times\; S_\lambda(\omega_{01}).$$

To increase the relaxation time (T_1), one could either

1. **Shrink the matrix element** $\langle 0|\hat{A}|1\rangle$ by engineering the wave function of qubit state appropriately
2. **Reduce the noise** S_B at ω_{01} through materials and microwave engineering (e.g. low-loss dielectrics, Purcell filters).

1.3 Pure dephasing T_ϕ

The diagonal element of the coupling operator $\hat{A} = \partial\hat{H}_q/\partial\lambda$ that commute with $\hat{H}_q$ can cause pure dephasing by randomly changing the qubit frequency. By the Hellmann–Feynman theorem the diagonal elements of $\partial\hat{H}_q/\partial\lambda$ are the derivatives of the eigenenergies, so their difference is the frequency sensitivity, $\langle 1|\hat{A}|1\rangle - \langle 0|\hat{A}|0\rangle = \frac{\partial(E_1-E_0)}{\partial\lambda} = \hbar\,\frac{\partial\omega_{01}}{\partial\lambda}$. Therefore, the qubit frequency fluctuates as

$$\omega_{01}(t) = \omega_{01} + \delta\omega_{01}(t), \qquad \delta\omega_{01}(t) = \frac{\partial\omega_{01}}{\partial\lambda}\,\delta\lambda(t). \tag{10}$$

The derivative $\partial\omega_{01}/\partial\lambda$ is the qubit's *sensitivity* to the noisy parameter λ (gate charge, flux, etc.).

The standard method to probe the coherence is a Ramsey experiment: prepare the superposition $(|0\rangle + |1\rangle)/\sqrt{2}$, let it evolve freely for a time t, and read out the coherence. Because the qubit frequency fluctuates, the relative phase accumulated during the free evolution,

$$\varphi(t) = \int_0^t \delta\omega_{01}(t')\,\mathrm{d}t' = \frac{\partial\omega_{01}}{\partial\lambda}\int_0^t \delta\lambda(t')\,\mathrm{d}t', \tag{11}$$

is a random variable. The off-diagonal coherence carries this accumulated phase, $\rho_{01}(t) = \rho_{01}(0)\,\mathrm{e}^{\mathrm{i}\varphi(t)}$. Since a measurement averages over many repetitions and hence over the noise, the observed coherence is the noise-averaged phase factor,

$$\frac{\rho_{01}(t)}{\rho_{01}(0)} = \big\langle \mathrm{e}^{\,\mathrm{i}\varphi(t)} \big\rangle. \tag{12}$$

For any random variable, the exponetial average can be expanded using its *cumulants* $\langle \varphi^n \rangle_c$,

$$\big\langle \mathrm{e}^{\,\mathrm{i}\varphi} \big\rangle = \exp\Big[\mathrm{i}\langle\varphi\rangle_c - \tfrac{1}{2}\langle\varphi^2\rangle_c - \tfrac{\mathrm{i}}{6}\langle\varphi^3\rangle_c + \cdots\Big], \tag{13}$$

the first two of which are the mean $\langle\varphi\rangle_c = \langle\varphi\rangle$ and the variance $\langle\varphi^2\rangle_c = \langle\varphi^2\rangle - \langle\varphi\rangle^2$. Assuming that $\varphi(t)$ is Gaussian with zero mean, the average of $\mathrm{e}^{\mathrm{i}\varphi}$ is fixed entirely by its variance,

$$\big\langle \mathrm{e}^{\,\mathrm{i}\varphi(t)} \big\rangle = \mathrm{e}^{-\frac{1}{2}\langle\varphi^2(t)\rangle}, \tag{14}$$

since all cumulants beyond the second vanish. The decay of coherence is therefore governed by the phase variance,

$$\langle\varphi^2(t)\rangle = \left(\frac{\partial\omega_{01}}{\partial\lambda}\right)^2 \int_0^t\!\!\int_0^t \big\langle \delta\lambda(t_1)\,\delta\lambda(t_2)\big\rangle\,\mathrm{d}t_1\,\mathrm{d}t_2. \tag{15}$$

It is evident from the above equation that the phase variance in turn is set by the two-time autocorrelation of the noise. For a stationary process the autocorrelation is the Fourier transform of the power spectral density (the Wiener–Khinchin theorem),

$$\big\langle \delta\lambda(t_1)\delta\lambda(t_2)\big\rangle = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, S_\lambda(\omega)\,\mathrm{e}^{-\mathrm{i}\omega(t_1-t_2)}. \tag{16}$$

. Inserting this and carrying out the two time integrals gives

$$\langle\varphi^2(t)\rangle = \left(\frac{\partial\omega_{01}}{\partial\lambda}\right)^2 \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, S_\lambda(\omega)\,\frac{\sin^2(\omega t/2)}{(\omega/2)^2}. \tag{17}$$

For noise with a short correlation time compared to the measurement ($t_c \ll t$), the spectral density is approximately flat over the filter function's bandwidth, $S_\lambda(\omega) \approx S_\lambda(0)$, and the frequency integral gives a contribution linear in t. The coherence then decays exponentially,

$$\rho_{01}(t) \simeq \exp\Big[-\tfrac{1}{2}\Big(\frac{\partial\omega_{01}}{\partial\lambda}\Big)^2 S_\lambda(0)\,|t|\Big], \tag{18}$$

defining a pure-dephasing time

$$\frac{1}{T_\phi} = \frac{1}{2}\Big(\frac{\partial\omega_{01}}{\partial\lambda}\Big)^2 S_\lambda(0). \tag{19}$$

This form holds for noise spectra that are regular at low frequency; for spectra singular at $\omega = 0$, such as $1/f$ noise, it leads to non-exponential decay [3, 4].

The kernel $\sin^2(\omega t/2)/(\omega/2)^2$ is an example of the *filter function* [5, 6] of the free-evolution (Ramsey) sequence: the dephasing is the overlap of the noise spectrum with this filter. It is peaked at $\omega = 0$ with width $\sim 1/t$, so free evolution is sensitive to *low-frequency* noise—precisely where $1/f$ noise is strongest. The appearance of a frequency-domain filter function is not special to free evolution. In a Ramsey sequence, every instant contributes equally to the accumulated phase. This is no longer the case, if one apply a π pulse in the middle of the evolution that interchanges $|0\rangle$

and $|1\rangle$ and so reverses the sign of all subsequently accumulated phase (Hahn echo). A general sequence therefore weights the integrand by a *switching function* $f(t') \in \{+1, -1\}$,

$$\varphi(t) = \frac{\partial\omega_{01}}{\partial\lambda} \int_0^t f(t')\,\delta\lambda(t')\,\mathrm{d}t', \tag{20}$$

with $f = +1$ throughout for Ramsey, and f flipping sign at each π pulse. Repeating the variance calculation with f in place gives the same structure as before,

$$\langle\varphi^2(t)\rangle = \left(\frac{\partial\omega_{01}}{\partial\lambda}\right)^2 \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, S_\lambda(\omega) \left|\tilde{f}(\omega, t)\right|^2, \qquad \tilde{f}(\omega, t) = \int_0^t f(t')\,\mathrm{e}^{\mathrm{i}\omega t'}\,\mathrm{d}t', \tag{21}$$

where the filter function $\left|\tilde{f}(\omega, t)\right|^2$ is simply the power spectrum of the switching function. Dephasing is thus always an overlap integral between the noise spectrum and a filter that the experimenter controls through the pulse sequence. Therefore, by designing a pulse sequences one could reduce the effect of the low-frequency noises.

Finally, if the qubit operates at a "sweet spot" where the first-order sensitivity $\partial\omega_{01}/\partial\lambda$ vanishes, the residual dephasing is then governed by the second-order coupling, $\frac{1}{2}(\partial^2\omega_{01}/\partial\lambda^2)\,\delta\lambda^2$, which requires a separate treatment of the quadratic noise [7].

In order to increase the dephasing time, there could be three possibilities for engineering the qubit.

1. **Reduce the frequency sensitivity** $\partial\omega_{01}/\partial\lambda$ — *dispersion engineering.* For example, one can operate at a sweet spot where the qubit frequency is first-order insensitive to the noisy parameter ($\partial\omega_{01}/\partial\lambda = 0$) or up to any order.

2. **Reduce the low-frequency noise** $S_\lambda(\omega)$ — *materials engineering*

3. **Shape the filter function** $\left|\tilde{f}(\omega, t)\right|^2$— *pulse engineering* to suppress its weight where the noise is strongest. For example, the Hahn echo and CPMG sequences push the filter's sensitive band away from $\omega = 0$, reducing the effect of low-frequency noises.

2 The transmon

The transmon is the most widely used superconducting qubit. This section presents its Hamiltonian, the parameter choices that suppresses charge noise, the dephasing introduced by dispersive readout, the use of a Purcell filter to control relaxation through the readout channel, and the materials choice to increase the relaxation time.

2.1 The Hamiltonian

The transmon is a Cooper-pair box: a Josephson junction shunted by a capacitor, with Hamiltonian

$$H = 4E_C\,(\hat{n} - n_g)^2 - E_J \cos\hat{\varphi}, \tag{22}$$

where $\hat{n}$ is the number of Cooper pairs transferred across the junction, $\hat{\varphi}$ the conjugate phase ($[\hat{\varphi}, \hat{n}] = \mathrm{i}$), $E_C = e^2/2C_\Sigma$ the charging energy, E_J the Josephson energy, and n_g the offset charge set by the electrostatic environment. The offset charge fluctuates due to stray charges in the substrate and on surface.

The transmon is defined by the regime

$$\frac{E_J}{E_C} \gg 1, \qquad E_J/E_C \sim 20\text{–}100, \tag{23}$$

obtained by making the shunt capacitor large, which lowers E_C. In this regime the phase is localized in the cosine well and the low-lying spectrum is that of a weakly anharmonic oscillator. Expanding (22) about $\hat{\varphi} = 0$ gives

$$\hbar\omega_{01} \simeq \sqrt{8E_J E_C} - E_C, \qquad \alpha \equiv \hbar(\omega_{12} - \omega_{01}) \simeq -E_C, \tag{24}$$

so the anharmonicity is set by E_C. A finite anharmonicity is required to address the $|0\rangle \leftrightarrow |1\rangle$ transition without exciting $|1\rangle \leftrightarrow |2\rangle$, so it cannot be arbitrarily lowered.

2.2 Charge-noise insensitivity

The dephasing due to the charge noise is governed by the frequency sensitivity $\partial\omega_{01}/\partial n_g$. The eigenvalue problem of (22) is Mathieu's equation, and its solution shows that the dependence of the qubit frequency on the offset charge (the charge dispersion) is exponentially suppressed in the transmon regime [2],

$$\epsilon_{01}(n_g) \equiv \omega_{01}(n_g = \tfrac{1}{2}) - \omega_{01}(n_g = 0) \;\propto\; \exp\big(-\sqrt{8E_J/E_C}\,\big). \tag{25}$$

The sensitivity $\partial\omega_{01}/\partial n_g$ is bounded by this dispersion and is therefore also exponentially small. As E_J/E_C increases, the charge dispersion decreases exponentially while the anharmonicity relative to the transition frequency decreases only as a power law, so charge insensitivity is gained at a small cost in anharmonicity. The transmon suppresses the n_g-dependence of the qubit frequency at all values of n_g, so no biasing or feedback is required. This is what makes the transmon insensitive to charge noise, leading to a long coherence time.d

2.3 Dispersive readout and photon shot noise

The transmon qubit is typically measured through a readout resonator of frequency ω_r, coupled with strength g. In the dispersive regime $|\Delta| = |\omega_{01} - \omega_r| \gg g$, the interaction shifts the resonator frequency by an amount that depends on the qubit state,

$$H_{\text{disp}} = \hbar\omega_r a^\dagger a + \frac{\hbar\omega_{01}}{2}\sigma_z + \hbar\chi\, a^\dagger a\, \sigma_z, \tag{26}$$

with dispersive shift χ. Measuring the resonator frequency reads out the qubit state without exchanging energy with the qubit.

The dispersive interaction term in (26) can also be read as a shift of the qubit frequency proportional to the resonator photon number, $\delta\omega_{01} = 2\chi\, a^\dagger a$. The photon number is therefore a longitudinal noisy parameter in the sense of §1.3, with sensitivity $\partial\omega_{01}/\partial\bar{n} = 2\chi$. Fluctuations in the resonator population, from residual thermal photons or from measurement photons, dephase the qubit. In the limit $\chi \ll \kappa$, where κ is the resonator decay rate, the measurement induced dephasing rate is [8]

$$\Gamma_\phi^{\text{ph}} \propto \frac{\chi^2}{\kappa}\,\bar{n}_{\text{th}}, \tag{27}$$

where $\bar{n}_{\text{th}}$ is the average thermal photon number. This noise is intrinsic to dispersive readout and could be reduced by lowering $\bar{n}_{\text{th}}$.

2.4 Purcell decay and the Purcell filter

The readout resonator is also a relaxation channel. The dressed qubit state in the dispersive limit contains a photon components of order g/Δ, so the qubit can decay through the resonator at the Purcell rate [3]

$$\Gamma_1^{\text{Purcell}} = \left(\frac{g}{\Delta}\right)^2 \kappa, \tag{28}$$

where κ is the decay rate of the readout resonator. In order to decrease $\Gamma_1^{\text{Purcell}}$, one has to make either $\frac{g}{\Delta}$ or κ small, but it leads to longer measurement time required.

A Purcell filter [9, 10] is developed to solve this problem. By placing additional filter resonator between the readout resonator and the feedline, the effective decay rate can be made frequency dependent: $\kappa(\omega_r)$ remains large for fast readout while $\kappa(\omega_{01})$ is suppressed for long T_1. This can be viewed as the filter shapes the bath spectral density seen by the qubit, reducing $S_\lambda(\omega_{01})$ while leaving $S_\lambda(\omega_r)$ unchanged. In filtered devices the Purcell-limited lifetime exceeds the measured T_1, so Purcell decay is not the limiting channel [11].

2.5 The materials limit on T_1

With charge noise suppressed and Purcell decay controlled by filtering, the transmon lifetime is set by the intrinsic environmental noise $S_\lambda(\omega_{01})$, which in current devices is dominated by dielectric loss. Recent effort to utilize a tantalum on a silcon substrate [11] aims to reduce such dielectric loss. Without changes to the qubit design, they obtained T_1 up to 1.68 ms.

3 The fluxonium

We now turn to another promising superconducting qubit, the fluxonium. A range of parameter regimes are being explored in literature; rather than surveying them, we take the experiments of Nguyen *et al.* [12], operated at the half-flux sweet spot, as a representative example and examine briefly what makes them good qubits.

3.1 The Hamiltonian

The fluxonium is a Josephson junction shunted by a large inductance, whose Hamiltonian reads

$$H = 4E_C\,\hat{n}^2 + \tfrac{1}{2}E_L\,\hat{\varphi}^2 - E_J\cos(\hat{\varphi} - \varphi_{\text{ext}}), \tag{29}$$

where $\hat{\varphi}$ is the phase across the inductance, $\hat{n}$ the conjugate charge with $[\hat{\varphi}, \hat{n}] = \mathrm{i}$, $E_C = e^2/2C$ the charging energy, $E_L = (\hbar/2e)^2/L$ the inductive energy, and $\varphi_{\text{ext}} = 2\pi\Phi_{\text{ext}}/\Phi_0$ the reduced external flux biasing the loop. The parameters satisfy $E_L \ll E_J$ and $1 \lesssim E_J/E_C \lesssim 10$ [12]. The inductance is a *superinductance*, realized as a chain of $N \sim 10^2$ Josephson junctions, since a geometric inductor cannot reach the required values.

Note that the $\frac{1}{2}E_L\hat{\varphi}^2$ term breaks the 2π-periodicity of the cosine, so $\hat{\varphi}$ is a non-compact variable on the real line rather than a phase on a circle. The fluxonium spectrum is independent of the offset charge n_g, making it robust against charge-offset noise; the control parameter is instead the external flux φ_{ext}.

3.2 The half-flux sweet spot

The devices are operated at the half-flux point $\varphi_{\text{ext}} = \pi$. There the Josephson term becomes $-E_J \cos(\hat{\varphi} - \pi) = +E_J \cos\hat{\varphi}$, so the potential $V(\varphi) = \frac{1}{2}E_L\varphi^2 + E_J \cos\varphi$ is symmetric about $\varphi = 0$ and, for E_J larger than E_L, forms a double well. The lowest two eigenstates $|0\rangle, |1\rangle$ are the symmetric and antisymmetric combinations of states localized in the two wells—the tunnel splitting of the otherwise degenerate classical ground state [12]. The transition frequency at half flux is about an order of magnitude below its value at zero flux, reaching $\omega_{01}/2\pi \sim 0.5\,\text{GHz}$ for these devices, while the higher transition stays large, giving a strong anharmonicity $\omega_{12}/\omega_{01} \approx 3$–$10$. The strong anharmonicity is useful for a qubit control. The device parameters span $E_J \sim 2$–$5\,\text{GHz}$, $E_C \sim 1\,\text{GHz}$, and $E_L \sim 0.5$–$1\,\text{GHz}$ [12]. There are a few features of this operating point that gives long coherence time.

Low transition frequency for long T_1: The dominant relaxation channel is dielectric loss. Its rate is, determined by the Fermi's golden rule that we discuss in the first section, given by [12]

$$\frac{1}{T_1(\omega)} = \frac{1}{(2e)^2} \left| \langle 0|\hat{\varphi}|1\rangle \right|^2 S_{\text{diel}}(\omega), \qquad S_{\text{diel}}(\omega) = 2\hbar\omega\, \operatorname{Re} Y_C(\omega), \tag{30}$$

where $Y_C(\omega) = \omega C/Q_{\text{diel}}$ is the admittance of the lossy capacitance and $Q_{\text{diel}} = 1/\tan\delta_C$. Since $S_{\text{diel}} \propto \omega^2 \tan\delta_C$, the noise sampled at ω_{01} grows steeply with frequency. Lowering the transition frequency by about a factor of ten (around $0.5\,\text{GHz}$) therefore reduces the power spectral density, with no change in materials.

First-order flux insensitivity for long T_ϕ: The fluxonium dephases through $1/f$ flux noise in φ_{ext}, with spectral density $S_\Phi(\omega) = 2\pi A^2/\omega$ and an amplitude $A \approx 2 \times 10^{-6}\,\Phi_0$ at $1\,\text{Hz}$, typical of superconducting loops [12]. The dephasing rate is proportional to the flux sensitivity $\partial\omega_{01}/\partial\varphi_{\text{ext}}$, which vanishes at half flux by the symmetry of the potential,

$$\left.\frac{\partial\omega_{01}}{\partial\varphi_{\text{ext}}}\right|_{\varphi_{\text{ext}}=\pi} = 0. \tag{31}$$

Half flux is thus a sweet spot at which the qubit is first-order insensitive to flux noise. The residual second-order coupling $\propto (\partial^2\omega_{01}/\partial\varphi_{\text{ext}}^2)A^2$, which bounds T_ϕ above $10\,\text{ms}$ for these devices [7, 12]. Moreover, the second order flux sensitivity scales with E_L^2, namely, $1/N^2$ with N being the number of junctions.

Large inductance for long T_1: The flux noise can also induce relaxation, coupling to the qubit through the persistent current $\hat{I}_p = \hat{\Phi}/L$ with rate [12]

$$\Gamma_{ij}^{\Phi} = \frac{1}{(2e)^2}\frac{1}{L^2} \left| \langle j|\hat{\varphi}|i\rangle \right|^2 S_\Phi(\omega_{ij}), \tag{32}$$

where $\hat{\varphi} = (2e/\hbar)\hat{\Phi}$. The factor $1/L^2$ means the relaxation time due to flux noise grows as L^2; for the large chain inductance here it reaches hundreds of milliseconds and is not a limiting channel. This is in contrast to typical flux-tunable qubits, whose T_1 at the low-frequency sweet spot is often limited by flux noise.

3.3 Coherence time

With these features, Nguyen *et al.* [12] measured $T_2 > 100\,\mu\text{s}$ across eight devices at half flux, with the best device exceeding $400\,\mu\text{s}$, and found the coherence limited by energy relaxation rather than by flux noise. The relaxation in turn was attributed to dielectric loss. More recent experiment [13] reported a Ramsey coherence time $T_2^* = 1.48\,\text{ms}$ and single-qubit gate fidelity 0.9999, with the coherence still limited by dielectric absorption. The fluxonium thus reaches the millisecond coherence time through low-frequency operation at a flux sweet spot.

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