Hadron Physics in Light-Front Dynamics

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Analysis of virtual meson production in a (1 + 1)-dimensional scalar field model

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PHYSICAL REVIEW D **103,** 076002 (2021)

Light-front dynamic analysis of the longitudinal charge density using the solvable scalar field model in (1 + 1) dimensions

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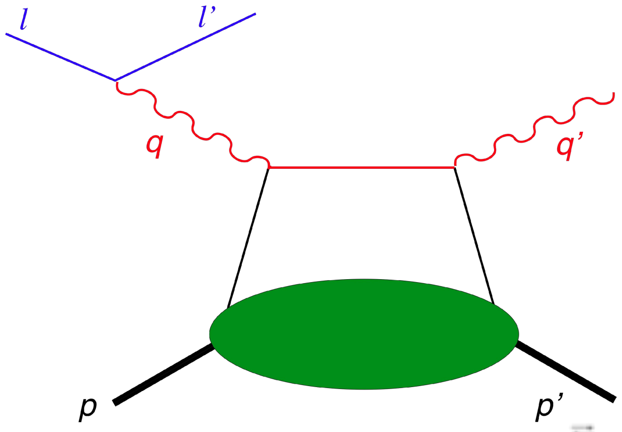
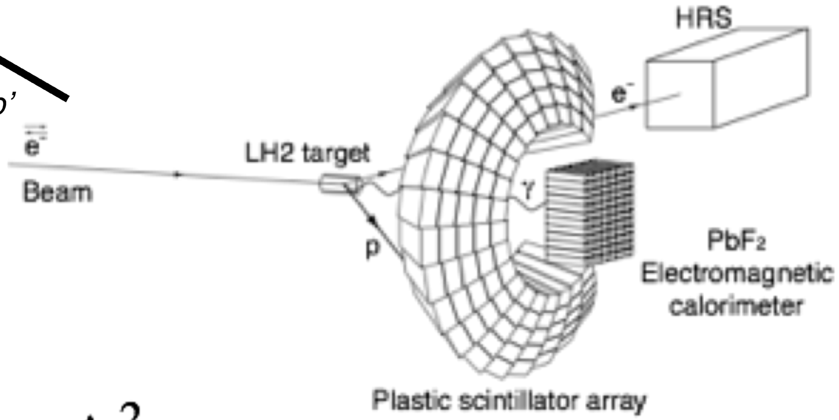
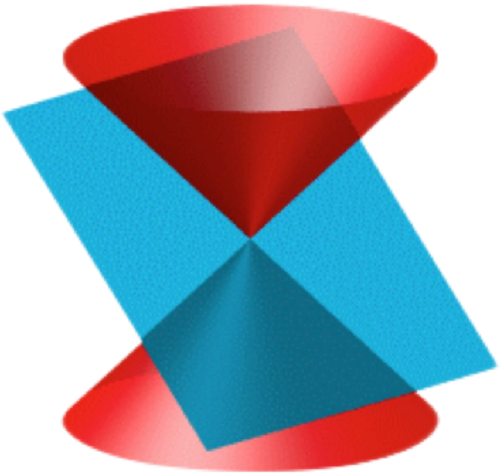
Better Work in Forward Direction



GPD





LFD

$$t = \Delta^2 = -\frac{\zeta^2 M^2 + \Delta_\perp^2}{1-\zeta};\Delta^+(\equiv \Delta^0 + \Delta^3) = \zeta P^+;\Delta_\perp^2 > \Delta_{\perp\min}^2 \neq 0$$

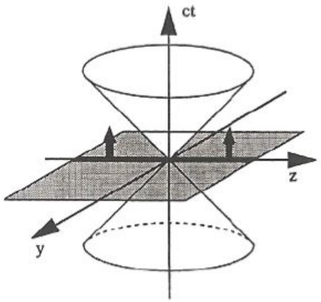
Outline

- Rationale for the Light-Front Dynamics in Hadron Physics

  Dirac's Proposition for Relativistic Dynamics

  Instant Form Dynamics(IFD) vs. Light-Front Dynamics(LFD)
- Benchmarking GPD Applicability in Forward Direction
- GPD Sum Rule and Valence/Nonvalence Decomposition
- Conclusion and Outlook

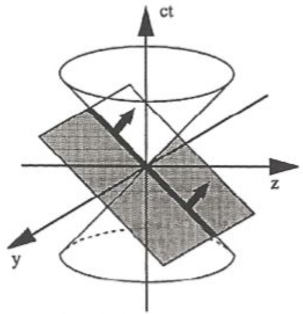
Dirac's Proposition for Relativistic Dynamics



The instant form

Equal t 1949 *Equal τ*

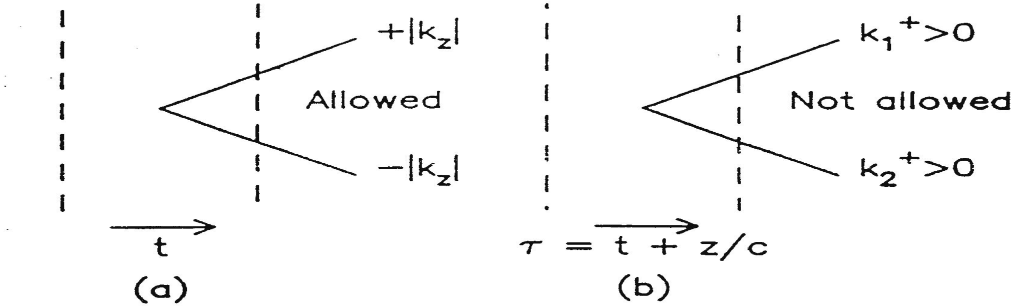
$$p^0 \leftrightarrow p^- = p^0 - p^3$$
$$(p^1, p^2) \leftrightarrow \vec{p}_\perp$$
$$p^3 \leftrightarrow p^+ = p^0 + p^3$$



The front form

Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2} \qquad p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



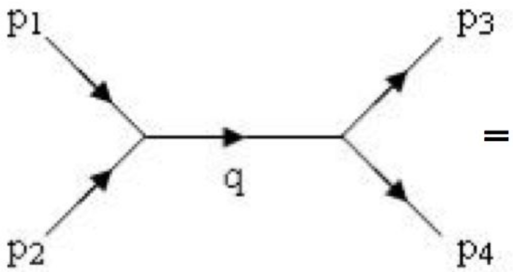
Except zero-modes

$$k_1^+ = k_2^+ = 0$$

IFD
Instant Form Dynamics

LFD
Light-Front Dynamics

"$e^+e^- \to \mu^+\mu^-$"



$$= \frac{1}{q^2 - m^2} = \frac{1}{s - m^2} \qquad q^2 = (p_1 + p_2)^2 \neq m^2$$

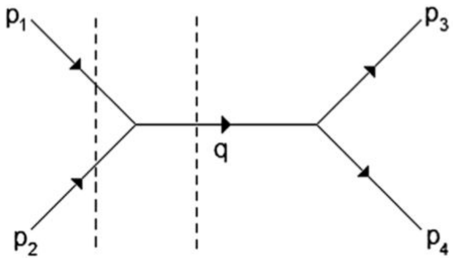
Four-momentum conservation but off-mass-shell

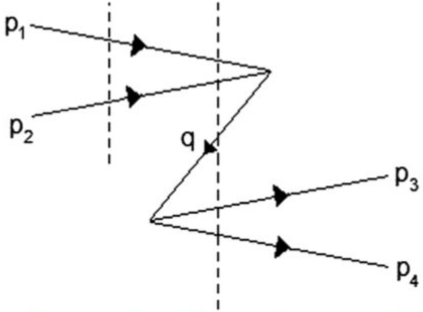
Feynman Diagram: Invariant under all 10 Poincaré generators

t→ (time evolution; time ordered process in QFT; Energy is not conserved within Δt)

$$(\Delta E)(\Delta t) \sim \hbar$$

Three-momentum conservation but on-mass-shell

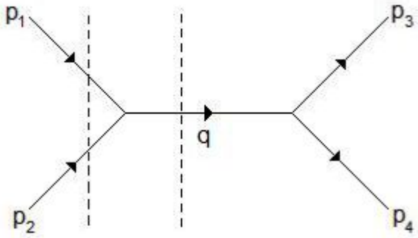




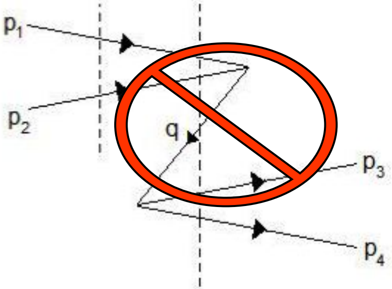
Individual Time-Ordered Diagrams: Invariant only under translation and rotation (6 kinematic generators)

However, in LFD, (b) drops for any reference frame (not just for IMF)

τ (= t+z/c) →



(a)



(b)

Invariant under 7 kinematic generators including the longitudinal boost.

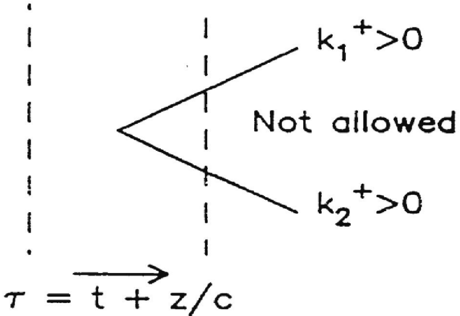
$$\Sigma^a_{LFD} + \Sigma^b_{LFD} = \frac{1}{q^+}\left(\frac{1}{p_1^- + p_2^- - q^-} + 0\right)$$

$$= \frac{1}{q^+\left(\frac{(p_1+p_2)^2 + (\vec{p}_{1\perp} + \vec{p}_{2\perp})^2}{(p_1+p_2)^+} - \frac{m^2 + \vec{q}_\perp^2}{q^+}\right)}$$

$$= \frac{1}{(p_1+p_2)^2 - m^2}$$

$$= \frac{1}{s - m^2}$$

$$p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



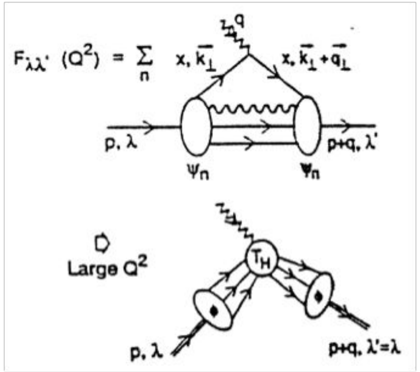
Except zero-mode

$$k_1^+ = k_2^+ = 0$$

Applications to Hadron Phenomenology

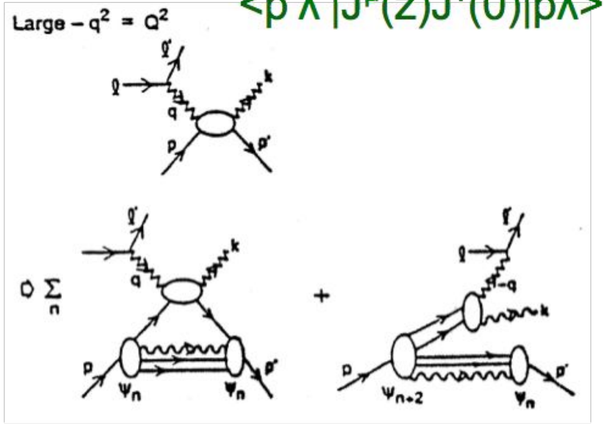
Form Factors $l\ p \to l'\ p'$

$\langle p'\ \lambda' | J^{+}(0) | p\ \lambda \rangle$

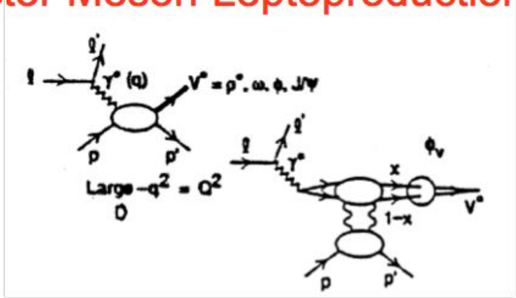


Virtual Compton $\gamma^{*}\ p \to \gamma'\ p'$

$\langle p' \lambda' | J^{\mu}(z) J^{\nu}(0) | p \lambda \rangle$

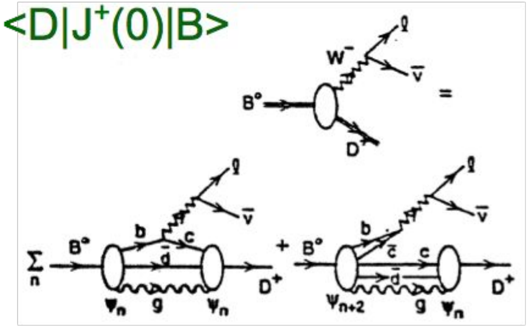


Vector Meson Leptoproduction $\gamma^{*} p \to V^{*} p'$



Weak Decay

$\langle D | J^{+}(0) | B \rangle$

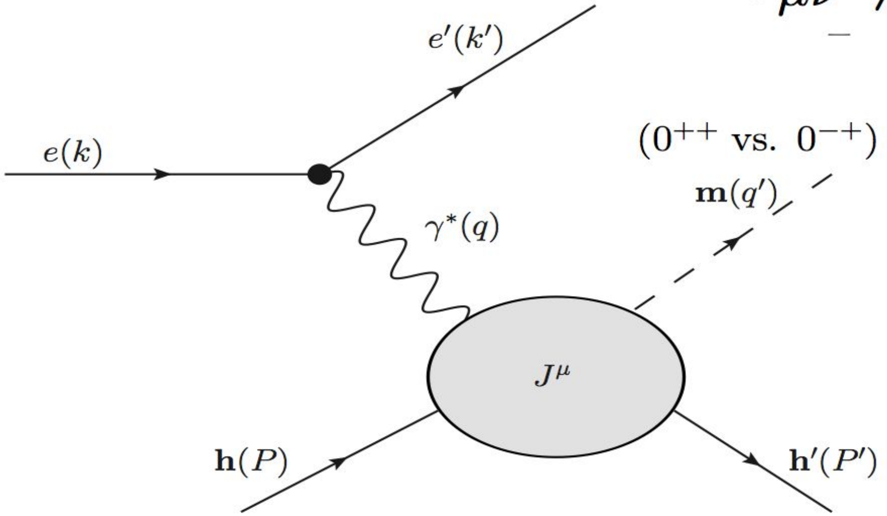


$$\langle|\mathcal{M}|^2\rangle = \left(\frac{e^2}{q^2}\right)^2 \mathcal{L}^{\mu\nu}\mathcal{H}_{\mu\nu}$$

$$\mathcal{L}^{\mu\nu} = q^2\left[g^{\mu\nu} + \frac{2}{q^2}(k^\mu k'^\nu + k'^\mu k^\nu)\right] + 2ih\epsilon^{\mu\nu\alpha\beta}k_\alpha k'_\beta$$

$$\mathcal{H}_{\mu\nu} = J_\mu^\dagger J_\nu$$

$$\mathcal{H}_{\mu\nu} \neq \mathcal{H}_{\nu\mu}$$



Salient Features of Meson Leptoproduction

- No interference with the Bethe-Heitler process
- Consistency between our benchmark BSA prediction for 0^{-+} meson production off the scalar target with the data of the exclusive coherent electroproduction of the π^0 off ^{4}He measured at JLab Hall B

C.Ji, H.-M.Choi, A.Lundeen, B.Bakker, PRD99,116008(2019)

- General formulation of hadronic amplitudes in Meson Production off the Scalar Target (0^{++} vs. 0^{-+})
- Comparison/Contrast with the leading twist GPD formulation.

Pseudoscalar(0^{-+}) Meson vs. Scalar(0^{++}) Meson

$$\epsilon^{\mu\nu\alpha\beta} \quad \text{vs.} \quad d^{\mu\nu\alpha\beta} = g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta}$$

C.Ji & B.Bakker, PoS QCDEV2017,038(2017);
B.Bakker & C.Ji, Few Body Syst. 58,no.1,8(2017)

$$F_{PS}(Q^2, t, x)$$

$$J_S^\mu = (S_q q_\alpha + S_{\bar{P}} \bar{P}_\alpha) d^{\mu\nu\alpha\beta} q_\beta \Delta_\nu$$

$$J_{PS}^\mu = F_{PS} \epsilon^{\mu\nu\alpha\beta} q_\nu \bar{P}_\alpha \Delta_\beta$$

$$F_1 = S_q - S_{\bar{P}}$$

$$F_2 = S_{\bar{P}}$$

$$F_1(Q^2, t, x) \qquad F_2(Q^2, t, x)$$

$$J_S^\mu = F_1(q^2 \Delta^\mu - q^\mu q \cdot \Delta) + F_2[(\bar{P} \cdot q + q^2)\Delta^\mu - (\bar{P}^\mu + q^\mu) q \cdot \Delta]$$

$$q \;\; ; \;\; \bar{P} = P + P' \;\; ; \;\; \Delta = P - P' = q' - q$$

$$\boxed{J^{\mu}_{PS} = F_{PS}\epsilon^{\mu\nu\alpha\beta}q_{\nu}\bar{P}_{\alpha}\Delta_{\beta}}$$

$$\begin{aligned} \mathcal{H}_{\mu\nu} &= J^{\dagger}_{\mu}J_{\nu} \\ &= |F_{PS}|^2\epsilon_{\mu\alpha\beta\gamma}\epsilon_{\nu\alpha'\beta'\gamma'}q^{\alpha}\bar{P}^{\beta}\Delta^{\gamma}q^{\alpha'}\bar{P}^{\beta'}\Delta^{\gamma'} \\ &= \mathcal{H}_{\nu\mu} \end{aligned}$$

$$\boxed{\epsilon^{\mu\nu\alpha\beta}k_{\alpha}k'_{\beta}\mathcal{H}_{\mu\nu} = 0}$$

$$\frac{d\sigma^{PS}_{h=+1} - d\sigma^{PS}_{h=-1}}{d\sigma^{PS}_{h=+1} + d\sigma^{PS}_{h=-1}} = 0$$

Beam-Spin Asymmetry of Exclusive Coherent Electroproduction of the π^0 Off ^{4}He

Frank Thanh Cao, Ph.D.

University of Connecticut, 2019

To understand the partonic structure of nucleons in nuclei, extracting the beam spin asymmetry (BSA) from exclusive processes is an important measurement to get at the so-called Generalized Parton Distributions (GPDs) that describe the partons behavior inside the nucleon. In particular, BSA in Deeply Virtual Meson Production (DVMP) can offer valuable constraints on the transverse GPDs which are not accessible through Deeply Virtual Compton Scattering (DVCS).

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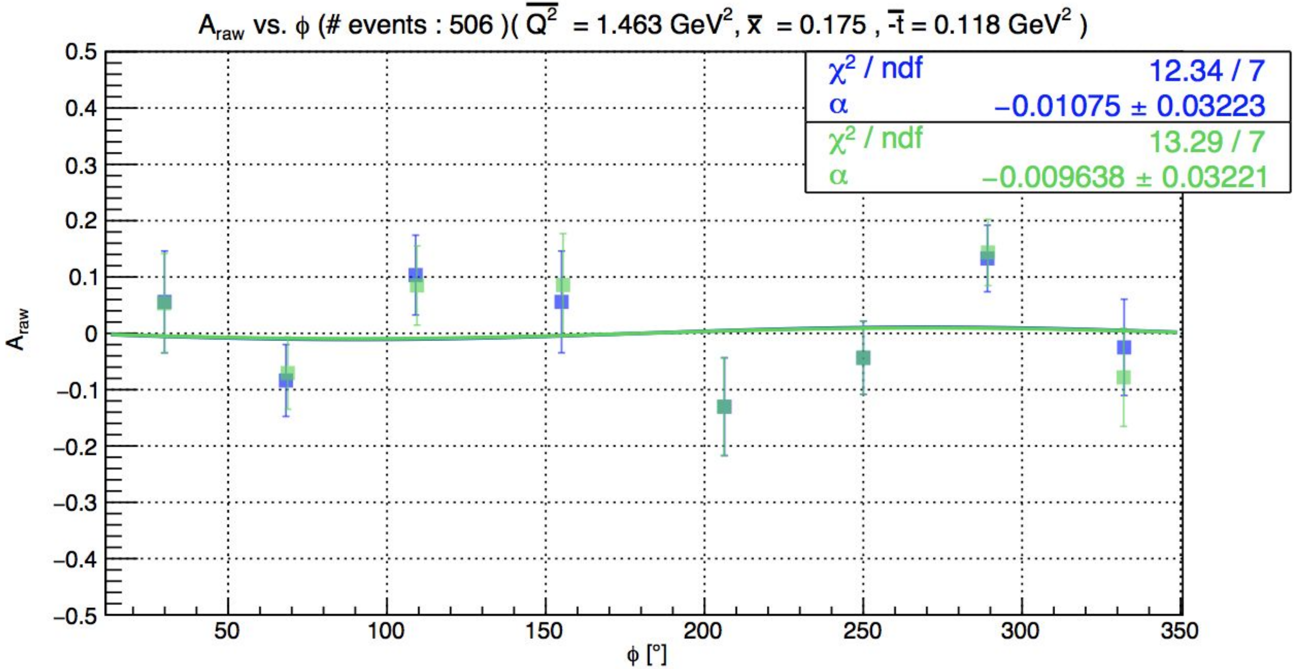
This benchmark measurement is in agreement with symmetry arguments presented in a recent theoretical formulation [2] that offers a framework complementary to that of the GPDs and gives confidence in the assumptions made for future studies of exclusive nuclear processes.



$$A_{LU}\left(\phi\right) = A_{LU}^{90^\circ}\sin\phi$$

$$A_{LU}^{90^\circ} = -1.08 \pm 3.22\,(\text{stat.}) \pm 2.83\,(\text{sys.})\,\%$$

4-26-2019

Beam-Spin Asymmetry of Exclusive Coherent Electroproduction of the π0 Off 4He

Frank Thanh Cao

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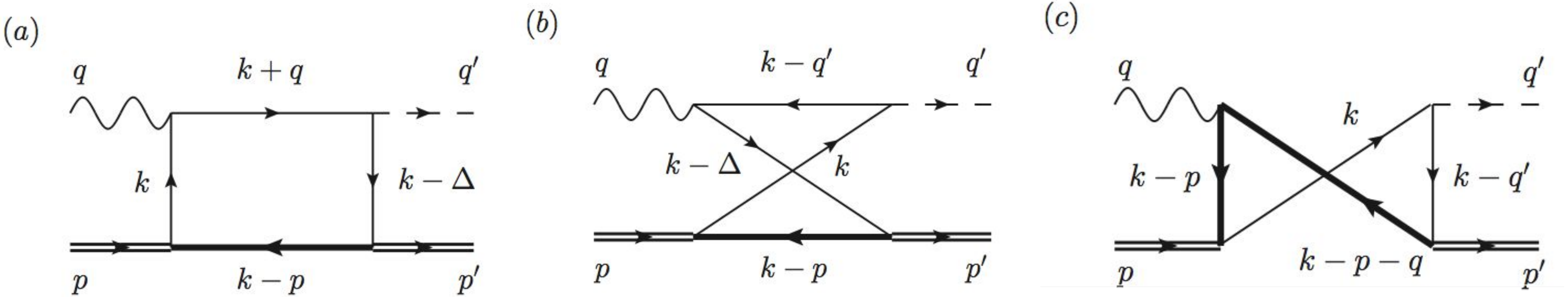
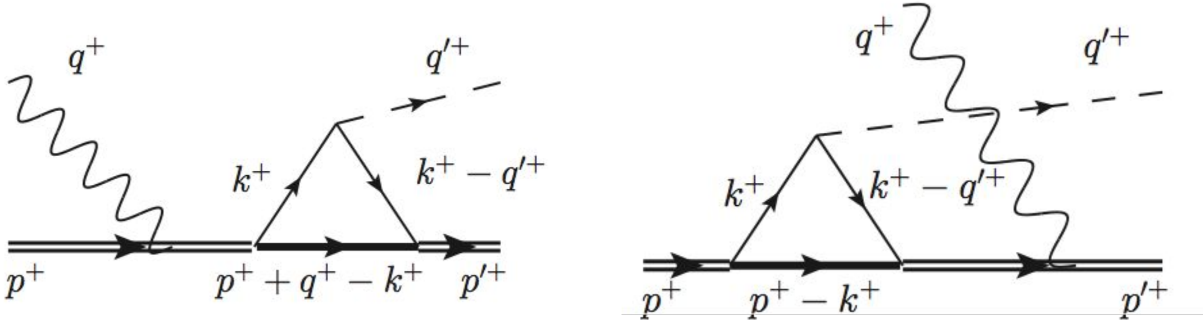
Scalar Field Model Simulation of VMP in Forward Direction

$$\text{“}\gamma^* + {}^4\text{He} \to f_0(980) + {}^4\text{He}\text{”}$$

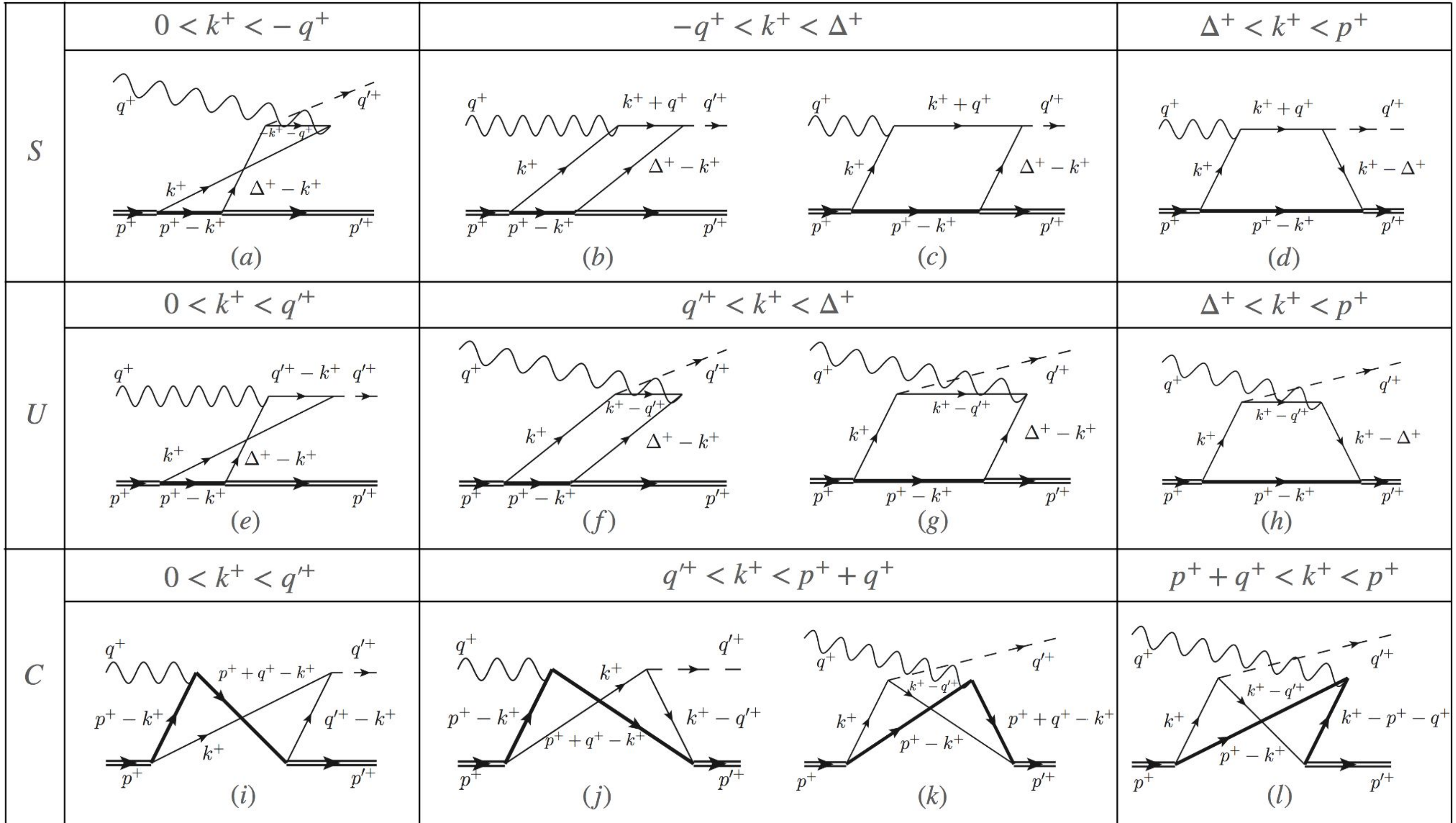
$$\mathcal{M}^{\mu(1+1)}_{\text{tot}} = [(\Delta \cdot q)q^\mu - q^2\Delta^\mu]\mathcal{F}$$



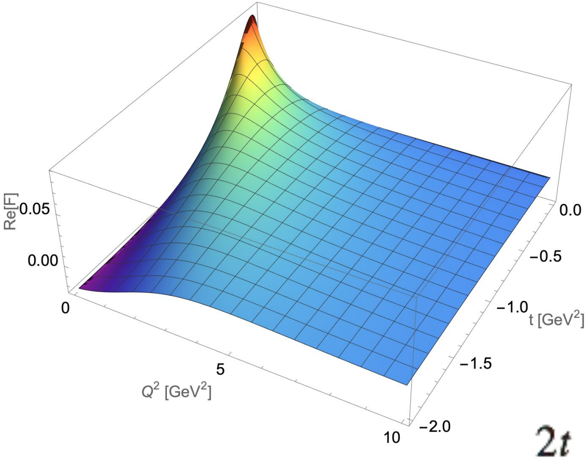
Two more amplitudes for the charged target, but not for the neutral target



Light-Front Time-Ordered Amplitudes

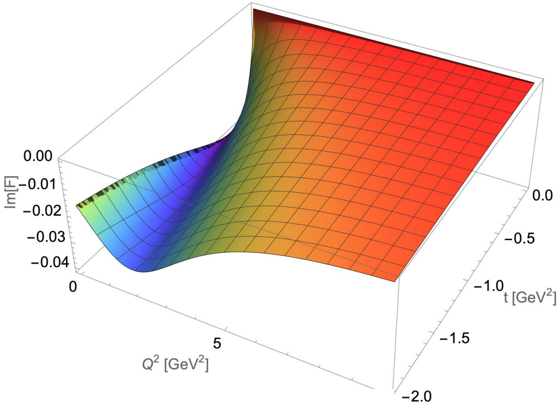


Compton Form Factor



$$\mathcal{M}_{\text{tot}}^{\mu(1+1)} = [(\Delta \cdot q)q^\mu - q^2\Delta^\mu]\mathcal{F}$$

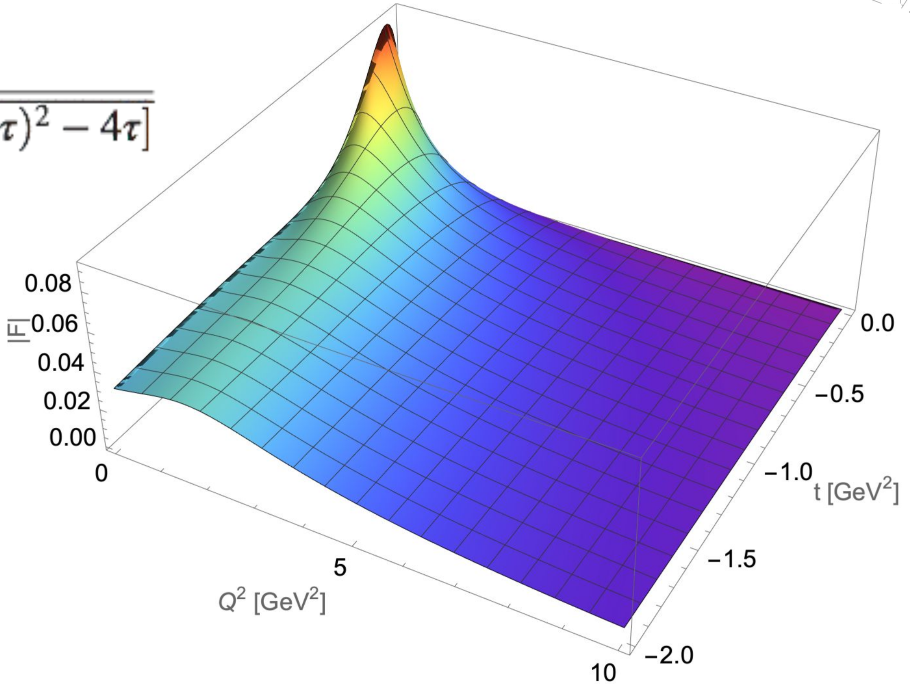
$$\mathcal{F}_c^{\text{VMP}}(Q^2, t) = \frac{\mathcal{M}_{\text{charged}}^\mu}{(\Delta \cdot q)q^\mu - q^2\Delta^\mu}$$

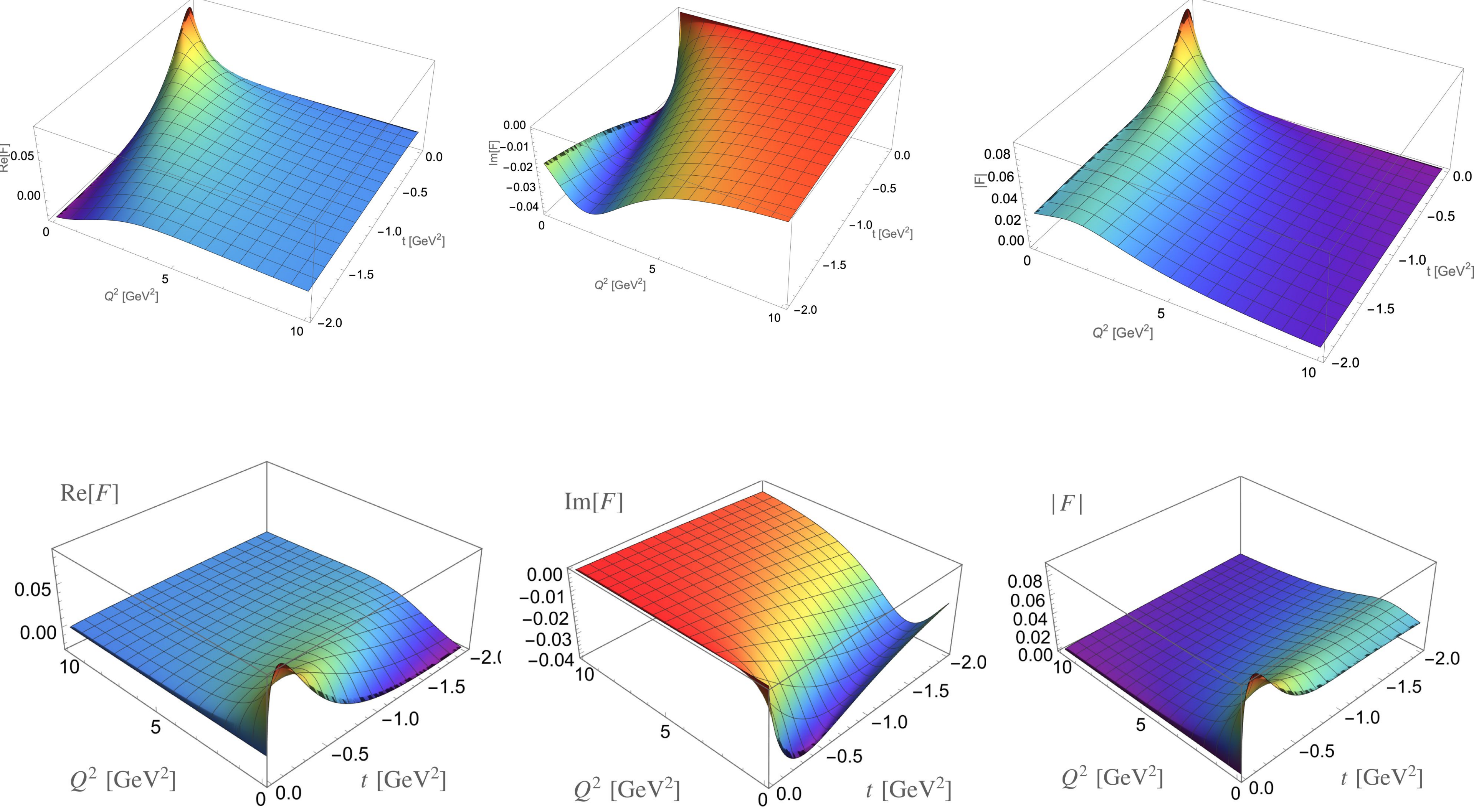


$$x_{\text{Bj}} = \frac{2t}{t(1+\mu_s+\tau) - \sqrt{t(t-4M_T^2)[(1+\mu_s+\tau)^2 - 4\tau]}}$$

$$\zeta = \frac{1}{2M_T^2}\left(t + \sqrt{t^2 - 4tM_T^2}\right)$$

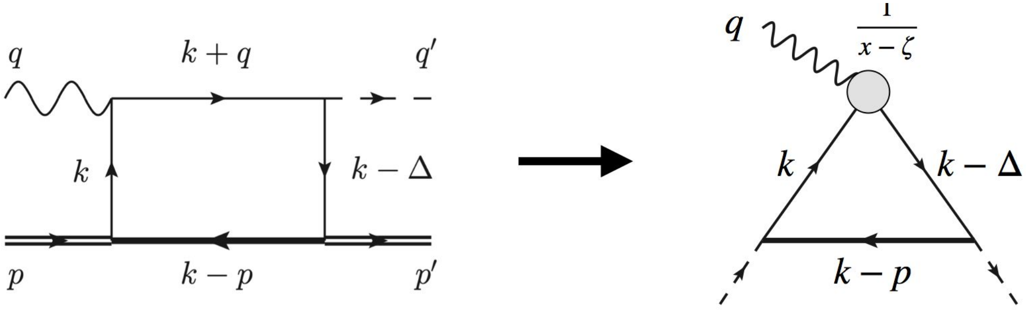
$$\mu_s = M_S^2/Q^2 \quad \text{and} \quad \tau = -t/Q^2$$





Re[F]
0.05
0.00
0
5
10
Q^2 [GeV^2]
0.0
−0.5
−1.0
−1.5
−2.0
t [GeV^2]
Im[F]
0.00
−0.01
−0.02
−0.03
−0.04
0
5
10
Q^2 [GeV^2]
0.0
−0.5
−1.0
−1.5
−2.0
t [GeV^2]
|F|
0.08
0.06
0.04
0.02
0.00
0
5
10
Q^2 [GeV^2]
0.0
−0.5
−1.0
−1.5
−2.0
t [GeV^2]
Re[F]
0.05
0.00
10
5
0
Q^2 [GeV^2]
0.0
−0.5
−1.0
−1.5
t [GeV^2]
Im[F]
0.00
−0.01
−0.02
−0.03
−0.04
10
5
0
Q^2 [GeV^2]
0.0
−0.5
−1.0
−1.5
−2.0
t [GeV^2]
|F|
0.08
0.06
0.04
0.02
0.00
10
5
0
Q^2 [GeV^2]
0.0
−0.5
−1.0
−1.5
−2.0
t [GeV^2]

DVMP Reduction to GPD in S-channel with + Current



$$\mathcal{M}_s^\mu \sim \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2 - m^2} \boxed{\frac{2k^\mu + q^\mu}{(k+q)^2 - m^2}} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2}$$

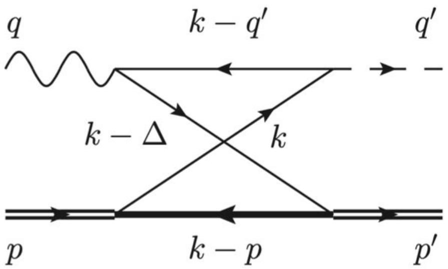
For a plus current of a virtual photon,

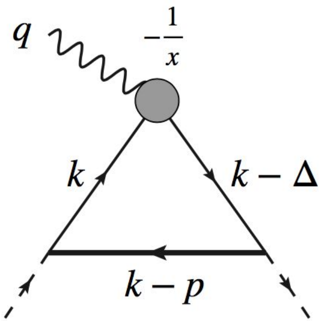
$$\boxed{\frac{2k^+ + q^+}{(k^+ + q^+)(k^- - k_t^-)}} \frac{1}{k^2 - m^2} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2}$$ **where** $$k_t^- = -q^- + \frac{m_{Q_1}^2}{k^+ + q^+} - i\frac{\epsilon}{k^+ + q^+}$$

For large Q^2 : $$\boxed{} \simeq \frac{1}{(x-\zeta)} \frac{\zeta'}{Q^2} (2x - \zeta) + \mathcal{O}\left(\frac{1}{Q^4}\right)$$

$$= \boxed{\frac{1}{x-\zeta}} \frac{\zeta'}{Q^2} \boxed{\frac{1}{k^2 - m^2} \frac{2k^+ - \Delta^+}{p^+} \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2}}$$

DVMP Reduction to GPD in U-channel with + Current





$$\mathscr{M}_u^\mu \sim \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2-m^2} \boxed{\frac{2k^\mu - \Delta^\mu - q'^\mu}{(k-q')^2-m^2}} \frac{1}{(k-\Delta)^2-m^2} \frac{1}{(k-p)^2-M^2}$$

For a plus current of a virtual photon,

$$\boxed{\frac{2k^+ - \Delta^+ - q'^+}{(k^+-q'^+)(k^- - k_u^-)}} \frac{1}{k^2-m^2} \frac{1}{(k-\Delta)^2-m^2} \frac{1}{(k-p)^2-M^2}$$ **where** $$k_u^- = q'^- + \frac{m_{Q_1}^2}{k^+-q'^+} - i\frac{\epsilon}{k^+-q'^+}$$

For large Q^2 : $\boxed{} \simeq -\frac{1}{x}\frac{\zeta'}{Q^2}(2x-\zeta) + \mathcal{O}\left(\frac{1}{Q^4}\right)$

$$= \boxed{-\frac{1}{x}} \frac{\zeta'}{Q^2} \boxed{\frac{1}{k^2-m^2} \frac{2k^+ - \Delta^+}{p^+} \frac{1}{(k-\Delta)^2-m^2} \frac{1}{(k-p)^2-M^2}}$$

$$\mathscr{M}^{+}_{Leading} = \frac{1}{4\pi}\,\frac{\zeta'}{Q^2}\left(\frac{1}{x-\zeta} - \frac{1}{x}\right)\frac{1}{k^2 - m^2}\,\frac{2k^+ - \Delta^+}{p^+}\,\frac{1}{(k-\Delta)^2 - m^2}\,\frac{1}{(k-p)^2 - M^2}$$

$$A^{+} = (\Delta\cdot q)q^{+} - q^2\Delta^{+} = \frac{1}{2}Q^2\zeta p^{+}\left[1 + \frac{t}{Q^2} + \cdots\right]$$

$$\frac{\mathscr{M}^{+}_{Leading}}{A^{+}_{Leading}} = \frac{1}{2\pi}\,\frac{1}{Q^4}\left(\frac{1}{x-\zeta} - \frac{1}{x}\right)\frac{1}{k^2 - m^2}\,\frac{2k^+ - \Delta^+}{p^+}\,\frac{1}{(k-\Delta)^2 - m^2}\,\frac{1}{(k-p)^2 - M^2}$$

DVMP Reduction to GPD with - Current works as well.

$$\frac{\mathscr{M}^{-}_{Leading}}{A^{-}_{Leading}} = \frac{1}{2\pi}\,\frac{1}{Q^4}\left(\frac{1}{x-\zeta} - \frac{1}{x}\right)(2x-\zeta)\,\frac{1}{k^2 - m^2}\,\frac{1}{(k-\Delta)^2 - m^2}\,\frac{1}{(k-p)^2 - M^2}$$

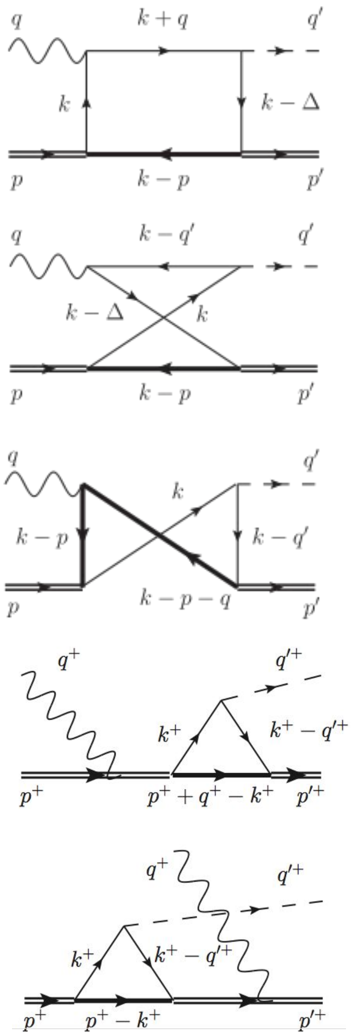
$$M^{\mu} = A^{\mu} F$$

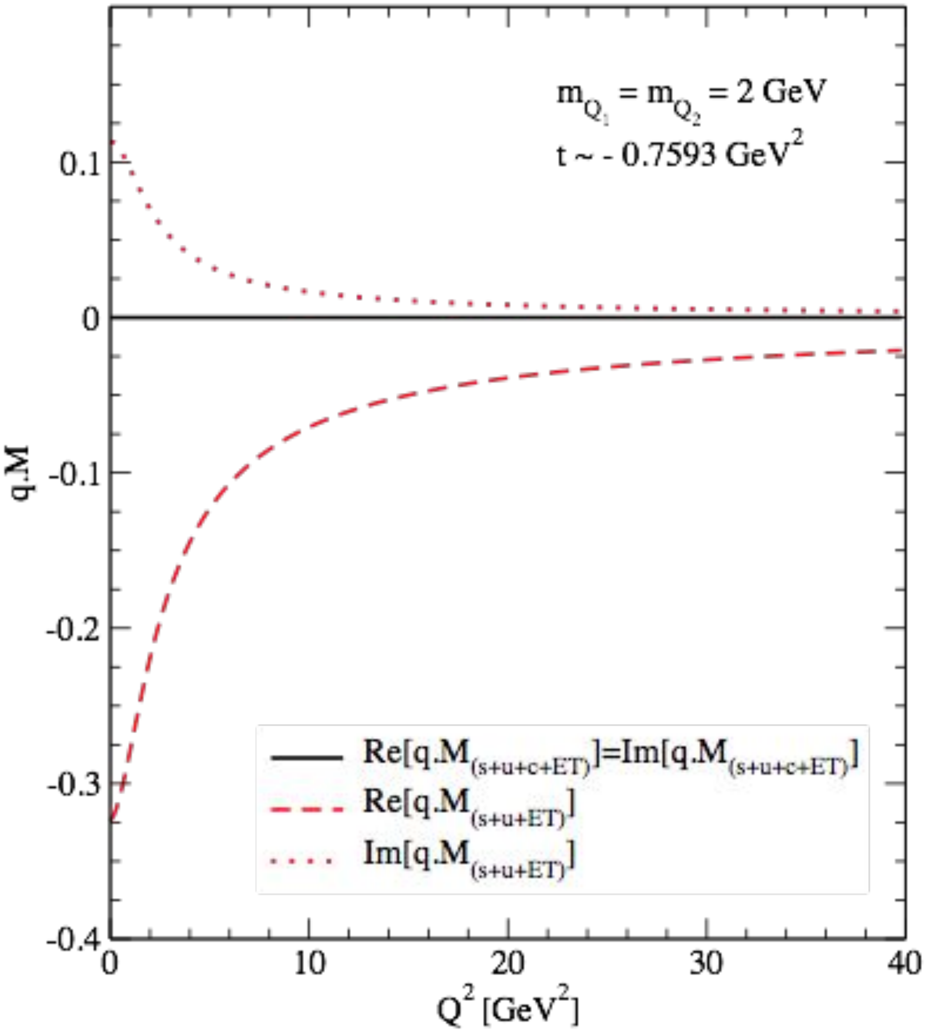
$$\mathcal{M}^{\mu}_{Leading} \sim \frac{q^{\mu} - 2q'^{\mu}}{q^2} \left(\frac{1}{x-\zeta} - \frac{1}{x} \right) \frac{1}{k^2 - m^2} (2x - \zeta) \frac{1}{(k-\Delta)^2 - m^2} \frac{1}{(k-p)^2 - M^2}$$

$$(\Delta \cdot q)\, q^{\mu} - q^2\, \Delta^{\mu} \longrightarrow \frac{Q^2}{2} (2\, q'^{\mu} - q^{\mu})$$

Gauge Invariance works asymptotically:

$$q' \cdot q \rightarrow q^2 / 2$$

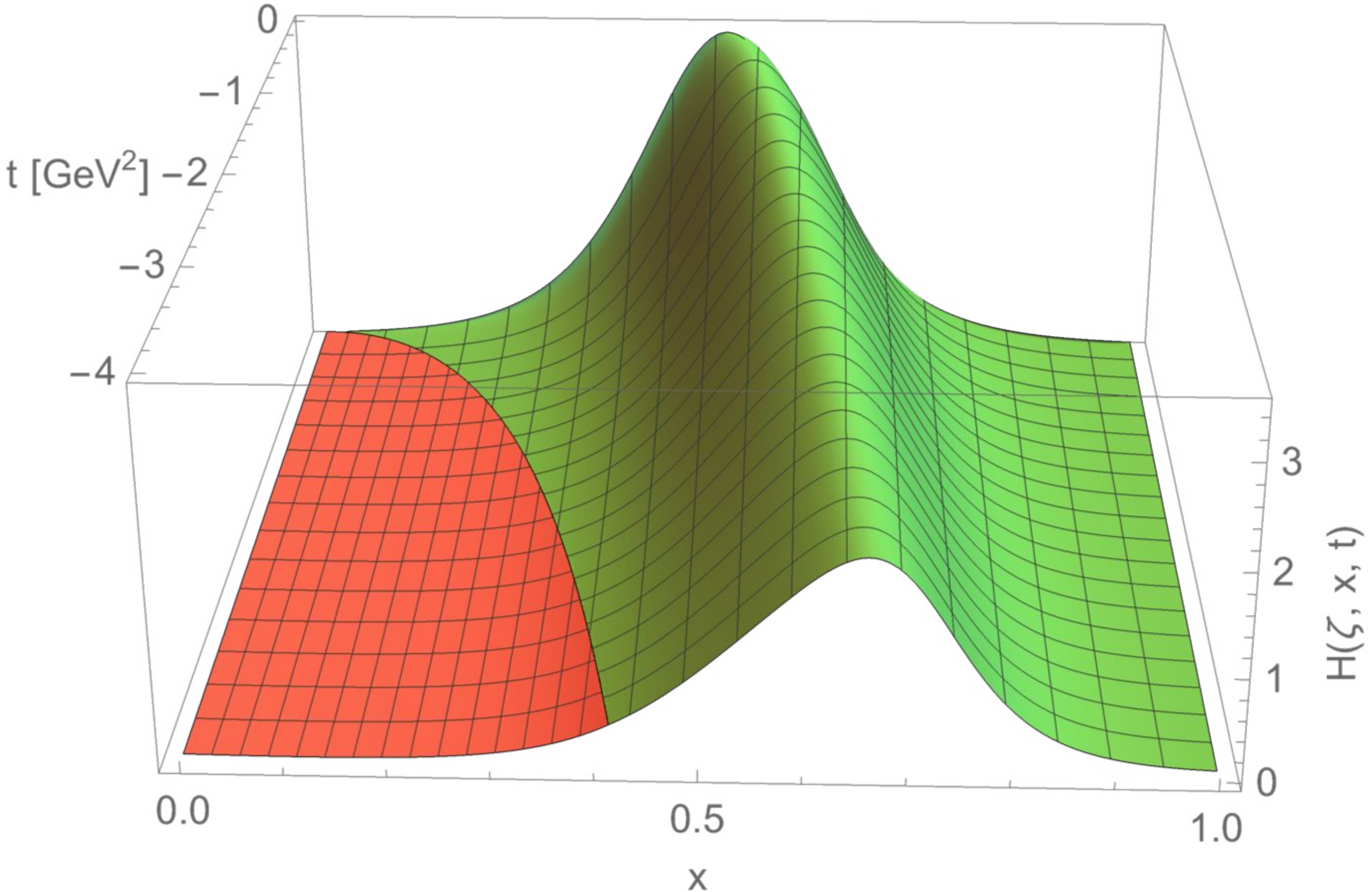




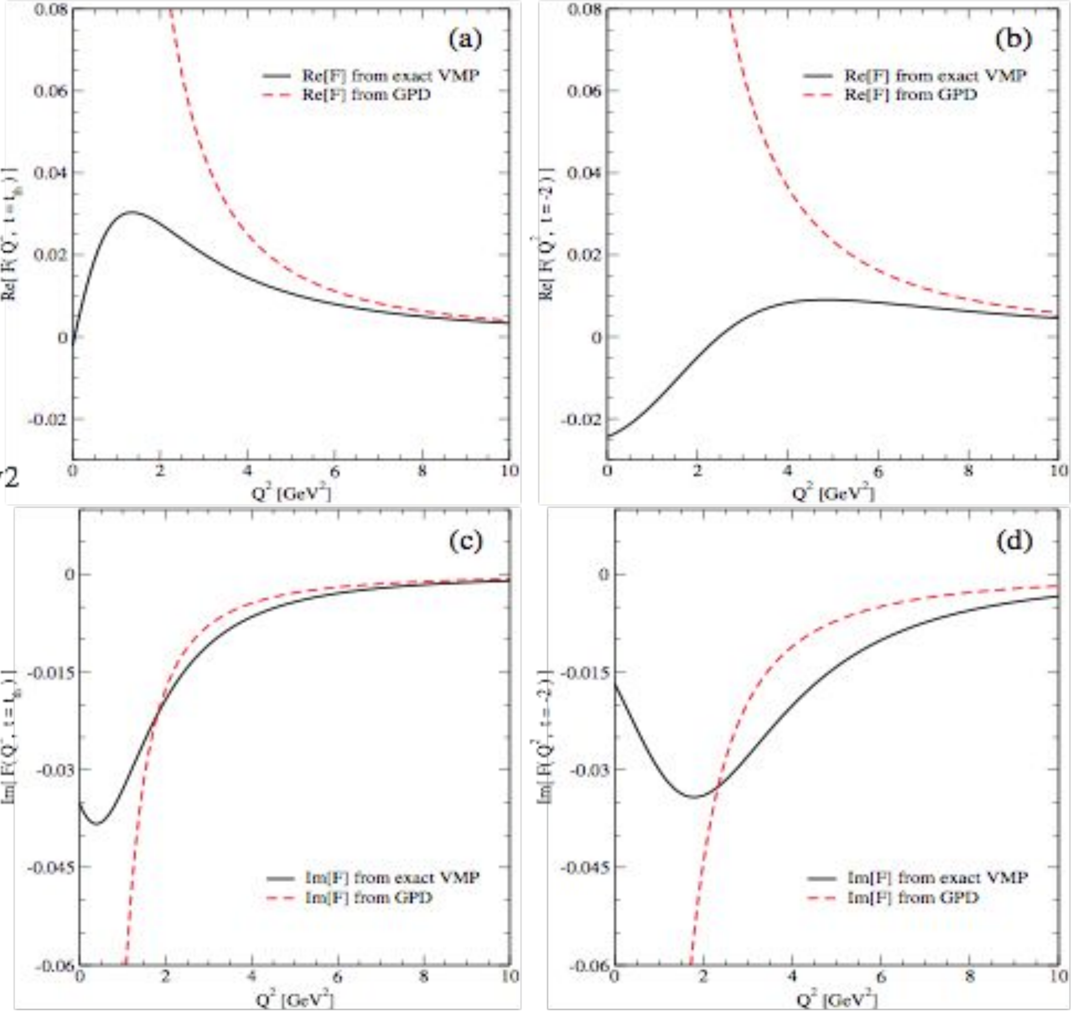
$$(\Delta \cdot q)\, q^\mu - q^2\, \Delta^\mu$$

$$\frac{Q^2}{2}\,(2\, q'^\mu - q^\mu)$$

$$\mathcal{M}^{\pm\text{DVMP}}_{s+u} = \frac{e_{Q_1}\zeta}{4\pi Q^2}\int_0^1 dx\left(\frac{1}{x-\zeta}-\frac{1}{x}\right)H(\zeta,x,t)$$



$t = t_{\rm th} \simeq -0.7593\,\mathrm{GeV}^2$

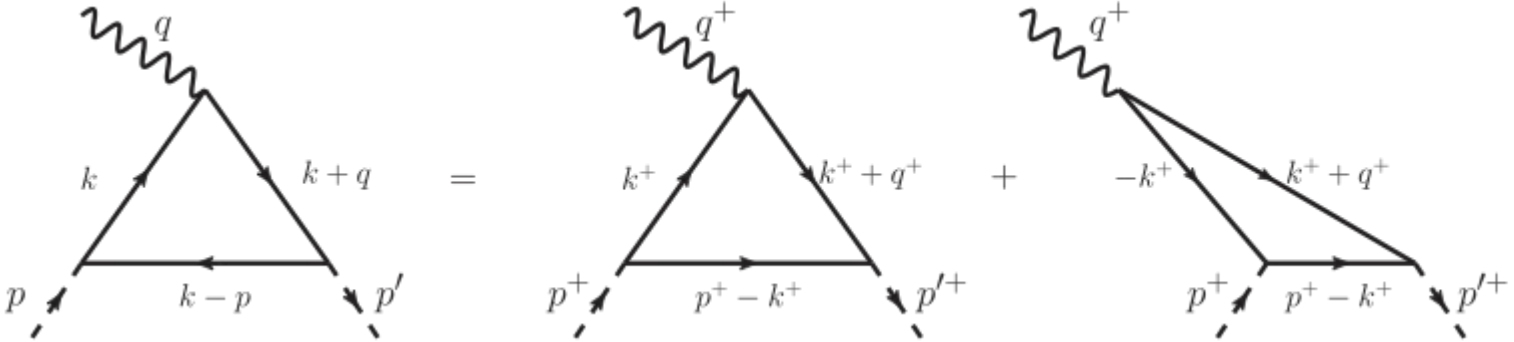


$t = -2\,\mathrm{GeV}^2$

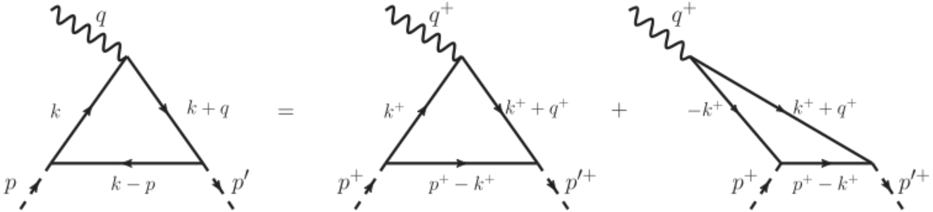
$-t/Q^2 \lesssim 0.1$

$$F_{\mathcal{M}}(t) = \int_0^1 \frac{dx}{1-\zeta/2} H(\zeta, x, t).$$

$$H(\zeta, x, t) = \begin{cases} H_{\mathrm{ERBL}}(\zeta, x, t), & \text{for } 0 \le x \le \zeta, \\ H_{\mathrm{DGLAP}}(\zeta, x, t), & \text{for } \zeta \le x \le 1 \end{cases}$$



$$J_S^{\mu}(0) = (p + p')^{\mu} F_{\mathcal{M}}^{S}(q^2)$$


q
k
k + q
p
k − p
p′
=
q+
k+
k+ + q+
p+
p+ − k+
p′+
+
q+
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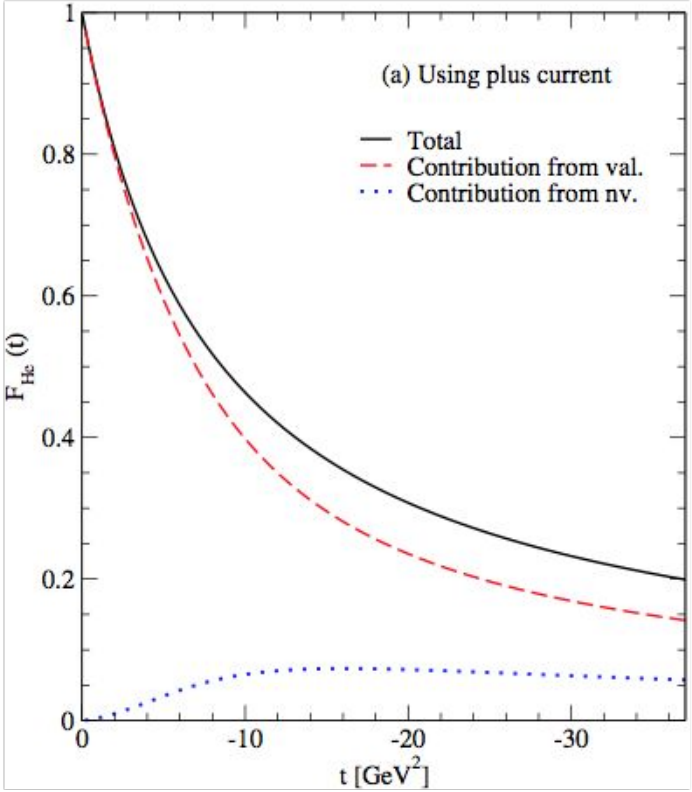
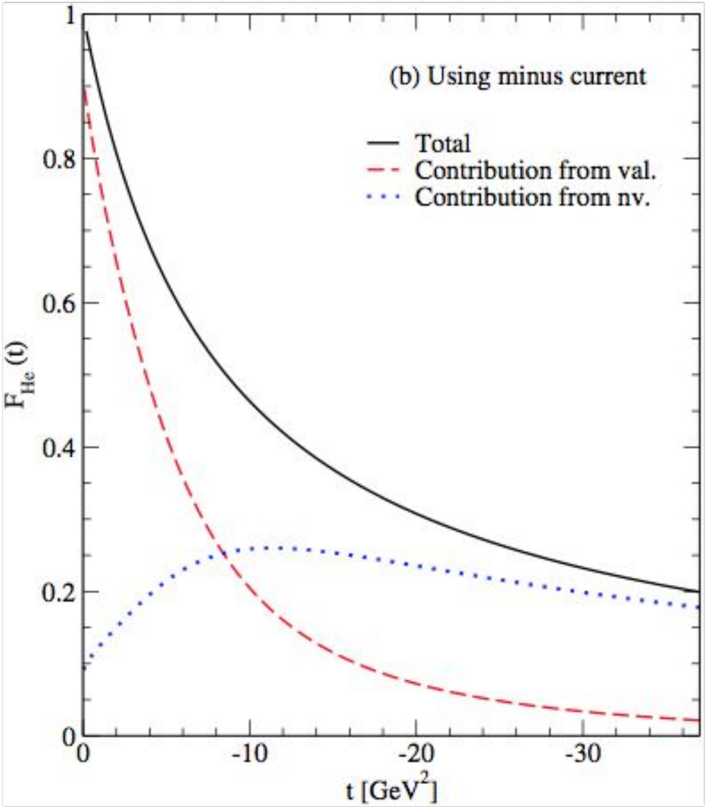
k+ + q+
p+
p+ − k+
p′+


(a) Using plus current
Total
Contribution from val.
Contribution from nv.
F_He (t)
t [GeV^2]
1
0.8
0.6
0.4
0.2
0
0
-10
-20
-30


(b) Using minus current
Total
Contribution from val.
Contribution from nv.
F_He (t)
t [GeV^2]
1
0.8
0.6
0.4
0.2
0
0
-10
-20
-30

Decomposition of the Form Factor

$$\sum_{i=V,NV} J_i^\mu(t) = (P+P')^\mu \sum_{i=V,NV} F_i(t) = (2P^\mu - \Delta^\mu) \sum_{i=V,NV} F_i(t)$$

$$J_V^\mu(t) = \int_{\Delta^+}^{P^+} dk^+ \int dk^- \frac{2k^\mu - \Delta^\mu}{D_V}$$

$$J_{NV}^\mu(t) = \int_0^{\Delta^+} dk^+ \int dk^- \frac{2k^\mu - \Delta^\mu}{D_{NV}}$$

$$\frac{2k^\mu - \Delta^\mu}{2P^\mu - \Delta^\mu} = \frac{2x - \zeta}{2 - \zeta} \quad only \quad if \quad \Delta^\mu = \zeta P^\mu \quad or \quad q^\mu = q'^\mu + \zeta P^\mu$$

Note here that $(q-q')^2 = \Delta^2 = t = \zeta^2 M^2 > 0$

while $t < 0$ in DVMP.

Conclusion and Outlook

- The advantage of LFD is maximized in 1+1D.
- GPD formulation is most applicable in the forward direction.
- LFD application in 1+1D provides a good benchmark analysis of the GPD application.
- Unless small $|t|/Q^2$, "Cat's ears" contribution should not be neglected.
- Sum rule correspondence between DGLAP/ERBL GPDs and Valence/Nonvalence contributions to the form factor works only for a certain current component.
- Form factor decomposition depends on the current component although the form factor itself is independent of the choice of the current component.(Democracy in current components)
- Application to the energy-momentum tensor decomposition appears feasible.

Conclusion and Outlook

- 3+1 D extension with BSA investigation is underway.
- Interpolation between IFD and LFD of 1+1D QCD was successful and 3+1D QCD extension looks also feasible.