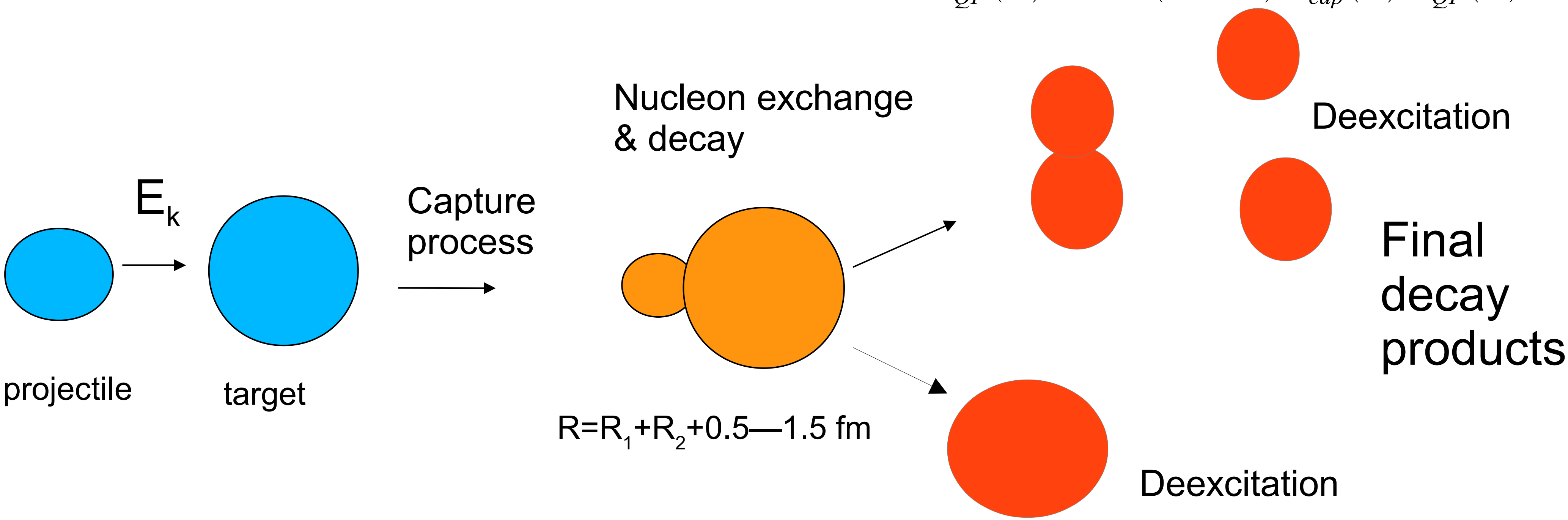
Excitations functions of evaporation residues in heavy ion reactions leading to compound nuclei with Z=80-90

Sh. A. Kalandarov, G.G. Adamian, N.V. Antonenko

Bogolyubov Lab. of Theoretical Physics
Joint Institute for Nuclear Research
Dubna, Russia

Dinuclear system conception

$$\sigma_{QF}(J) = \pi \bar{\lambda}^2 (2J+1) P_{cap}(J) P_{QF}(J)$$



$$\sigma_{ER}(J) = \pi \bar{\lambda}^2 (2J+1) P_{cap}(J) P_{CN}(J) W^{sur}(J)$$

E*~10-100 MeV
Ek~ up to 10-15 MeV/nucleon

Reaction stages

$$\sigma_{ER}(J) = \pi \lambda\!\!\!^{-2} (2J+1) P_{cap}(J) P_{CN}(J) W^{sur}(J)$$

I. Capture
II. Nucleon exchange(fusion) & decay(QF)
III. Survival probability of CN

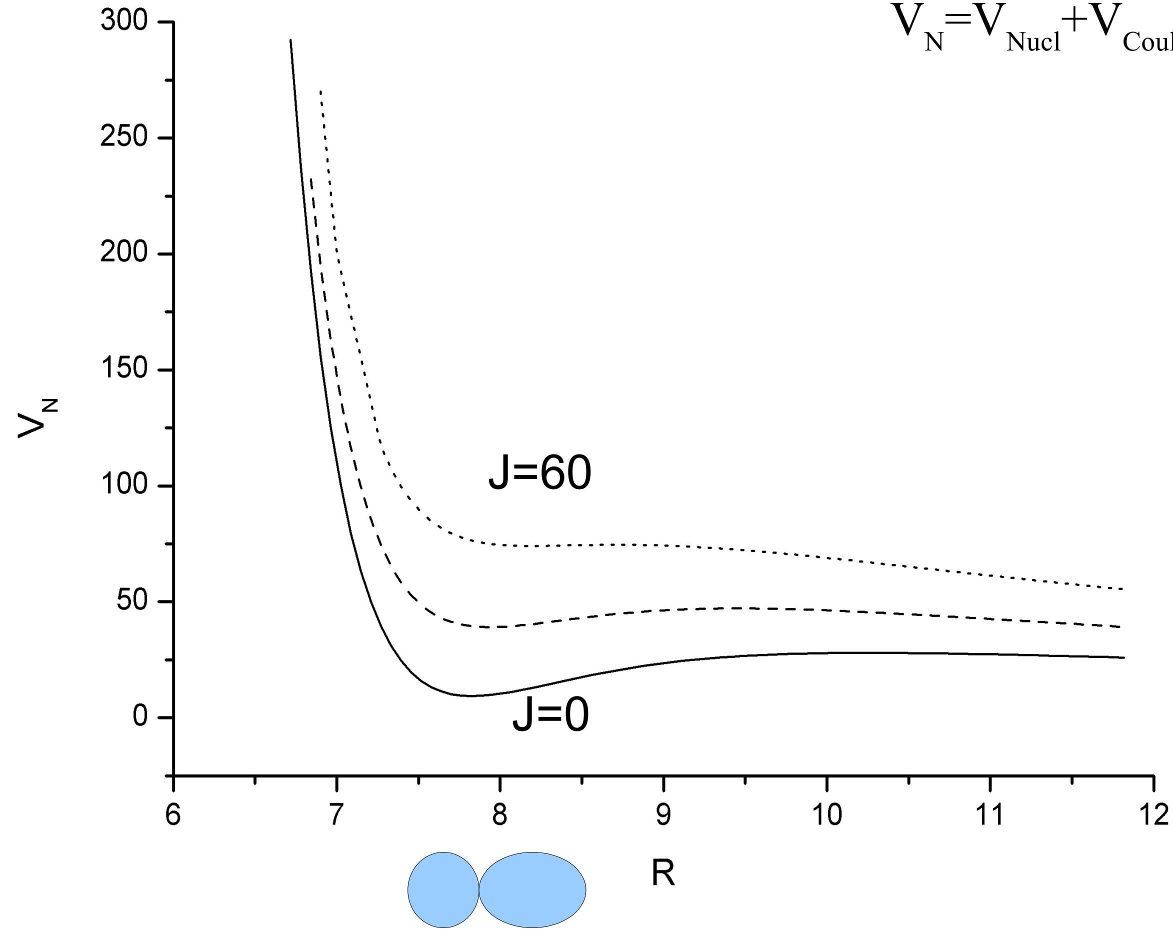
Capture

«transition through barrier» approach

$$P_{cap}(E_{c.m.}, J) = \frac{1}{1 + e^{2\pi (V_b(J) - E_{c.m.})/h\omega(J)}}$$

with $J = 0 - J_{max}$, where $J_{max} = min(J_{kin}, J_{cr})$

$$V_N = V_{Nucl} + V_{Coul} + V_{rot}$$



$$R_{min} = R_1 + R_2 + 0.5\text{—}1.5 \text{ fm}$$

Quasiclassical dynamical approach

$$\frac{d(\mu(R,\alpha_1,\alpha_2)\dot{R})}{dt} + \gamma_R(R,\alpha_1,\alpha_2)\dot{R}(t) = F(R),$$

$$F(R,\alpha_1,\alpha_2) = -\frac{\partial V(R,\alpha_1,\alpha_2)}{\partial R} - \dot{\mathbf{R}}^2\frac{\partial \mu(R)}{\partial R},$$

$$\frac{dL}{dt} = \gamma_\theta(R,\alpha_1,\alpha_2)R(t)\big(\dot{\theta}R(t) - \dot{\theta}_1 R_{1\mathrm{eff}} - \dot{\theta}_2 R_{2\mathrm{eff}}\big),$$

$$L_0 = J_R(R,\alpha_1,\alpha_2)\dot{\theta} + J_1\dot{\theta}_1 + J_2\dot{\theta}_2,$$

$$E_{\mathrm{rot}} = \frac{J_R(R,\alpha_1,\alpha_2)\dot{\theta}^2}{2} + \frac{J_1\dot{\theta}_1^2}{2} + \frac{J_2\dot{\theta}_2^2}{2},$$

Nasirov et. al., Nuclear Physics A 759 (2005) 342–369

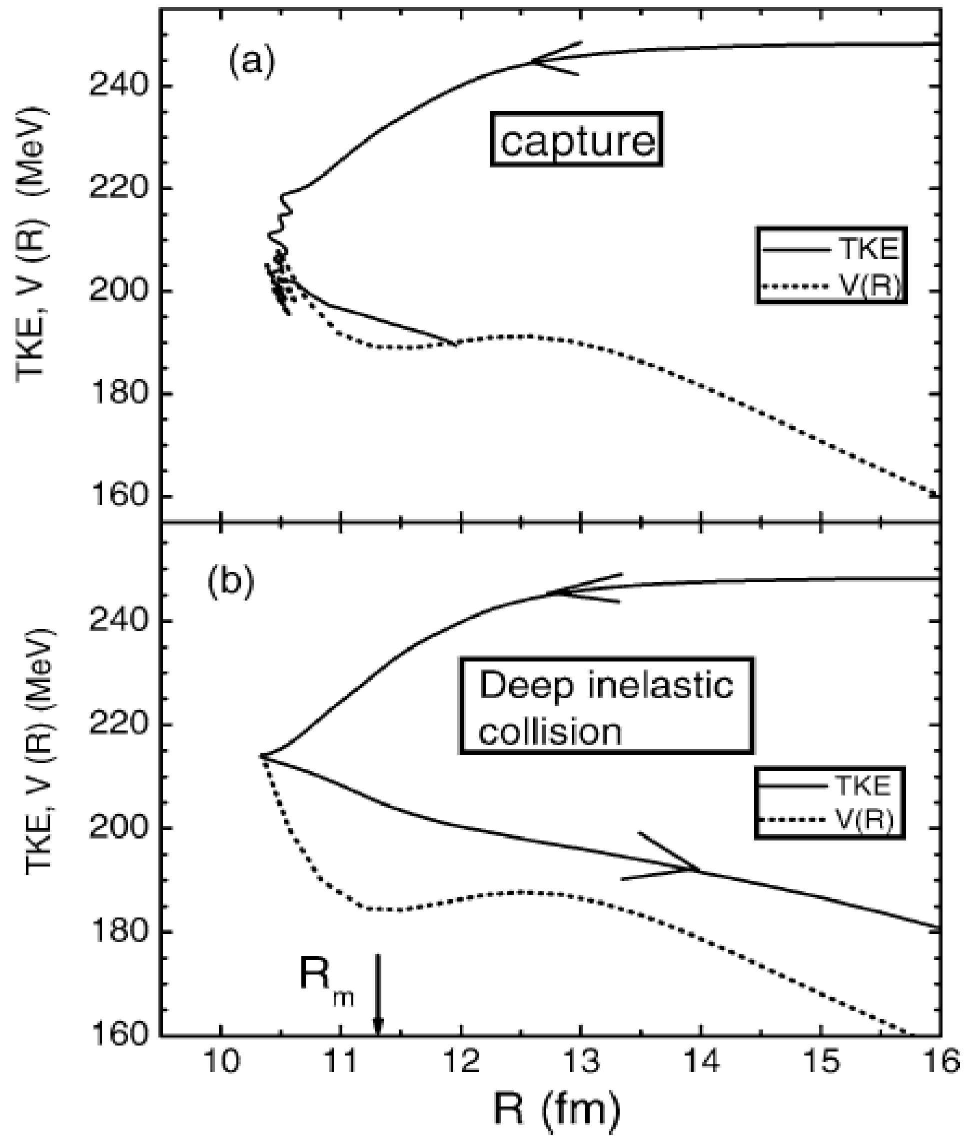


Fig. 2. Illustration of capture (a) and deep inelastic collision (b) at heavy ion collisions. The kinetic energy of the relative motion and the part of nucleus–nucleus potential are shown by solid and dotted curves, respectively.

Potential energy of DNS

$$U(R,Z,A,J) = B_1 + B_2 + V(R,Z,A,\beta_1,\beta_2,J) - [B_{12} + E_{12}^{rot}(J)],$$

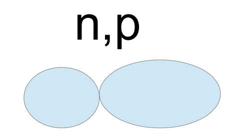
$$V(R,Z,A,\beta_1,\beta_2,J) = V_C(R,Z,A,\beta_1,\beta_2) + V_N(R,Z,A,\beta_1,\beta_2) + \frac{\hbar^2 J(J+1)}{2\Im(R,A,\beta_1,\beta_2)}$$

$$V_N = \int \rho_1(\mathbf{r_1})\rho_2(\mathbf{R}-\mathbf{r_2})F(\mathbf{r_1}-\mathbf{r_2})d\mathbf{r_1}d\mathbf{r_2},$$

where $F(\mathbf{r_1}-\mathbf{r_2}) = C_0[F_{\text{in}}\frac{\rho_0(\mathbf{r_1})}{\rho_{00}} + F_{\text{ex}}(1-\frac{\rho_0(\mathbf{r_1})}{\rho_{00}})]\delta(\mathbf{r_1}-\mathbf{r_2})$ is the Skyrme-type density-depending effective nucleon-nucleon interaction, which is known from the theory of finite Fermi systems [28], and $\rho_0(\mathbf{r}) = \rho_1(\mathbf{r}) + \rho_2(\mathbf{R}-\mathbf{r})$, $F_{\text{in,ex}} = f_{\text{in,ex}} + f'_{\text{in,ex}}\frac{(N-Z)(N_2-Z_2)}{(N+Z)(N_2+Z_2)}$. Here, $\rho_1(\mathbf{r_1})$ and $\rho_2(\mathbf{r_2})$, and N_2 (Z_2) are the nucleon densities of, respectively, the light and the heavy nuclei of the DNS, and neutron (charge) number of the heavy nucleus of the DNS.

Nucleon exchange(fusion) & decay(QF)

Nucleon exchange and decay of DNS is described within master equation, which is large system of differential equations for given J



$$\frac{dP_{Z_1,A_1,Z_2,A_2}}{dt}=\Delta^{(-,0)}_{Z_1+1,A_1+1,Z_2-1,A_2-1}P_{Z_1+1,A_1+1,Z_2-1,A_2-1}(t)+\Delta^{(+,0)}_{Z_1-1,A_1-1,Z_2+1,A_2+1}P_{Z_1-1,A_1-1,Z_2+1,A_2+1}(t)$$
$$+\Delta^{(0,-)}_{Z_1,A_1+1,Z_2,A_2-1}P_{Z_1,A_1+1,Z_2,A_2-1}(t)+\Delta^{(0,+)}_{Z_1,A_1-1,Z_2,A_2+1}P_{Z_1,A_1-1,Z_2,A_2+1}(t)$$
$$-\left(\Delta^{(-,0)}_{Z_1,A_1,Z_2,A_2}+\Delta^{(+,0)}_{Z_1,A_1,Z_2,A_2}+\Delta^{(0,-)}_{Z_1,A_1,Z_2,A_2}+\Delta^{(0,+)}_{Z_1,A_1,Z_2,A_2}+\Lambda^{d}_{Z_1,A_1,Z_2,A_2}\right)P_{Z_1,A_1,Z_2,A_2}(t)$$

And then the yields for quasifission products: $$Y_{Z_1,A_1,Z_2,A_2}=\Lambda^{d}_{Z_1,A_1,Z_2,A_2}\int_0^t P_{Z_1,A_1,Z_2,A_2}(t)\,dt$$

For CN formation probability: $$P_{CN}=\int_0^t P_{Z_0,A_0,Z_{CN},A_{CN}}(t)\,dt$$

G.G. Adamian et.al., Phys. Rev. C **68**, 034601 (2003)

With the microscopical transport coefficients:

$$\Delta_{Z,N}^{(\pm,0)}(\Theta)=\frac{1}{\Delta t}\sum_{P,T}^{Z}|g_{PT}|^2 n_P^T(\Theta)[1-n_T^P(\Theta)]$$
$$\times\frac{\sin^2[\Delta t(\epsilon_P-\epsilon_T)/2\hbar]}{(\epsilon_P-\epsilon_T)^2/4},$$

$$\Delta_{Z,N}^{(0,\pm)}(\Theta)=\frac{1}{\Delta t}\sum_{P,T}^{N}|g_{PT}|^2 n_P^T(\Theta)[1-n_T^P(\Theta)]$$
$$\times\frac{\sin^2[\Delta t(\epsilon_P-\epsilon_T)/2\hbar]}{(\epsilon_P-\epsilon_T)^2/4},$$

$$g_{PT}(R)=\frac{1}{2}\int d\mathbf{r}\psi_T^*(\mathbf{r})[U_T(\mathbf{r})+U_P(\mathbf{r}-\mathbf{R})]\psi_P(\mathbf{r}-\mathbf{R})$$

$$\Lambda_{Z,N}^{qf}(\Theta)=\sum_n \Lambda_{Z,N}^{qf}(n)\Phi_{Z,N}(n,\Theta),$$

$$\Lambda_{Z,N}^{fis}(\Theta)=\sum_n \Lambda_{Z,N}^{fis}(n)\Phi_{Z,N}(n,\Theta).$$

$$\Lambda_{Z,N}^{qf}(\Theta)=\frac{\omega}{2\pi\omega^{B_{qf}}}\left(\sqrt{\left(\frac{\Gamma}{2\hbar}\right)^2+(\omega^{B_{qf}})^2}-\frac{\Gamma}{2\hbar}\right)$$
$$\times\exp\left(-\frac{B_{qf}(Z,N)}{\Theta(Z,N)}\right),$$

G.G. Adamian et.al., Phys. Rev. C **68**, 034601 (2003)

Or phenomenological transport coefficients:

$$\Delta = 2\pi k \frac{R_1 R_2}{R_1 + R_2} \sqrt{\frac{\rho'_{DNS}}{\rho_{DNS}}}$$

k=0.5x 10^{21} 1/(fm s)

$$\rho_{DNS} = \int_0^{Ex_{DNS}-\epsilon} \int_0^{Ex_{DNS}} \rho_1(Ex_1)\,\rho_2(Ex_{DNS}-\epsilon-Ex_1)\,d\epsilon\,dEx_1$$

$$\Lambda^{d}_{Z_1, A_1, Z_2, A_2} = \frac{\int_0^{Ex_{DNS}-B_d-\epsilon'} \int_0^{Ex_{DNS}-B_d} \rho_1(Ex_1)\,\rho_2(Ex_{DNS}-B_d-\epsilon'-Ex_1)\,d\epsilon'\,dEx_1}{h\int_0^{Ex_{DNS}-\epsilon} \int_0^{Ex_{DNS}} \rho_1(Ex_1)\,\rho_2(Ex_{DNS}-\epsilon-Ex_1)\,d\epsilon\,dEx_1}$$

h=4.1356 10^{-21} MeV s

L.G. Moretto and J.S. Sventek, Physics Letters B 58, 26 (1975)

Survival probability

Survival probability of excited CN depends on competition between evaporation and fission channels.

Particle emission width from excited nucleus for given J:

$$\Gamma_i = \frac{m_i R^2}{\pi \hbar^2 \rho_{CN}(E^*_{CN})} \int_0^{E^*_{CN}-B_i} \rho_{res}(E^*_{CN} - B_i - \epsilon)\epsilon d\epsilon,$$

Fission of excited nucleus for given J:

$$\Gamma_f = \frac{1}{2\pi\rho(E_{ex})} \int_{E_{ex}}^{E_{ex}-B_f} \rho(E_{ex} - B_f - \varepsilon) d\varepsilon$$

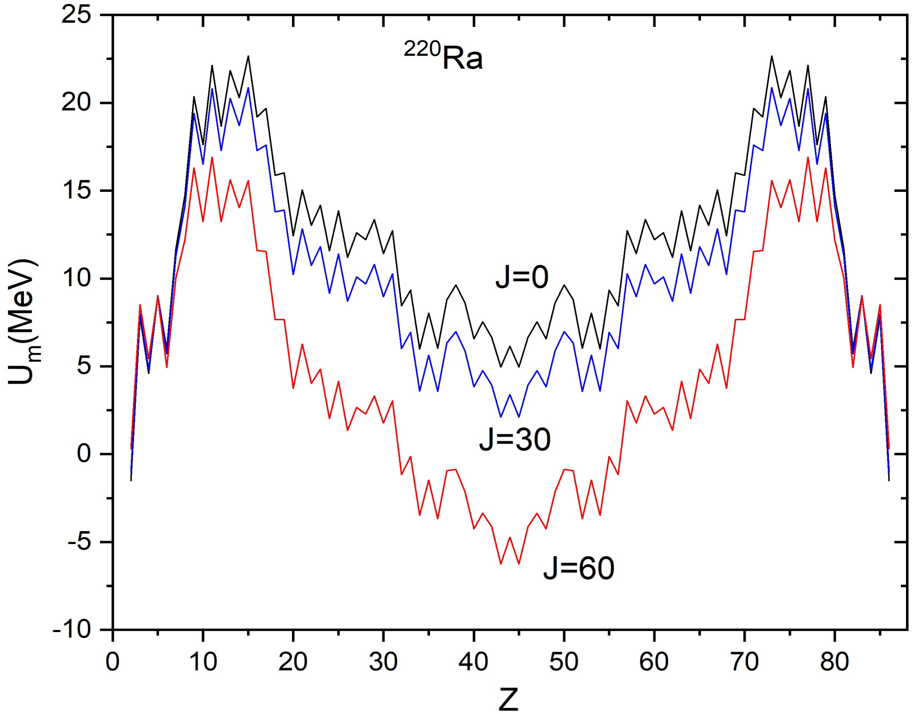
where fission barrier B_f is taken from misroscopic-macroscopic models.

Formation-decay approach

If nucleon exchange time is much shorter then the decay time of DNS, then we can use stationary solution of master equation. In stationary solution of master equation, the probability of finding system in a given charge and mass asymmetry is proportional to the relevant level density.

At high excitation energy limit, system is distributed in CN and DNS configurations as :

$$P_{Z,A}(E^*_{CN},J) = \frac{\exp[-U(R_m,Z,A,J)/T_{CN}(J)]}{1+\sum_{Z'=2,A'}\exp[-U(R_m,Z',A',J)/T_{CN}(J)]}.$$



Excitation energies of CN and DNS:

$$E^*_{\mathrm{CN}}(J) = E_{\mathrm{c.m.}} + Q - E^{\mathrm{rot}}_{12}(J), \qquad E^*_{Z,A}(J) = E^*_{\mathrm{CN}}(J) - U(R_m, Z, A, J).$$

Particle emission width from CN

$$\Gamma_i = \frac{m_i R^2}{\pi \hbar^2 \rho_{CN}(E^*_{CN})} \int_0^{E^*_{CN} - B_i} \rho_{res}(E^*_{CN} - B_i - \epsilon)\epsilon d\epsilon,$$

and decay width of DNS configurations:

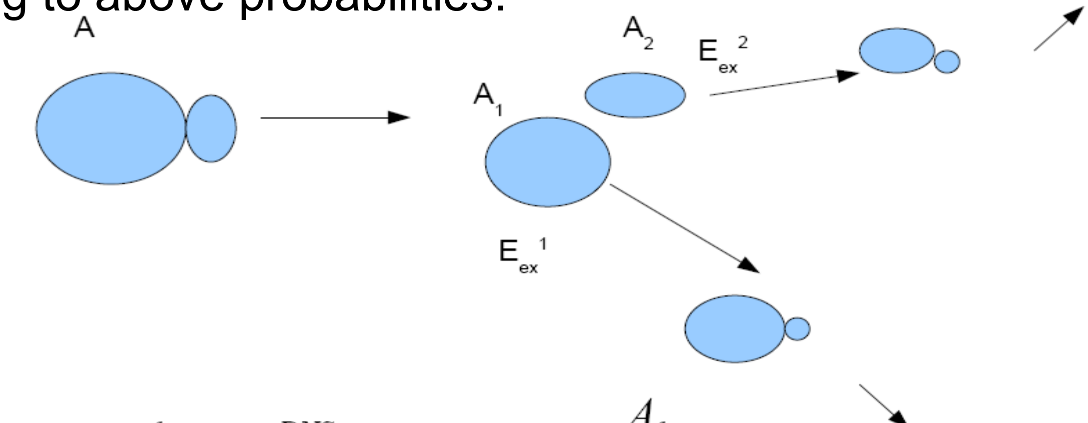
$$\Gamma_{Z,A} = \frac{1}{2\pi \rho_{DNS}} \int_{E_{ex}}^{E_{ex}-B_d} \rho_{DNS}(E*_{Z,A} - B_d(Z,A) - \varepsilon) d\varepsilon$$

$$\rho_{DNS} = \int_0^{Ex_{DNS}-\epsilon} \int_0^{Ex_{DNS}} \rho_1(Ex_1)\rho_2(Ex_{DNS} - \epsilon - Ex_1) d\epsilon \, dEx_1$$

Normalized probabilities for any given decay channels are(at the limit of high excitation energy):

$$W_{Z,A}(E_{CN},J)=\frac{P_{Z,A}P_{Z,A}^{d}}{\sum P_{Z,A}P_{Z,A}^{d}}$$

Cascade decay process of excited system is generated by Monte-Carlo method according to above probabilities:



$$E_{ex}^{1}=(E_{ex}^{DNS}-(U_{b}-U_{min}))\frac{A_{1}}{A}$$

$$E_{ex}^{2}=(E_{ex}^{DNS}-(U_{b}-U_{min}))\frac{A_{2}}{A}$$

Partial formation cross sections for residual nuclei

$$\sigma_{Z,A}(J)=\sigma_{cap}(J)W^{sur}_{Z,A}(J)$$

where $$W^{sur}_{Z,A}(J)=\frac{N_{Z,A}}{N_{it}}$$

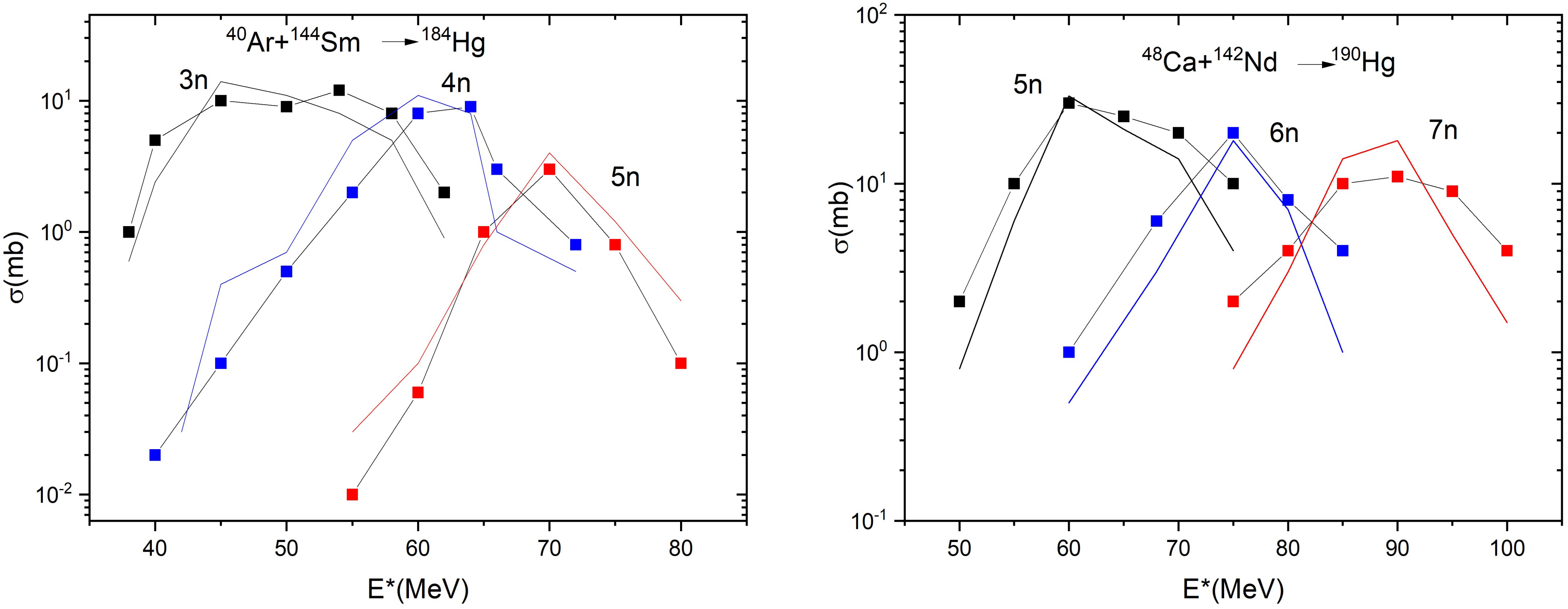
$N_{Z,A}$ — number of given nucleus which is formed in the end of deexcitation cascade, N_{it} - number of iterations

Number of iterations is chosen according to smallest cross sections to be described.

Coupled formation-decay describes evaporation, quasifission and fusion fission channels in a unique way all within DNS model

Results

Excitation functions in xn channels for Hg(Z=80) isotopes



Experimental results are from D. Camas et al., Phys. Rev. C 105, 044612(2022)

Excitation functions in xn channels for Po(Z=84) isotopes

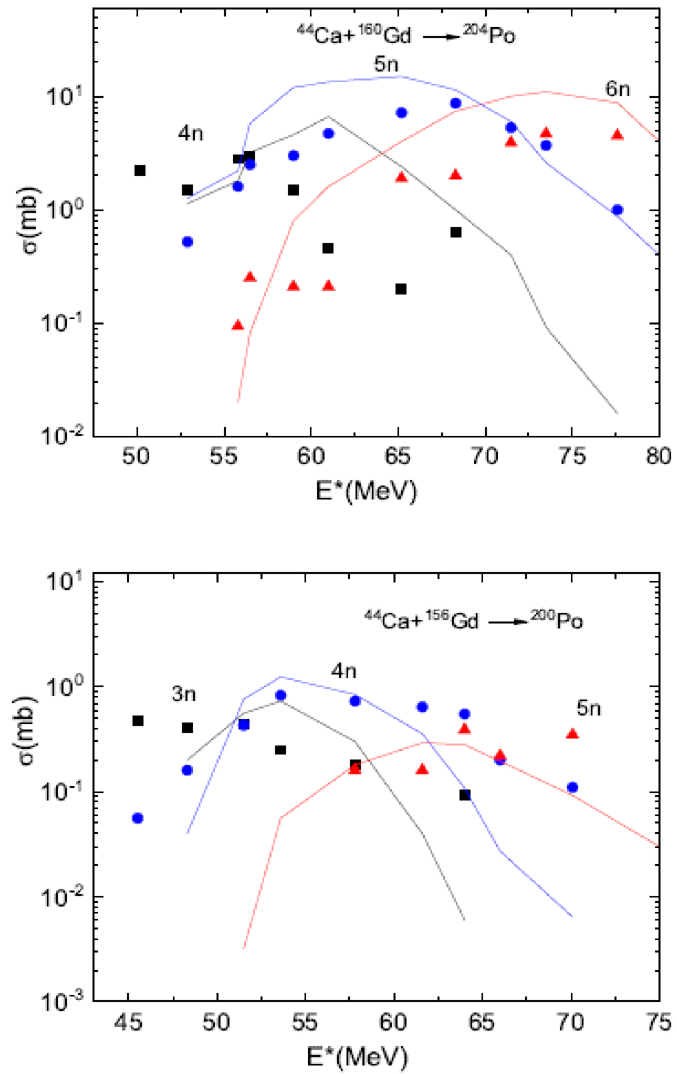


FIG. 5: Comparison of calculated (lines) and experimental (symbols) data for the excitation functions in xn channels of the reactions ^{44}Ca+^{160}Gd and ^{44}Ar+^{156}Gd leading to different isotopes of polonium. The experimental data are taken from Ref. [18].

Excitation functions in pxn and alpha xn channels for Po(Z=84) isotopes

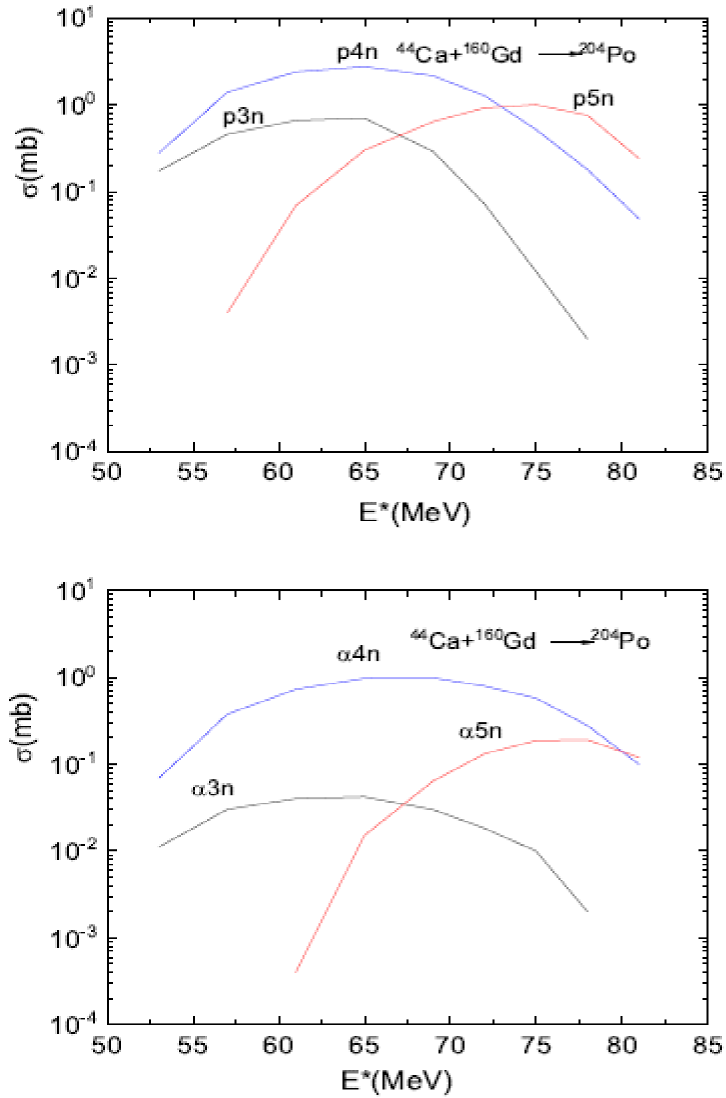
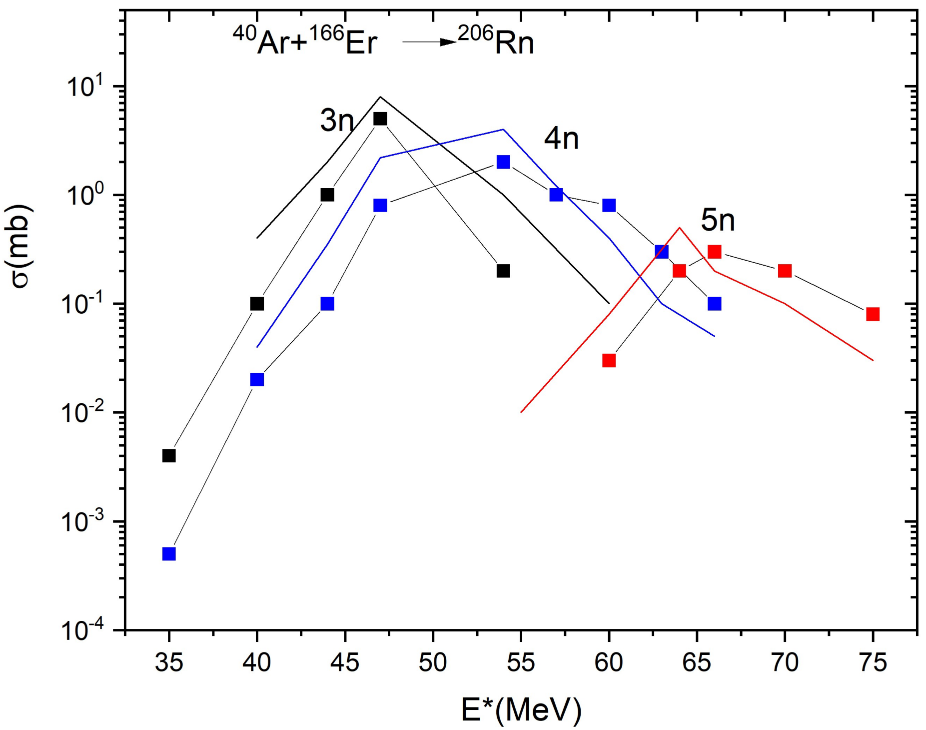
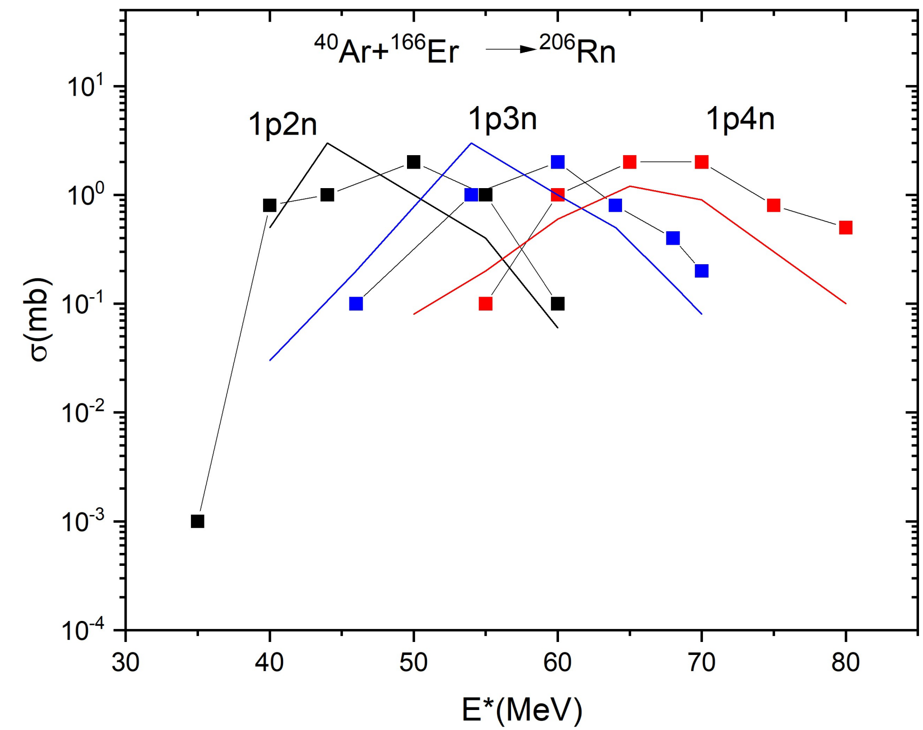


FIG. 6: Calculated excitation functions in pxn and αxn channels of the reactions ^{44}Ca+^{160}Gd and ^{44}Ar+^{156}Gd leading to different isotopes of polonium as CN.

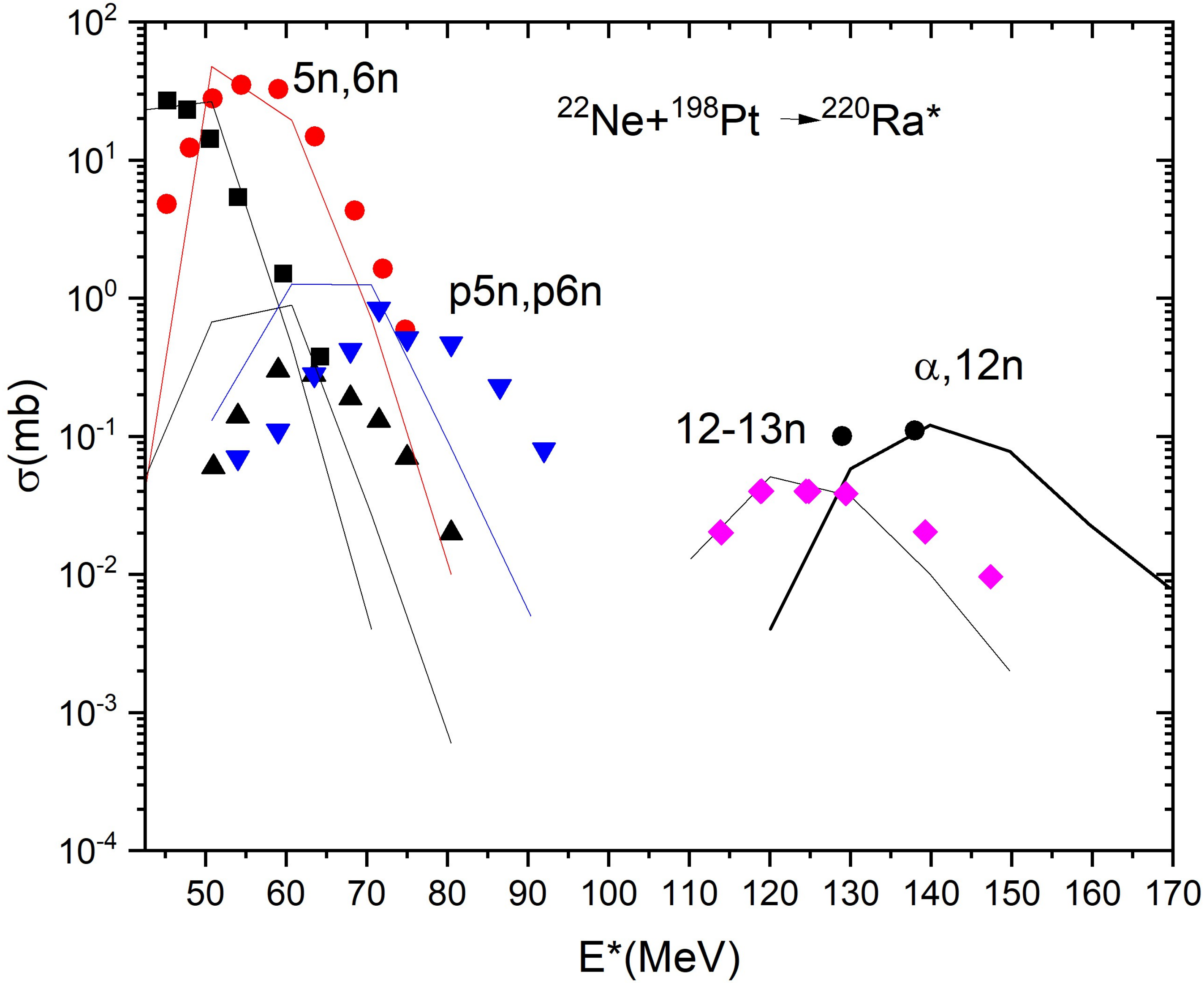
Excitation functions in xn and pxn channels for $^{206}Rn(Z=86)$





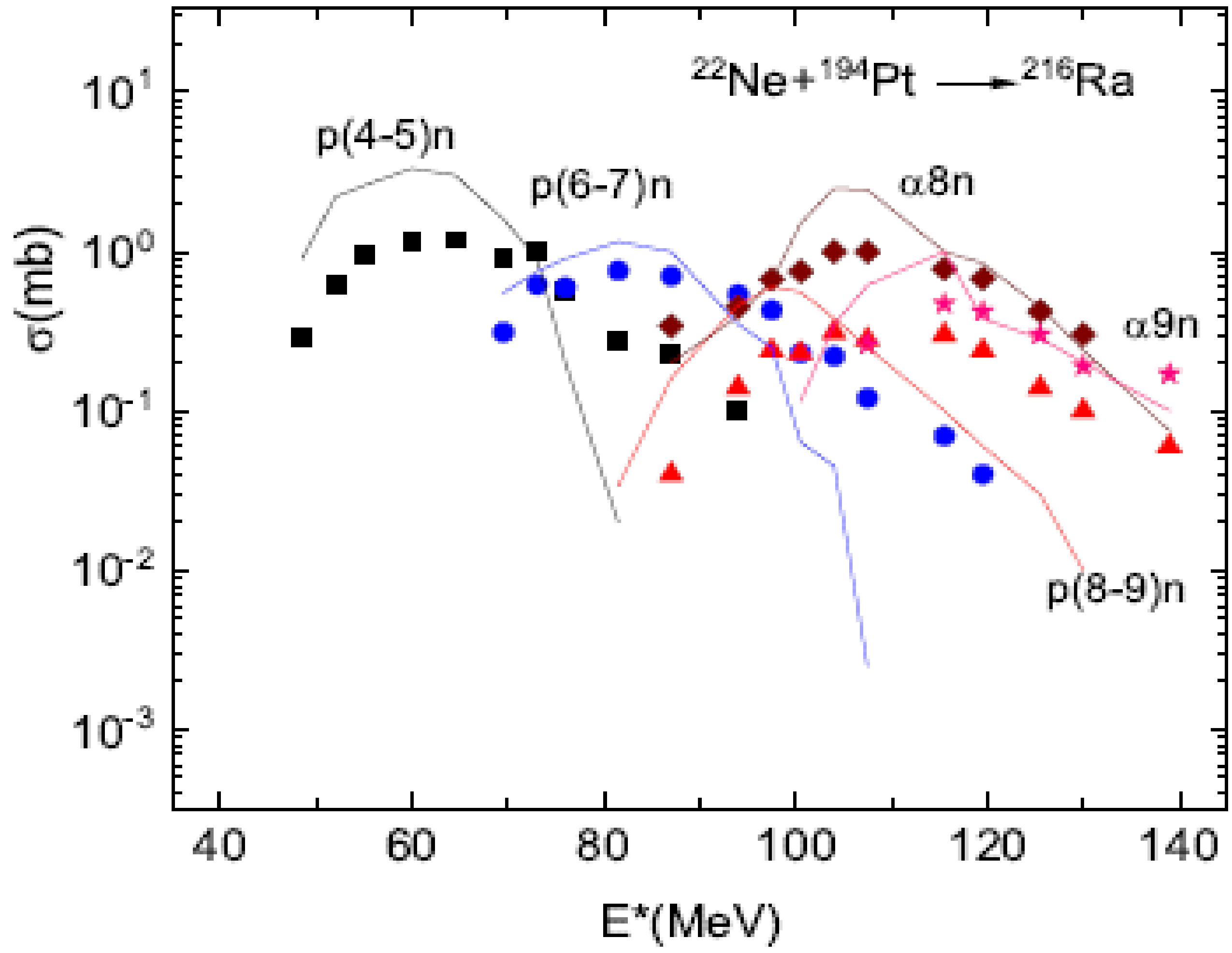
Experimental results are from D. Camas et al., Phys. Rev. C 105, 044612(2022)

Excitation functions for ^{220}Ra in xn, pxn, αxn channels (Z=88)

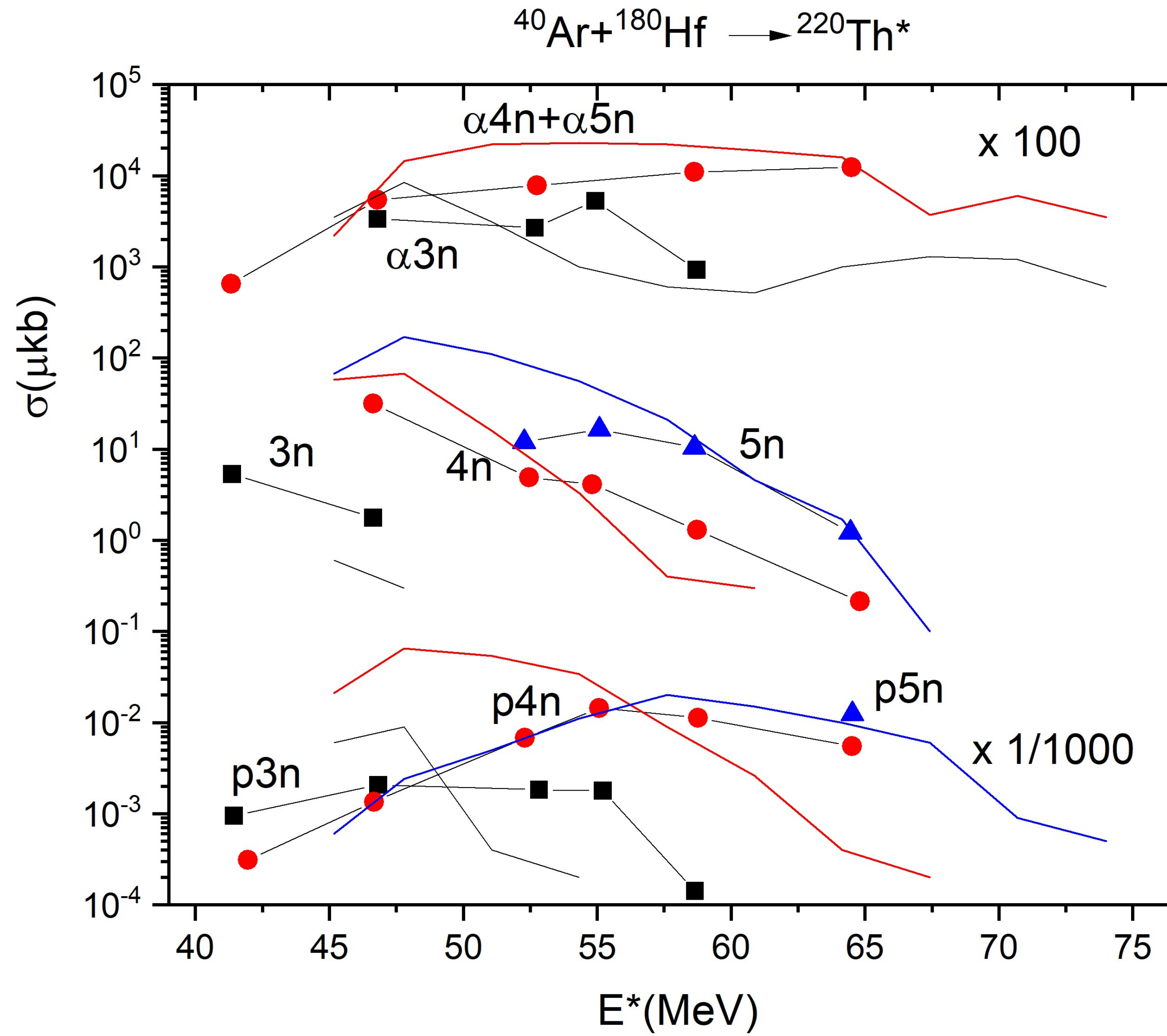


Experimental results are from A.N. Andreyev, Nucl. Phys. A 620, 229(1997)

Excitation functions for ^{216}Ra in xn, pxn, αxn channels (Z=88)



Excitation functions for ^{220}Th in xn, pxn and αxn channels (Z=90)



Experimental results are from C.C. Sahm et al.,, Nucl. Phys. A 441, 316(1985)

Summary

In the presented DNS model all decay channels including evaporation of light particles, cluster emission, quasifission and fission channels are described in a unique way.

Excitation functions in xn, pxn and αxn channels are described for reactions leading to CN with Z=80-90. It is shown, that pxn and αxn channels are as strong enough as xn evaporation channel, so that survival probablities of ER's strongly dependent on charged particle emission channels as well.

Good agreement with experimental data allows us to conclude that DNS model can be successfully applied for both fusion and fission processes.