

Density dependence of the heavy light meson distribution amplitude

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In collaboration with:

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14th APCTP-BLTP JINR Joint Workshop-Memorial Workshop in Honor of Prof. Yongseok Oh: Modern problems in nuclear and elementary particle physics, 9-14 July 2023, PIC-APCTP, Pohang

My memoriam with Prof. Yongseok Oh

- I joined the Asia Pacific Center for Theoretical Physics (APCTP) as APEC-YST Postdoctoral
- In my contract letter, it was obviously mentioned that I must work with Prof. Yongseok Oh (50%) and with APCTP (50%)
- During working with Prof. Yongseok Oh, we published 5 papers

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Impact of medium modifications of the nucleon weak and electromagnetic form factors on the neutrino mean free path in dense matter

Parada T. P. Hutaaruk, Yongseok Oh, and K. Tsushima
Phys. Rev. D **98**, 013009 – Published 30 July 2018

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Electroweak properties of pions in a nuclear medium

Parada T. P. Hutaaruk, Yongseok Oh, and K. Tsushima
Phys. Rev. C **99**, 015202 – Published 10 January 2019

JPS Conf. Proc. 26, 031031 (2019) [4 pages]
Proceedings of the 8th International Conference on Quarks and Nuclear Physics (QNP2018)

Pion Structure in a Nuclear Medium

Abstract References

Full text: PDF (eReader) / PDF (Download) (356 kB)

Parada T. P. Hutaaruk¹, Yongseok Oh^{1,2}, and Kazuo Tsushima^{1,3}

JPS Conf. Proc. 26, 024031 (2019) [4 pages]
Proceedings of the 8th International Conference on Quarks and Nuclear Physics (QNP2018)

Effects of Medium Modifications of Nucleon Form Factors on Neutrino Scattering in Dense Matter

Abstract References

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Valence-quark distributions of pions and kaons in a nuclear medium

Parada T. P. Hutaaruk, J. J. Cobos-Martínez, Yongseok Oh, and K. Tsushima
Phys. Rev. D **100**, 094011 – Published 13 November 2019

Outline



- * **Light Front Quark Model (LFQM)**

- Weak decay constants

- Distribution amplitudes (DAs)

- * **A nuclear medium effect from the quark meson coupling (QMC) model**

- Effective Lagrangian of the QMC model

- Quark masses in a nuclear medium

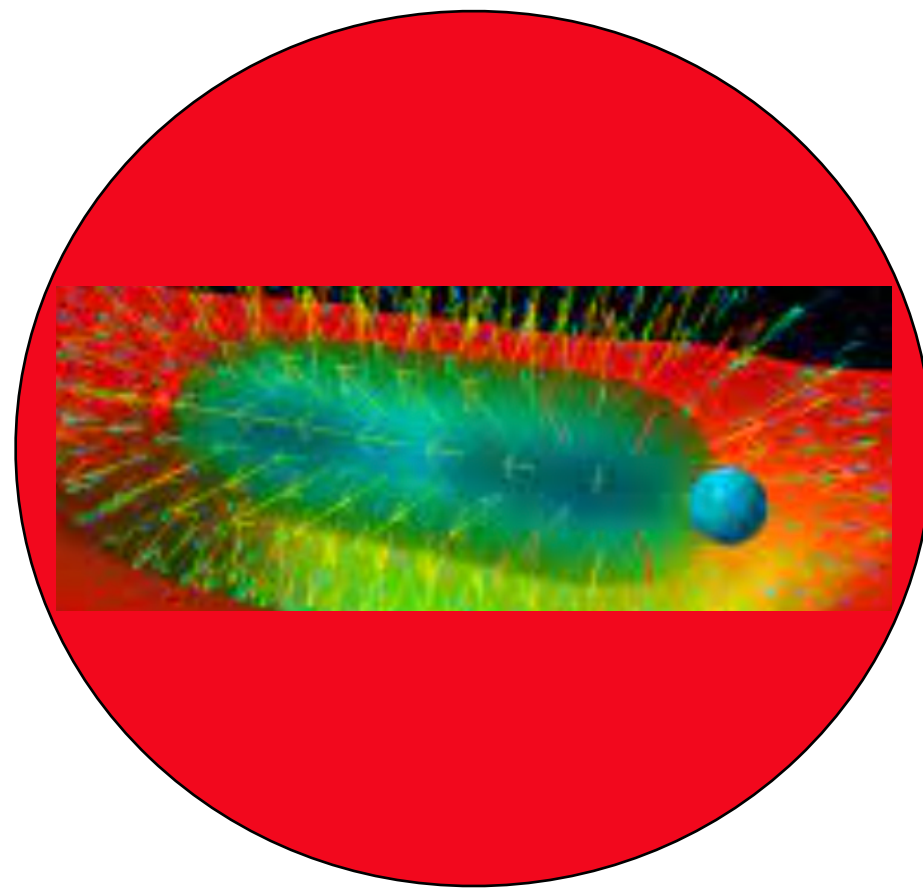
- * **DAs in a nuclear medium**

- * **Summary and outlook**

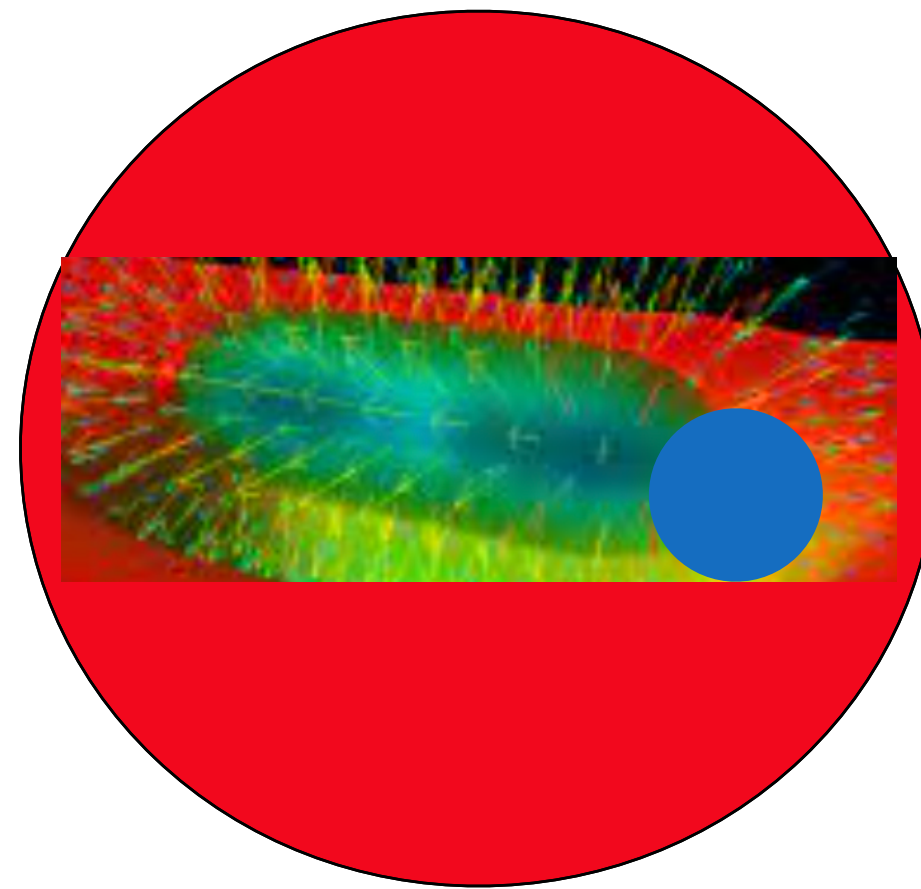
Meson contents

✱ Light-Light and Heavy-light mesons quark contents:

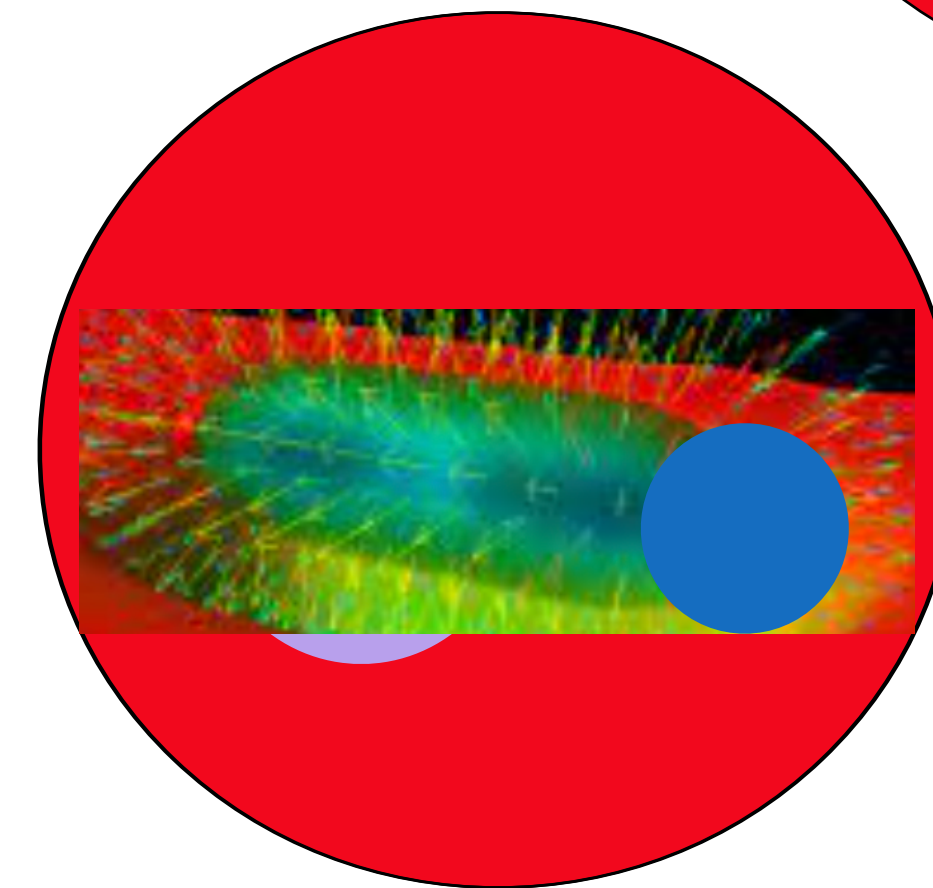
- Pseudoscalar mesons: π^+ ($u\bar{d}$), K^+ ($u\bar{s}$), D^+ ($c\bar{d}$), and B^+ ($u\bar{b}$)
- Vector mesons: ρ^+ ($u\bar{d}$), K^{*+} ($u\bar{s}$), B^{*+} ($u\bar{b}$), and D^{*+} ($c\bar{d}$)



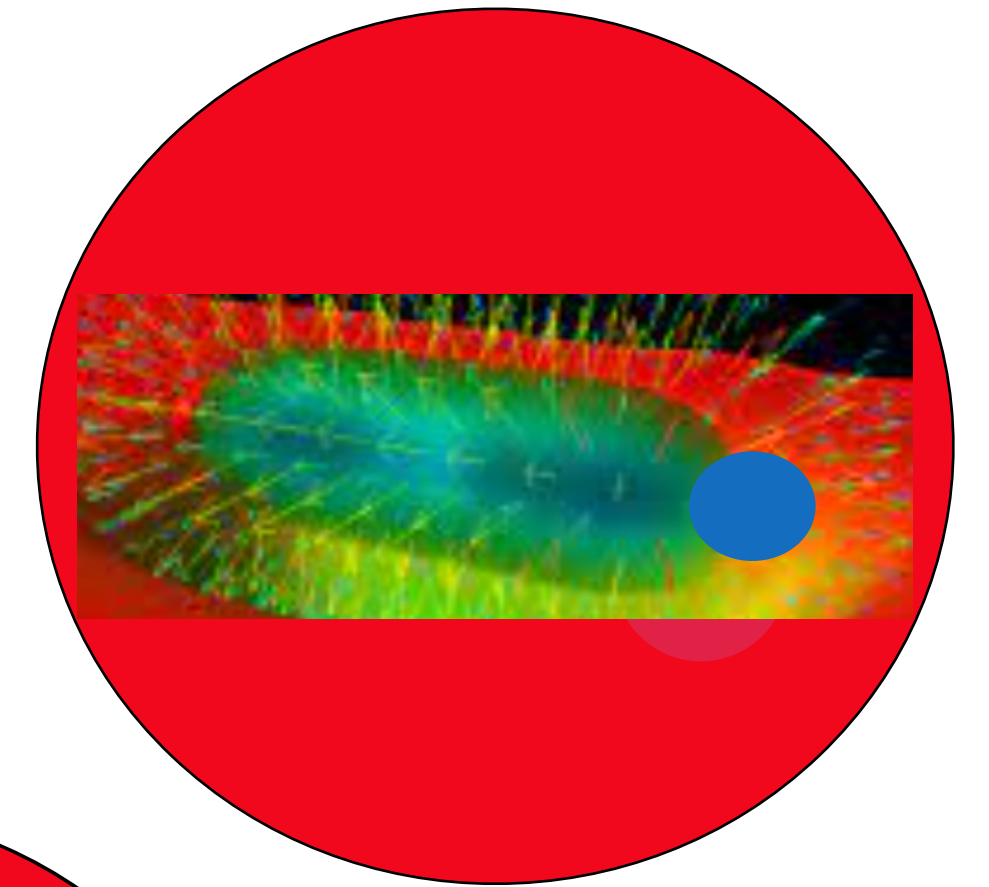
Pion (π)



D meson



B meson



Kaon (K)

Light Front Quark Model (LFQM)

- In the LFQM, the meson state—**bound state of the constituent quark and antiquark pair in the noninteracting representation**—Bakamjian Thomas (BT) Construction—the interaction is included in the meson mass operator—satisfy the Poincare group structure [\[Bakamjian and Thomas, PR 92 \(1953\) 1300\]](#)
- Interaction encoded in the mass eigenfunctions—**applying the variational principle**—to deal with the mass eigenvalues problem—trial wave function in the Gaussian basis [\[Arifi, HM Choi, and C-R.Ji, PRD 107 \(2023\) 053003 and References therein\]](#)
- The LFWF of the ground state meson in momentum space is given

$$\Psi_{\lambda_q \lambda_{\bar{q}}}^{JJ_z}(x, \mathbf{k}_\perp) = \underbrace{\Phi(x, \mathbf{k}_\perp)}_{\text{Radial wave function}} \underbrace{\mathcal{R}_{\lambda_q \lambda_{\bar{q}}}^{JJ_z}(x, \mathbf{k}_\perp)}_{\text{Spin-orbit wave function}},$$

Light Front Quark Model (LFQM)

- The spin-orbit wave functions are defined by

$$\mathcal{R}_{\lambda_q \lambda_{\bar{q}}}^{JJ_z} = \frac{1}{\sqrt{2}\tilde{M}_0} \bar{u}_{\lambda_q}(p_q) \Gamma_M v_{\lambda_{\bar{q}}}(p_{\bar{q}}), \quad \tilde{M}_0 \equiv \sqrt{M_0^2 - (m_q - m_{\bar{q}})^2},$$

M—Pseudo-scalar or vector mesons

- The invariant meson mass is defined [\[Arifi, HM Choi, and C-R.Ji, PRD 107 \(2023\) 053003 and References therein\]](#)

$$M_0^2 = \frac{\mathbf{k}_{\perp}^2 + m_q^2}{x} + \frac{\mathbf{k}_{\perp}^2 + m_{\bar{q}}^2}{1-x}.$$

- The vertices for the **pseudoscalar** and **vector mesons**

$$\Gamma_P = \gamma_5, \quad \Gamma_V = -\not{\epsilon}(J_z) + \frac{\epsilon \cdot (p_q - p_{\bar{q}})}{M_0 + m_q + m_{\bar{q}}},$$

Light Front Quark Model (LFQM)

- The trial radial wave function in the Gaussian basis [\[Arifi, HM Choi, and C-R.Ji, PRD 107 \(2023\) 053003\]](#)

$$\Phi_{1S}(x, \mathbf{k}_\perp) = \frac{4\pi^{3/4}}{\beta^{3/2}} \sqrt{\frac{\partial k_z}{\partial x}} e^{-\mathbf{k}^2/2\beta^2}, \quad \frac{\partial k_z}{\partial x} = \frac{M_0}{4x(1-x)} \left[1 - \frac{(m_q^2 - m_{\bar{q}}^2)^2}{M_0^4} \right],$$

- The LFWF is normalized

$$\int \frac{dx d^2\mathbf{k}_\perp}{2(2\pi)^3} |\Psi(x, \mathbf{k}_\perp)|^2 = 1.$$

- Computing the eigenvalue—BT construction—using the meson mass operator

$$(H_0 + V_{q\bar{q}})|\Psi_{q\bar{q}}\rangle = M_{q\bar{q}}|\Psi_{q\bar{q}}\rangle, \quad H_0 = \sqrt{m_q^2 + \mathbf{p}_q^2} + \sqrt{m_{\bar{q}}^2 + \mathbf{p}_{\bar{q}}^2},$$

Light Front Quark Model (LFQM)

- The quark-antiquark potentials

$$V_{\text{Conf}} = a + br, \quad V_{\text{Hyp}} = \frac{32\pi\alpha_s \langle \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} \rangle}{9m_q m_{\bar{q}}} \delta^3(r),$$

$$V_{\text{Coul}} = -\frac{4\alpha_s}{3r},$$

$$V_{q\bar{q}} = V_{\text{Conf}} + V_{\text{Coul}} + V_{\text{Hyp}},$$

- The LFWF and mass of the meson ground state—variational analysis

[Arifi, HM Choi, and C-R.Ji, PRD 107 (2023) 053003]

$$M_{q\bar{q}} = \langle \bar{\Psi}_{q\bar{q}} | H_{q\bar{q}} | \Psi_{q\bar{q}} \rangle = \langle \phi_{1S} | H_{q\bar{q}} | \phi_{1S} \rangle,$$

- Analytic mass formula

$$M_{q\bar{q}} = \frac{\beta}{\sqrt{\pi}} \sum_{i=q,\bar{q}} z_i e^{z_i/2} K_1\left(\frac{z_i}{2}\right) + a + \frac{2b}{\beta\sqrt{\pi}}$$

$$- \frac{8\alpha_s\beta}{3\sqrt{\pi}} + \frac{32\alpha_s\beta^3 \langle \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} \rangle}{9\sqrt{\pi}m_q m_{\bar{q}}},$$

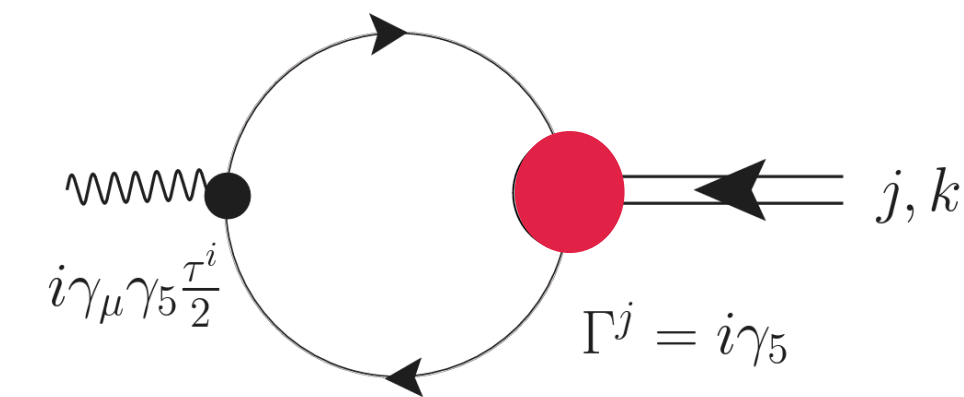
Light Front Quark Model (LFQM)

- The weak decay constant for the **pseudoscalar** and **vector mesons**

[Arifi, HM Choi, and C-R.Ji, PRD 107 (2023) 053003]

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 q | P(P) \rangle = i f_P P^\mu,$$

$$\langle 0 | \bar{q} \gamma^\mu q | V(P, J_z) \rangle = f_V M_V \epsilon^\mu(J_z),$$



- The weak decay constant expressions in the LFQM

$$f_M = 2\sqrt{6} \int_0^1 dx \int \frac{d^2 \mathbf{k}_\perp}{2(2\pi)^3} \frac{\Phi(x, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}^2 + \mathbf{k}_\perp^2}} \mathcal{O}_M,$$

$$\mathcal{O}_P = \mathcal{A}, \quad \mathcal{O}_V = \mathcal{A} + \frac{2\mathbf{k}_\perp^2}{D_0},$$

$$\mathcal{A} = (1-x)m_q + xm_{\bar{q}} \quad D_0 = M_0 + m_q + m_{\bar{q}}.$$

- The leading twist DAs for the **pseudoscalar** and **vector mesons**—plus component currents

$$A_P^+ = \langle 0 | \bar{q}(z) \gamma^+ \gamma_5 q(-z) | P(P) \rangle,$$

$$= i f_P P^+ \int_0^1 dx e^{i\zeta P \cdot z} \phi_P(x) |_{z^+ = z_\perp = 0},$$

$$A_V^+ = \langle 0 | \bar{q}(z) \gamma^+ q(-z) | V(P, 0) \rangle,$$

$$= f_V M_V \epsilon^+(0) \int_0^1 dx e^{i\zeta P \cdot z} \phi_V(x) |_{z^+ = z_\perp = 0},$$

Light Front Quark Model (LFQM)

○ In the LFQM model, the final expression for the LFWF

$$\phi_M(x) = \frac{2\sqrt{6}}{f_M} \int \frac{d^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{\Phi(x, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}^2 + \mathbf{k}_\perp^2}} \mathcal{O}_M.$$

○ DAs must satisfy the normalization

$$\int_0^1 \phi_M(x) dx = 1.$$

Quark-Meson Coupling (QMC) Model

[PAM Guichon, PLB 200 (1988) 235-240]

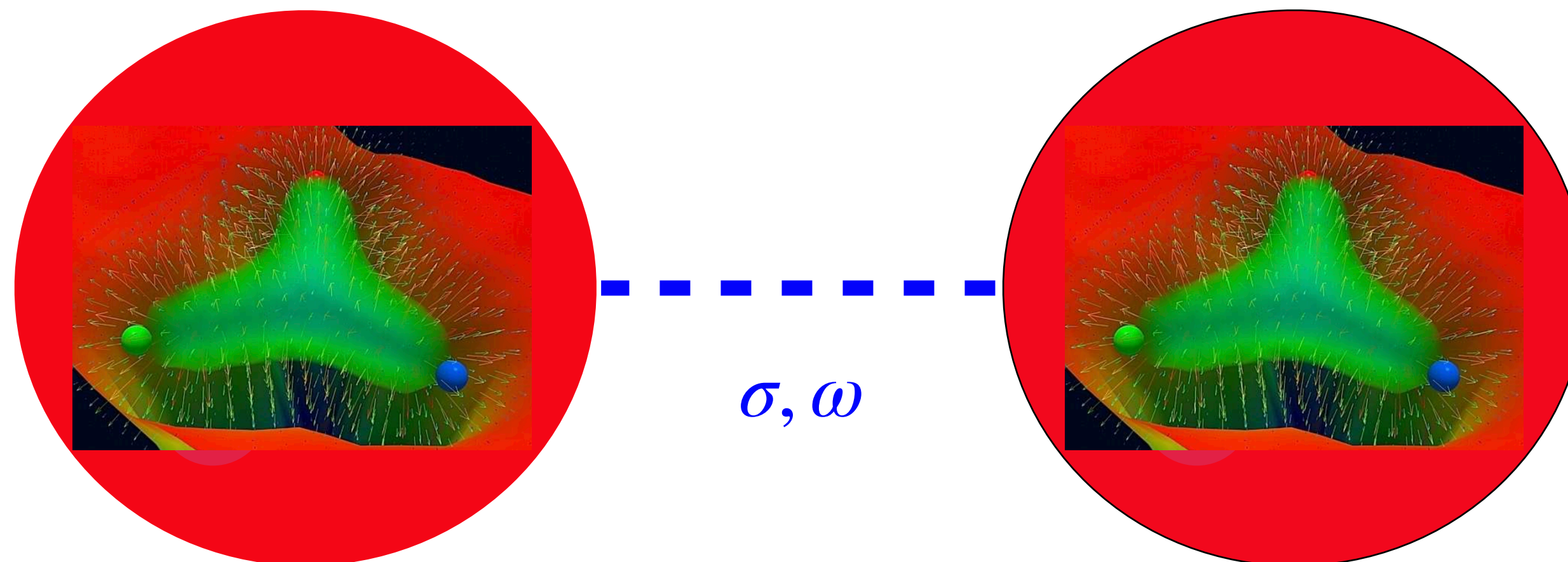
○ Effective Lagrangian of the QMC model for the symmetric nuclear matter (SNM)

$$\mathcal{L}_{\text{QMC}} = \bar{\psi}_N [i\gamma \cdot \partial - M_N^*(\sigma) - g_\omega \omega^\mu \gamma_\mu] \psi_N + \mathcal{L}_{\text{meson}},$$

$$M_N^*(\sigma) = M_N - g_\sigma(\sigma)\sigma.$$

$$\mathcal{L}_{\text{meson}} = \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{2} \partial_\mu \omega_\nu (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu.$$

[PAM Guichon, Koichi Saito, Rodionov, A. William, NPA 601 (1996) 349-379]



Nonoverlapping baryon

Quark-Meson Coupling (QMC) Model

[PAM Guichon, JR Stone, AW Thomas, PPNP 100 (2018) 262-297]

- In the mean-field approach, the nucleon Fermi momentum is related to the baryon density and scalar density

$$\rho_B = \frac{\gamma}{(2\pi)^3} \int dk \Theta(k_F - |k|) = \frac{\gamma k_F^3}{3\pi^2},$$
$$\rho_s = \frac{\gamma}{(2\pi)^3} \int dk \Theta(k_F - |k|) \frac{M_N^*(\sigma)}{\sqrt{M_N^{*2}(\sigma) + k^2}},$$

- In the QMC model, the nuclear matter is described as a collection of **non-overlapping MIT bags of nucleon** [PAM Guichon, JR Stone, AW Thomas, PPNP 100 (2018) 262-297]

- The Dirac equations for the quark and antiquarks in the bag

$$\left[i\gamma \cdot \partial_x - (m_l - V_\sigma^q) \mp \gamma^0 \left(V_\omega^q + \frac{1}{2} V_\rho^q \right) \right] \begin{pmatrix} \psi_u(x) \\ \psi_{\bar{u}}(x) \end{pmatrix} = 0, \quad \left[i\gamma \cdot \partial_x - (m_l - V_\sigma^q) \mp \gamma^0 \left(V_\omega^q - \frac{1}{2} V_\rho^q \right) \right] \begin{pmatrix} \psi_d(x) \\ \psi_{\bar{d}}(x) \end{pmatrix} = 0, \quad [i\gamma \cdot \partial_x - m_s] \begin{pmatrix} \psi_s(x) \\ \psi_{\bar{s}}(x) \end{pmatrix} = 0,$$

Quark-Meson Coupling (QMC) Model

- The effective in-medium quark masses [PAM Guichon, JR Stone, AW Thomas, PPNP 100 (2018) 262-297]

$$m_l^* \equiv m_l - V_\sigma^q,$$

- For the strange, charm, and bottom quarks, the effective mass in the medium is equal to that in the vacuum—decoupled from the scalar and vector potential in nuclear matter
- The scalar and vector mean-field potentials felt by the light quarks in SNM

$$V_\sigma^q \equiv g_\sigma^q \sigma = g_\sigma^q \langle \sigma \rangle,$$
$$V_\omega^q \equiv g_\omega^q \omega = g_\omega^q \delta^{\mu,0} \langle \omega^\mu \rangle,$$

- The effective mass of the hadron in the nuclear medium

$$m_h^* = \sum_{j=l,\bar{l},s,\bar{s}} \frac{n_j \Omega_j^* - z_h}{R_h^*} + \frac{4}{3} \pi R_h^{*3} B,$$

Which is determined by satisfying the stability condition

$$\left. \frac{dm_h^*}{dR_h} \right|_{R_h=R_h^*} = 0,$$

Quark-Meson Coupling (QMC) Model

- The scalar and vector meson mean fields at the hadron level are related to baryon and scalar densities

$$\sigma = \frac{4g_\sigma^N C_N(\sigma)}{(2\pi)^3 m_\sigma^2} \int d\mathbf{k} \Theta(k_F - |\mathbf{k}|) \frac{M_N^*(\sigma)}{\sqrt{M_N^{*2}(\sigma) + k^2}}, \quad \omega = \frac{g_\omega \rho_B}{m_\omega^2}, \quad C_N(\sigma) = \frac{-1}{g_\sigma^N} \left[\frac{\partial M_N^*(\sigma)}{\partial \sigma} \right],$$

$$= \frac{4g_\sigma^N C_N(\sigma)}{(2\pi)^3 m_\sigma^2} \rho_s,$$

- $C_N(\sigma) = 1$ for the point-like nucleon—the QMC model is similar to the QHD (Walecka) model

- The energy total per nucleon

$$E^{\text{tot}}/A = \frac{4}{(2\pi)^3 \rho_B} \int d\mathbf{k} \Theta(k_F - |\mathbf{k}|) \sqrt{M_N^{*2}(\sigma) + k^2} + \frac{m_\sigma^2 \sigma^2}{2\rho_B} + \frac{g_\omega^2 \rho_B}{2m_\omega^2}.$$

Quark-Meson Coupling (QMC) Model

[PAM Guichon, JR Stone, AW Thomas, PPNP 100 (2018) 262-297]

○ The total energy per nucleon

$$E^{\text{tot}}/A = \frac{4}{(2\pi)^3 \rho_B} \int dk \Theta(k_F - |\mathbf{k}|) \sqrt{M_N^{*2}(\sigma) + k^2} + \frac{m_\sigma^2 \sigma^2}{2\rho_B} + \frac{g_\omega^2 \rho_B}{2m_\omega^2}.$$

○ The coupling constants are taken by fitting to energy at saturation density

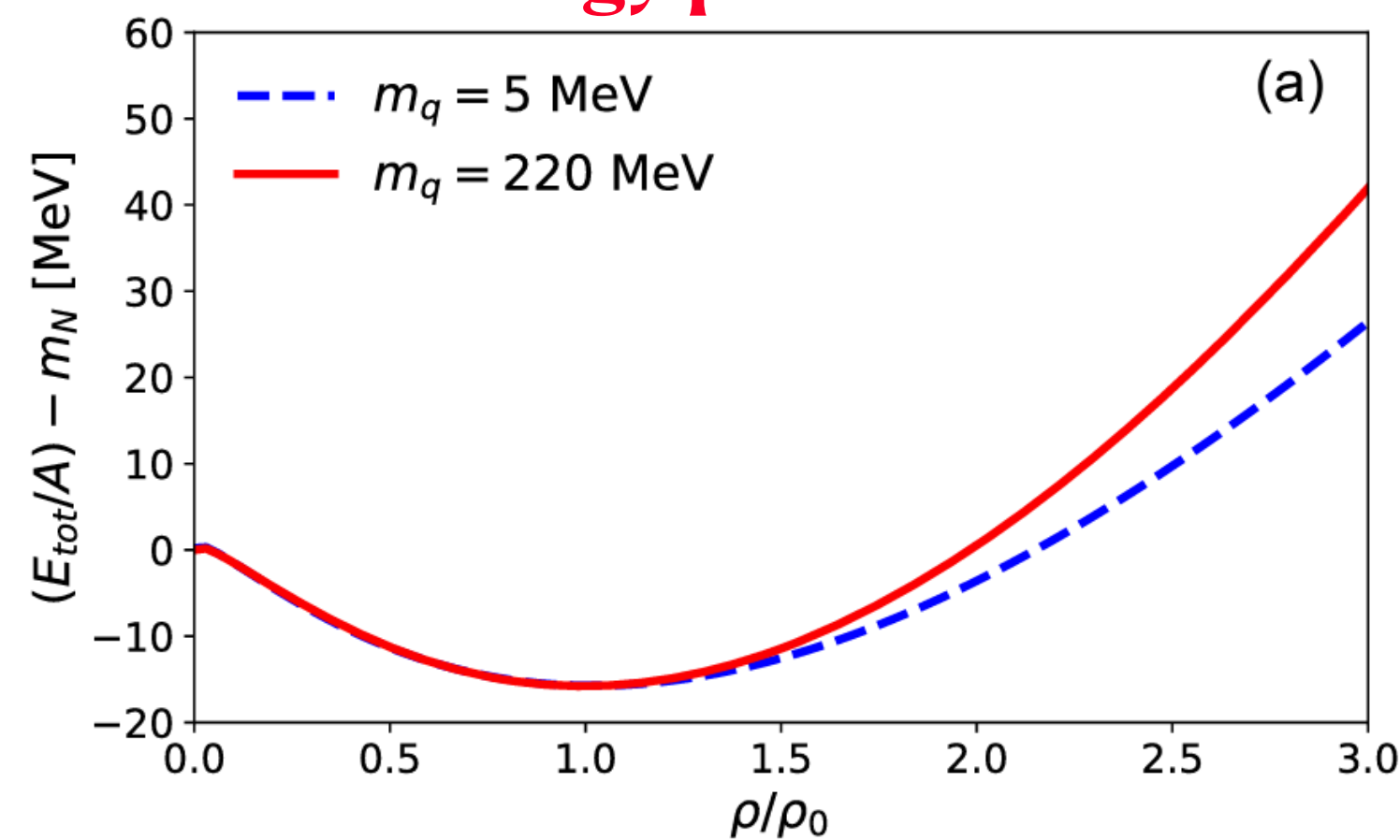
m_q [MeV]	$(g_\sigma^N)^2/4\pi$	$(g_\omega^N)^2/4\pi$	m_N^* [MeV]	K [MeV]
5	5.39	5.30	755	279
220	6.40	7.57	699	321

Quark-Meson Coupling (QMC) Model

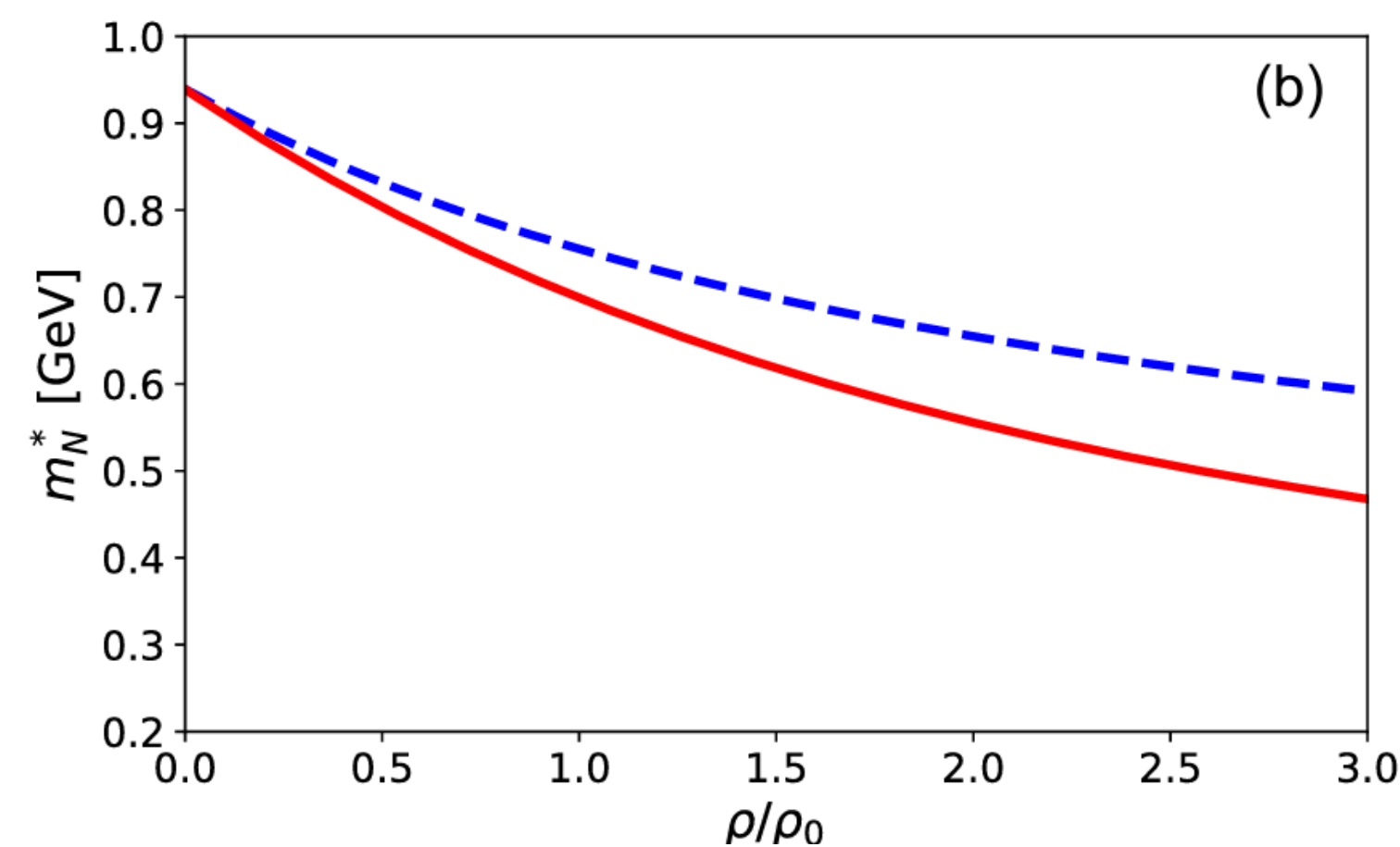
[Ahmad Jafar Arifi, PTPH, Kazuo Tsushima, PRD 107 (2023) 114010]

○ Numerical results for the QMC model

Energy per nucleon



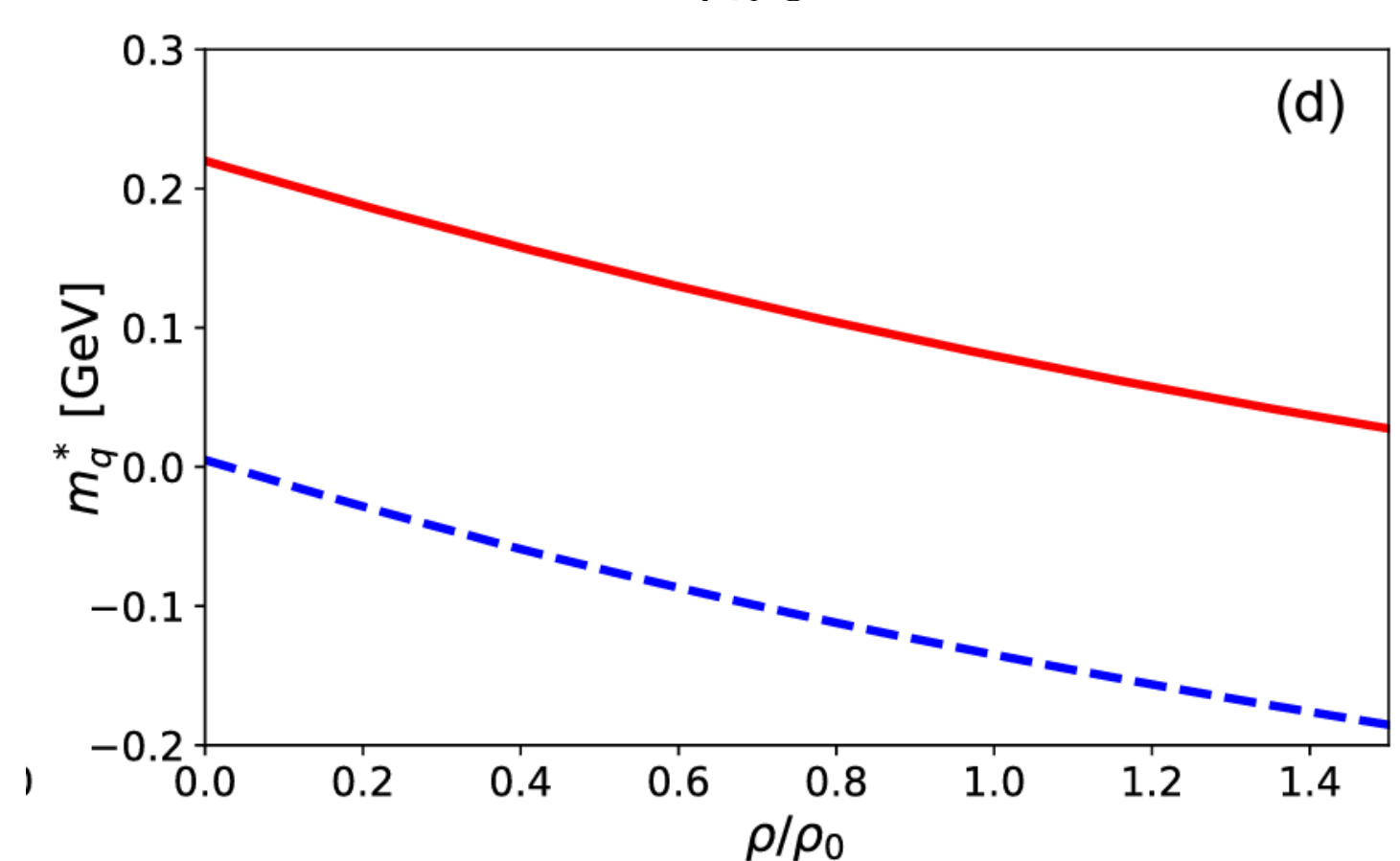
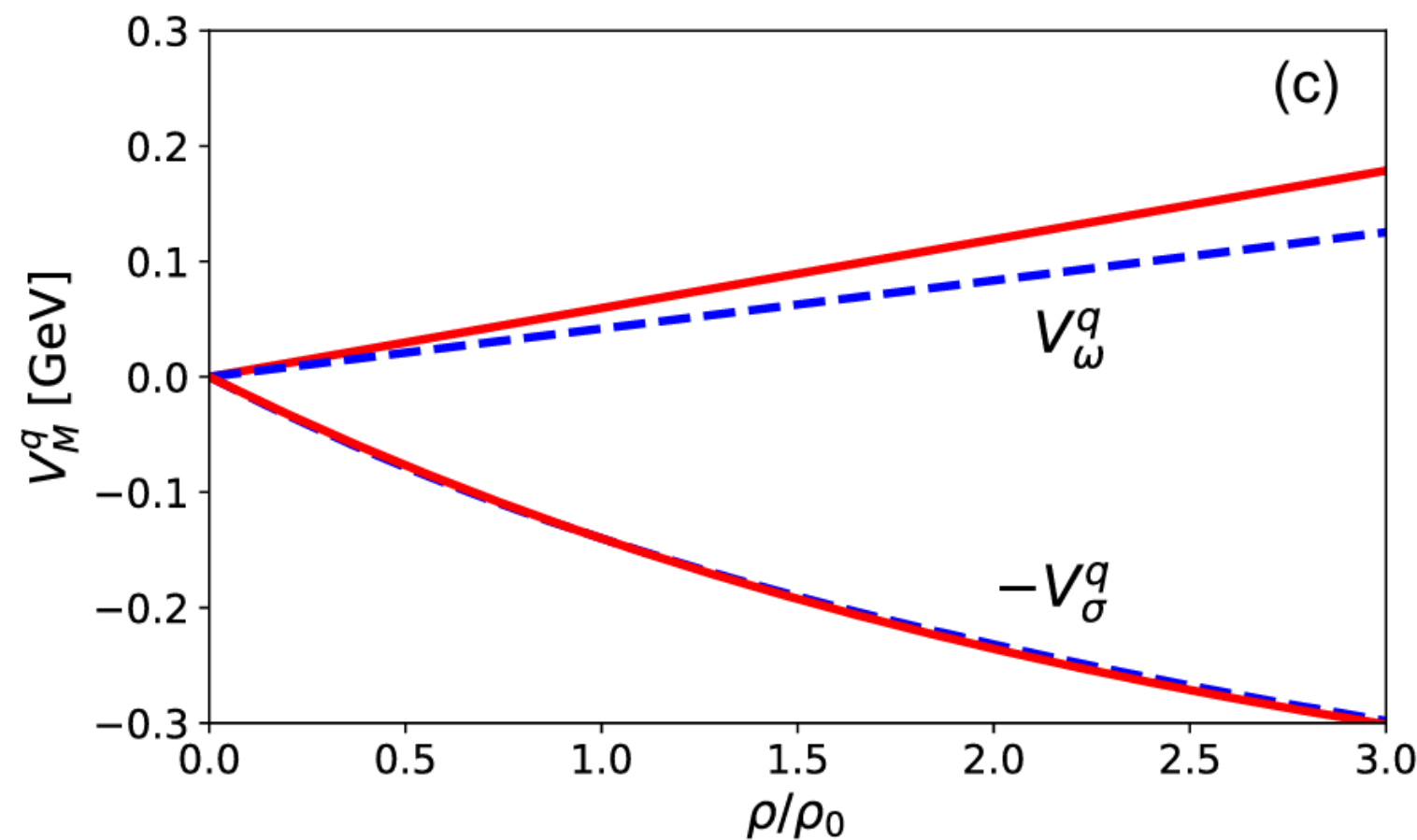
Effective nucleon mass



— The energy total per nucleon for different quark masses produces the binding energy at saturation density

— Effective nucleon mass decreases as density increases — consistent with other models

Potentials



Quark masses

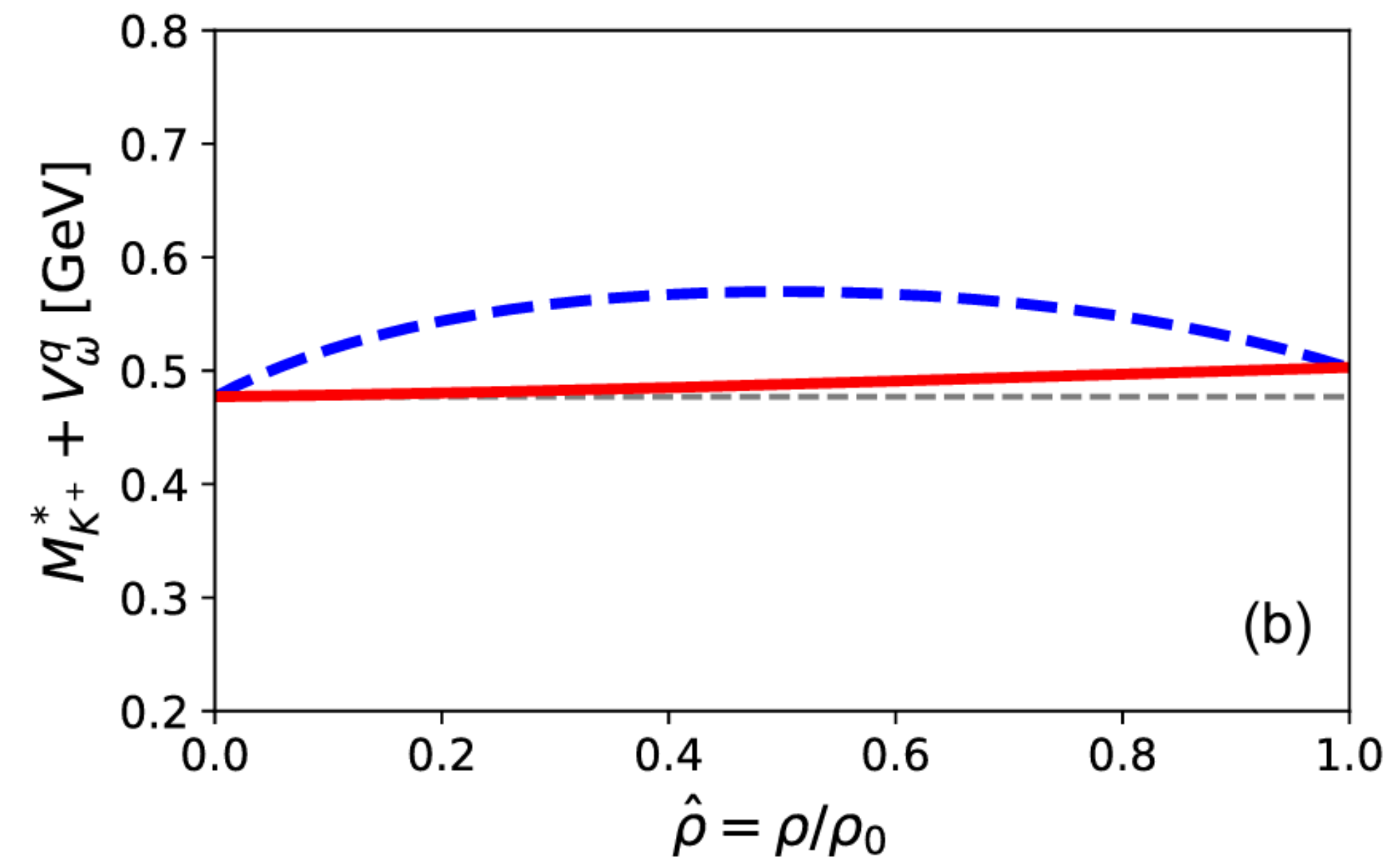
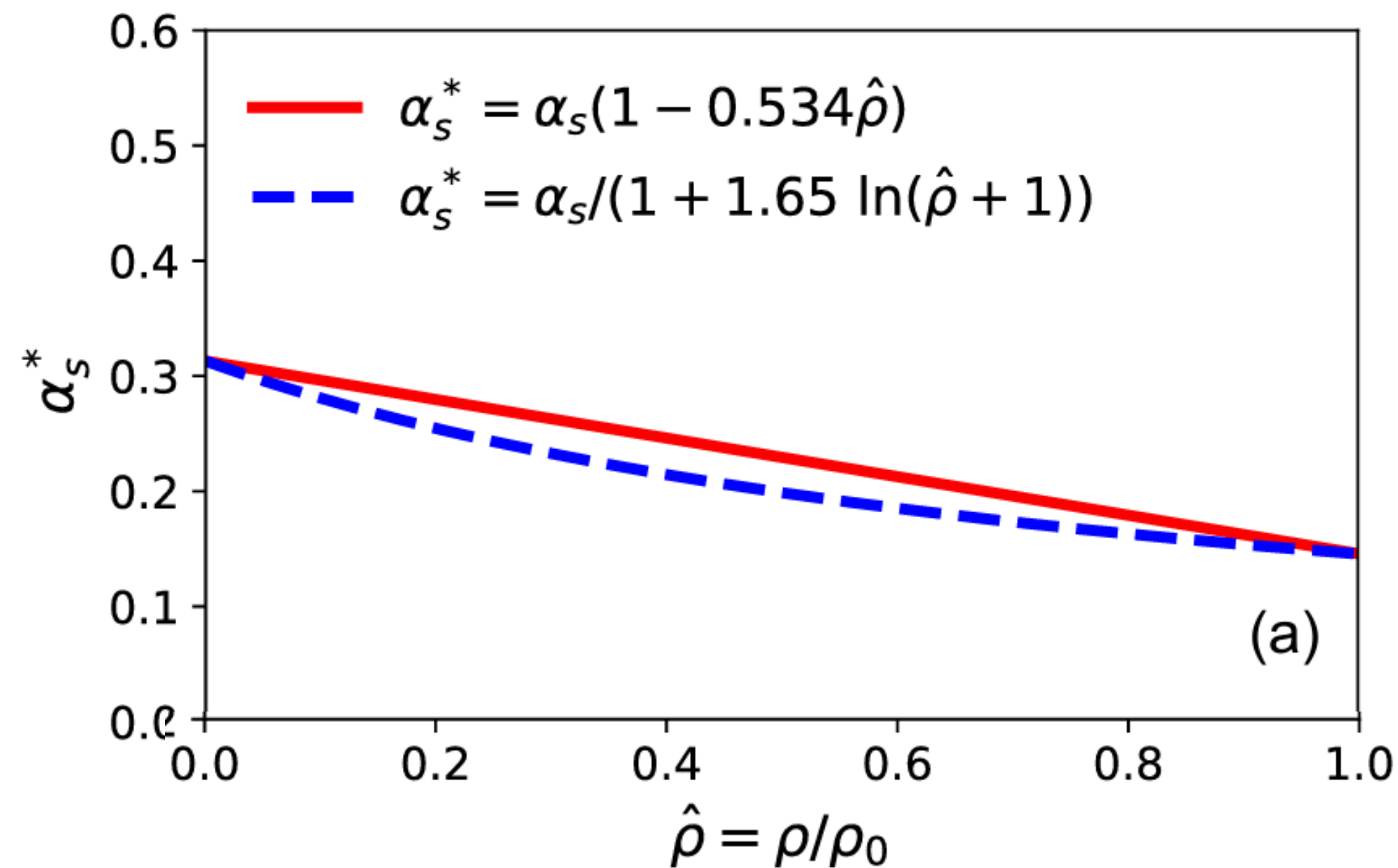
— Quark mass decreases as the density increases

Medium Modifications in the LFQM+QMC model

[Ahmad Jafar Arifi, PTPH, Kazuo Tsushima, PRD 107 (2023) 114010]

○ The strong coupling constant is modified in nuclear medium

$$\alpha_s^* \equiv \begin{cases} \alpha_{s(1)}^*(\rho) = \alpha_s(1 - b_1 \hat{\rho}), \\ \alpha_{s(2)}^*(\rho) = \frac{\alpha_s}{1 + b_2 \ln(\hat{\rho} + 1)}, \end{cases}$$



Medium Modifications in the LFQM+QMC model

- The light quark and antiquark energies are modified in a nuclear medium by the vector potential

$$p_i^{*0} = \begin{cases} E_q^* + V_\omega^q, & \text{for light quark,} \\ E_{\bar{q}}^* - V_\omega^q, & \text{for light antiquark,} \end{cases}$$

- The total meson energies

$$P^{*0} = \begin{cases} E_M^*, & \text{for } (q\bar{q}), \\ E_M^* + V_\omega^q, & \text{for } (q\bar{Q}), \\ E_M^* - V_\omega^q, & \text{for } (Q\bar{q}), \end{cases}$$

Medium Modifications in the LFQM+QMC model

- The Bjorken variable x —defined by the plus-component ratio of the quark to the meson momenta in free space

$$x = \frac{p_q^+}{P^+} = \frac{p_q^0 + p_q^3}{P^0 + P^3} = \frac{E_q + p_q^3}{E_M + P^3}, \quad \longrightarrow \quad x^* \equiv \frac{p_q^{*+}}{P^{*+}},$$

- For the quark-antiquark mesons $p_q^{*+} = p_{\bar{q}}^{*+} \equiv E_q^* + p_q^{*3}, \quad P^{*+} \equiv E_M^* + P^{*3},$

- In a nuclear medium, the Bjorken variable has a new definition

$$x \rightarrow \tilde{x}^* = \frac{p_q^{*+} + V_\omega^q}{P^{*+}} = x^* + \frac{V_\omega^q}{P^{*+}},$$

- By applying a change variable definition—to shift the integral limit of the Bjorken variable in **calculating the weak decay constant for the light mesons**

Medium Modifications in the LFQM+QMC model

○ The weak-decay constant for the light meson in a nuclear medium is given by

$$f_M^* = 2\sqrt{6} \int_{-\frac{v^q}{p^{*+}}_1}^{1-\frac{v^q}{p^{*+}}_1} dx^* \int \frac{d^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{\Phi(\tilde{x}^*, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}(\tilde{x}^*)^2 + \mathbf{k}_\perp^2}} \mathcal{O}_M(\tilde{x}^*, \mathbf{k}_\perp).$$

○ A final expression of the weak decay constant for the light meson in a nuclear medium

$$f_M^* = 2\sqrt{6} \int_0^1 d\tilde{x}^* \int \frac{d^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{\Phi(\tilde{x}^*, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}(\tilde{x}^*)^2 + \mathbf{k}_\perp^2}} \mathcal{O}_M(\tilde{x}^*, \mathbf{k}_\perp).$$

Medium Modifications in the LFQM+QMC model

[Ahmad Jafar Arifi, PTPH, Kazuo Tsushima, PRD 107 (2023) 114010]

- For the light-heavy meson— $q\bar{Q}$ or $\bar{q}Q$ contents—Both scalar and vector potential contribute to the weak decay constant and DAs in a nuclear medium
- The longitudinal momentum for the quark and antiquark in a nuclear medium are given

$$x \rightarrow \begin{cases} \tilde{x}^* = \frac{p_q^{*+} + V_\omega^q}{P^{*+} + V_\omega^q} = \frac{x^* + V_\omega^q/P^{*+}}{(1 + V_\omega^q/P^{*+})}, & \text{for } (q\bar{Q}), \\ \tilde{x}^* = \frac{p_q^{*+} - V_\omega^q}{P^{*+} - V_\omega^q} = \frac{x^* - V_\omega^q/P^{*+}}{(1 - V_\omega^q/P^{*+})}, & \text{for } (Q\bar{q}). \end{cases} \quad dx^* = (1 \pm V_\omega^q/P^{*+})d\tilde{x}^*,$$

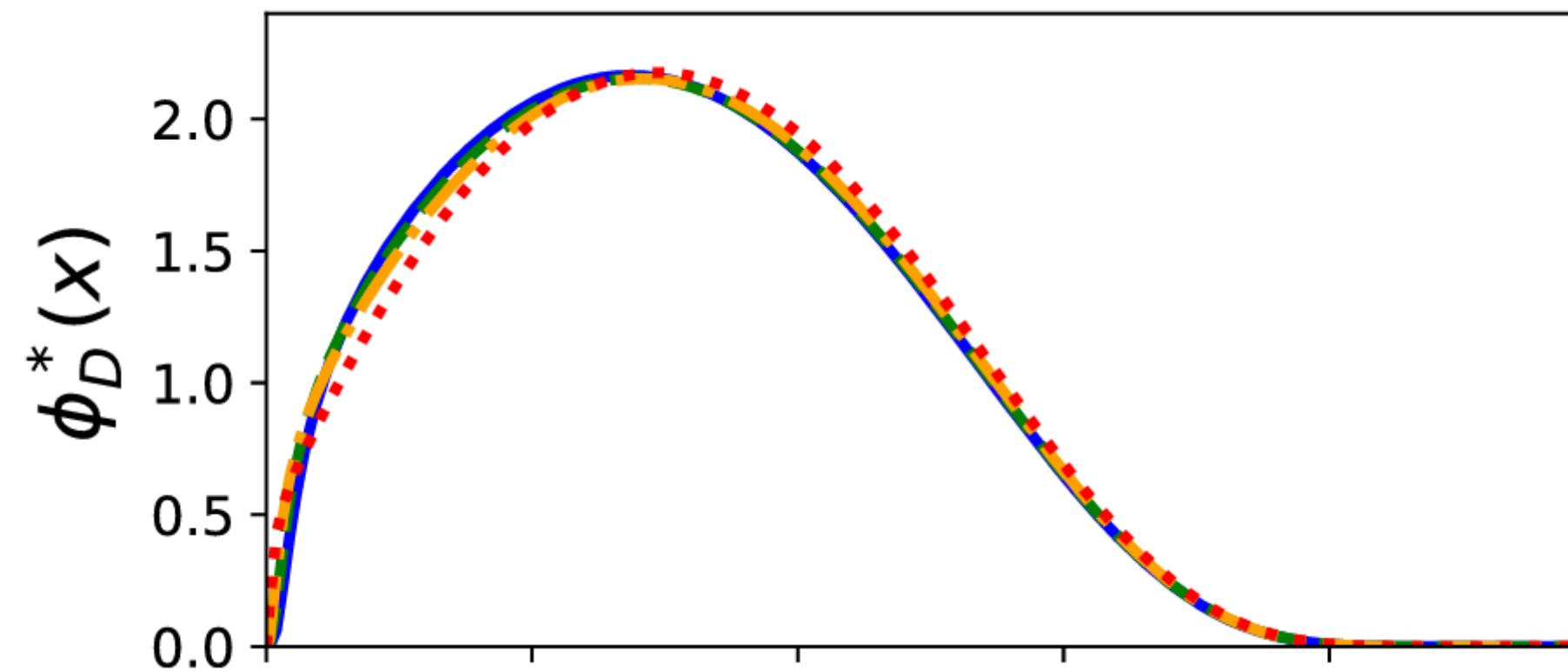
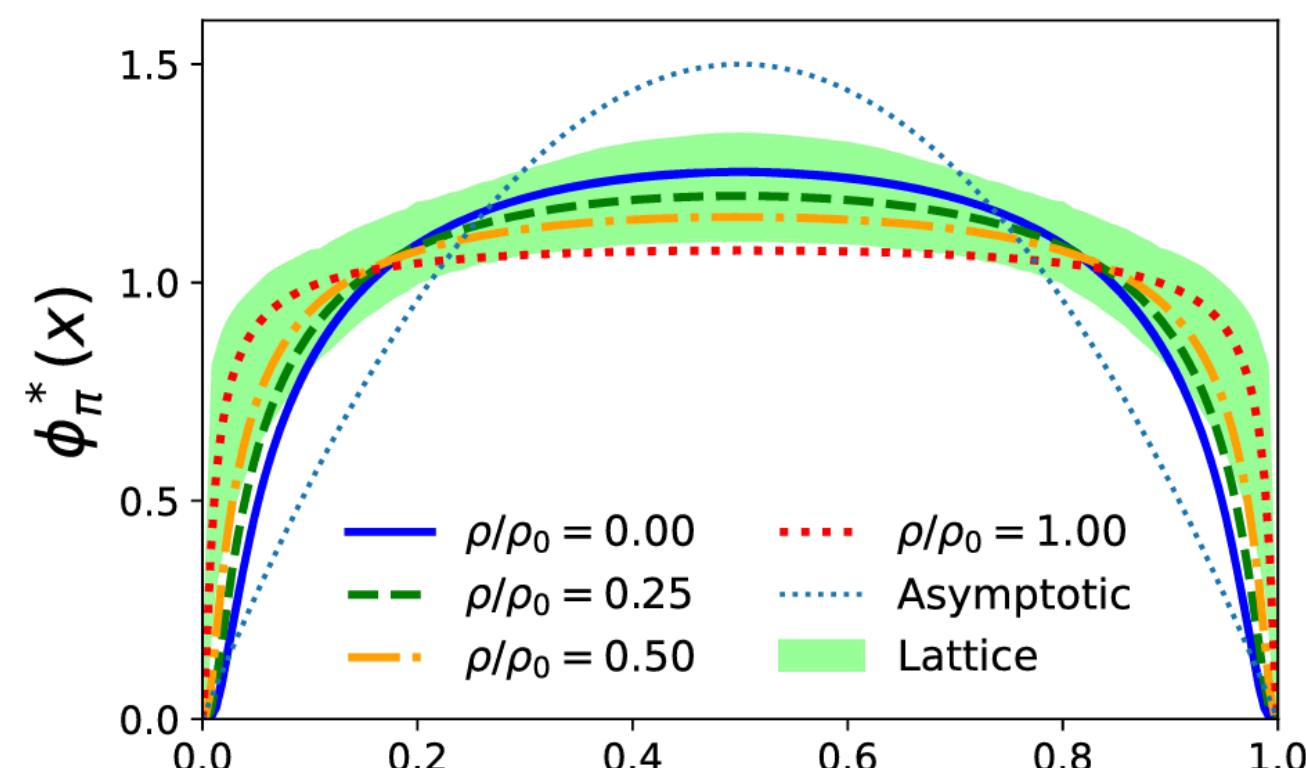
- A final expression for the light-heavy meson decay weak constant in a nuclear medium

$$f_M^* = 2\sqrt{6} \int_0^1 d\tilde{x}^* \int \frac{d^2\mathbf{k}_\perp}{2(2\pi)^3} \left(1 \pm \frac{V_\omega^q}{P^{*+}}\right) \times \frac{\Phi(\tilde{x}^*, \mathbf{k}_\perp)}{\sqrt{\mathcal{A}(\tilde{x}^*)^2 + \mathbf{k}_\perp^2}} \mathcal{O}_M(\tilde{x}^*, \mathbf{k}_\perp).$$

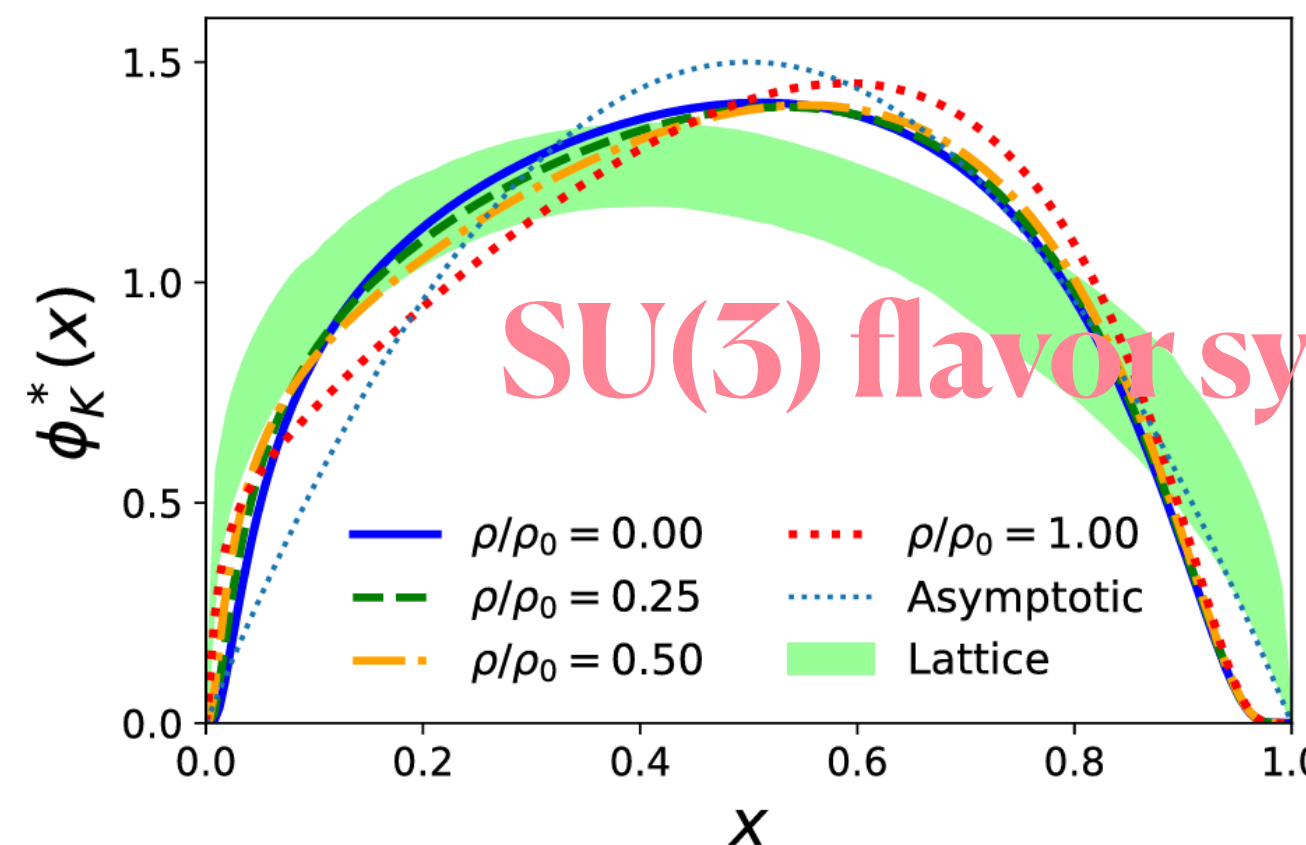
DAs in Nuclear Medium

[Ahmad Jafar Arifi, PTPH, Kazuo Tsushima, PRD 107 (2023) 114010]

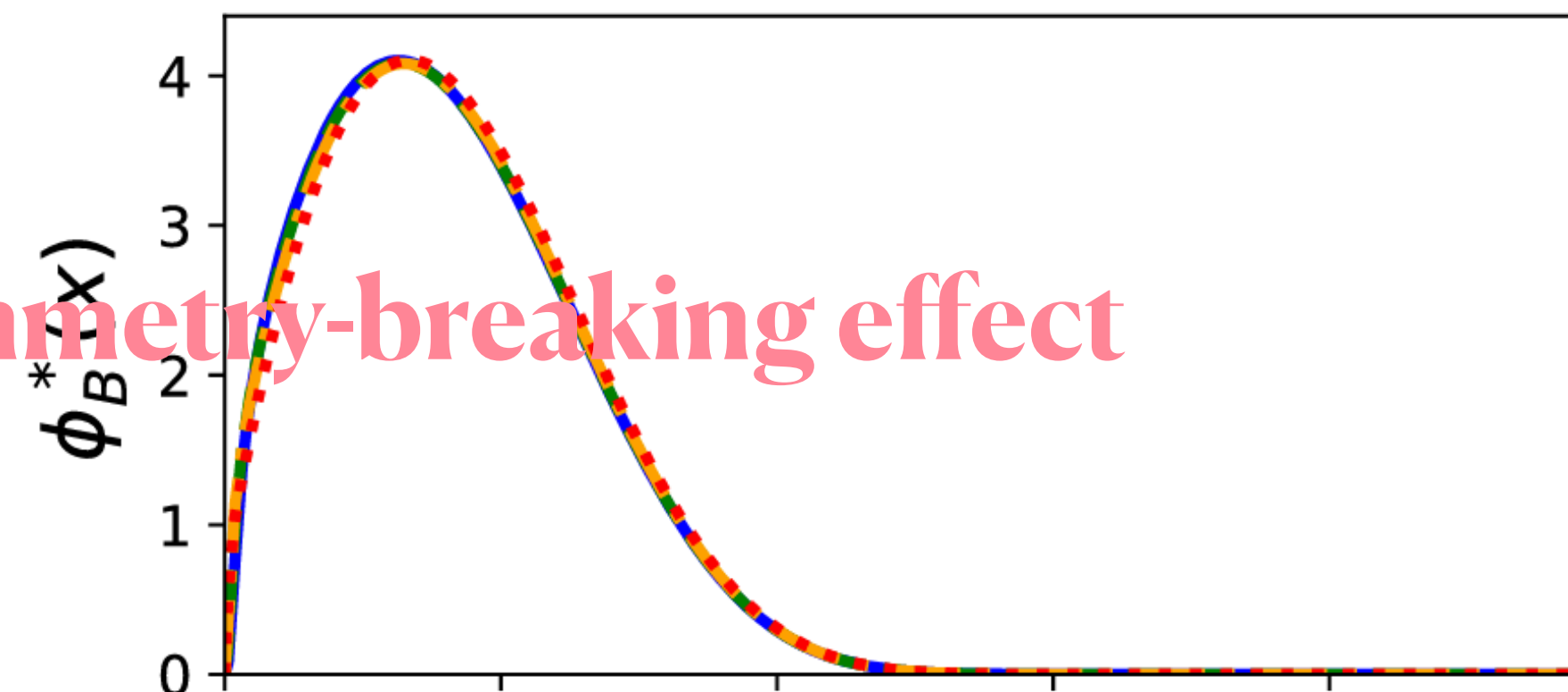
- Numerical results for DAs for the pseudoscalar mesons



—Pion DA is consistent with the lattice QCD results in free space



SU(3) flavor symmetry-breaking effect



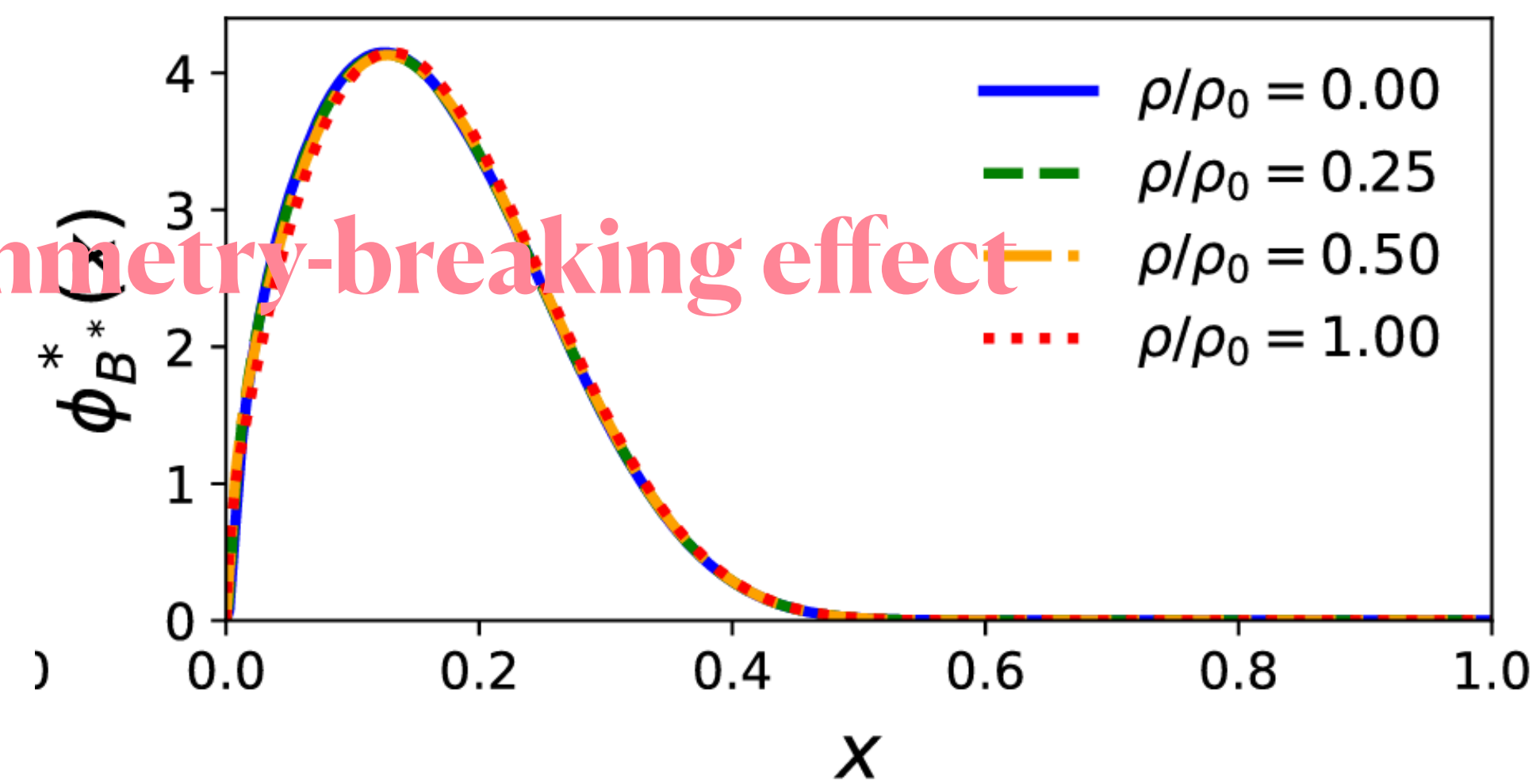
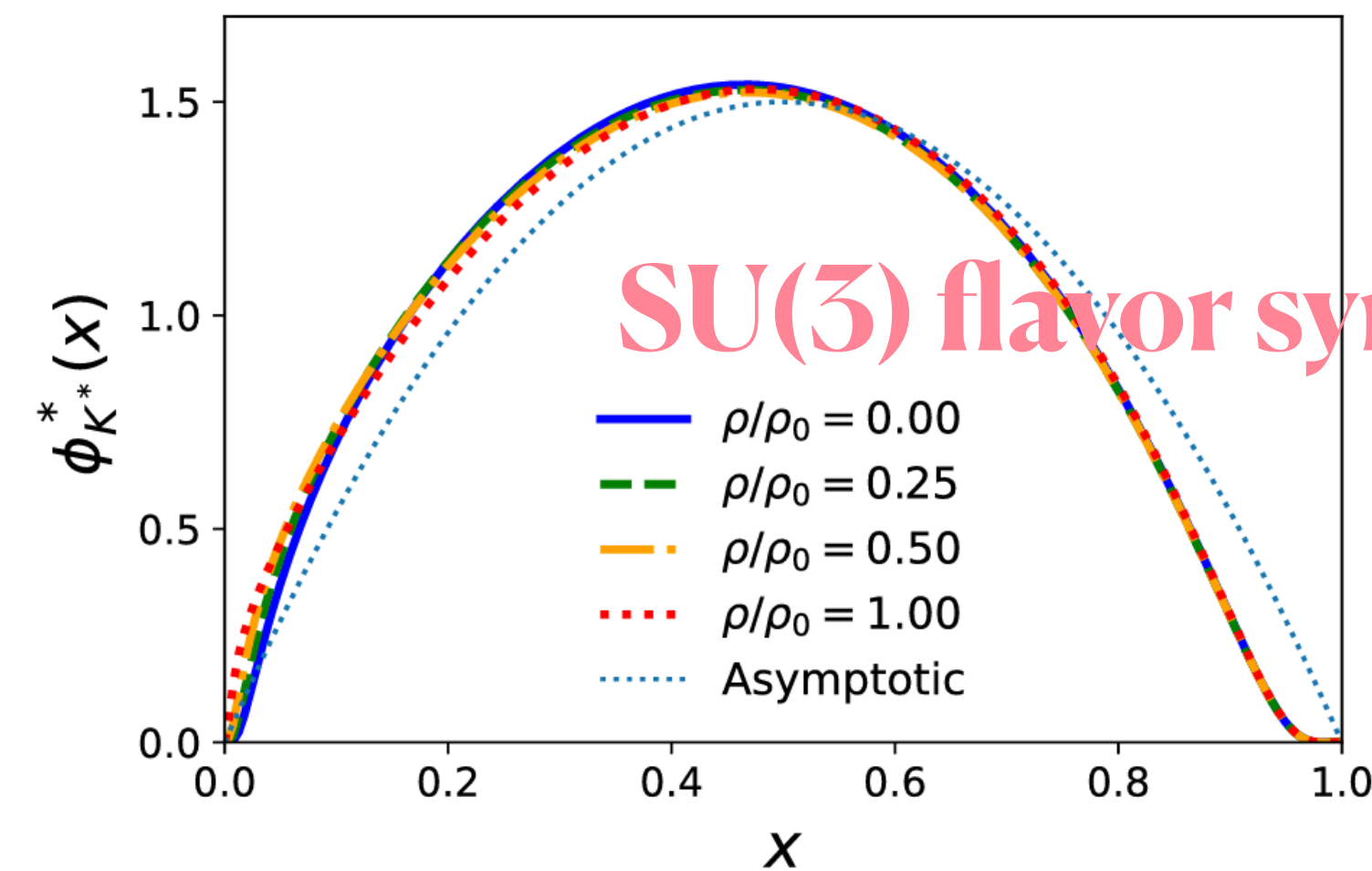
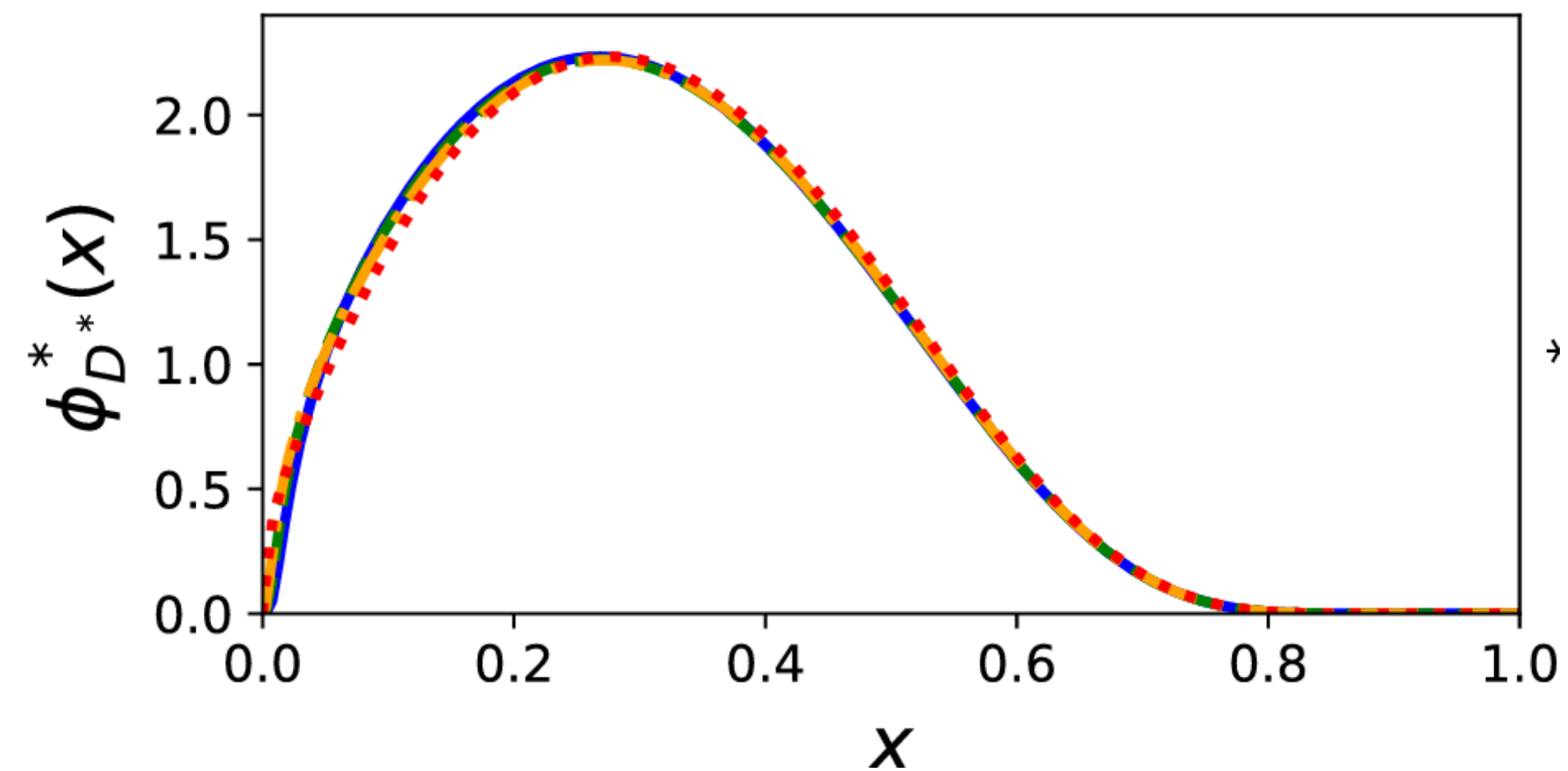
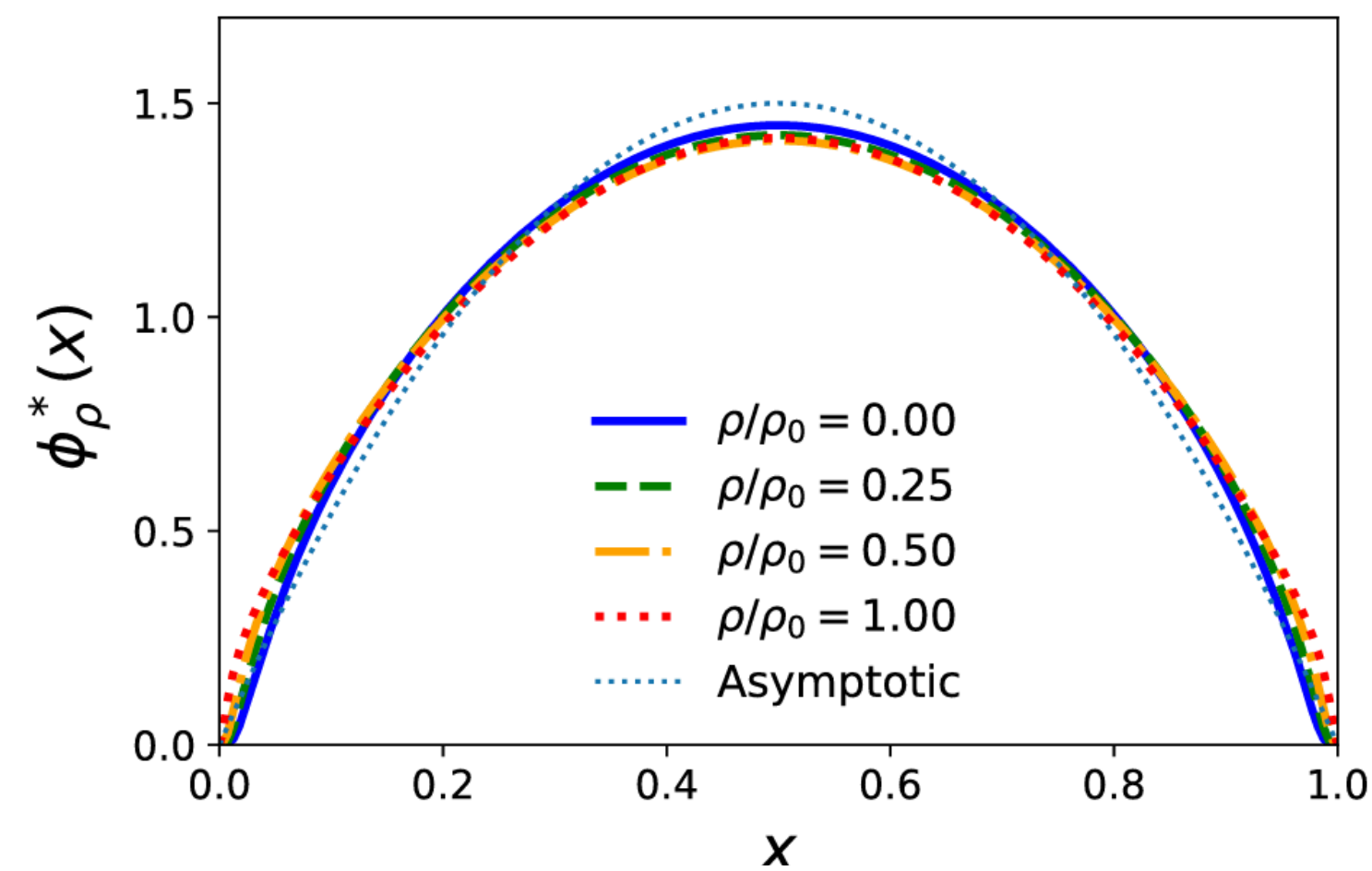
—Kaon DA is consistent with the lattice QCD results in free space at low $-x$ but at $x > 0.45$ is rather different

[Lattice Parton Collaboration, PRL 129 (2022) 132001]

DAs in Nuclear Medium

[Ahmad Jafar Arifi, PTPH, Kazuo Tsushima, PRD 107 (2023) 114010]

- Numerical results for DAs for the vector mesons



SU(3) flavor symmetry-breaking effect

Summary and Outlook

- We have demonstrated the DAs for the light-light and heavy-light mesons in free space and nuclear medium—**Pion and Kaon DAs (low-x) results are consistent with the lattice QCD data**—Unfortunately no (exp+lattice) data available for the heavy-light mesons
- Results for this calculation look promising—**we are going to extend our investigation/studying on the medium effects on the heavy-light meson form factors**—Heavy-light meson is a very interesting object/system because they have different quark contents—light and heavy quarks —different symmetries
- Other indirect observables could be possible to observe heavy-light mesons—**to observe the sensitiveness of the structure and properties of meson to the nuclear medium effects**—PDF, GPD, etc

Thank you very much for your attention

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