

# The double gamma decay of the quadrupole state of spherical nuclei

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A. P. Severyukhin, N. N. Arsenyev, N. Pietralla, Phys. Rev. C **104**, 024310 (2021)

A. P. Severyukhin, N. N. Arsenyev, Physics of Atomic Nuclei **85**, 919 (2022)

The  $\gamma\gamma$ -decay reactions are formally analogous to neutrinoless double- $\beta$  decay processes where in the latter two  $\beta$ -particles and in the former two  $\gamma$ -quanta appear in the final state and share the total transition energy. This paper reports on the situation, in which the  $\gamma\gamma$ -decay of the low-energy quadrupole state occurs in a nuclear transition which could proceed by a single- $\gamma$  decay in competition. To describe the  $\gamma\gamma$ -decay, a formalism relates the electromagnetic interaction up to second order in the electromagnetic operators and two-quantum processes in atomic nuclei. The coupling between one-, two- and three-phonon terms in the wave functions of excited nuclear states is taken into account within the microscopic model based on the Skyrme energy density functional. As our test case we considered  $^{48}\text{Ca}$  for which its dipole polarizability has recently been measured.

## Outline

- Introduction: The nuclear double- $\gamma$  decay  $\longrightarrow$
- The competitive double- $\gamma$  decay of the first  $2^+$  state  $\longrightarrow$
- The electric dipole polarizability  $\longrightarrow$
- The interaction of the one- and two-phonon configurations  $\longrightarrow$
- The case of  $^{48}\text{Ca}$   $\longrightarrow$

# Über Elementarakte mit zwei Quantensprüngen

Von Maria Göppert-Mayer

(Eingegangen 7. Dezember 1930)

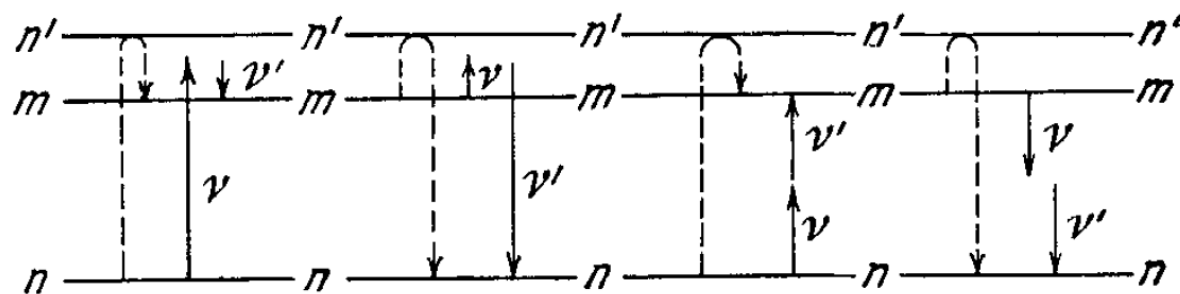
(Göttinger Dissertation)

(Mit 5 Figuren)

## Einleitung

Der erste Teil dieser Arbeit beschäftigt sich mit dem Zusammenwirken zweier Lichtquanten in einem Elementarakt. Mit Hilfe der Diracschen Dispersionstheorie<sup>1)</sup> wird die Wahrscheinlichkeit eines dem Ramaneffekt analogen Prozesses, nämlich der Simultanemission zweier Lichtquanten, berechnet. Es zeigt sich, daß eine Wahrscheinlichkeit dafür besteht, daß ein angeregtes Atom seine Anregungsenergie in zwei Lichtquanten aufteilt, deren Energien in Summe die Anregungsenergie er-

licht, des  
frequenz  
Doppeltem  
Lichtquar  
aufteilt.  
lichkeit di



Die punktierten Linien bedeuten das Verhalten des Atoms, aufwärtsgerichtete Pfeile absorbierte, abwärtsgerichtete emittierte Lichtquanten

Figg. 1—4

NUCLEAR TWO-PHOTON DECAY IN  $0^+ \rightarrow 0^+$  TRANSITIONS

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**Abstract:** The two-photon decay of the first excited  $0^+$  state of  $^{16}\text{O}$  has been measured using the Heidelberg-Darmstadt crystal ball. A branching ratio of  $\Gamma_{\gamma\gamma}/\Gamma_{\text{tot}} = (6.6 \pm 0.5) \cdot 10^{-4}$  was obtained. As in the cases of  $^{40}\text{Ca}$  and  $^{90}\text{Zr}$  previous experimental matrix elements and properties of the  $0_1^+$  and  $0_2^+$  states in  $^{16}\text{O}$ ,  $^{40}\text{Ca}$  and  $^{90}\text{Zr}$  E1 and M1 transitions of similar strength angular correlation. The ratio of the matrix values  $(-6.2 \pm 1.5)$  or  $(-0.16 \pm 0.04)$ .

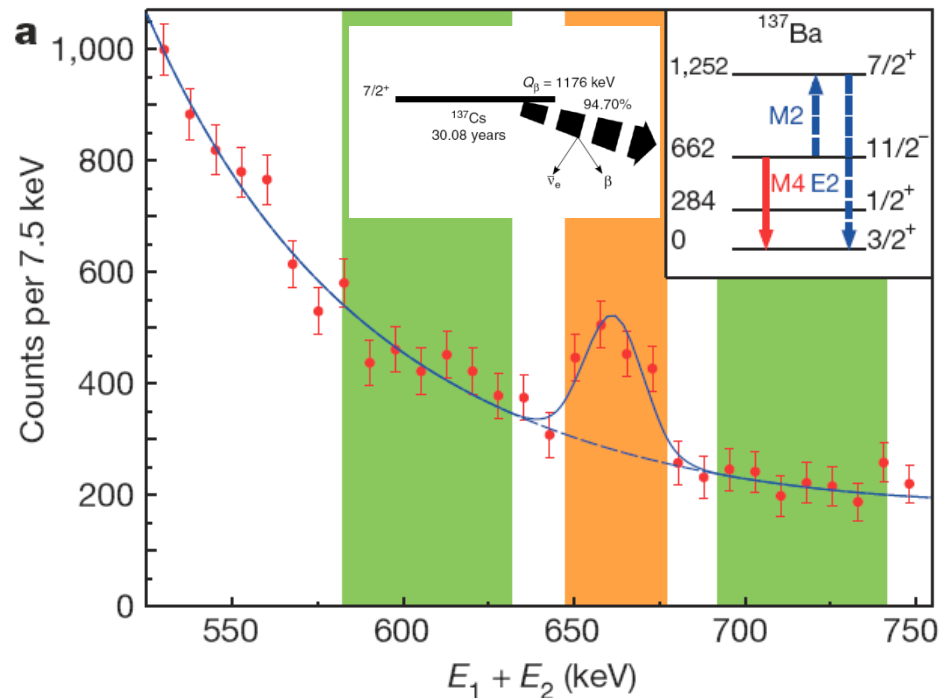
An interpretation of  $2\gamma$  matrix elements polarizabilities and magnetic susceptibility decay mode.

Nucleus	$^{16}\text{O}$	$^{40}\text{Ca}$	$^{90}\text{Zr}$
$\Delta E_{12} = E_2 - E_1 [\text{MeV}]$	6.049	3.352	1.761
$T_{1/2} [\text{ns}]$	0.067	2.1	61
$(\Gamma_{\gamma\gamma}/\Gamma_{\text{tot}}) \cdot 10^{-4}$	$6.6 \pm 0.5$	$4.5 \pm 1.0^{\text{d)}$	$1.8 \pm 0.3^{\text{a)}$
$\alpha_{E1}^{12} [10^{-3} \text{fm}^3]$	$16.9 \pm 4.3$	$7.8 \pm 1.9$	$20.1 \pm 10.9$
$\chi^{12} [10^{-3} \text{fm}^3]$	$-2.7 \pm 0.7$ $(-16.9 \pm 4.3)$	$-18.3 \pm 4.5$	$-10.4 \pm 5.7$
$\alpha_{E2}^{12} [\text{fm}^5]$	$\leq 120$	$\leq 670$	$\leq 4000$
$\langle 0_1^+   \bar{r}^2   0_2^+ \rangle [ \text{fm}^2 ]$	$3.55 \pm 0.21^{\text{b)}$	$2.6 \pm 0.1^{\text{c)}$	$1.71 \pm 0.06^{\text{d)}$
$\alpha_{E1}^{11} [10^{-3} \text{fm}^3]$	$585^{\text{e)}$	$2230^{\text{e)}$	$6330^{\text{e)}$
$\chi_{\text{P}}^{11} [10^{-3} \text{fm}^3]$	$1.78^{\text{f)}$	$5.65^{\text{f)}$	$14.5^{\text{f)}$
$\bar{E}_{E1} - E_1 [\text{MeV}]$	$24^{\text{e)}$	$20.2^{\text{e)}$	$16.7^{\text{g)}$
$\bar{E}_{M1} - E_1 [\text{MeV}]$	$17^{\text{h)}$	$10^{\text{i)}$	$9^{\text{j)}$

# Observation of the competitive double-gamma nuclear decay

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The double-gamma ( $\gamma\gamma$ )-decay of a quantum system in an excited state is a fundamental second-order process of quantum electrodynamics. In contrast to the well-known single-gamma ( $\gamma$ )-decay, the  $\gamma\gamma$ -decay is characterized by the simultaneous emission of two  $\gamma$  quanta, each with a continuous energy spectrum. In nuclear physics, this exotic decay mode has only been observed for transitions between states with spin-parity quantum numbers  $J^\pi = 0^+$  (refs 1–3). Single-gamma decays—the main experimental obstacle to observing the  $\gamma\gamma$ -decay—are strictly forbidden for these  $0^+ \rightarrow 0^+$  transitions. Here we report the observation of the  $\gamma\gamma$ -decay of an excited nuclear state ( $J^\pi = 11/2^-$ ) that is directly competing with an allowed  $\gamma$ -decay (to ground state  $J^\pi = 3/2^+$ ). The branching ratio of the competitive  $\gamma\gamma$ -decay of the  $11/2^-$  isomer of  $^{137}\text{Ba}$  to the ground state relative to its single  $\gamma$ -decay was determined to be  $(2.05 \pm 0.37) \times 10^{-6}$ . From the measured angular correlation and the shape of the energy spectra of the individual  $\gamma$ -rays, the contributing combinations of multiplicities of the  $\gamma$  radiation were determined. Transition matrix elements calculated using the quasiparticle-phonon model reproduce our measurements well. The  $\gamma\gamma$ -decay rate gives access to so far unexplored important nuclear structure information, such as the generalized (off-diagonal) nuclear electric polarizabilities and magnetic susceptibilities<sup>3</sup>.



**Figure 2 | Energy-sum spectrum and energy-gated time spectra of the  $72^\circ$ -group.** **a**, Energy-sum spectrum  $E_1 + E_2$  after subtraction of the random coincidences (requiring the energy condition  $|E_1 - E_2| < 300$  keV). The spectrum is fitted with a superposition of a Gaussian and an exponential function to describe the peak and the background, respectively. The orange ( $647$  keV  $< E_1 + E_2 < 677$  keV) and green areas ( $582$  keV  $< E_1 + E_2 < 632$  keV and  $692$  keV  $< E_1 + E_2 < 742$  keV) represent the energy conditions employed

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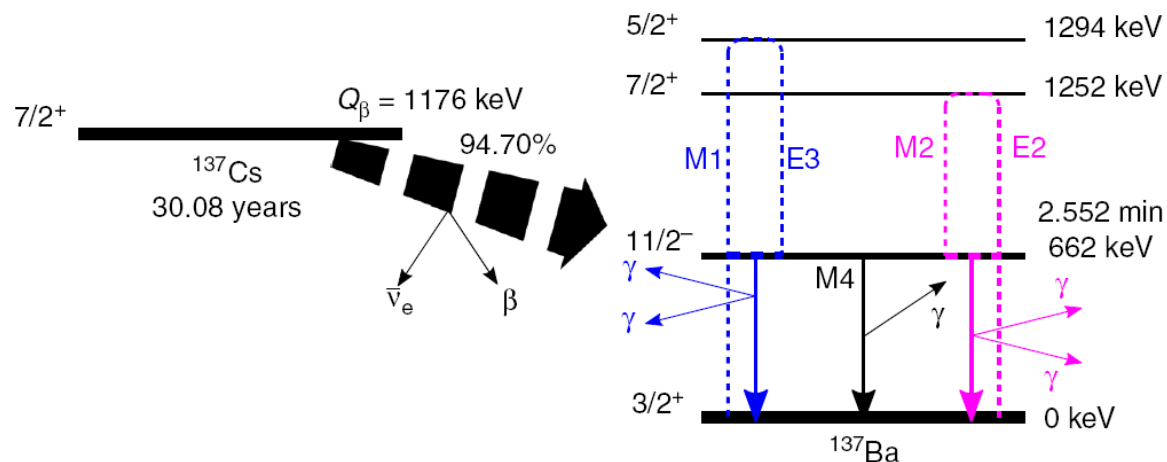
<https://doi.org/10.1038/s41467-020-16787-4>

OPEN

# Electromagnetic character of the competitive $\gamma\gamma/\gamma$ -decay from $^{137m}\text{Ba}$

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Second-order processes in physics is a research topic focusing attention from several fields worldwide including, for example, non-linear quantum electrodynamics with high-power lasers, neutrinoless double- $\beta$  decay, and stimulated atomic two-photon transitions. For the electromagnetic nuclear interaction, the observation of the competitive double- $\gamma$  decay from  $^{137m}\text{Ba}$  has opened up the nuclear structure field for detailed processes through the manifestation of off-diagonal nuclear this observation with an  $8.7\sigma$  significance, and an improvement versus single-photon branching ratio as  $2.62 \times 10^{-6}(30)$ . Our conclusions from the original experiment, where the decay  $\nu$  by a quadrupole-quadrupole component. Here, we find a : energy distribution consistent with a dominating octupole-dip quadrupole-quadrupole component in the decay, hindered d nuclear structure. The implied strongly hindered double-phot the possibility of the double-photon branching as a feasible t on off-diagonal polarisability in nuclei where this hindrance



## TWO-QUANTUM TRANSITIONS OF ATOMIC NUCLEI (II)

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**Abstract:** The analysis, made by the author in a previous paper<sup>1)</sup>, of two-quantum processes in atomic nuclei is continued. Nuclear  $\gamma\gamma$ -transitions are considered taking into account the interference of all channels allowed by the selection rules with respect to the angular momentum and parity of nuclear states  $j_1 \pi_1$  and  $j_2 \pi_2$  between which the transition occurs.

General formulae of the differential probability  $dW_{\gamma\gamma}(\omega_1 \mathbf{k}_1; \omega_2 \mathbf{k}_2)$  for the radiation of quanta with energies  $\omega_1$  and  $\omega_2$  and wave vectors  $\mathbf{k}_1$  and  $\mathbf{k}_2$ , and formulae for the quantum spectra and for the total probability of the  $\gamma\gamma$ -transition are obtained. The functions determining the angular quantum correlation are tabulated for  $\gamma\gamma$ -transitions competing with the one-quantum  $ML_0$  and  $EL_0$  processes at  $L_0 = 2.3.4.5$ .

### 2.2. GENERAL FORM OF TWO-PHOTON MATRIX ELEMENTS

Let us consider the matrix element of the two-photon radiation of the nucleus in the transition  $j_1 \mu_1 \rightarrow j_2 \mu_2$ . According to perturbation theory in the second order in  $e^2$  we have for this element

$$\begin{aligned} & \langle j_2 \mu_2 | H_{\gamma\gamma}(\omega_1 \omega_2) | j_1 \mu_1 \rangle \\ &= - \left\{ \sum_{ij_i \mu_i} \frac{\langle j_2 \mu_2 | H_{\gamma}(\omega_1) | j_i \mu_i \rangle \langle j_i \mu_i | H_{\gamma}(\omega_2) | j_1 \mu_1 \rangle}{E_i - \omega_1} + \right. \\ & \quad \left. + \sum_{sj_s \mu_s} \frac{\langle j_2 \mu_2 | H_{\gamma}(\omega_2) | j_s \mu_s \rangle \langle j_s \mu_s | H_{\gamma}(\omega_1) | j_1 \mu_1 \rangle}{E_s - \omega_2} \right\}. \end{aligned} \quad (12)$$

Using  $\hbar = c = 1$ , the  $\gamma$ -decay width of the  $2_1^+$  state of and even-even nuclei is related to its reduced electric quadrupole transition strength,  $B(E2; 2_1^+ \rightarrow 0_{gs}^+)$ , via

$$\Gamma_\gamma = \frac{4\pi}{75} \left( E_{2_1^+} \right)^5 B(E2; 2_1^+ \rightarrow 0_{gs}^+).$$

To describe the  $\gamma\gamma$ -decay between the  $2_1^+$  and  $0_{gs}^+$  states, we use a formalism that explicitly relates the electromagnetic interaction up to second order in the electromagnetic operators and two-quantum processes in atomic nuclei. Thus, the  $\gamma\gamma$ -decay width can be estimated as

$$\Gamma_{\gamma\gamma'} = \frac{64\pi}{42525} \left( E_{2_1^+} \right)^7 (\alpha_{E1E1})^2 (1 + \delta),$$

with

$$\alpha_{E1E1} = \sum_i \frac{\langle 0_{gs}^+ || M(E1) || 1_i^- \rangle \langle 1_i^- || M(E1) || 2_1^+ \rangle}{E_{1_i^-} - 0.5 E_{2_1^+}},$$

$$\delta = \left( \frac{\alpha_{M1M1}}{\alpha_{E1E1}} \right)^2 + \frac{3}{11} 10^{-4} \left( \frac{\alpha_{E2E2}}{\alpha_{E1E1}} \right)^2 (E_{2_1^+})^4 + \dots$$

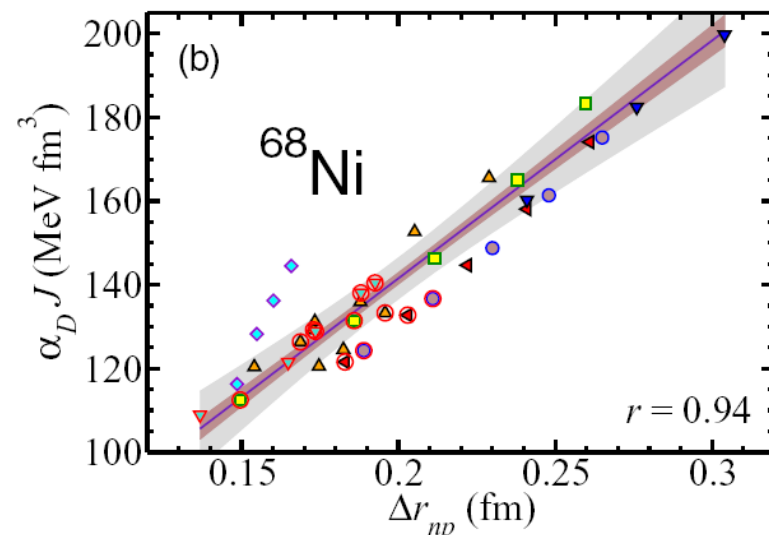
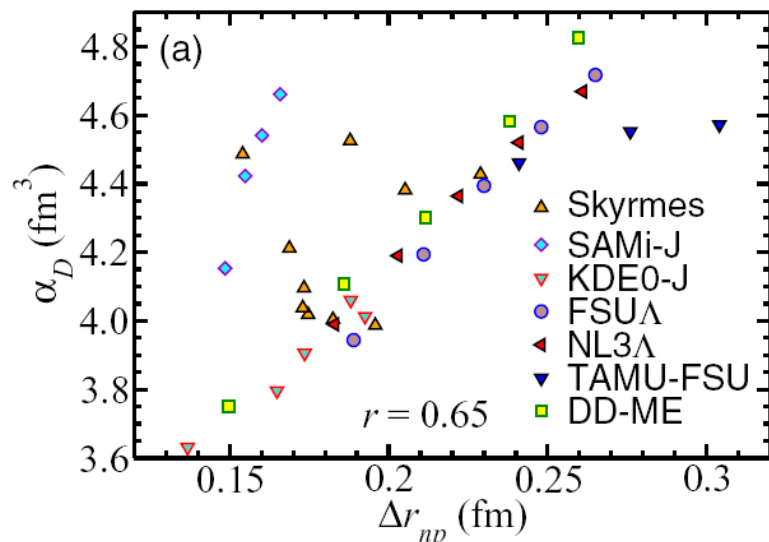
The  $\gamma\gamma$ -decay width is dominated by the  $E1E1$  contribution, i.e.,  $\delta \ll 1$ . The case of  $^{48}\text{Ca}$ :  $\delta(M1) = 1.7 \times 10^{-3}$  and  $\delta(E2) = 1.9 \times 10^{-7}$ .



The electric dipole polarizability,

$$\alpha_D = \frac{8\pi}{9} \sum_i \frac{\langle 0_{gs}^+ || M(E1) || 1_i^- \rangle \langle 1_i^- || M(E1) || 0_{gs}^+ \rangle}{E_{1_i^-}},$$

have played an important role, in particular, its value has strong implications in constraining the symmetry energy  $J$  as well as its density dependence and slope parameter  $L$ . The symmetry energy also plays an important role in nuclei, where it contributes to the formation of neutron skins in the presence of a neutron excess.



Calculations based on energy density functionals (EDFs) pointed out that J and L can be correlated with isovector collective excitations of the nucleus, such as pygmy dipole resonances (PDRs) and giant dipole resonances (GDRs), thus suggesting that the neutron skin thickness, the difference of the neutron and proton root-mean-square radii, could be constrained by studying properties of collective isovector observables at low energy.

Since the electric dipole polarizability,

$$\alpha_D = \frac{8\pi}{9} \sum_i \frac{\langle 0_{gs}^+ || M(E1) || 1_i^- \rangle \langle 1_i^- || M(E1) || 0_{gs}^+ \rangle}{E_{1_i^-}},$$

and  $\alpha_{E1E1}$

$$\alpha_{E1E1} = \sum_i \frac{\langle 0_{gs}^+ || M(E1) || 1_i^- \rangle \langle 1_i^- || M(E1) || 2_1^+ \rangle}{E_{1_i^-} - 0.5E_{2_1^+}},$$

are, although challenging, but in principle accessible observables, it is useful to compare their values.

# Two-phonon structures

The coupling between one-, two- and three-phonon terms in the wave functions of excited states are taken into account

$$\begin{aligned} \Psi_\nu(JM) = & \left( \sum_i R_i(J\nu) Q_{JM_i}^+ \right. \\ & + \sum_{\substack{\lambda_1 i_1 \\ \lambda_2 i_2}} P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu) [Q_{\lambda_1 i_1}^+ Q_{\lambda_2 i_2}^+]_{JM} \\ & \left. + \sum_{\substack{\lambda_1 i_1 \\ \lambda_2 i_2 \\ \lambda_3 i_3 J'}} T_{\lambda_3 i_3}^{\lambda_1 i_1 \lambda_2 i_2}(J\nu) \left[ [Q_{\lambda_1 i_1}^+ Q_{\lambda_2 i_2}^+]_{J'} Q_{\lambda_3 i_3}^+ \right]_{TM} \right) |0\rangle \end{aligned}$$

$\lambda^\pi = 1^-, 2^+, 3^-, \text{ and } 4^+$

The equations have the same form as the equations of the quasiparticle-phonon model (QPM), but the single-particle spectrum and the parameters of the residual interaction are calculated with the Skyrme EDF.

M. Grinberg , Ch. Stoyanov, Nucl. Phys. A. 573, 231 (1994)

V.Yu. Ponomarev, Ch. Stoyanov, N. Tsoneva, M. Grinberg, Nucl. Phys. A 635, 470(1998)

# Details of calculations

Making use of the finite rank separable approximation for the residual interaction enables one to perform the calculations in very large configuration spaces

Nguyen Van Giai, Ch. Stoyanov, V. V. Voronov, Phys. Rev. C57,1204 (1998).

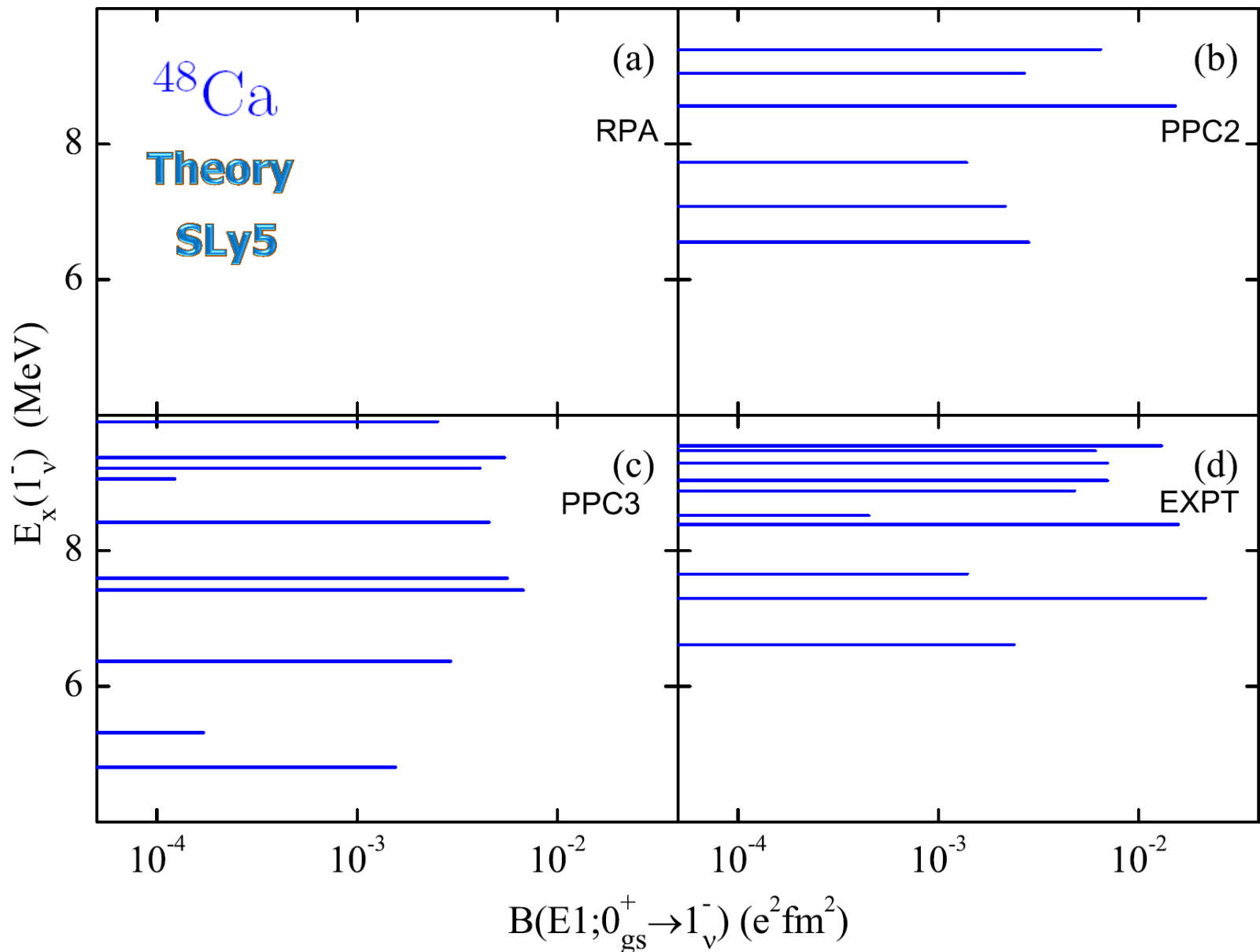
The rank of the set of linear equations is equal to the number of one-, two- and three-phonon configurations included in the wave function. Its solution requires to compute the matrix elements of the quasiparticle-phonon interaction.

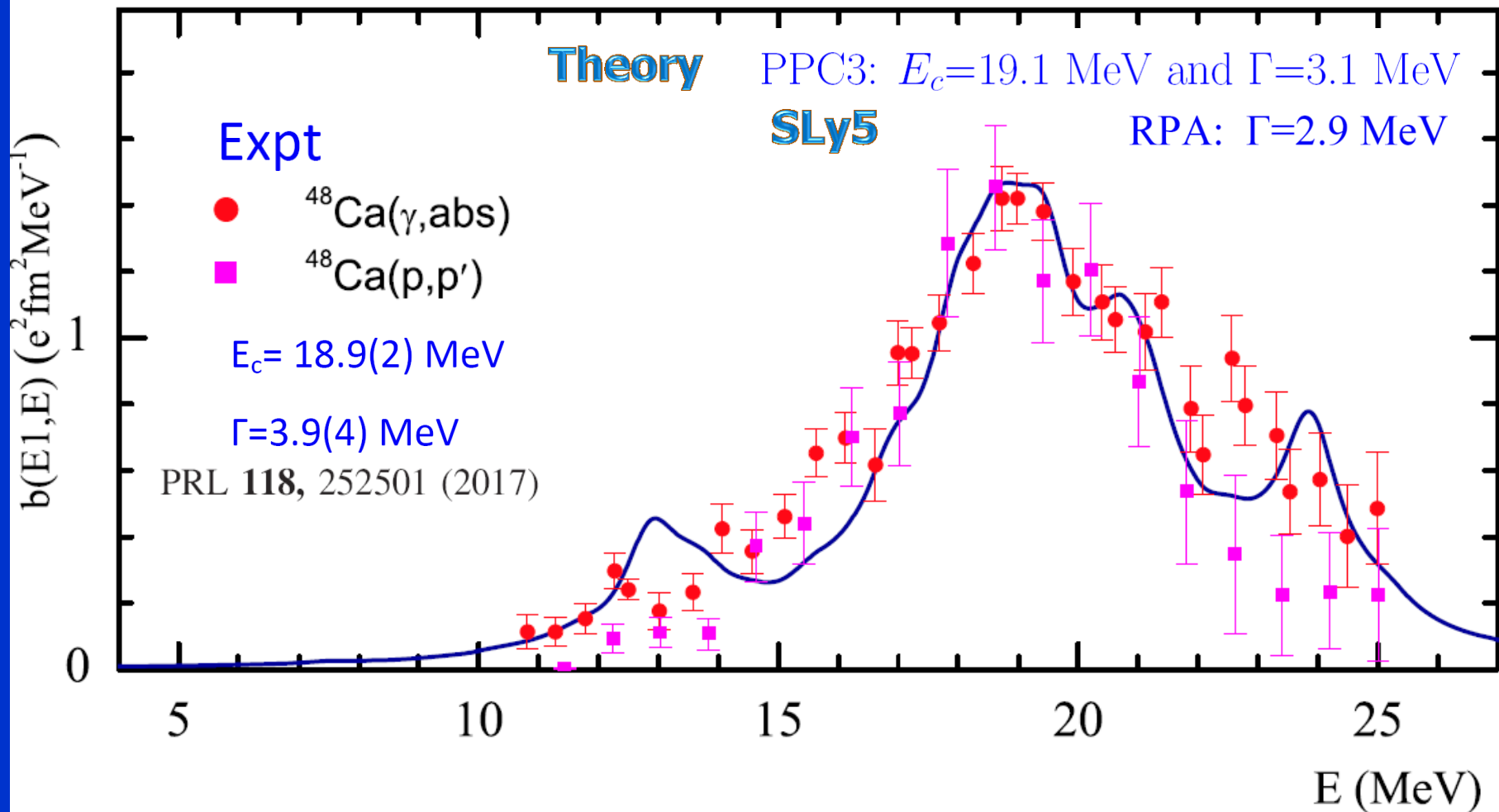
$$\langle 0 | Q_{\lambda i} H [Q_{\lambda_1 i_1}^+ Q_{\lambda_2 i_2}^+]_{\lambda} | 0 \rangle$$

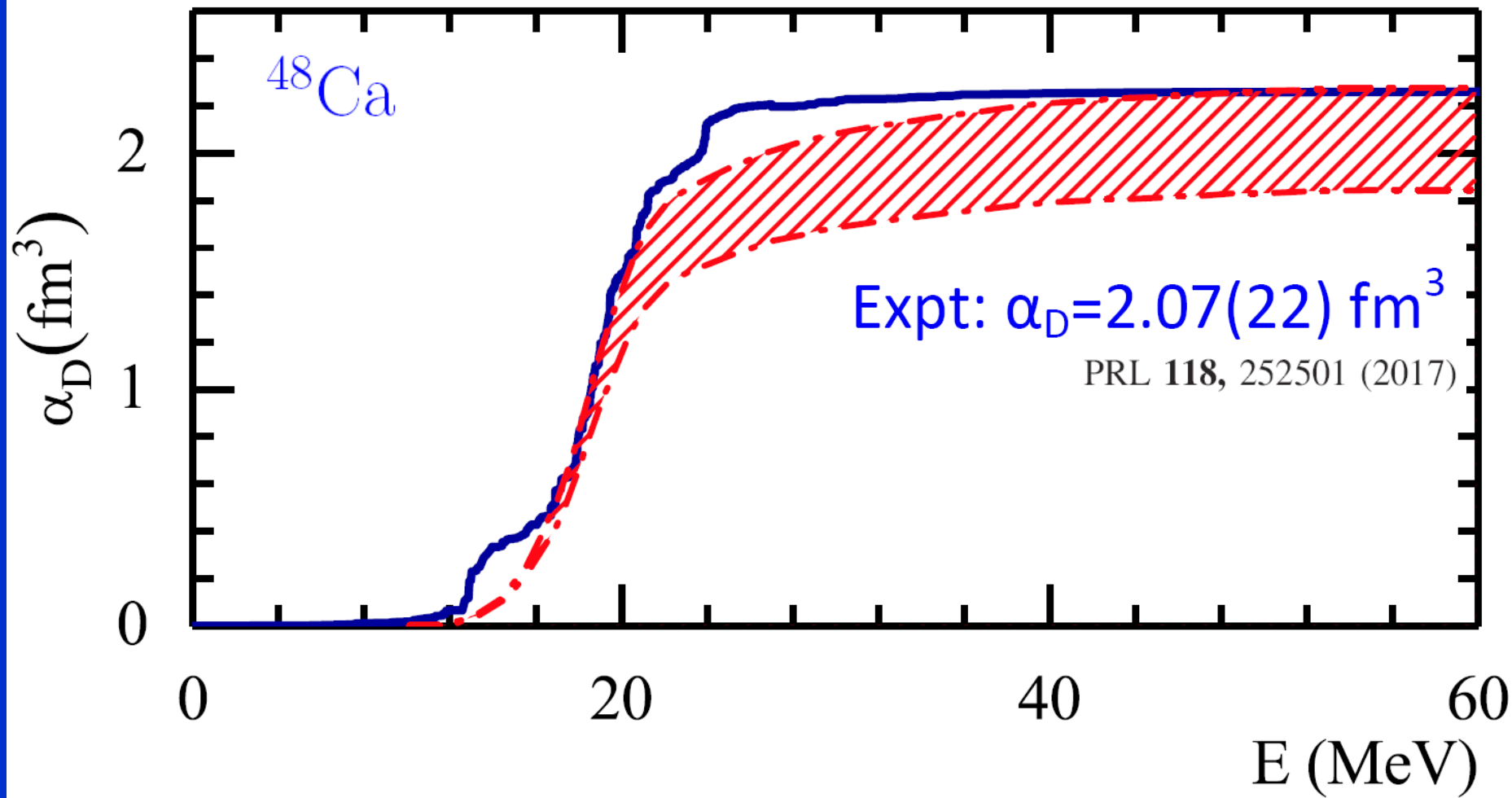
$$\langle 0 | [Q_{\lambda'_2 i'_2} Q_{\lambda'_1 i'_1}]_{\lambda} H [[Q_{\lambda_1 i_1}^+ Q_{\lambda_2 i_2}^+]_{\lambda'} Q_{\lambda_3 i_3}^+]_{\lambda} | 0 \rangle$$

A.P.S., V.V. Voronov, and Nguyen Van Giai, Eur. Phys. J. A22, 397 (2004).

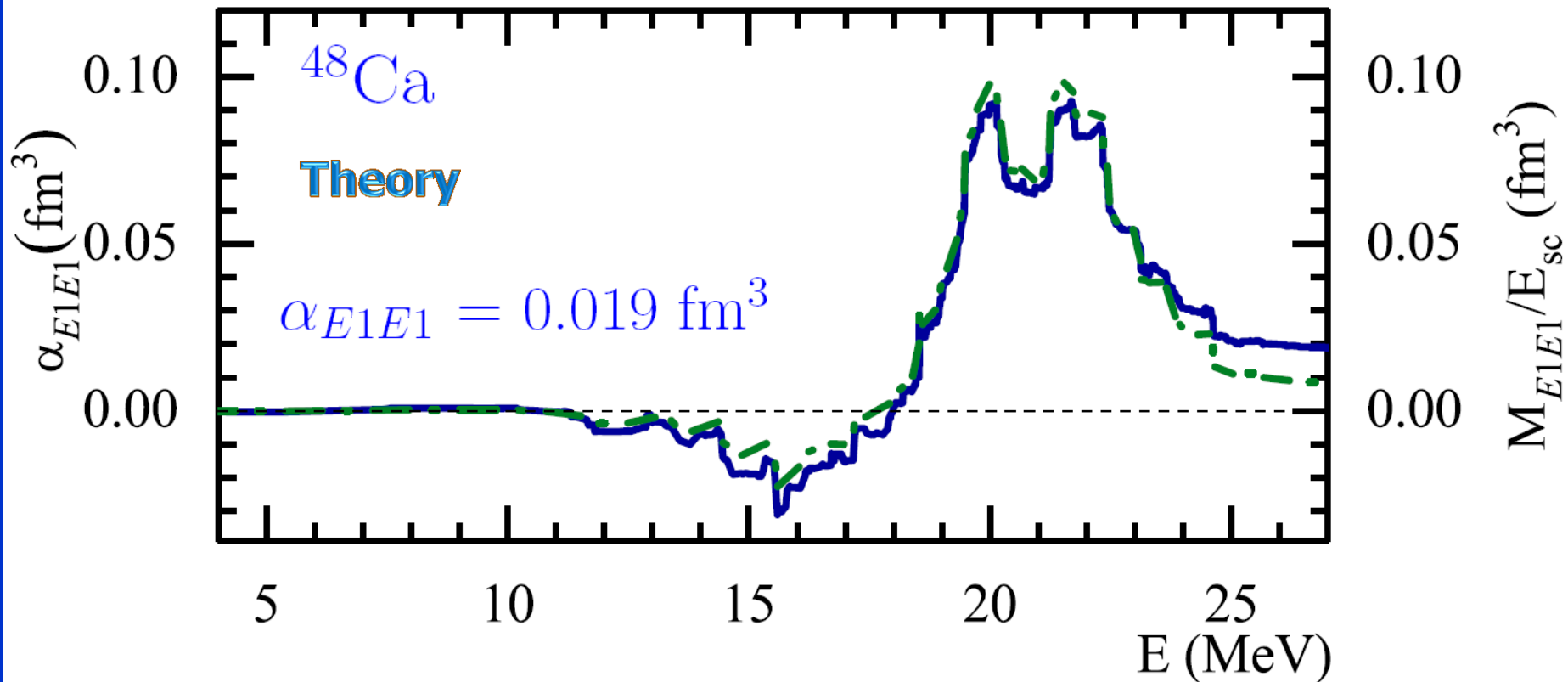
# The pygmy dipole resonance







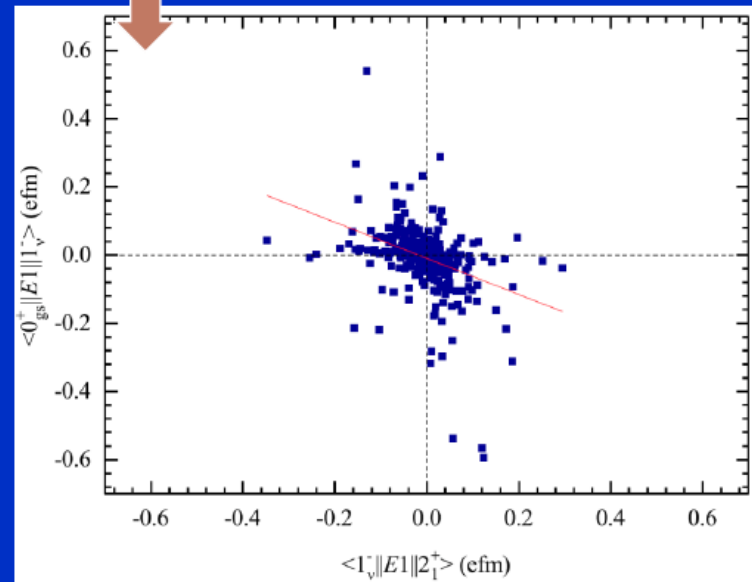
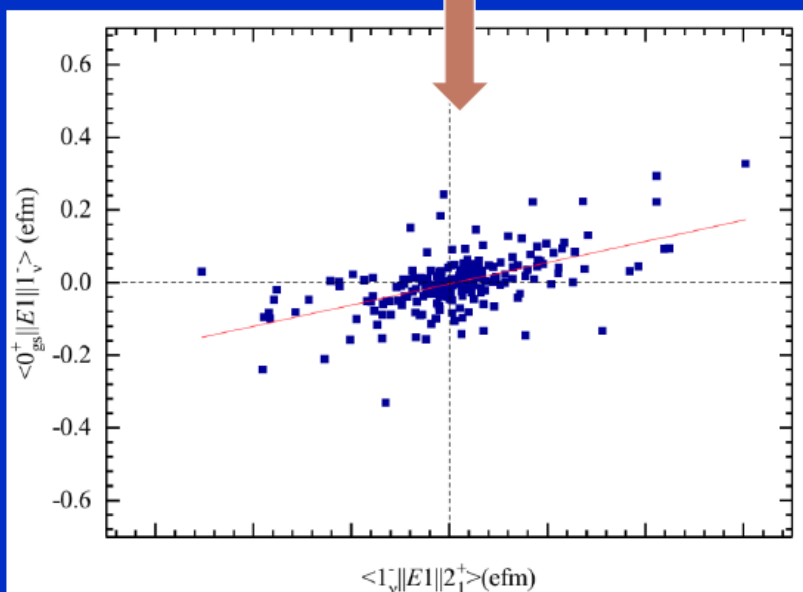
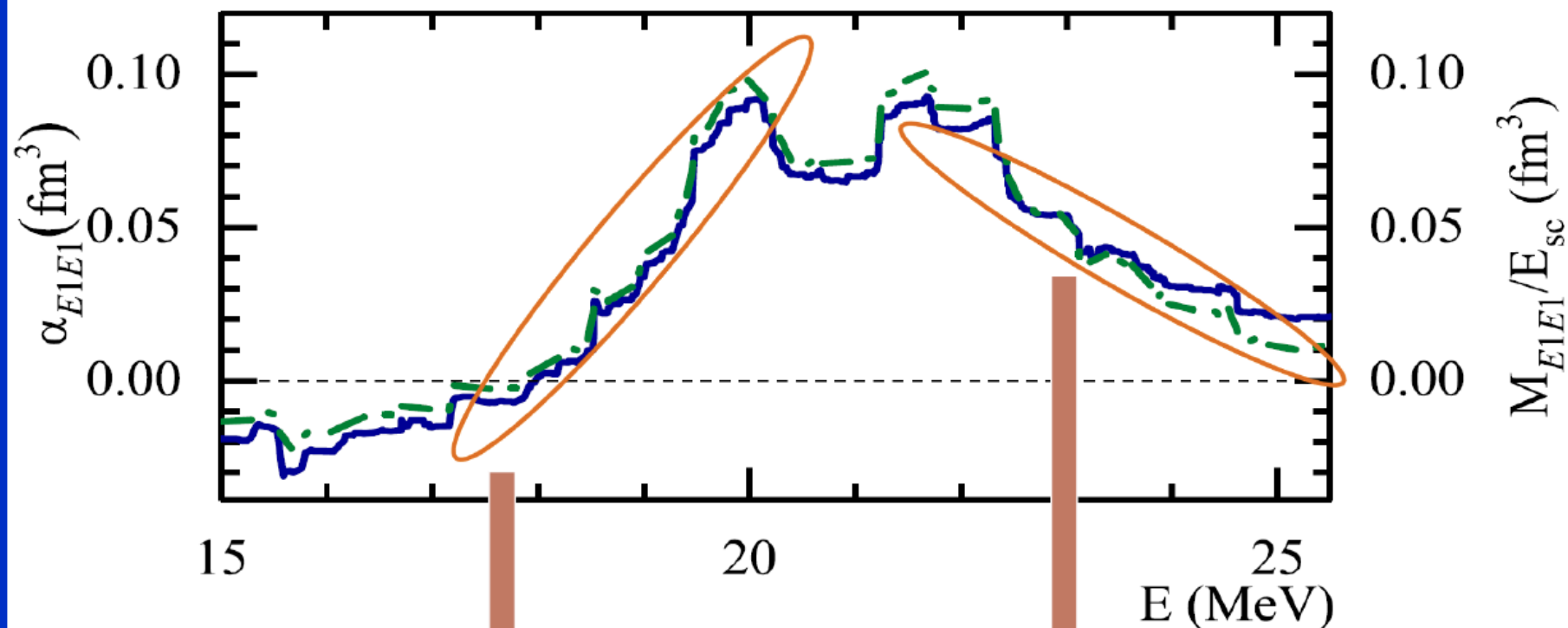
$$\alpha_D = \frac{8\pi}{9} \sum_i \frac{\langle 0_{gs}^+ || M(E1) || 1_i^- \rangle \langle 1_i^- || M(E1) || 0_{gs}^+ \rangle}{E_{1_i^-}}$$

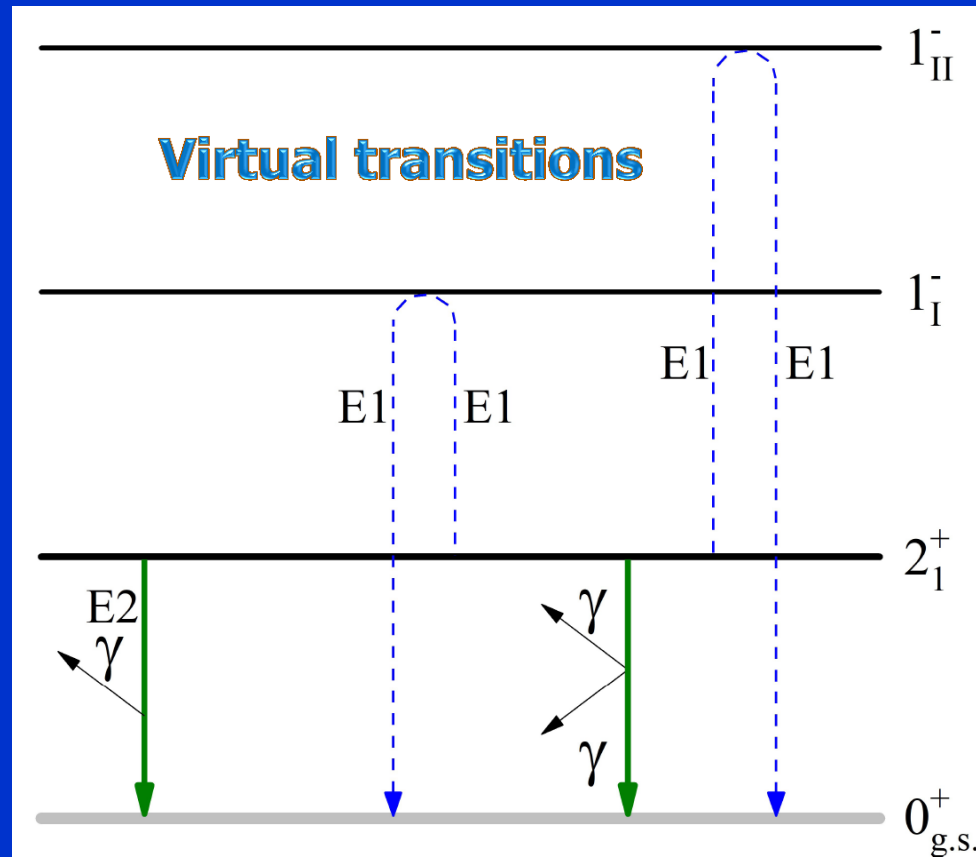


$$\alpha_{E1E1} = \sum_i \frac{\langle 0_{gs}^+ || M(E1) || 1_i^- \rangle \langle 1_i^- || M(E1) || 2_1^+ \rangle}{E_{1_i^-} - 0.5E_{2_1^+}}$$

$$M_{E1E1} = \sum_i \langle 0_{gs}^+ || M(E1) || 1_i^- \rangle \langle 1_i^- || M(E1) || 2_1^+ \rangle$$







We construct a two-state mixing scheme by considering the  $GDR$  state, the coupled  $GDR \otimes 2_1^+$  state, and the interaction  $V$  between them. The relative phases of amplitudes are opposite in the perturbed states I and II, i.e.,

$$|1_I^-\rangle = \alpha|GDR\rangle + \beta|GDR \otimes 2_1^+\rangle,$$

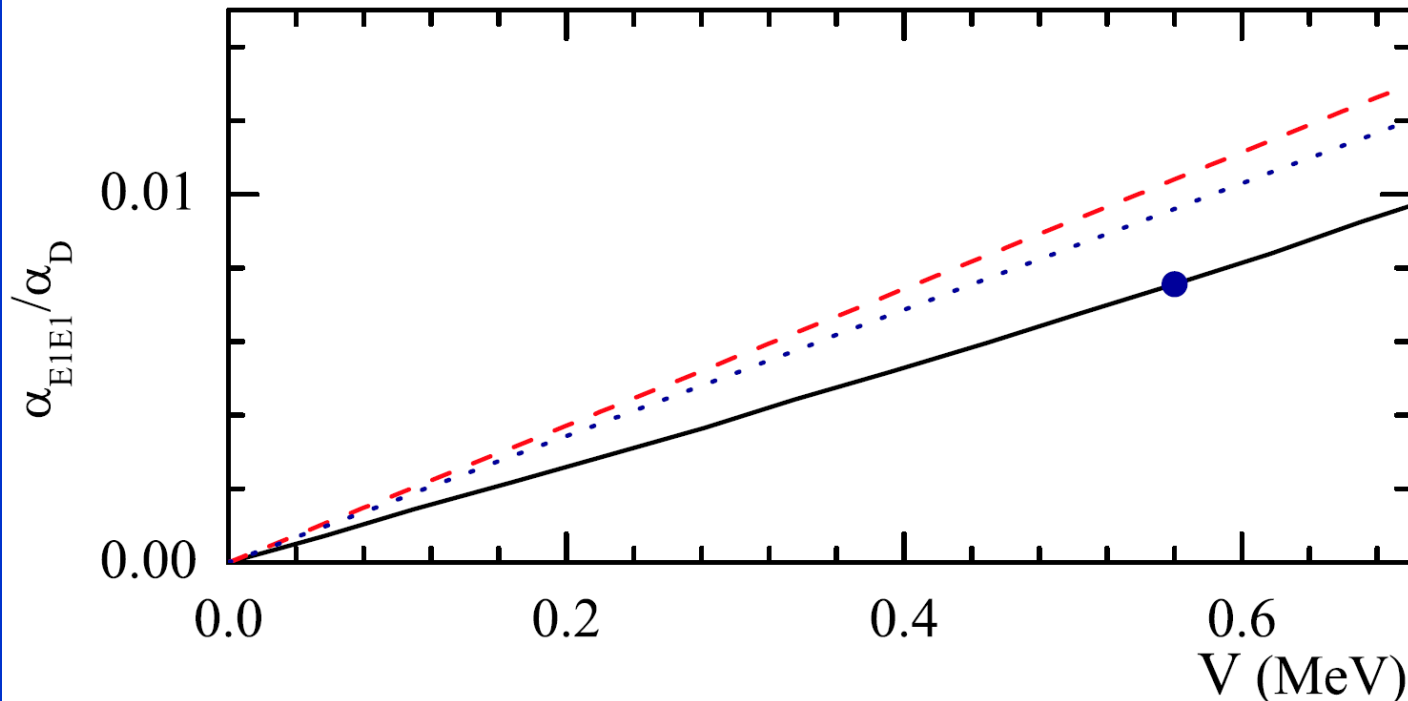
$$|1_{II}^-\rangle = -\beta|GDR\rangle + \alpha|GDR \otimes 2_1^+\rangle.$$

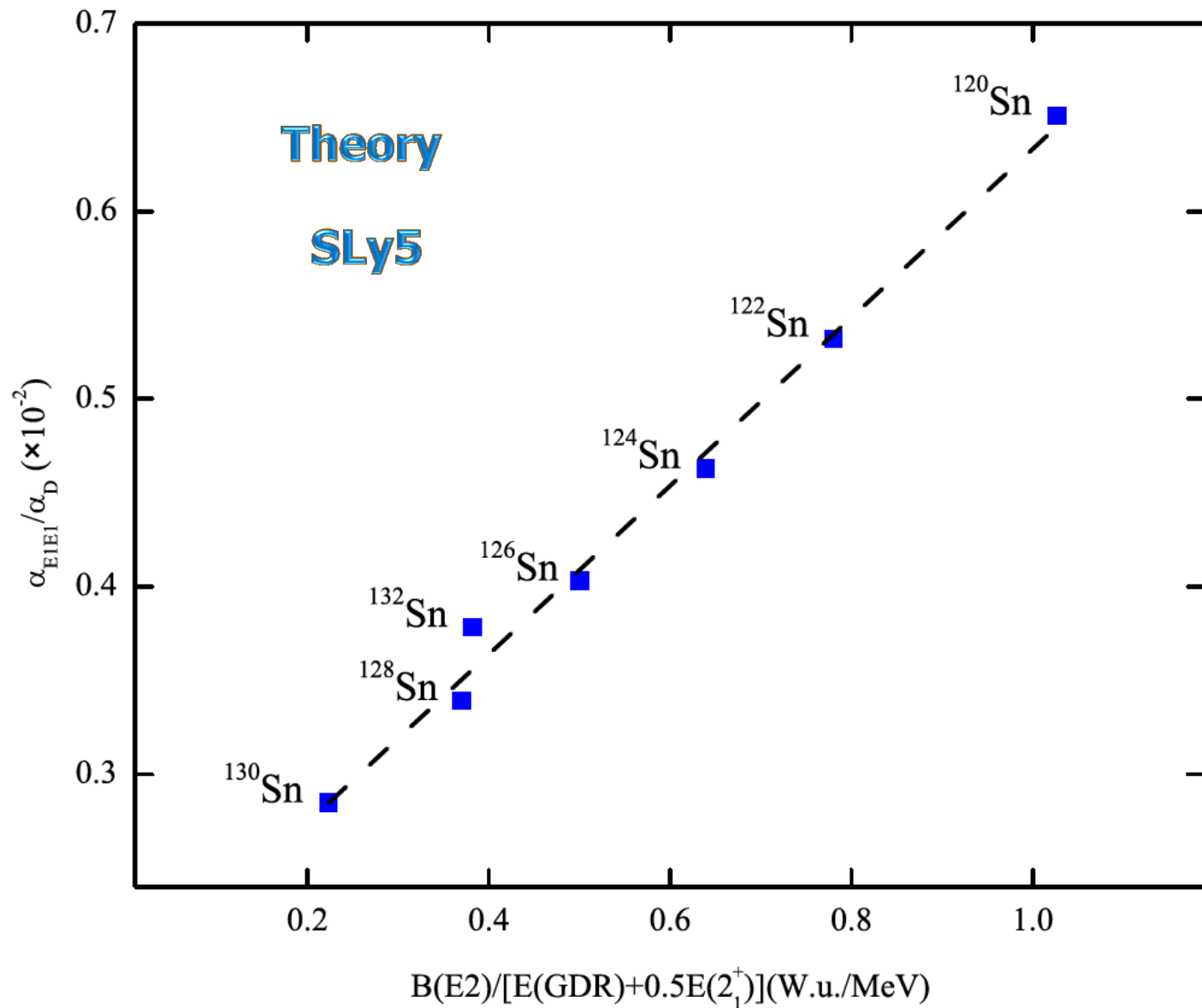
Notice that  $\alpha_{E1E1}/\alpha_D=0$  if the basis configurations do not mix. For two limiting cases of weak or strong mixing, we obtain

$$\frac{\alpha_{E1E1}}{\alpha_D} = \frac{9}{8\pi} \frac{V}{E_{GDR} + 0.5\Delta E_u}.$$

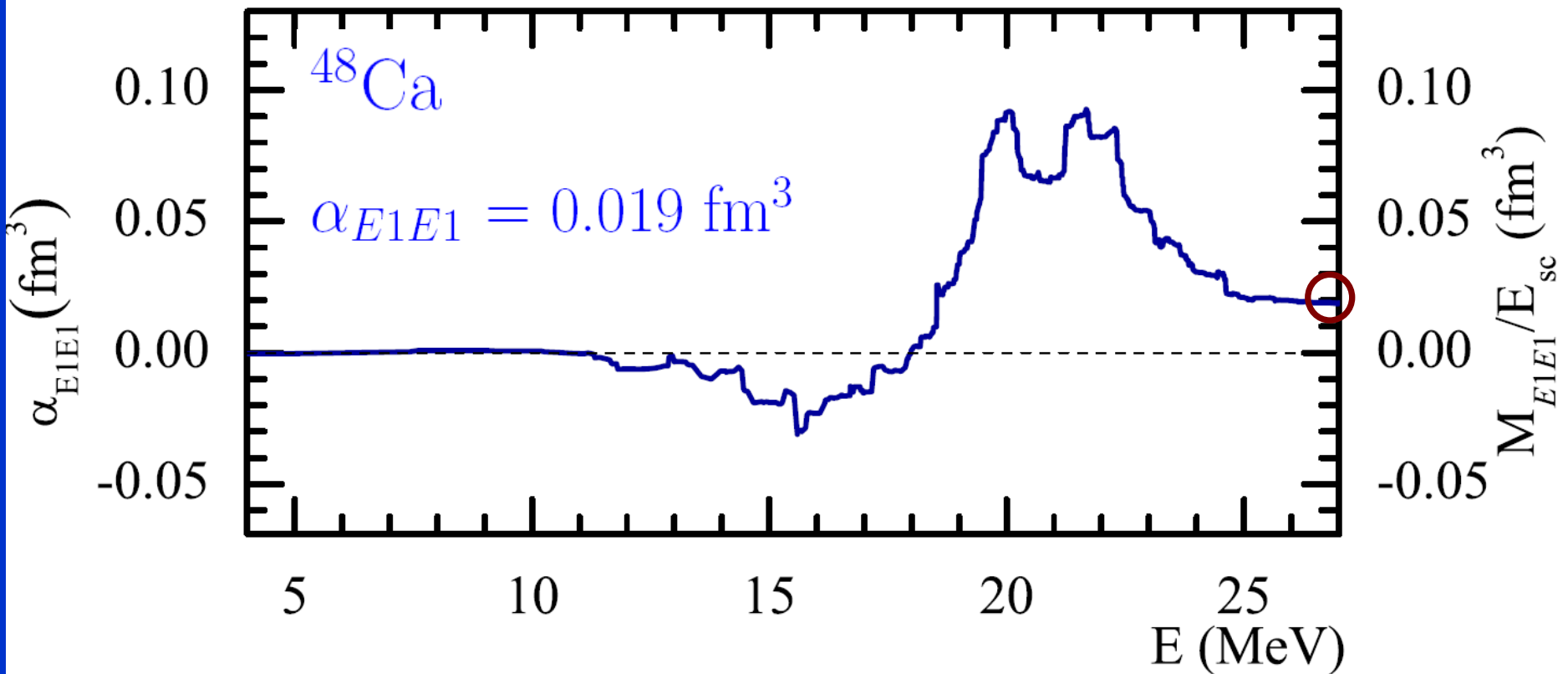
We can simulate the full diagonalization reasonably well by using a two-state mixing. The *GDR* state built on the most collective RPA states, the two-phonon  $GDR \otimes 2_1^+$  state and the interaction

$$V = \frac{1}{\sqrt{N_{1ph}N_{2ph}}} \sum_{i=1}^{N_{1ph}} \sum_{k=1}^{N_{2ph}} \langle 0 | Q_{1i} H [Q_{21}^+ Q_{1k}^+]_1 | 0 \rangle,$$





**Figure 1.** The ratio between polarizabilities  $\alpha_{E1E1}$  and  $\alpha_D$  as a function of  $B(E2; 2_1^+ \rightarrow 0_{gs}^+)$  value with respect to the energy centroid of the GDR ( $E(GDR)$ ) and the  $2_1^+$  energy ( $E(2_1^+)$ ). The  $\alpha_{E1E1}/\alpha_D$  evolution of neutron-rich tin isotopes (squares) is calculated taking into account the phonon-phonon coupling based on the SLy5 EDF. The dashed line corresponds to the linear fit predicted Eq. (6).



the  $2_1^+$  state of  $^{48}\text{Ca}$ :

**Theory**

$E_x = 3.19 \text{ MeV}$   $B(E2) = 1.3 \text{ W.u.}$   $\Gamma_\gamma = 3.5 \times 10^{-3} \text{ eV}$

$\Gamma_{\gamma\gamma} = 1.0 \times 10^{-10} \text{ eV}$   $\frac{\Gamma_{\gamma\gamma}}{\Gamma_\gamma} = 3 \times 10^{-8}$

# Conclusion

- Starting from Skyrme mean-field calculations we have studied for the first time the  $\gamma\gamma/\gamma$  decay of the first  $2^+$  state of an even-even nucleus. As our test case we considered  $^{48}\text{Ca}$  for which its dipole polarizability has recently been measured.
- We use the Skyrme EDF SLy5 to create a single-particle spectrum and to analyze excited states of  $^{48}\text{Ca}$ . Our calculations take into account the coupling between one-, two- and three-phonon terms in the wave functions.
- It is shown that the  $\gamma\gamma$ -decay width is sensitive to the interaction of the one- and two-phonon configurations in the giant dipole resonance region.
- The maximal branching ratio of the competitive  $\gamma\gamma$  -decay relative to its single  $\gamma$ -decay is predicted for  $^{48}\text{Ca}$  as  $3 \times 10^{-8}$ . It is desirable to experimentally establish the  $\gamma\gamma$  decay of a first  $2^+$  state of an even-even nucleus.

Thanks for collaboration:

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*N. Pietralla – IKP, Technische Universität Darmstadt*

**Thank you for your attention!**