Heavy quarkonia in a bulk-viscous quark gluon plasma medium

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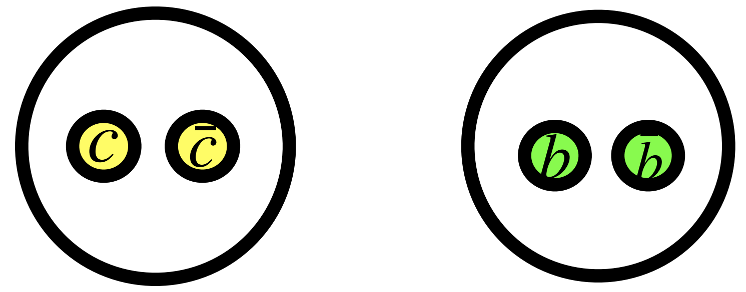
apctp asia pacific center for theoretical physics

Outline

- **Introduction**
- **Heavy quark potential in a bulk viscous medium**
- **Quarkonium spectral functions**
  - **Quarkonium properties**
- **Physical observables**
- **Summary**

Introduction

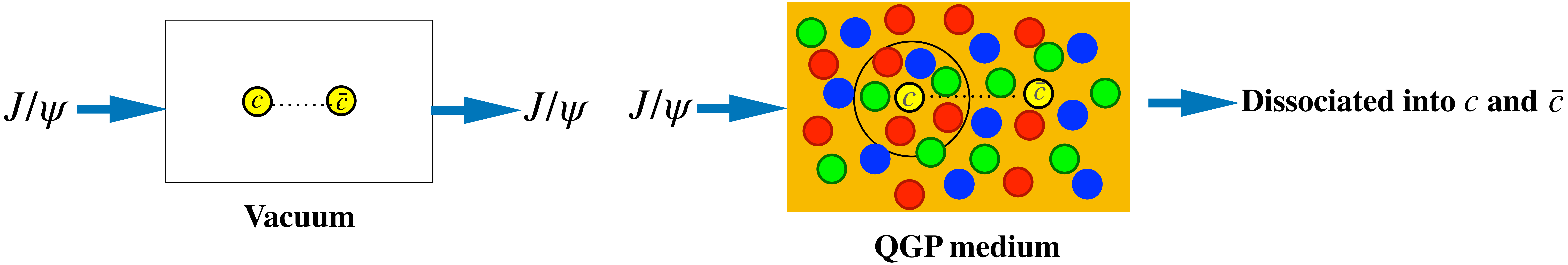
- **Quarkonia** are the bound state of heavy quarks and its own antiquarks



Charmonium Bottomonium

$$m_c, m_b \gg \Lambda_{QCD} \rightarrow \text{Non-relativistic bound states}$$

- Quarkonia masses higher than the QGP temperature $\rightarrow$ thermal production strongly suppressed
- **Quarkonium** $\rightarrow$ as a probe of the quark gluon plasma formed in heavy ion collision
- Weaker colour binding at high temperature plasma $\rightarrow$ Debye screening



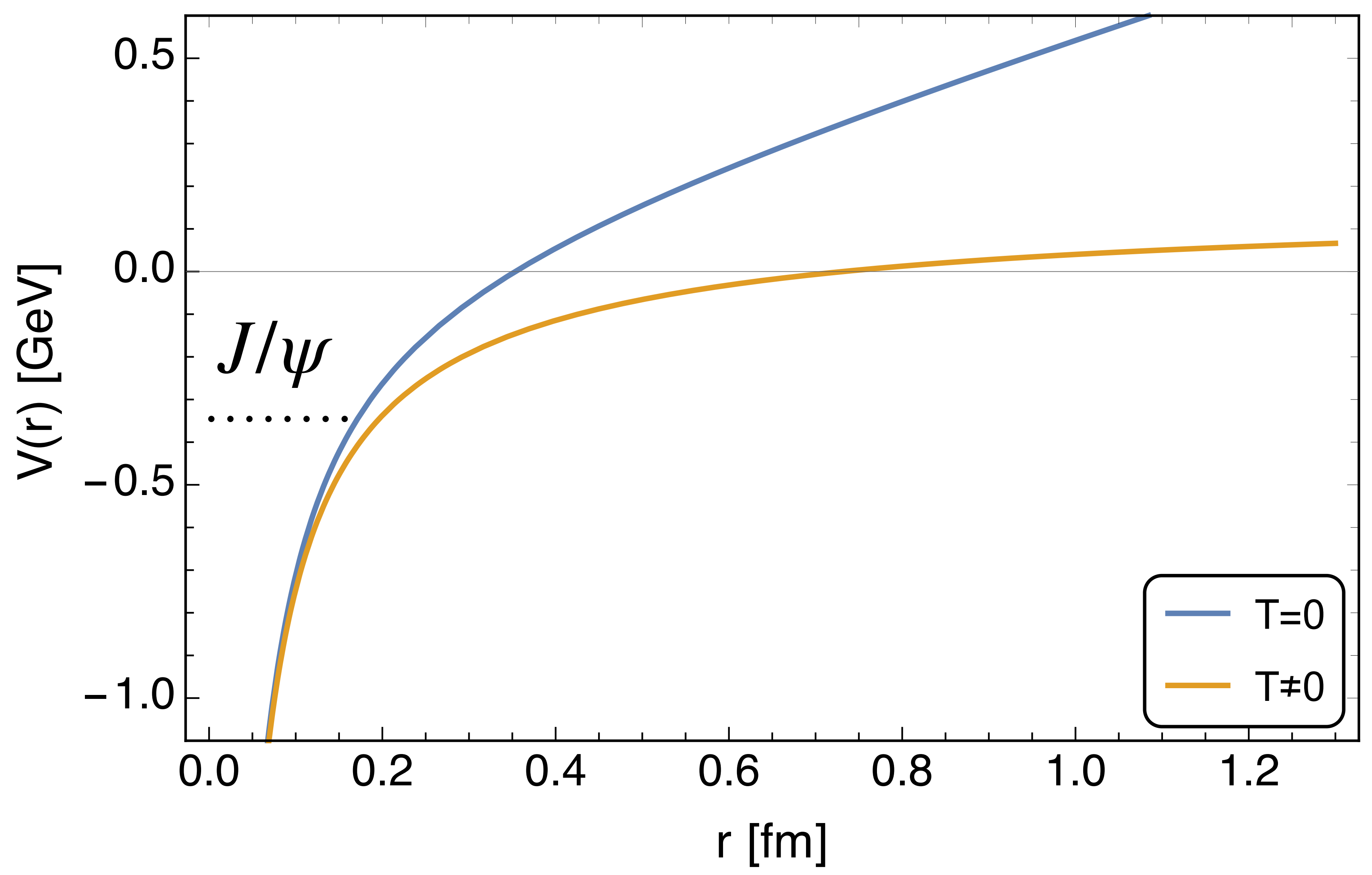
Matsui and H. Satz, Phys. Lett. B178, 416 (1986)

- Quarkonium suppression observed in many experiments of heavy-ion collisions : LHC, SPS, RHIC

Quarkonium properties

- ✓ Potential Models
- ✓ Effective Field Theory (EFT)
- ✓ Lattice QCD: Non-perturbative approach

- **Potential Models** serve as a basic tools to study the properties of quarkonium states



- In vacuum ($T = 0$) : Cornell potential

$$V(r, T = 0) = -\frac{\alpha}{r} + \sigma r$$

- In medium ($T \neq 0$) : Screened potential

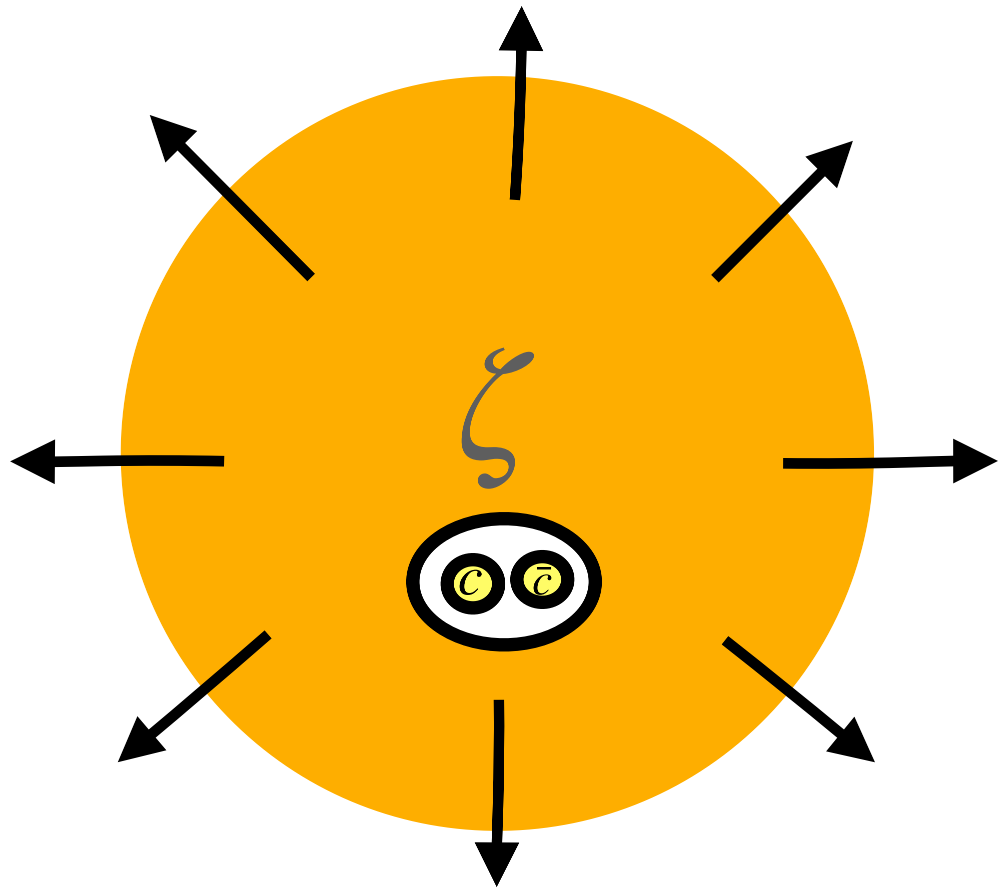
Schwinger model

$$V(r, T) = -\frac{\alpha}{r} e^{-m_D r} + \sigma r \{ \frac{1 - e^{-m_D r}}{m_D r} \}$$

Dixit, MPL A9 (1990)227

Question: heavy quarkonia as a probe of non-eq. QGP?

- QGP has many different **non-equilibrium properties:**
  - Dissipative effects
    - Shear viscosity (momentum anisotropy)
    - **Bulk viscosity**
  - Magnetic field
  - Moving medium
  - …..

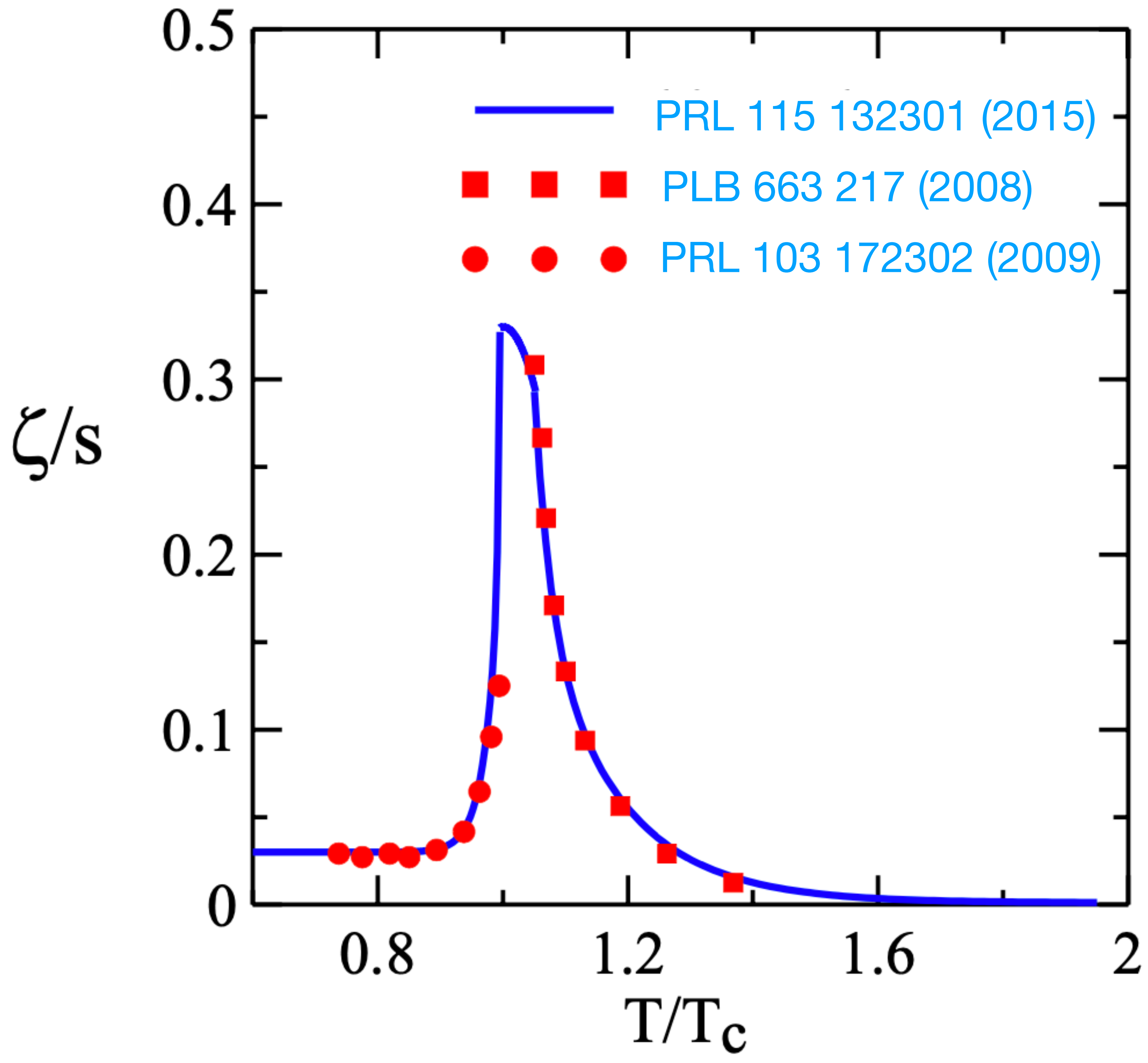


Resistance to isotropic expansion and compression

- QCD matter has non-zero bulk viscosity, which affects the evolution of the medium

- Bulk viscosity of the QCD matter is enhanced near the critical temperature

- Do **heavy quarkonia** work as an alternative probe for non-equilibrium nature of QGP?



Ryu et. al. PRL 115 132301 (2015)

Need to know

✔ **How sensitive are heavy quarkonia to the bulk viscous nature of the fluid ?**
✔ **How sensitive are physical observables?**

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How to incorporate bulk viscous correction

In-medium spectral functions

↑

In-medium heavy quark potential

$$V_{\text{in-medium}}(p) = \varepsilon^{-1}(p)\, V_{\text{Cornell}}(p)$$

↑

Dielectric permittivity $\varepsilon(p)$

$$\varepsilon^{-1}(p) = \lim_{p^0 \to 0} p^2 D^{00}(P)$$

↑

Propagators $D(p)$

$$D^{00}(p) = \frac{1}{2}(D_R + D_A + D_S)$$

↑

Self energies $\Pi(p)$

$$D_R(p) = \frac{1}{p^2 - \Pi_R(p)}$$

How to incorporate bulk viscous correction

Deformed distribution function in the presence of bulk viscous correction

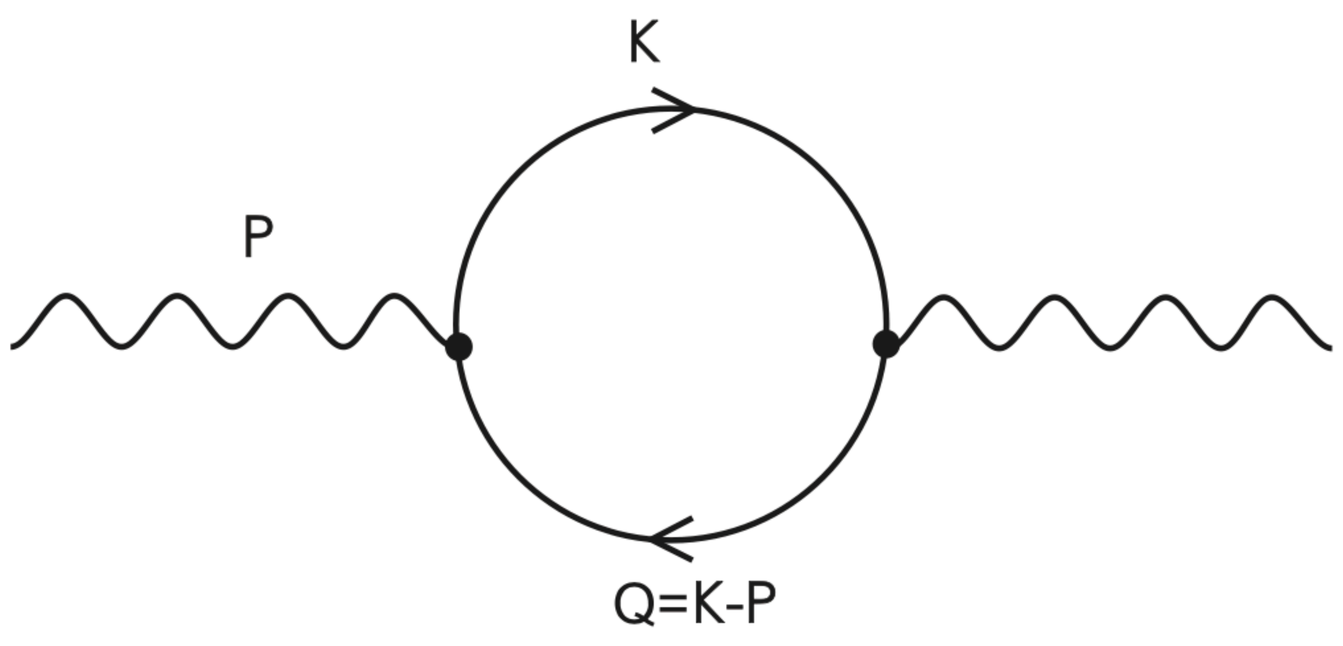
$$f(k) \approx f_{\rm id}(\tilde{k}) + \frac{m^2 \Phi}{2T\sqrt{k^2+m^2}} f_{\rm id}(\tilde{k})(1 \pm f_{\rm id}(\tilde{k}))$$

→ Non-equilibrium corrections from bulk viscosity

$$\Phi \propto \zeta \, \partial_\mu u^\mu$$

Du, Dumitru, Guo, Strickland, JHEP 01 (2017) 123

Quark contribution to the retarded gluon self energy in HTL approximation



$$\Pi_R^{\rm id,q}(P) = N_f \frac{g^2}{\pi^2} \int dk k \left(f_+^{\rm id}(\tilde{k}) + f_-^{\rm id}(\tilde{k}) \right) \left(\frac{p^0}{2p} \ln \frac{p^0 + p + i\epsilon}{p^0 - p + i\epsilon} - 1 \right)$$

$$\Pi_R^{\rm id,q}(P) = N_f \frac{g^2 T^2}{6} \left(1 + \frac{3\mu^2}{\pi^2 T^2} \right) f_q(\tilde{m}, \tilde{\mu}) \left(\frac{p^0}{2p} \ln \frac{p^0 + p + i\epsilon}{p^0 - p + i\epsilon} - 1 \right)$$

where $f_q(\tilde{m}, \tilde{\mu}) = 2\left[1 - \frac{3\tilde{m}\tilde{\mu} - 3\tilde{m}\ln[(1+e^{\tilde{m}+\tilde{\mu}})(1+e^{\tilde{\mu}-\tilde{m}})] - 3[\mathrm{Li}_2(-e^{\tilde{m}-\tilde{\mu}})] + \mathrm{Li}_2(-e^{\tilde{m}+\tilde{\mu}})}{\pi^2 + 3\tilde{\mu}^2} \right]$

Gluon contribution to the retarded gluon self energy

$$\Pi_R^{\mathrm{id,g}}(P) = 2N_c \frac{g^2T^2}{6} f_g(\tilde{m}) \left(\frac{p^0}{2p} \ln \frac{p^0 + p + i\epsilon}{p^0 - p + i\epsilon} - 1 \right)$$

where $$f_g(\tilde{m}) = \frac{1}{\pi^2} \left(3\tilde{m}^2 + 2\pi^2 - 6\tilde{m} \ln[e^{\tilde{m}} - 1] - 6\mathrm{Li}_2(e^{\tilde{m}}) \right)$$

Total retarded gluon self energy

$$\begin{aligned} \Pi_R^{\mathrm{id}}(P) &= \Pi_R^{\mathrm{id,q}}(P) + \Pi_R^{\mathrm{id,g}}(P) \\ &= m_{D,R}^2 \left(\frac{p^0}{2p} \ln \frac{p^0 + p + i\epsilon}{p^0 - p + i\epsilon} - 1 \right) \end{aligned}$$

$$m_{D,R}^2 = \frac{g^2T^2}{6} \left(N_f f_q(\tilde{m}, \tilde{\mu}) \ \left(1 + \frac{3\tilde{\mu}^2}{\pi^2} \right) + 2N_c f_g(\tilde{m}) \right)$$

Bulk viscous correction to quark contribution of gluon self energy

$$\delta_{\text{bulk}}\Pi_R^{(q)}(P) = N_f \frac{g^2T^2}{6}\frac{\tilde{m}^2}{\pi^2}\Phi\left(\frac{3}{1+e^{\tilde{m}-\tilde{\mu}}}+\frac{3}{1+e^{\tilde{m}+\tilde{\mu}}}\right)\left(\frac{p^0}{2p}\ln\frac{p^0+p+i\epsilon}{p^0-p+i\epsilon}-1\right)$$

Bulk viscous correction to gluon contribution

$$\delta_{\text{bulk}}\Pi_R^{g}(P) = 2N_c \frac{g^2T^2}{6}\frac{\tilde{m}^2}{\pi^2}\Phi\left(\frac{3}{e^{\tilde{m}}-1}\right)\left(\frac{p^0}{2p}\ln\frac{p^0+p+i\epsilon}{p^0-p+i\epsilon}-1\right)$$

Quasiparticle mass

$$m^2(T,\mu) = \frac{G^2(T)T^2N_c}{9}+\frac{G^2(T)T^2N_f}{18}\left(1+\frac{3\mu^2}{\pi^2T^2}\right)$$

Peshier, Kampfer, Pavlenko and Soff, PRD **54** (1996) 2399

Total retarded gluon self energy

$$\Pi_R(P) = \Pi_R^{\text{id}}(P) + \delta_{\text{bulk}}\Pi_R(P)$$

Retarded self energy

$$\Pi_R(P) = \widetilde{m}^2_{D,R}\left(\frac{p^0}{2p}\ln\frac{p^0+p+i\epsilon}{p^0-p+i\epsilon}-1\right)$$

Modified retarded Debye mass

$$\widetilde{m}^2_{D,R} = m^2_{D,R} + \delta m^2_{D,R}$$

Symmetric self energy

$$\Pi_S(P) \; = -2\pi i\, \widetilde{m}^2_{D,S}\, \frac{T}{p}\, \Theta(p^2 - p_0^2)$$

Modified symmetric Debye mass

$$\widetilde{m}^2_{D,S} = m^2_{D,S} + \delta m^2_{D,S}$$

Retarded propagator

$$\bar{D}_R(P) = \frac{1}{p^2 + \widetilde{m}^2_{D,R}}$$

Symmetric propagator

$$\bar{D}_S = -\frac{2\pi i T \widetilde{m}^2_{D,S}}{p(p^2 + \widetilde{m}^2_{D,R})^2}$$

Total propagator $$D^{00}(p) = \frac{1}{2}(D_R + D_A + D_S)$$

$$\mathrm{Re}\, D^{00} = \frac{1}{2}(D_R + D_A)$$

$$\mathrm{Im}\, D^{00} = \frac{1}{2} D_S$$

Dielectric permittivity $$\varepsilon^{-1}(p) = \lim_{p^0 \to 0} p^2 D^{00}(P)$$

$$\varepsilon^{-1}(p) = \frac{p^2}{p^2 + \widetilde{m}^2_{D,R}} - i \frac{\pi T p\, \widetilde{m}^2_{D,S}}{(p^2 + \widetilde{m}^2_{D,R})^2}$$

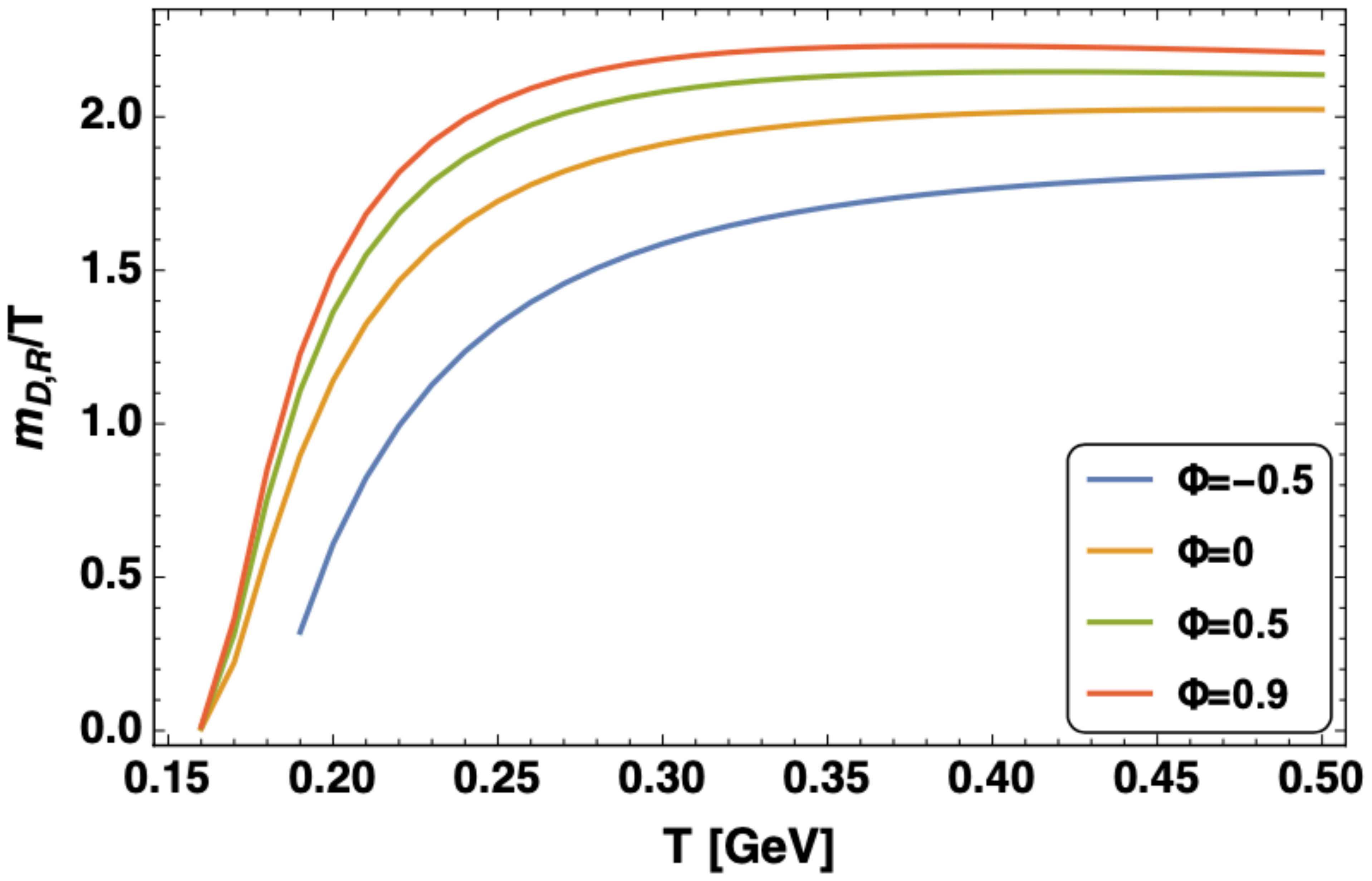
For without bulk viscous correction and massless case

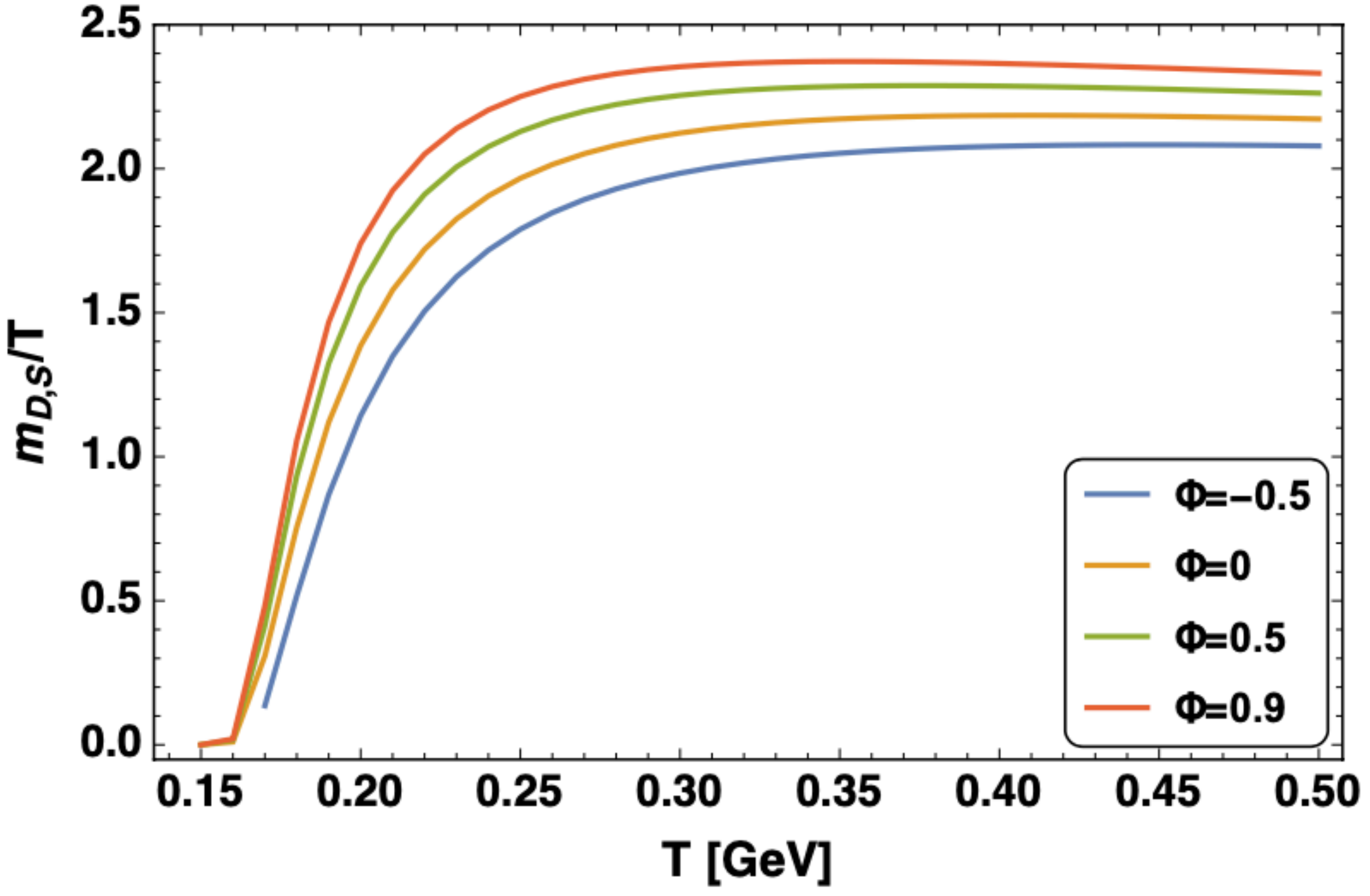
$$\widetilde{m}^2_{D,R} = \widetilde{m}^2_{D,S} = m_D^2 = \frac{g^2 T^2}{6}\left[2N_c + N_f\left(1 + \frac{3\tilde{\mu}^2}{\pi^2}\right)\right]$$

Equilibrium Debye mass

Dielectric permittivity $$\varepsilon^{-1}(p) = \frac{p^2}{p^2 + m_D^2} - i \frac{\pi T p\; m_D^2}{(p^2 + m_D^2)^2}$$

Debye masses in the presence of bulk viscous correction





- Debye screening increases as a function of Φ
- Non-linear behaviour of Debye masses with T ⟶ non-perturbative effects

Real and imaginary part of the potential

$$V_{\text{in-medium}}(p) = \varepsilon^{-1}(p)\, V_{\text{Cornell}}(p)$$

Real part of the potential

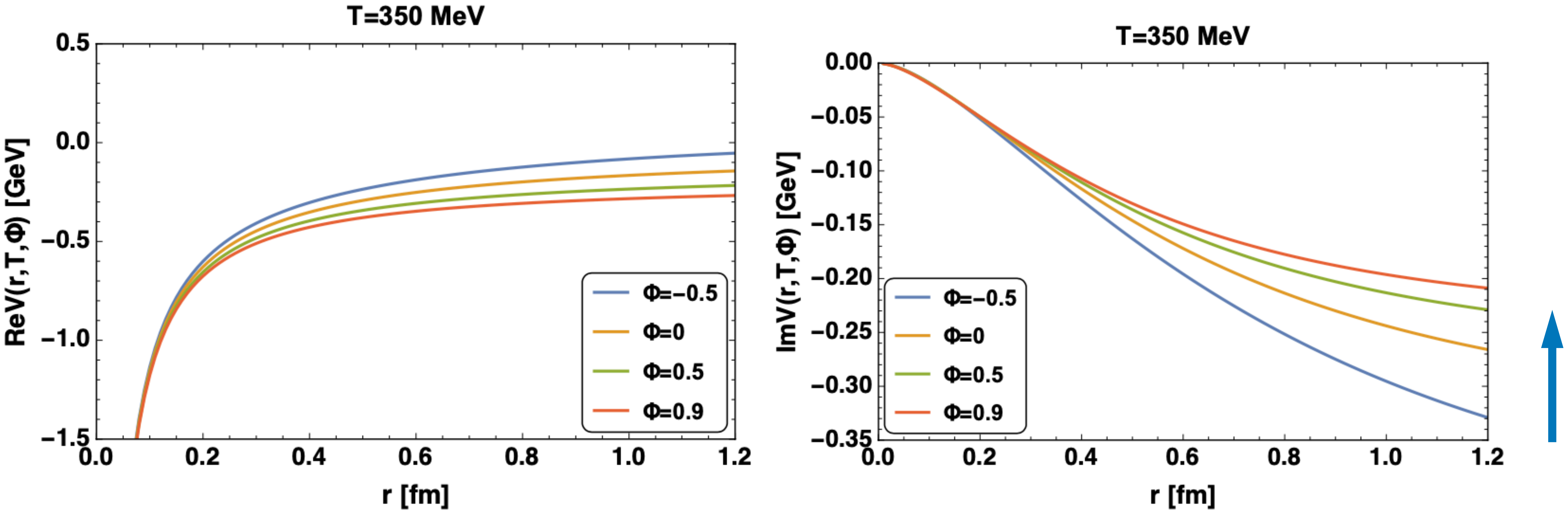
$$\mathrm{Re}\, V(r,T,\Phi) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} (e^{i\mathbf{p}\cdot\mathbf{r}} - 1) V_{\text{Cornell}}(p) \mathrm{Re}\, \varepsilon^{-1}(p)$$
$$= -\alpha\, \widetilde{m}_{D,R} \left(\frac{e^{-\widetilde{m}_{D,R}\, r}}{\widetilde{m}_{D,R}\, r} + 1 \right) + \frac{2\sigma}{\widetilde{m}_{D,R}} \left(\frac{e^{-\widetilde{m}_{D,R}\, r} - 1}{\widetilde{m}_{D,R}\, r} + 1 \right) + c$$

Imaginary part of the potential

$$\mathrm{Im}\, V(r) = \int \frac{d^3\mathbf{p}}{(2\pi)^{3/2}} (e^{i\mathbf{p}\cdot\mathbf{r}} - 1) V_{\text{Cornell}}(p)\, \mathrm{Im}\, \varepsilon^{-1}(p)$$
$$= -\alpha \lambda T\, \phi_2(\widetilde{m}_{D,R}\, r) - \frac{2\sigma T \lambda}{\widetilde{m}^2_{D,R}} \chi(\widetilde{m}_{D,R}\, r)$$

$$\phi_n(x) \equiv 2 \int_0^\infty dz \frac{z}{(z^2+1)^n} \left[1 - \frac{\sin(xz)}{xz} \right]$$
$$\chi(x) \equiv 2 \int_0^\infty \frac{dz}{z(z^2+1)^2} \left[1 - \frac{\sin(xz)}{xz} \right].$$

$$\lambda \equiv \frac{\widetilde{m}^2_{D,S}}{\widetilde{m}^2_{D,R}}$$



- Effect of bulk viscous corrections:
  - Larger screening
  - Suppression of $|\mathrm{Im}V|$ at large r

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Quarkonium spectral functions

s-wave vector channel spectral function

$$\rho^V(\omega) = \lim_{\mathbf{r},\mathbf{r}'\to 0} \frac{1}{2}\tilde{G}(\omega;\mathbf{r},\mathbf{r}')$$

$$\tilde{G}(\omega;\mathbf{r},\mathbf{r}') = \int_{-\infty}^{\infty} dt e^{i\omega t} G^{>}(t;\mathbf{r},\mathbf{r}')$$

Schrödinger equation

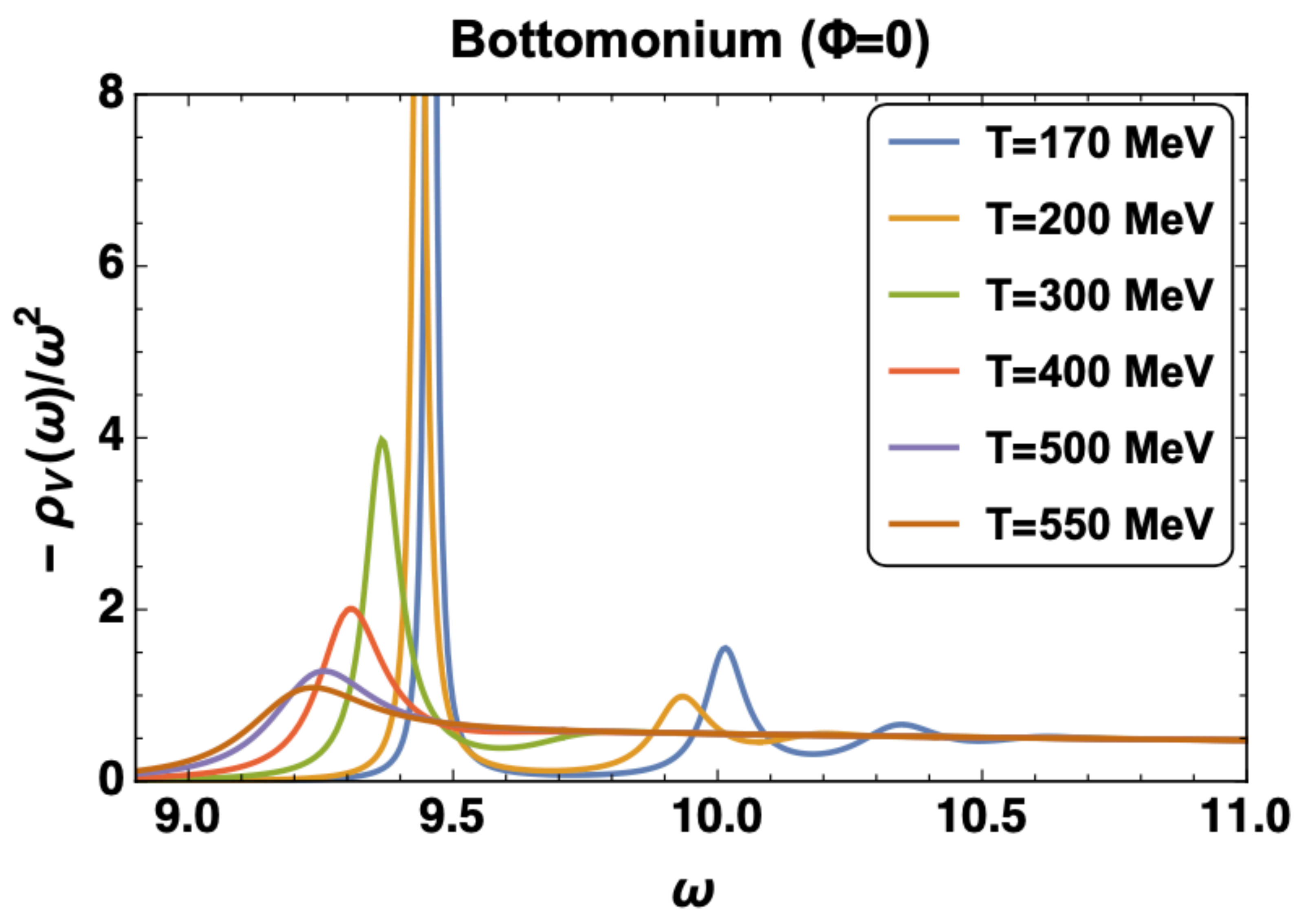
$$\left[\hat{H} \mp i|\mathrm{Im}V(r,T,\Phi)|\right] G^{>}(t;\mathbf{r},\mathbf{r}') = i\partial_t G^{>}(t;\mathbf{r},\mathbf{r}')$$

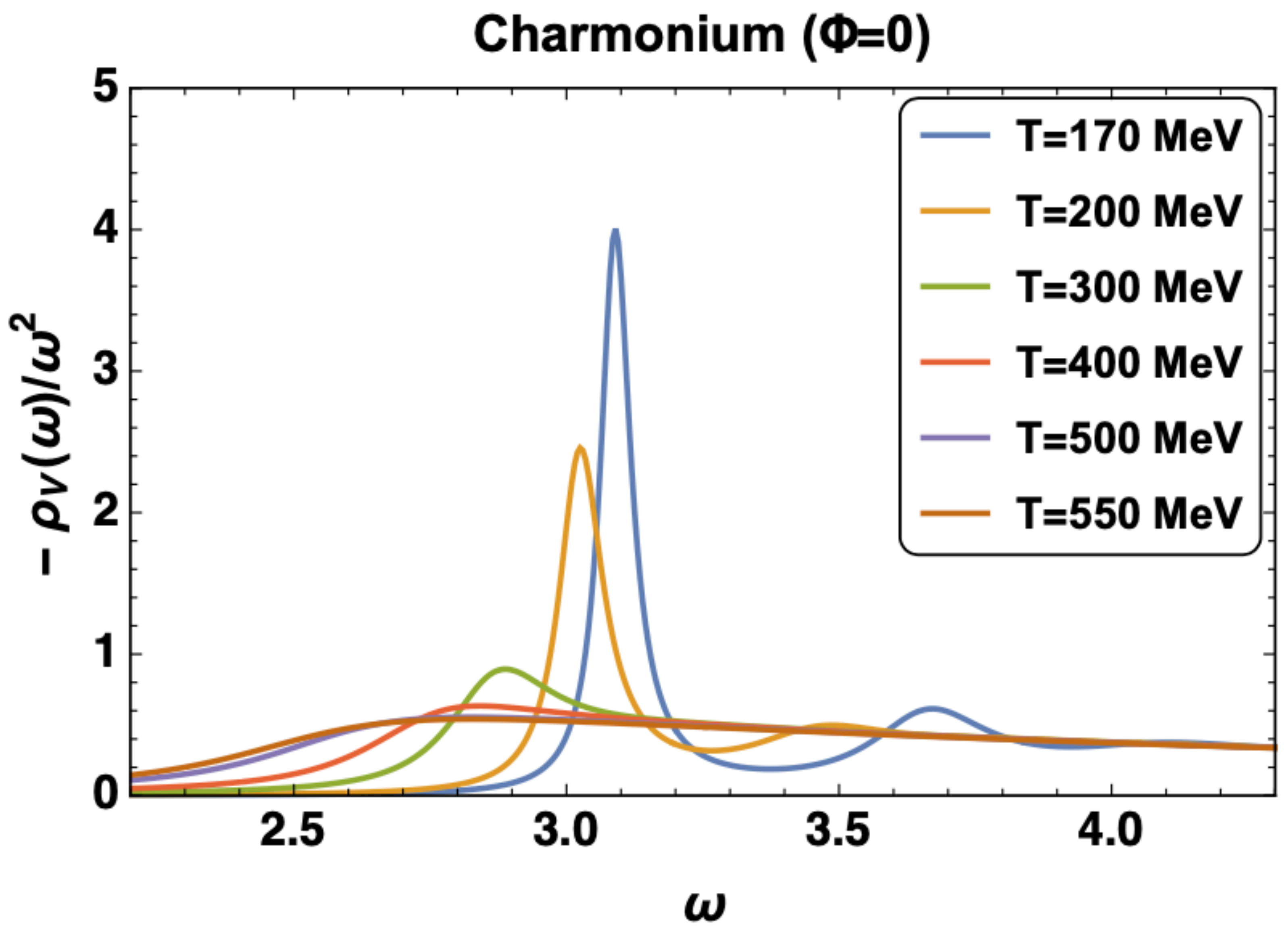
Burnier et. al., JHEP 01 (2008) 043

where $$\hat{H} = 2m_Q - \frac{\nabla_r^2}{m_Q} + \frac{l(l+1)}{m_Q r^2} + \mathrm{Re}\,V(r,T,\Phi)$$

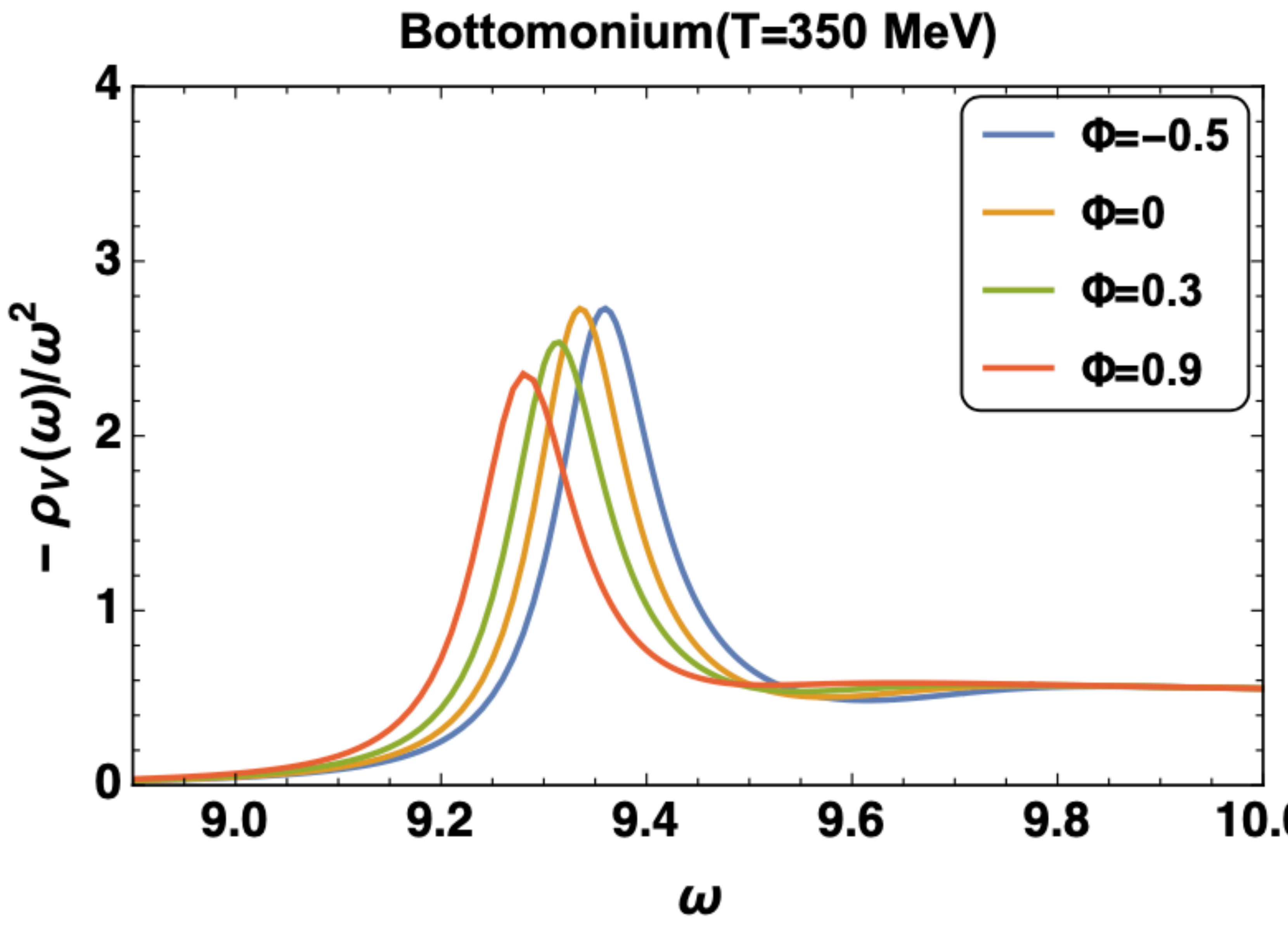
Quarkonium spectral functions

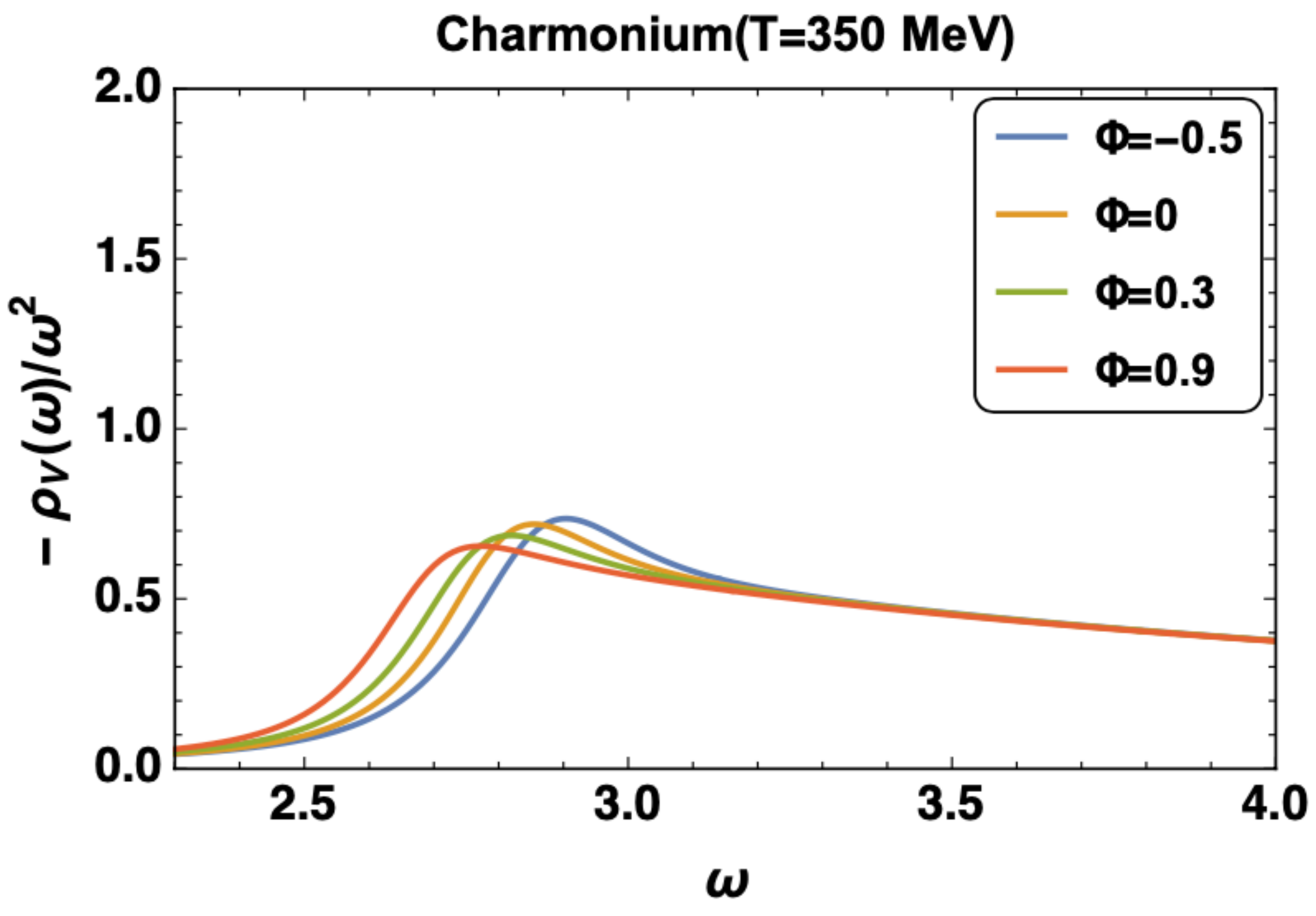
For $\Phi = 0$





For $\Phi \neq 0$

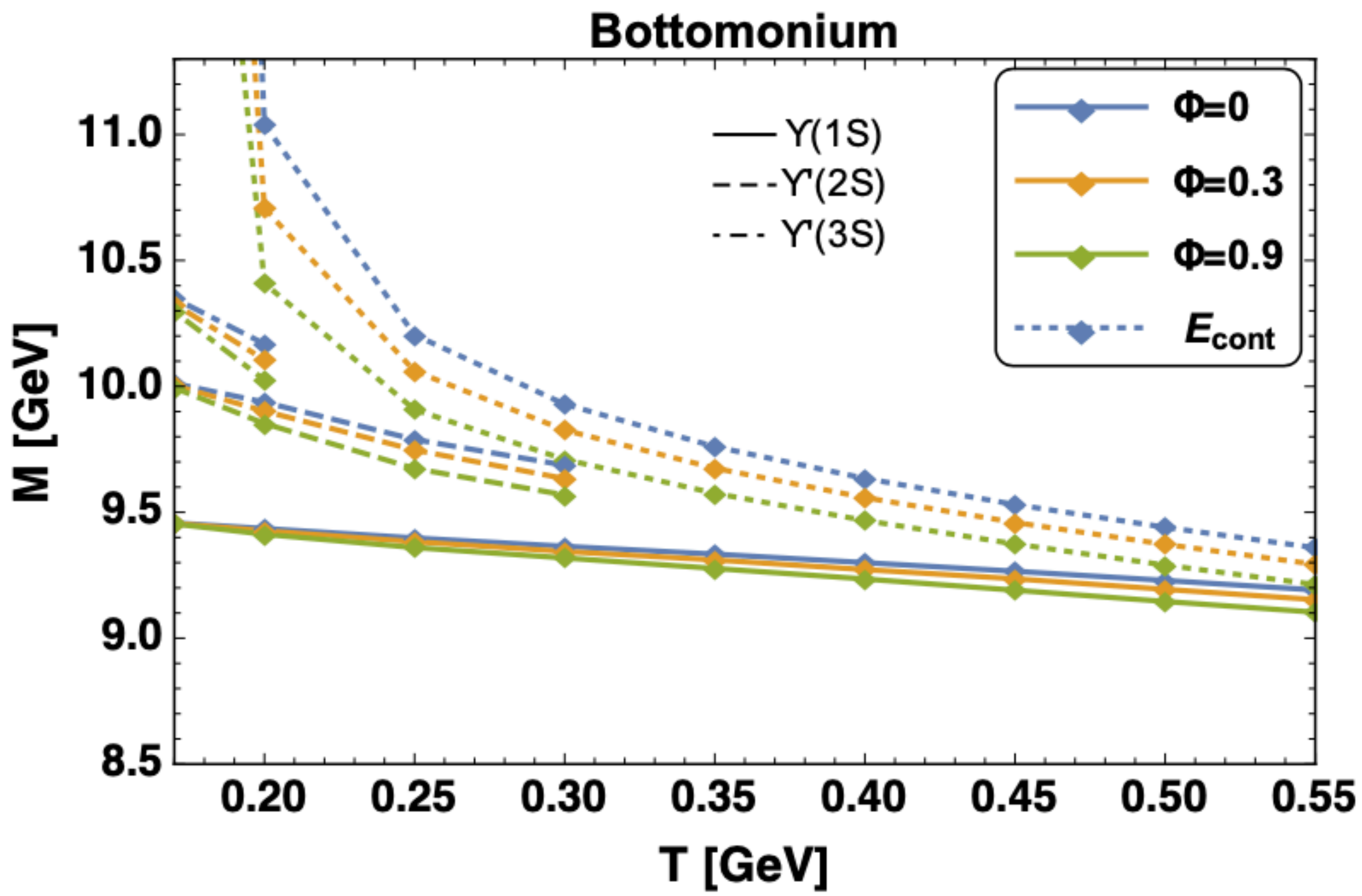


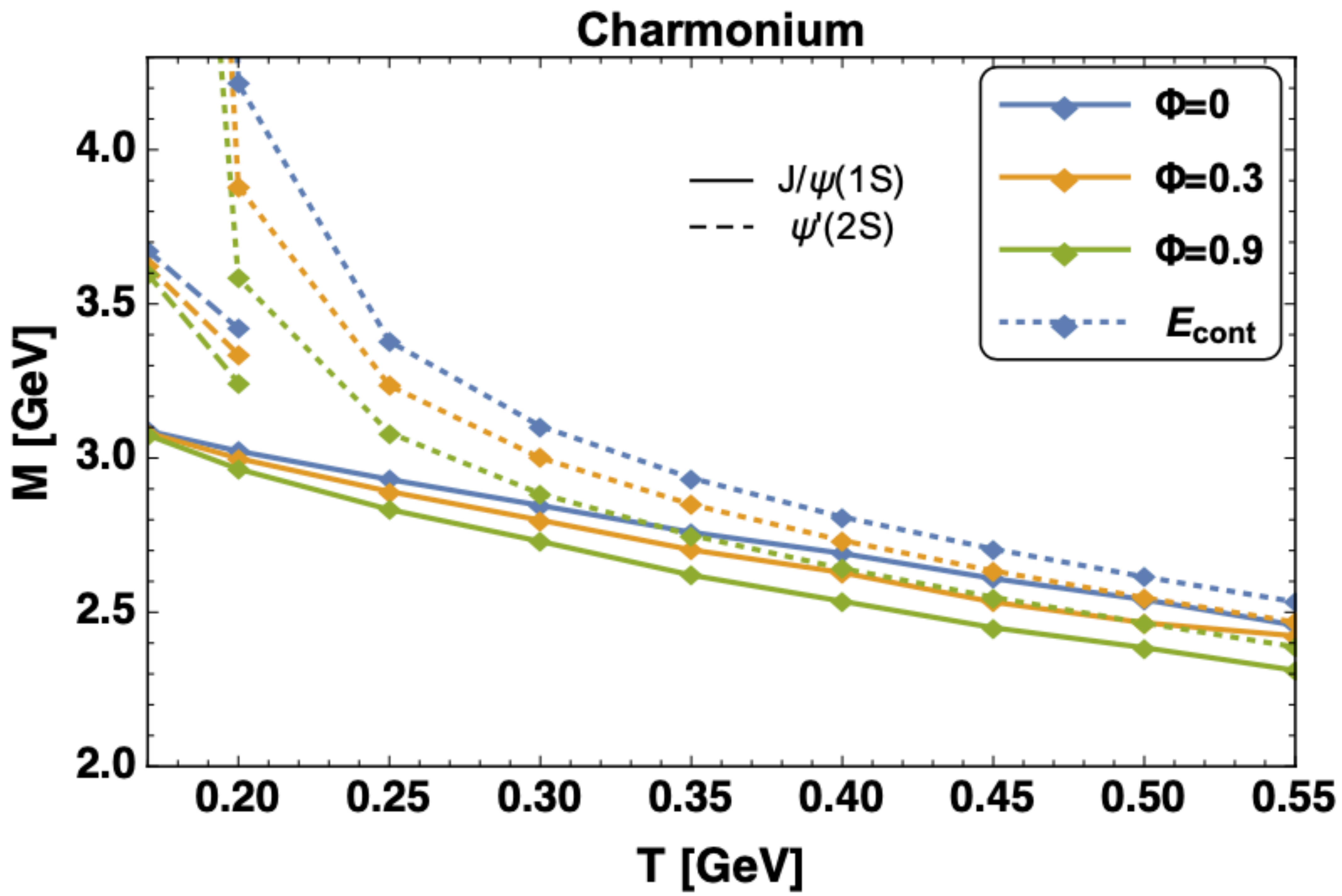


In-medium masses of quakonium states

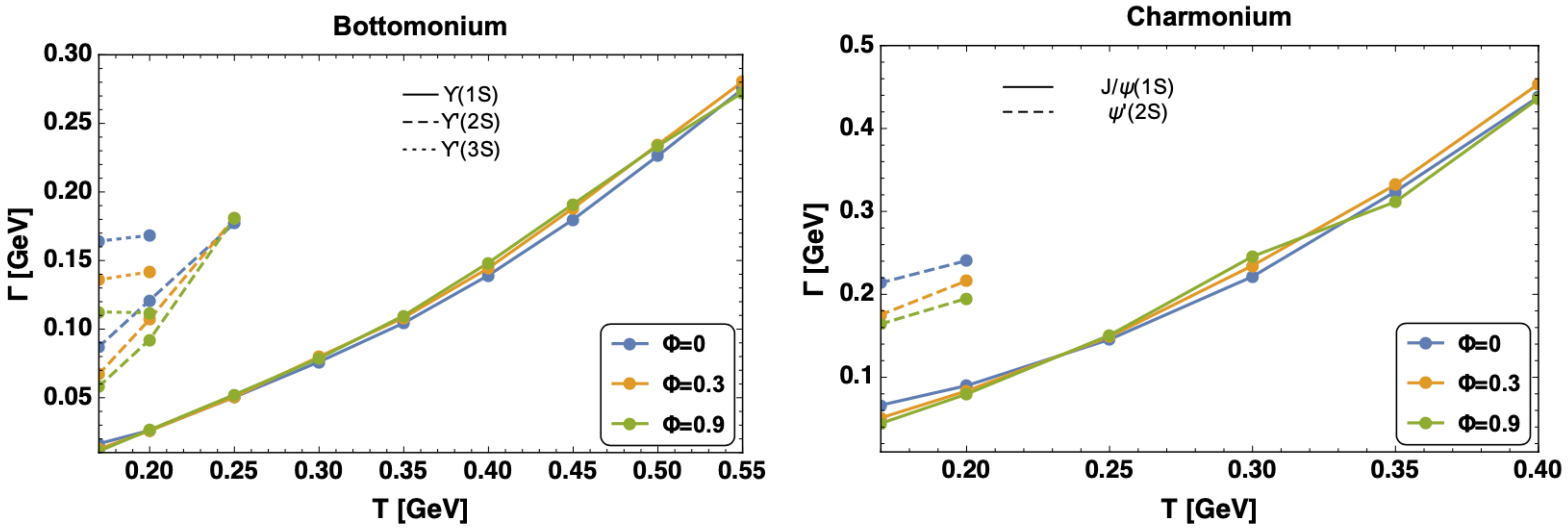
Fitting of the spectral function with the skewed Breit-Wigner form

$$\rho(\omega \approx E) = C\frac{(\Gamma/2)^2}{(\Gamma/2)^2 + (\omega - E)^2} + 2\delta\frac{(\omega - E)\Gamma/2}{(\Gamma/2)^2 + (\omega - E)^2} + A_1 + A_2(\omega - E) + O(\delta^2)$$





Decay widths of quarkonium states



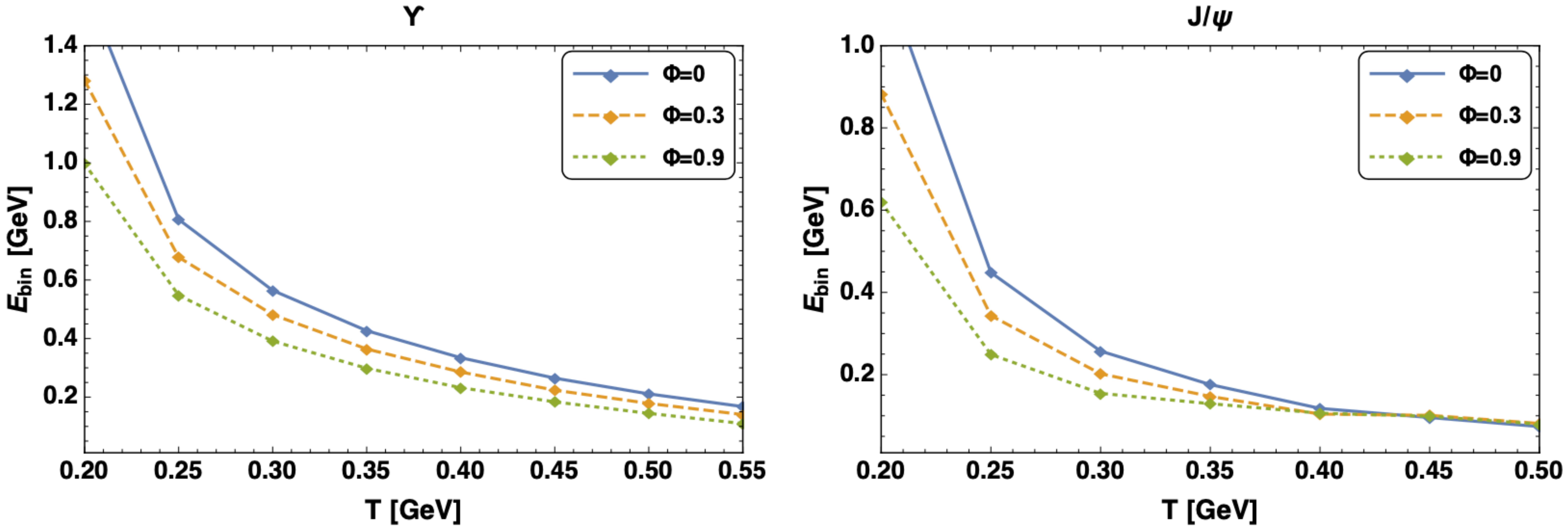
- Two competing effects
  - Spreading of wave functions $\longrightarrow$ increase decay widths
  - Suppression of $|\mathrm{Im}V|$ at large r $\longrightarrow$ decrease decay widths

$$\Gamma \approx \int \psi^* |\mathrm{Im}V| \psi$$

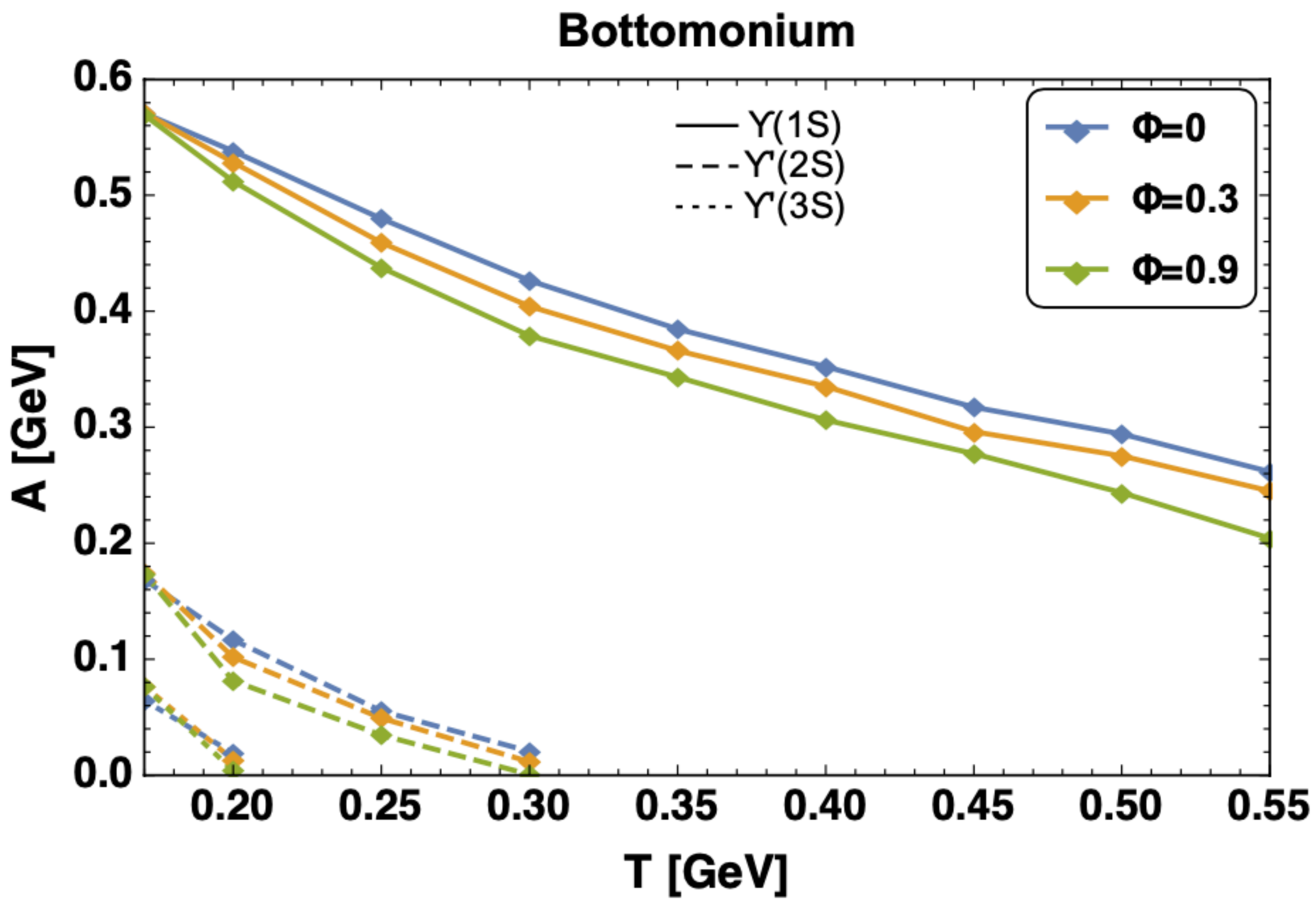
R_{AA} of excited states are more sensitive to bulk viscous corrections than R_{AA} of ground states

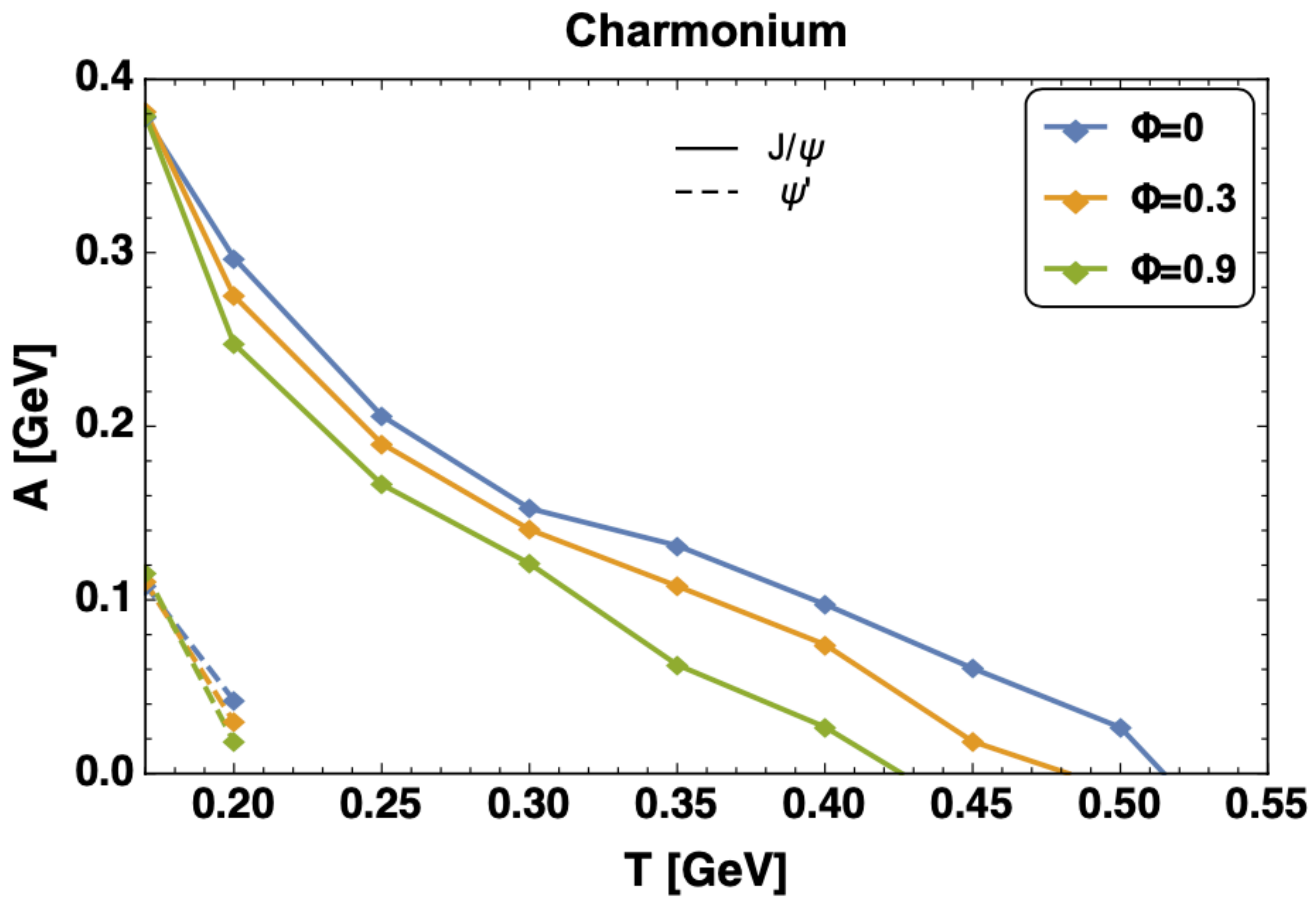
Binding energies of quarkonium states

$$E_{\text{bin}} = 2m_{c,b} + V(r \to \infty) - M$$



Area under the bound states peak





- Bound states peak area decreases as a function of bulk viscous correction

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Relative Production yield ψ' to J/ψ ratio

$$\frac{N_{\psi'}}{N_{J/\psi}} = \frac{R_{l\bar{l}}^{\psi'}}{R_{l\bar{l}}^{J/\psi}} \cdot \frac{M_{\psi'}^2 |\psi_{J/\psi}(0)|^2}{M_{J/\psi}^2 |\psi_{\psi'}(0)|^2}$$

G. T. Bodwin, E. Braaten, and G. P. Lepage, PRD 51 (1995)

Where $$R_{l\bar{l}} \propto \int d\omega d^3\mathbf{k}\, n_B(\sqrt{\omega^2 + \mathbf{k}^2}) \frac{\rho^V(\omega)}{\omega^2} \frac{\omega}{\sqrt{\omega^2 + \mathbf{k}^2}}$$

$$\rho^V(\omega)/\omega^2 \longrightarrow \sum_n A_n \delta(\omega - M_n)$$

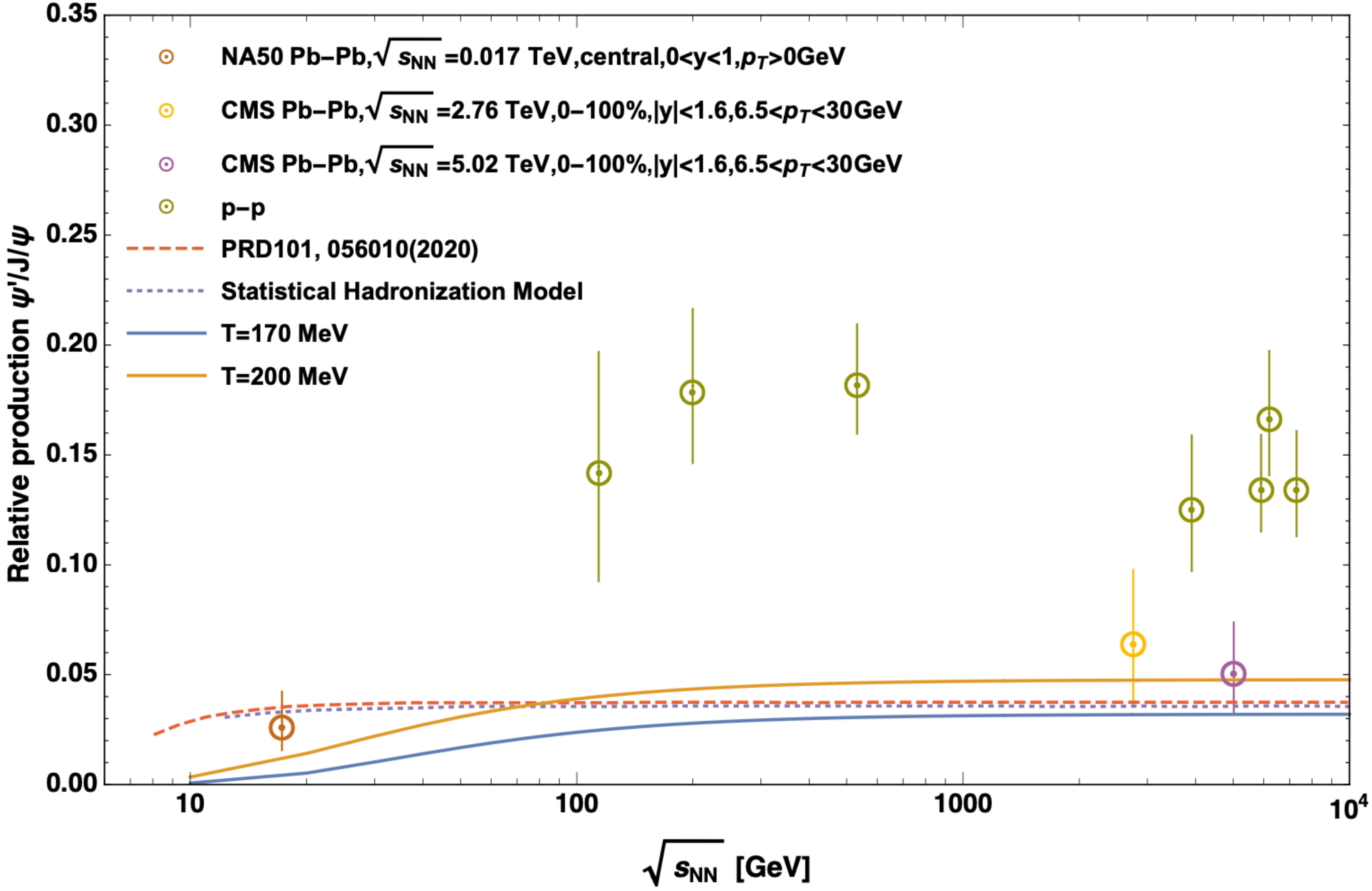
$$R_{l\bar{l}} \propto A_n \int d^3\mathbf{k}\, n_B(\sqrt{M_n^2 + \mathbf{k}^2}) \frac{M_n}{\sqrt{M_n^2 + \mathbf{k}^2}}$$

Area under the bound states peak (A_n)

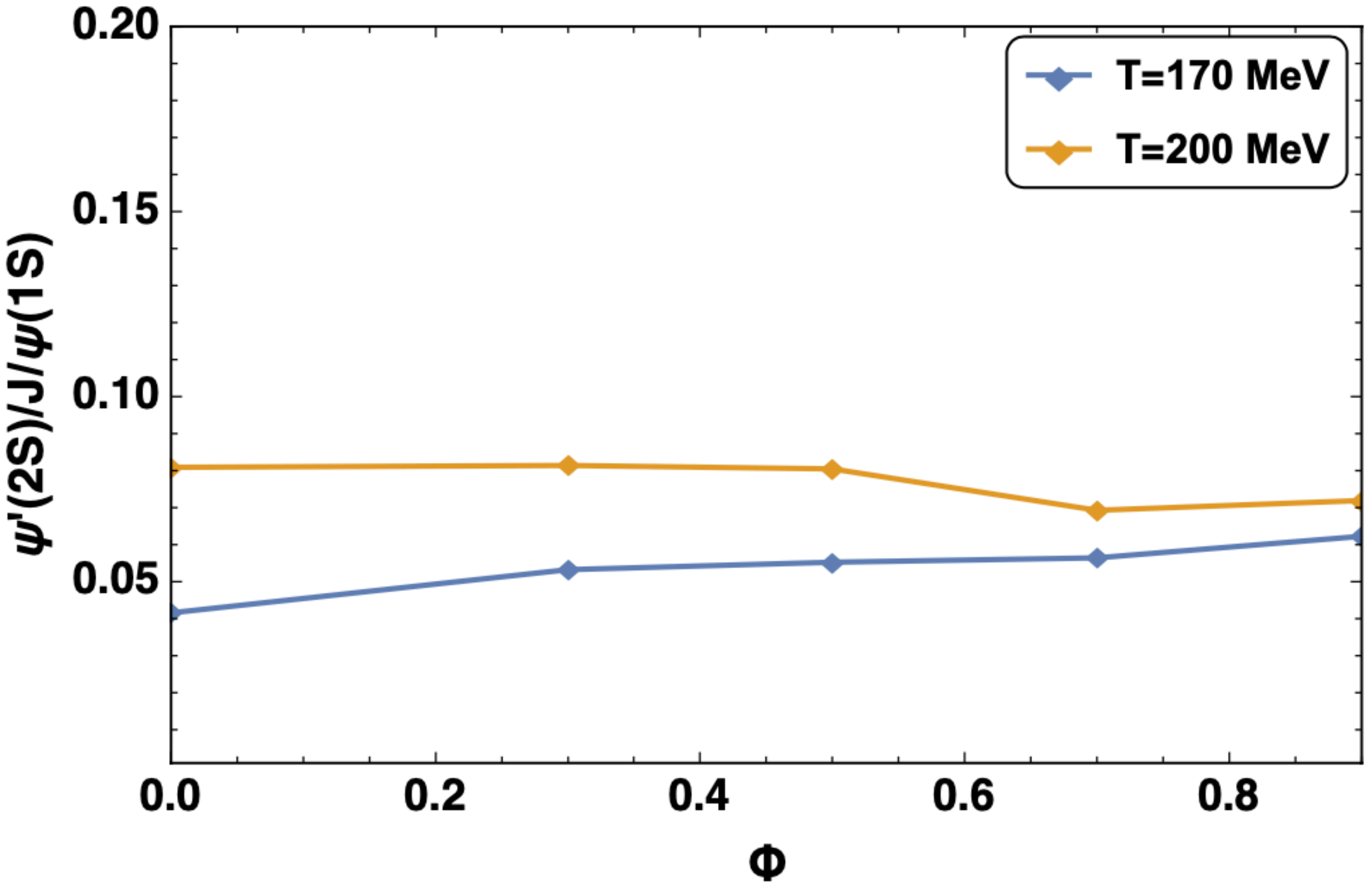
In-medium masses (M_n)

$\psi' / J/\psi$ ratio

$$\frac{N_{\psi'}}{N_{J/\psi}}$$



$\psi'/J/\psi$ ratio as a function of Φ



- $\psi'/J/\psi$ ratio is showing complicated dependence on the bulk viscous correction.

Nuclear modification factor R_{AA}

Time evolution of the probability of quarkonia in state n

$$\frac{\mathrm{d}}{\mathrm{d}\tau} p_n(\tau) = -\Gamma(T(\tau)) p_n(\tau)$$

Survival Probability $$S = p_n(\tau_f)$$

Initial temperature profile

$$T_0(\mathbf{b}, \mathbf{s}) = T_0(\mathbf{0}, \mathbf{0}) \left(\frac{T_A(\mathbf{s}) \left[1 - \left(1 - \frac{\sigma T_A(\mathbf{s}-\mathbf{b})}{A} \right)^A \right] + T_A(\mathbf{s}-\mathbf{b}) \left[1 - \left(1 - \frac{\sigma T_A(\mathbf{s})}{A} \right)^A \right]}{T_A(\mathbf{0}, \mathbf{0}) \left[1 - \left(1 - \frac{\sigma T_A(\mathbf{0})}{A} \right)^A \right] + T_A(\mathbf{0}, \mathbf{0}) \left[1 - \left(1 - \frac{\sigma T_A(\mathbf{0})}{A} \right)^A \right]} \right)^{1/4}$$

Fitting of decay width with the function $\Gamma = aT + bT^2$

$$S(\mathbf{b},\mathbf{s}) = e^{-1.5aT_0(\mathbf{b},\mathbf{s})\tau_0\left(\left(\frac{T_0(\mathbf{b},\mathbf{s})}{T_f}\right)^2-1\right)-3bT_0(\mathbf{b},\mathbf{s})^2\tau_0\left(\frac{T_0(\mathbf{b},\mathbf{s})}{T_f}-1\right)}$$

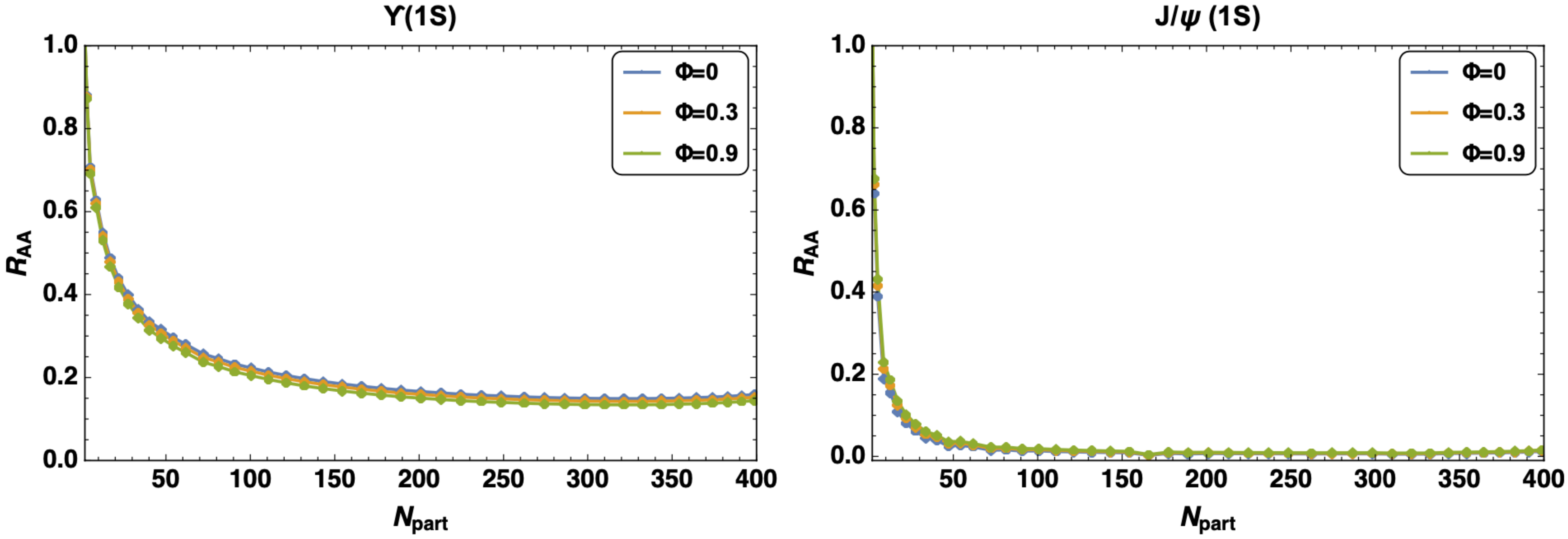
$$r(\mathbf{b}) = \frac{\int d^2\mathbf{s}T_A(\mathbf{s})T_A(\mathbf{s}-\mathbf{b})S(\mathbf{b},\mathbf{s})}{\int d^2\mathbf{s}T_A(\mathbf{s})T_A(\mathbf{s}-\mathbf{b})}$$

Nuclear modification factor

$$R_{AA} = \frac{r(\mathbf{b})}{r(\mathbf{b}=\mathbf{b}^*)}$$

When $N_{part} = 2$

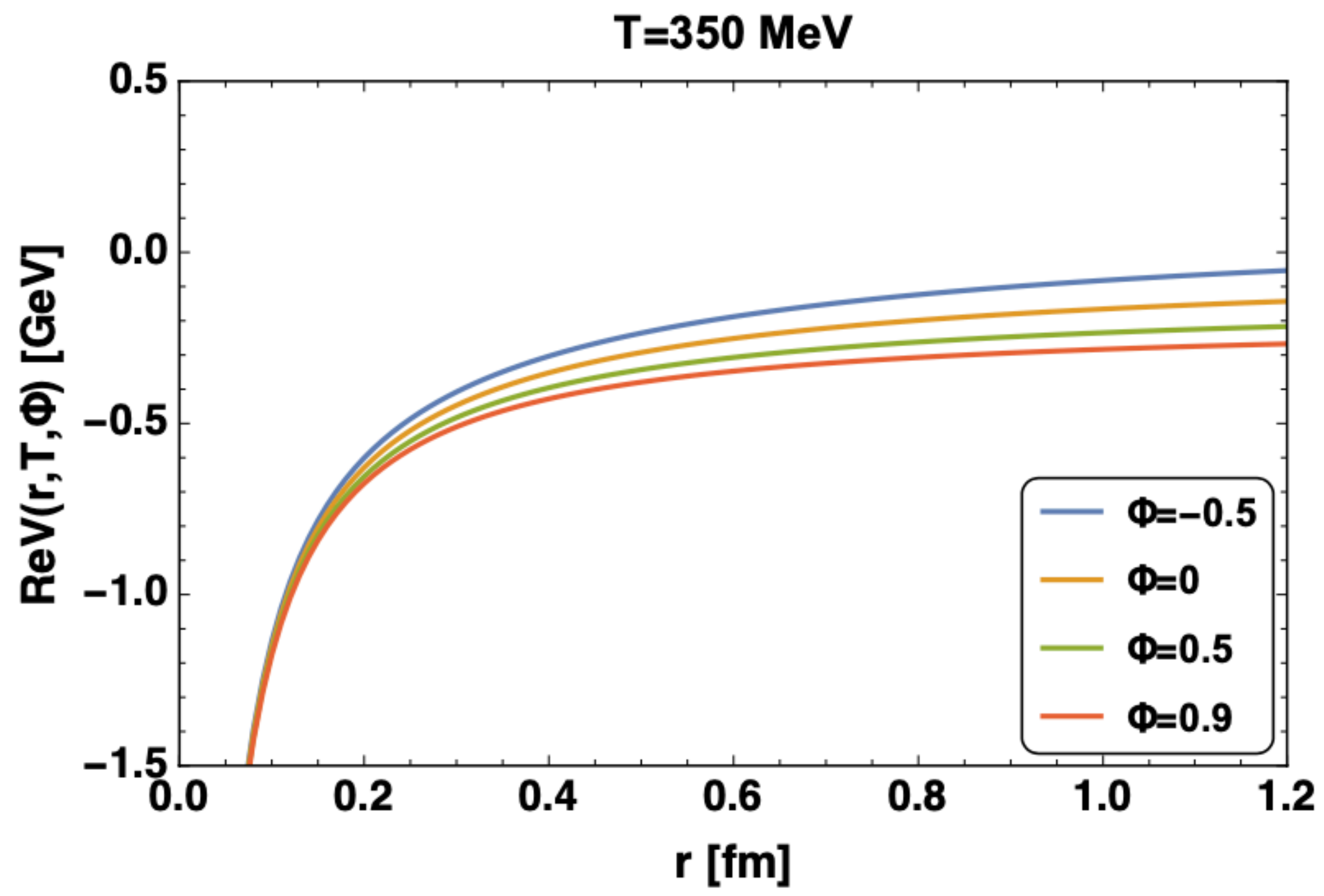
Nuclear modification factor R_{AA}

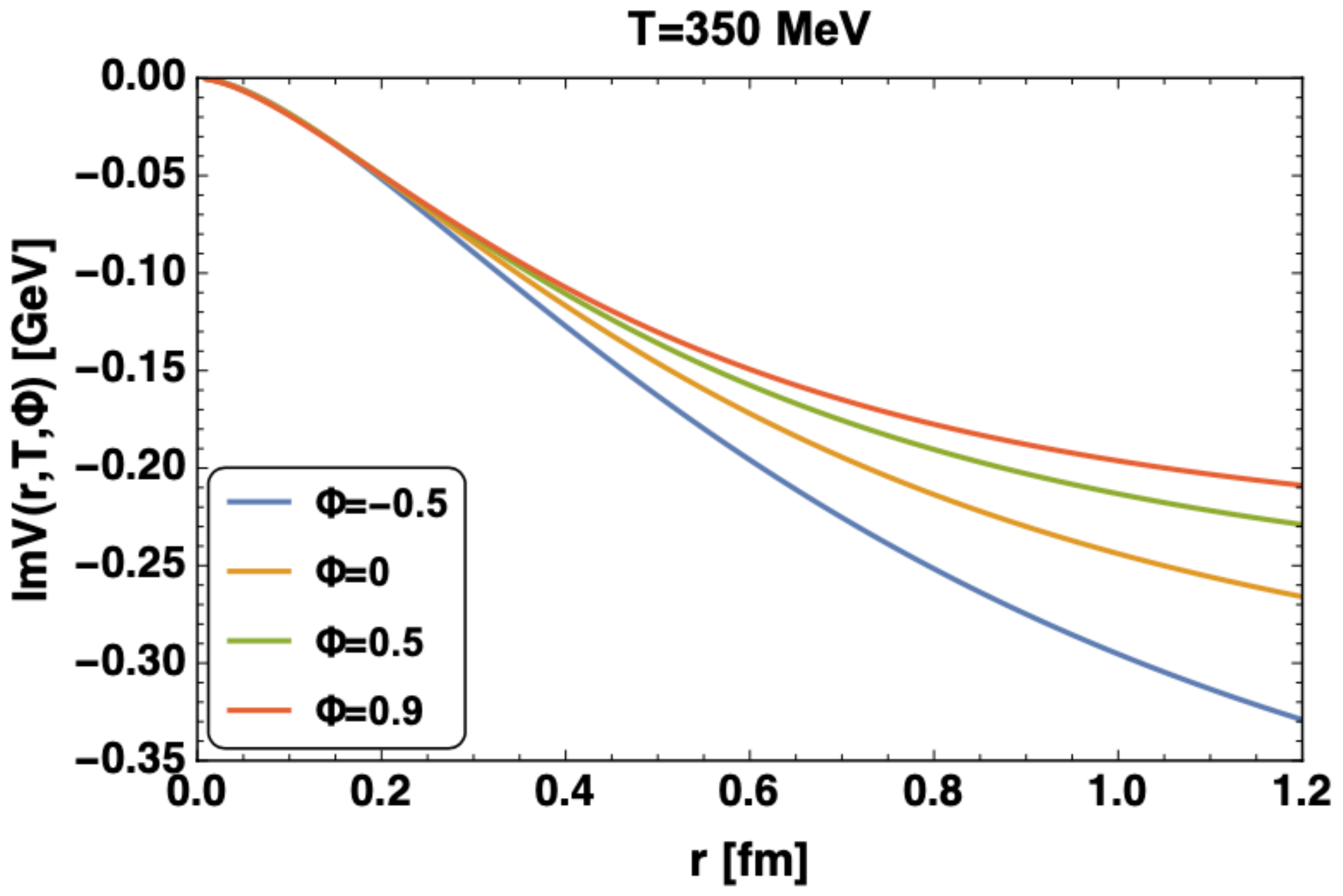


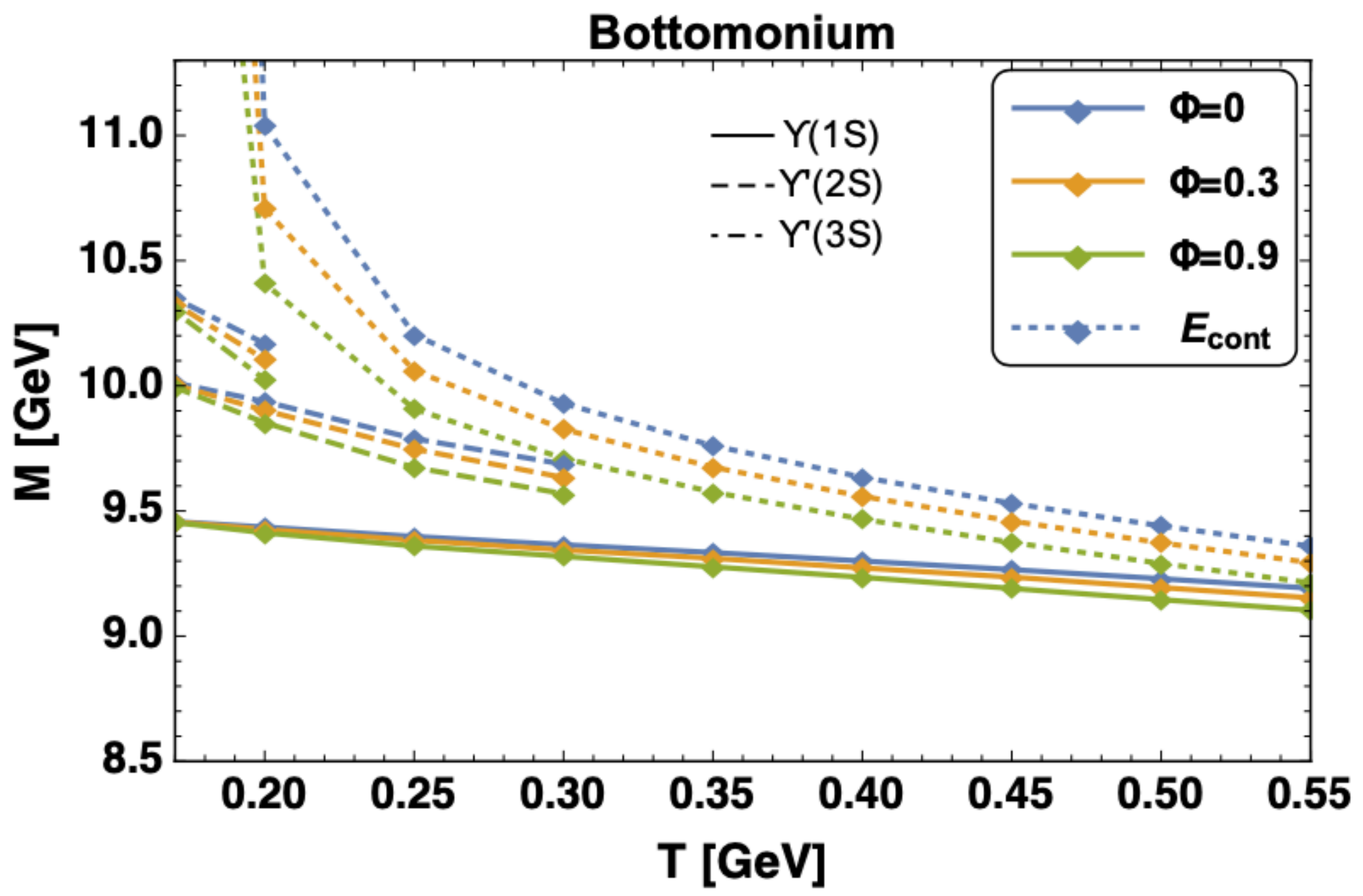
- R_{AA} of the ground states of quarkonia are not much affected by bulk viscous correction

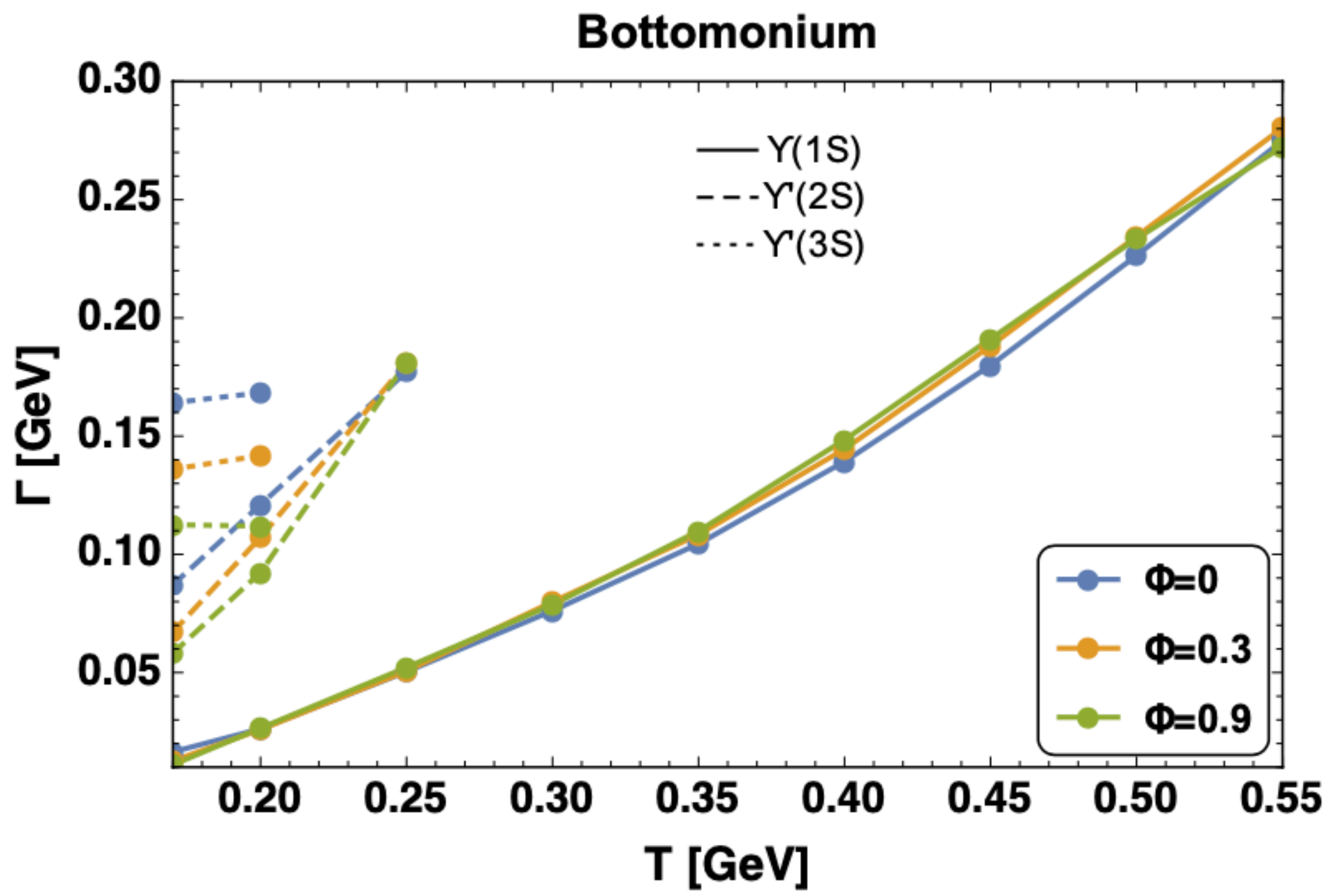
Summary

- **Heavy quarkonia properties in a bulk viscous plasma**
- **R_{AA} of excited states are more sensitive to bulk viscous corrections than R_{AA} of ground states**
  - **Potentially useful for critical point search: R_{AA} as a function of $\sqrt{s}$**









Thank you!

Backup slides

Parameters

- **Coupling constant**

$$\alpha_s = \frac{12\pi}{(11N_c - 2N_f)\ln\left(\frac{M^2}{\Lambda^2}\right)}$$

$M \approx 3.7T$ $\Lambda = 176$ MeV

$$c = -0.161 \text{ GeV}$$

$$\alpha = 0.513 \text{ GeV}$$

$$\sigma = (0.412 \text{ GeV})^2$$

$$m_b = 4.88 \text{ GeV}$$

$$m_c = 1.4692 \text{ GeV}$$

$$\Phi \propto \zeta\, \partial_\mu u^\mu$$

$$M_{\psi'} = 3.684 \text{ GeV and } M_{J/\psi} = 3.0969 \text{ GeV}$$

$$\psi_{J/\psi}(0) = 1.454 \text{ GeV}^3 \text{ and } \psi_{\psi'}(0) = 0.927 \text{ GeV}^3$$

$$T(\sqrt{s_{NN}}) = \frac{T_c}{1 + \exp(2.60 - \ln(\sqrt{s_{NN}})/0.45)}$$

Thickness function with uniform density inside the sphere

$$T_A(\mathbf{s}) = \frac{3}{2\pi r_0^3}(R_A^2 - \mathbf{s}^2)^{1/2}$$

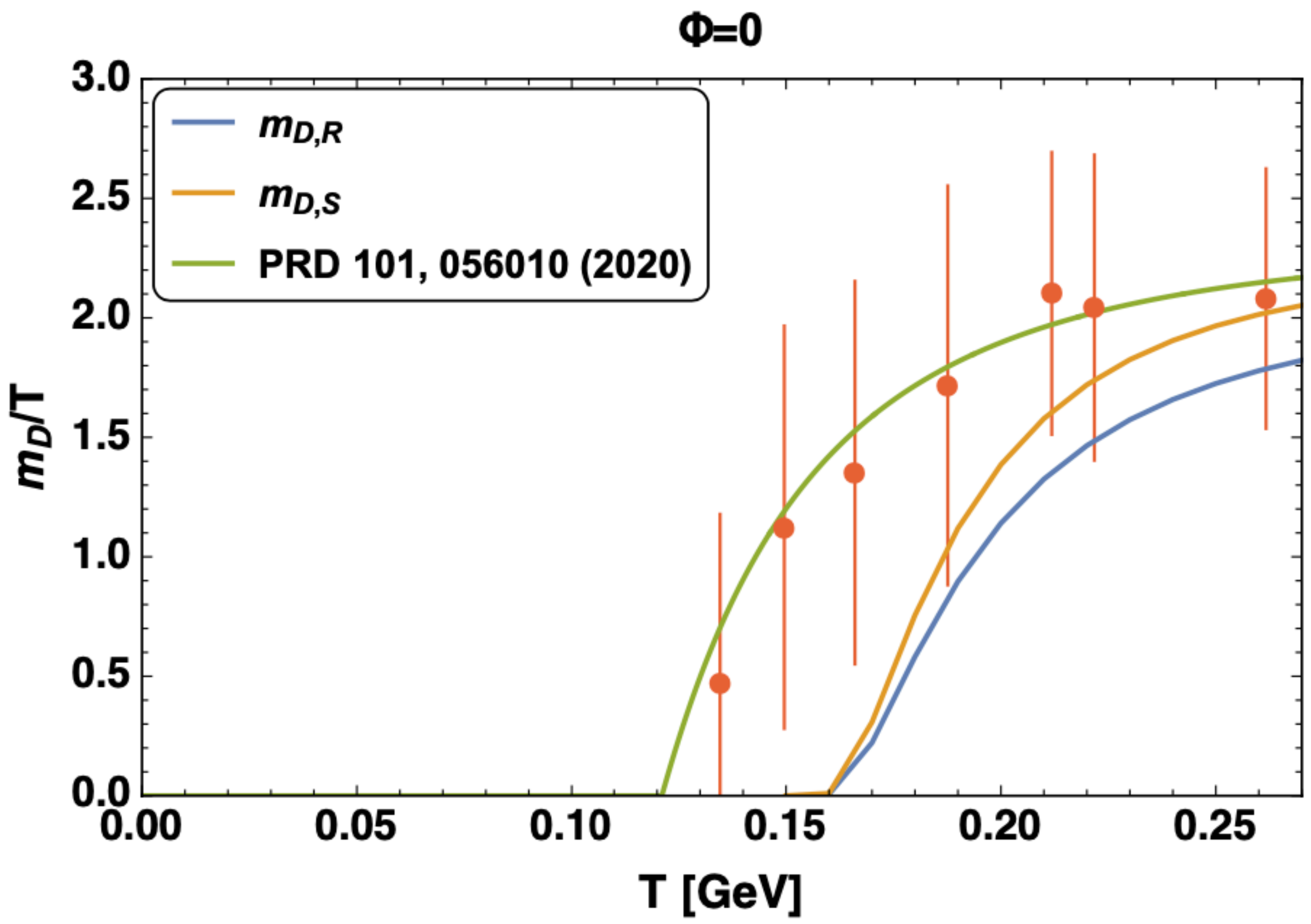
Time dependence of Temperature

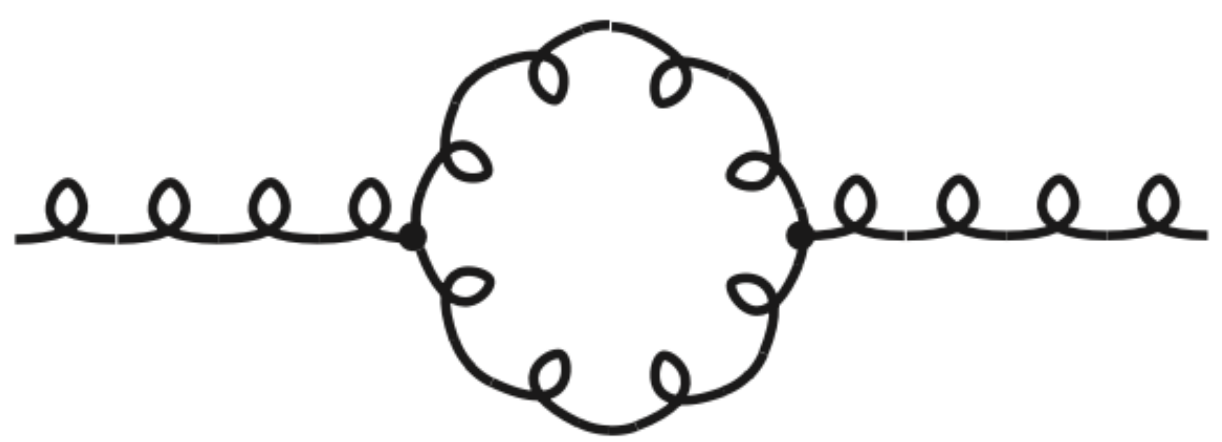
$$T(\tau, \mathbf{b}, \mathbf{s}) = T_0(\mathbf{b}, \mathbf{s}) \left(\frac{\tau_0}{\tau}\right)^{1/3}$$

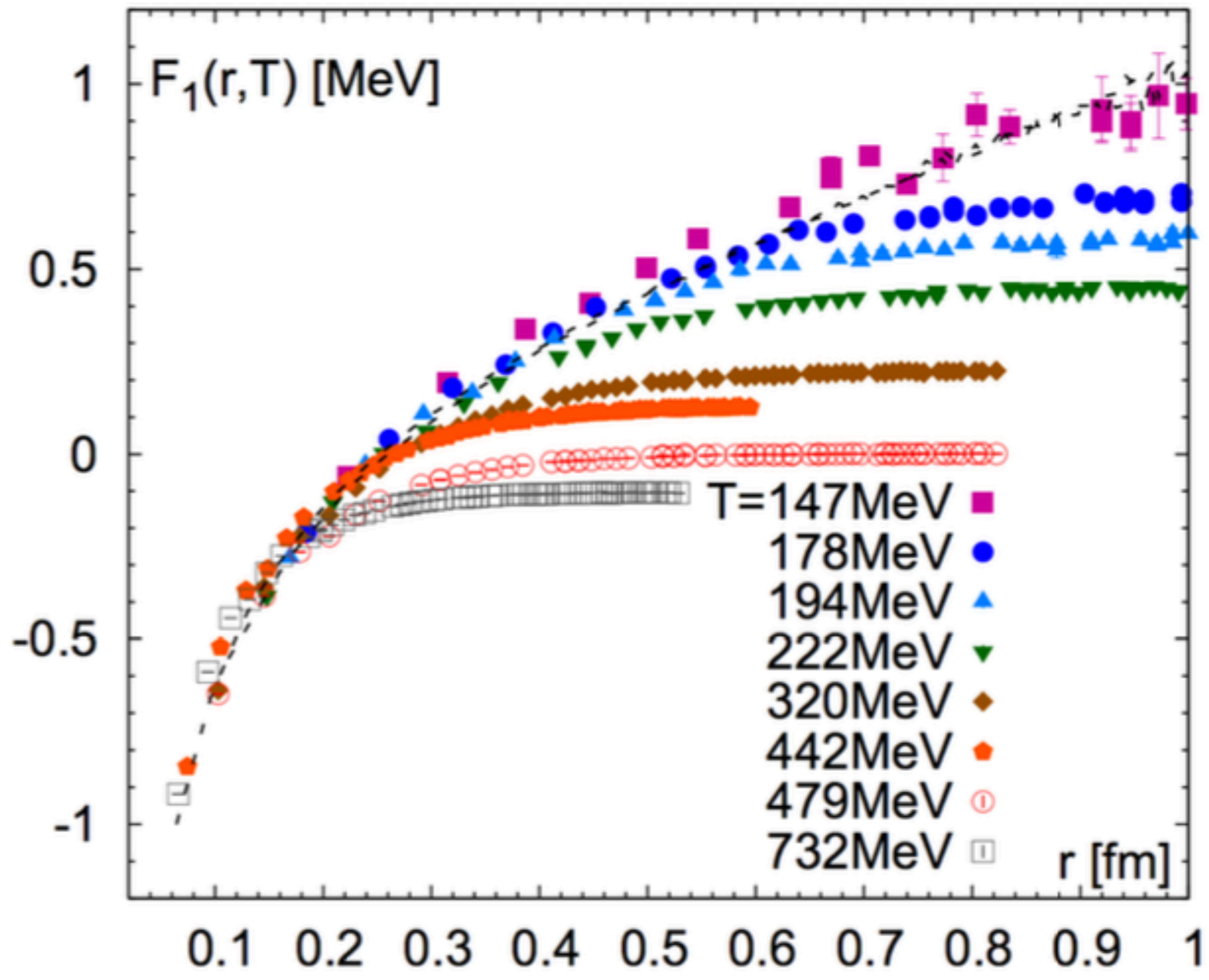
Total number of participating nucleon

$$N_{\text{part}}(\mathbf{b}) = \int d^2\mathbf{s} \left\{ T_A(\mathbf{s}) \left[1 - \left(1 - \frac{\sigma T_A(\mathbf{s} - \mathbf{b})}{A} \right)^A \right] + T_A(\mathbf{s} - \mathbf{b}) \left[1 - \left(1 - \frac{\sigma T_A(\mathbf{s})}{A} \right)^A \right] \right\}$$

Debye mass







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